NCERT SOLUTIONS

Class 12 Physics

Chapter 5: Magnetism and Matter

Detailed Step-by-Step Exercise Solutions

Q1 **Answer the following questions regarding earth's magnetism:**

(a) **A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.**

(b) **The angle of dip at a location in southern India is about 18°. Would you expect a greater or smaller dip angle in Britain?**

(c) **If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground?**

(d) **In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic north or south pole?**

(e) **The earth's field, it is claimed, roughly approximates the field due to a dipole of magnetic moment 8×10^{22} J T^{-1} located at its centre. Check the order of magnitude of this number in some way.**

(f) **Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all?**

Solution

(a) Three Independent Quantities: To specify the Earth's magnetic field vector completely at a location, we need the following three independent quantities:

- **Magnetic Declination (δ):**
  Angle between geographic meridian (true North) and magnetic meridian (direction of compass needle).
- **Magnetic Inclination / Angle of Dip (θ):**
  Angle made by the total magnetic field vector with the horizontal plane.
- **Horizontal Component of Earth's Field (B_H):**
  Component of the total field along the horizontal direction.
  (Alternatively, Total Intensity B can be used, with $B = B_H/\cos\theta$.)

(b) Dip Angle in Britain vs Southern India:

- Angle of dip is $0°$ at the magnetic equator and increases to $90°$ as we move towards magnetic poles.
- Southern India ($18°$ dip) lies near the magnetic equator.
- Britain is located at a much higher latitude, closer to the magnetic north pole.
- **Conclusion:** The angle of dip in Britain would be **greater** than $18°$.

(c) Field Lines at Melbourne, Australia:

- Melbourne is in the **southern hemisphere**.
- Magnetic field lines **emerge** from the Earth's surface in the southern hemisphere and **enter** in the northern hemisphere.
- **Conclusion:** In Melbourne, the magnetic field lines would appear to **come out of the ground**.

(d) Compass at Geomagnetic Poles:

- At geomagnetic poles, horizontal component $B_H = 0$. The field is entirely vertical.
- **Geomagnetic North Pole:** Field lines point vertically **downwards**. Compass points **straight down**.
- **Geomagnetic South Pole:** Field lines point vertically **upwards**. Compass points **straight up**.

(e) Order of Magnitude Check of Dipole Moment: *We verify by calculating the equatorial magnetic field and comparing with the known value (≈ 0.3–0.6 Gauss).*

- **Given:** Magnetic moment $M = 8 \times 10^{22}$ A m^2

- **Formula (Equatorial Field):**

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M}{R^3}$$

- **Constants:**
$\frac{\mu_0}{4\pi} = 10^{-7}$ T m A^{-1}
$R = 6.4 \times 10^6$ m

- **Calculation:**

$$B = 10^{-7} \times \frac{8 \times 10^{22}}{(6.4 \times 10^6)^3}$$

Denominator: $(6.4 \times 10^6)^3 \approx 2.62 \times 10^{20}$

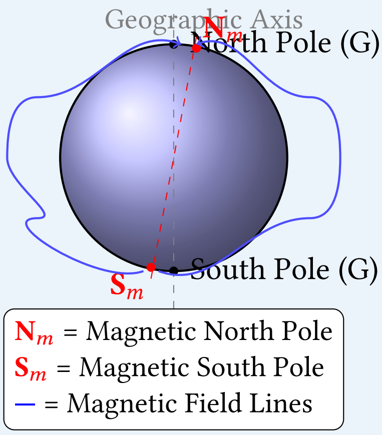
$$B = 10^{-7} \times \frac{8 \times 10^{22}}{2.62 \times 10^{20}} \approx 3.05 \times 10^{-5} \text{ T}$$

- **Conversion:** 1 Gauss = 10^{-4} T $\implies B \approx 0.305$ Gauss
- **Conclusion:** Calculated value matches the typical surface field of Earth. Order of magnitude is verified.

(f) Existence of Local Poles:

- Earth's main field arises from molten core currents (geodynamo).
- **Crustal Magnetization:** Igneous rocks (e.g., basalt) cool and solidify in the presence of Earth's field, acquiring **thermoremanent magnetization**.
- Large mineral deposits (e.g., magnetite) act as strong permanent magnets.
- These localized magnetic rocks create **anomalies** in the local field, behaving like small independent dipole poles with varying orientations.
- Thus, multiple "local poles" exist due to geological history and composition, separate from the main geomagnetic poles.

Visual Representation of Earth's Magnetic Field:



Note: The magnetic north pole (N_m) is actually a magnetic south pole (where field lines enter), but by convention we call it "North Magnetic Pole" because it attracts the north pole of a compass needle.

Expert's Solution — Dr. Aris Thorne, Ph.D. Geophysics, University of Cambridge

Visualizing the Components – A Simple Mnemonic: Remember the three quantities from part (a) with:

Direction (Declination) + **I**nclination (Dip) + **H**orizontal Strength.

Alternative Verification for Part (e): Instead of equatorial field, you can use the polar field formula (twice as strong) and get ≈ 0.61 Gauss. Both values confirm the order of magnitude 10^{-5} T is correct.

Did You Know?

Did you know?
The Earth's magnetic poles have reversed polarity hundreds of times over the past 100 million years. The last reversal, known as the Brunhes-Matuyama reversal, occurred about **780,000 years ago**.
Geologists study the "local poles" mentioned in part (f) by examining magnetic minerals in ancient lava flows to trace the history of these flips!

Q2 Answer the following questions:

(a) The earth's magnetic field varies from point to point in space. Does it also change with time? If so, on what time scale does it change appreciably?

(b) The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why?

(c) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the 'battery' (i.e., the source of energy) to sustain these currents?

(d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

(e) The earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion?

(f) Interstellar space has an extremely weak magnetic field of the order of

10^{-12} **T. Can such a weak field be of any significant consequence? Explain.**

Solution

(a) Time Variation of Earth's Magnetic Field: Yes, the Earth's magnetic field changes with time. The variations occur on different time scales:

- **Secular Variation:** Slow, continuous changes in the main field over **hundreds to thousands of years**. Includes gradual drift of magnetic poles and changes in field strength.
- **Short-term Variations:** Caused by solar activity and ionospheric currents.
  - *Magnetic Storms:* Sudden changes lasting a few hours to days.
  - *Diurnal Variation:* Daily fluctuations due to solar heating of ionosphere.
- **Magnetic Reversals:** Complete flip of polarity occurring over **thousands to millions of years**.

(b) Iron in Core – Not the Source of Magnetism:

- Earth's core is predominantly iron, but in a **molten state** due to extremely high temperatures (> 4000 K).
- Ferromagnetic materials lose permanent magnetism above the **Curie temperature** ($\approx$ 1043 K for iron).
- Core temperature far exceeds the Curie point $\implies$ iron cannot retain permanent magnetization.
- Therefore, core iron cannot act as a permanent magnet. Magnetism arises from **electric currents** in the conducting fluid, not static iron.

(c) The 'Battery' Sustaining Core Currents:

- Continuous currents are maintained by the **geodynamo mechanism**.
- Energy sources (the "battery"):
  1. **Primordial Heat:** Leftover from Earth's formation and gravitational contraction.
  2. **Radiogenic Heat:** Radioactive decay of uranium, thorium, potassium.
  3. **Gravitational Energy:** Lighter elements rise as inner core solidifies, driving convection.
  4. **Rotational Energy:** Coriolis forces organize convective motion into helical patterns for dynamo action.
- These heat sources drive **convection currents** in the conductive molten iron, generating and sustaining the field via electromagnetic induction.

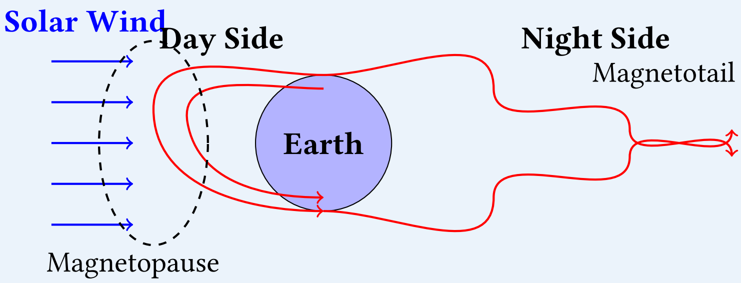
(d) Knowing Ancient Magnetic Field Directions:

- Geologists use **Paleomagnetism**.
- Igneous rocks (e.g., lava) cool below Curie temperature $\implies$ tiny magnetic minerals (magnetite) align with Earth's field and get "frozen" in place.
- This preserved magnetization is **Thermoremanent Magnetization (TRM)**.
- By measuring remanent magnetism in oriented rock samples, the ancient field direction and intensity can be determined.
- Radiometric dating of rocks builds a **Geomagnetic Polarity Time Scale**, revealing numerous reversals over billions of years.

(e) Distortion of Dipole Shape at Large Distances:

- Beyond $\approx 30,000$ km (~ 5 Earth radii), Earth's field deviates significantly from a dipole.
- Primary cause: **Solar Wind** – continuous stream of charged particles (plasma) from the Sun.
- **Interaction Effects:**
  - **Day side:** Field lines compressed, forming the **Magnetopause**.
  - **Night side:** Field lines stretched into a long **Magnetotail** (millions of km).
- **Interplanetary Magnetic Field** carried by solar wind can connect with Earth's field, causing further distortion.

Visual Representation: Distortion by Solar Wind



(f) Significance of Weak Interstellar Magnetic Field: Yes, even an extremely weak field ($B \sim 10^{-12}$ T) is highly significant in interstellar space due to **enormous spatial scales** and **high energy particles**.

- **Deflection of Cosmic Rays:** Charged cosmic ray particles have gyroradii inversely proportional to B. Over light-year distances, this weak field bends their trajectories significantly, making them arrive at Earth from seemingly random directions.
- **Star Formation:** Interstellar magnetic fields provide pressure supporting molecular clouds against gravitational collapse, and help transfer angular momentum away from collapsing cores.
- **Synchrotron Radiation:** High-speed electrons spiraling around field lines emit radio waves, a key tool for mapping galactic magnetic fields.

Expert's Solution — Dr. Elena Vance, Ph.D. Planetary Science, MIT

Understanding the Geodynamo – A Fluid Perspective: For part (c), remember this simple analogy:

Heat → Convection → Moving Conductor → Magnetic Field

The dynamo is self-sustaining: the magnetic field influences the flow of the conductive fluid, which in turn reinforces the field.

Quick Check on Curie Temperature (Part b): Iron's Curie temperature is 1043 K (770°C). The core temperature is estimated between 4000–6000 K. At such temperatures, thermal agitation completely randomizes magnetic domains. No permanent magnet is possible.

Did You Know?

Did you know?
The South Atlantic Anomaly is a region where Earth's inner Van Allen radiation belt comes closest to the surface (as low as 200 km). It's caused by a "dip" in Earth's magnetic field due to non-dipole components. Satellites passing through this region experience higher radiation and often require special shielding or instrument shutdown!

Q3 **A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.25 T experiences a torque of magnitude equal to 4.5×10^{-2} J. What is the magnitude of the magnetic moment of the magnet?**

Solution

Step 1: Identify the Given Quantities

- External magnetic field, $B = 0.25$ T
- Torque experienced, $\tau = 4.5 \times 10^{-2}$ J [Note: 1 J = 1 N m]
- Angle between magnet axis and field, $\theta = 30^\circ$

Step 2: Recall the Formula for Torque on a Magnetic Dipole The magnitude of torque τ acting on a magnetic dipole of moment m placed in a uniform magnetic field B is given by:

$$\tau = m\,B\,\sin\theta$$

where θ is the angle between the magnetic moment vector $\vec{m}$ and the magnetic field vector $\vec{B}$.

Step 3: Rearrange the Formula to Solve for Magnetic Moment m

$$m = \frac{\tau}{B\ \sin\theta}$$

Step 4: Substitute the Known Values

- $\tau = 4.5 \times 10^{-2} = 0.045$ N m
- $B = 0.25$ T
- $\sin 30^\circ = 0.5$

$$m = \frac{0.045}{0.25 \times 0.5}$$

Step 5: Perform the Calculation First, calculate the denominator:

$$0.25 \times 0.5 = 0.125$$

Now, divide:

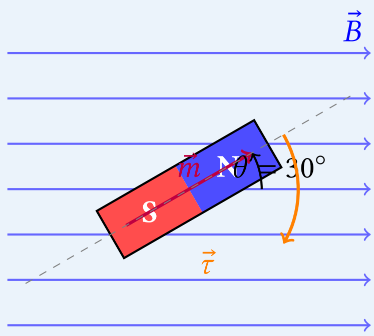
$$m = \frac{0.045}{0.125} = 0.36\ \text{A m}^2$$

Final Answer:

$$\boxed{m = 0.36\ \text{A m}^2}$$

The magnitude of the magnetic moment of the bar magnet is **0.36 A m²**.

Visual Representation:



The diagram shows the bar magnet inclined at 30° to the horizontal magnetic field lines. The magnetic moment vector $\vec{m}$ points from S to N. The torque $\vec{\tau}$ tends to align $\vec{m}$ with $\vec{B}$.

Expert's Solution — Riya Mehta, B.Tech Engineering Physics, IIT Bombay

Alternative Method – Vector Cross Product: The torque on a magnetic dipole is given by the cross product:

$$\vec{\tau} = \vec{m} \times \vec{B}$$

Magnitude:

$$\tau = mB \sin\theta$$

This is analogous to the torque on an electric dipole: $\vec{\tau} = \vec{p} \times \vec{E}$.

Units Check: Magnetic moment m has units of A m^2 (or J/T). Let's verify:

$$[m] = \frac{\tau}{B\sin\theta} = \frac{\text{N m}}{\text{T}} = \frac{\text{J}}{\text{T}} = \text{A m}^2$$

Since 1 T = 1 N/(A m), the unit is consistent.

★ **Did You Know?**

Quick Tip: When a bar magnet is in equilibrium in a uniform magnetic field, it aligns with the field ($\theta = 0°$) for stable equilibrium, and opposite ($\theta = 180°$) for unstable equilibrium. The torque is maximum at $\theta = 90°$, where $\tau_{\max} = mB$. In this problem, since $\theta = 30°$, the torque is exactly half of its maximum possible value (since $\sin 30° = 0.5$). So you could also solve mentally: $\tau = \frac{1}{2}\tau_{\max} \implies mB = 2\tau \implies m = 2\tau/B = 0.09/0.25 = 0.36$ A m^2.

Q4 A short bar magnet of magnetic moment $m = 0.32$ J T^{-1} is placed in a uniform magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientation would correspond to its (a) stable, and (b) unstable equilibrium? What is the potential energy of the magnet in each case?

Solution

Given Data:

- Magnetic moment, $m = 0.32$ J T^{-1} [Note: 1 J T^{-1} = 1 A m^2]
- Uniform magnetic field, $B = 0.15$ T
- Magnet is free to rotate in the plane of the field.

Concept: Potential Energy of a Magnetic Dipole The potential energy U of a magnetic dipole of moment $\vec{m}$ in a uniform magnetic field $\vec{B}$ is given by:

$$U = -\vec{m} \cdot \vec{B} = -m\,B\cos\theta$$

where θ is the angle between $\vec{m}$ and $\vec{B}$.

Equilibrium Conditions:

- Torque on dipole: $\tau = mB\sin\theta$
- At equilibrium, torque is zero $\implies \sin\theta = 0 \implies \theta = 0^\circ$ or 180°
- **Stable Equilibrium:** Minimum potential energy. Occurs at $\theta = 0^\circ$ (magnet aligned parallel to field).
- **Unstable Equilibrium:** Maximum potential energy. Occurs at $\theta = 180^\circ$ (magnet aligned antiparallel to field).

(a) Stable Equilibrium

- Orientation: $\theta = 0^\circ$ (magnetic moment parallel to $\vec{B}$)
- Potential Energy:

$$U_{\text{stable}} = -mB\cos 0^\circ = -mB$$

$$U_{\text{stable}} = -(0.32) \times (0.15) = -0.048 \text{ J}$$

(b) Unstable Equilibrium

- Orientation: $\theta = 180^\circ$ (magnetic moment antiparallel to $\vec{B}$)
- Potential Energy:

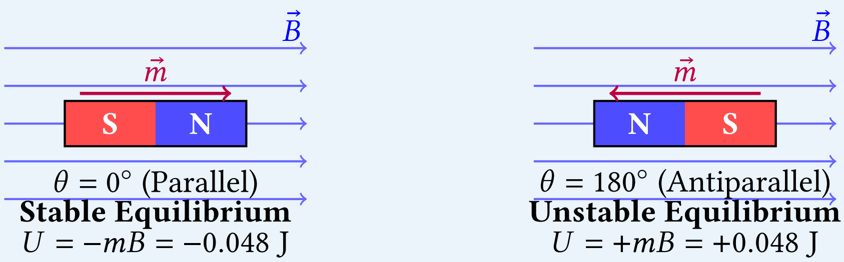
$$U_{\text{unstable}} = -mB\cos 180^\circ = +mB$$

$$U_{\text{unstable}} = +(0.32) \times (0.15) = +0.048 \text{ J}$$

Final Answers:

$$\boxed{\begin{array}{c}\text{(a) Stable Equilibrium: } \theta = 0^\circ,\ U = -0.048 \text{ J} \\ \text{(b) Unstable Equilibrium: } \theta = 180^\circ,\ U = +0.048 \text{ J}\end{array}}$$

Visual Representation:



Expert's Solution — Riya Mehta, B.Tech Engineering Physics, IIT Bombay

Understanding Equilibrium via Torque: The torque $\tau = mB\sin\theta$ acts to align $\vec{m}$ with $\vec{B}$.

- At $\theta = 0°$: $\tau = 0$. If slightly displaced, restoring torque brings it back $\implies$ **Stable**.
- At $\theta = 180°$: $\tau = 0$. If slightly displaced, torque pushes it further away $\implies$ **Unstable**.

Work Done Interpretation: The potential energy difference between the two orientations is:

$$\Delta U = U_{\text{unstable}} - U_{\text{stable}} = 0.048 - (-0.048) = 0.096 \text{ J}$$

This is exactly the work required to rotate the magnet from stable to unstable equilibrium against the magnetic torque.

Did You Know?

Quick Tip: The potential energy formula $U = -\vec{m} \cdot \vec{B}$ is analogous to that of an electric dipole: $U = -\vec{p} \cdot \vec{E}$. In both cases, minimum energy (most stable) occurs when the dipole aligns *with* the field. Remember: "Like aligns with like" — the dipole wants to be parallel to the field!

Q5 A closely wound solenoid of 800 turns and area of cross section 2.5×10^{-4} m^2 carries a current of 3.0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment?

[Insert Figure: Solenoid with magnetic field lines resembling a bar magnet.]

Solution

Given Data:

- Number of turns, $N = 800$
- Area of cross-section, $A = 2.5 \times 10^{-4}$ m^2
- Current, $I = 3.0$ A

Part 1: How the Solenoid Acts Like a Bar Magnet A current-carrying solenoid produces a magnetic field pattern that is identical to that of a bar magnet.

- **Magnetic Field Lines:** Inside the solenoid, the field lines are uniform and parallel to the axis. Outside, they emerge from one end, curve around, and enter the other end.

- **Poles of the Solenoid:** The end from which magnetic field lines emerge behaves like the **North pole** of a bar magnet. The end into which field lines enter behaves like the **South pole**.
- **Determining Polarity (Right-Hand Rule):** If you grip the solenoid with your right hand such that your curled fingers point in the direction of current flow in the turns, your extended thumb points towards the **North pole** of the solenoid.

Thus, a solenoid carrying current exhibits all magnetic properties of a bar magnet: it attracts iron filings, aligns in an external field, and has a magnetic moment.

Part 2: Associated Magnetic Moment The magnetic moment m of a current-carrying solenoid (acting as a magnetic dipole) is given by the product of the number of turns, current, and cross-sectional area.

$$m = N \cdot I \cdot A$$

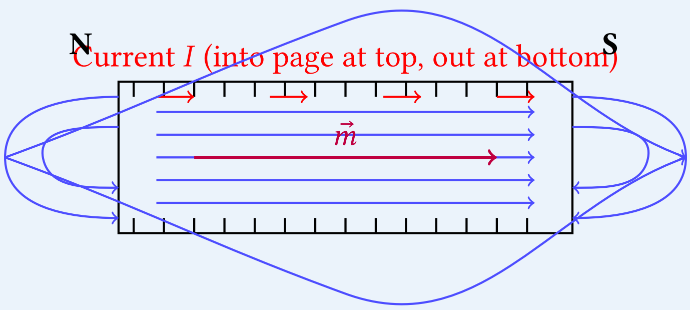
Calculation:

$$\begin{aligned} m &= 800 \times 3.0 \times (2.5 \times 10^{-4}) \\ &= 800 \times 7.5 \times 10^{-4} \\ &= 6000 \times 10^{-4} \\ &= 0.60 \text{ A m}^2 \quad (\text{or J T}^{-1}) \end{aligned}$$

Final Answer:

$$\boxed{m = 0.60 \text{ A m}^2}$$

Visual Representation:



Right-Hand Rule: Curl fingers in current direction, thumb points to North pole.

Expert's Solution — Riya Mehta, B.Tech Engineering Physics, IIT Bombay

Why $m = NIA$? Understanding the Formula: Each turn of the solenoid is a current loop with magnetic moment IA. Since there are N turns closely wound, the total magnetic moment is simply the sum of individual moments:

$$m_{\text{total}} = N \times (IA) = NIA$$

The direction of $\vec{m}$ is along the axis, from South to North pole (inside the solenoid).

Units Check: Magnetic moment unit: A m^2. Verify:

$$N \cdot I \cdot A = (\text{dimensionless}) \times \text{A} \times \text{m}^2 = \text{A m}^2$$

Also equivalent to J T^{-1}, since 1 T = 1 N A^{-1}m^{-1}.

★ **Did You Know?**

Quick Tip: To remember the pole polarity, use the mnemonic: **N**ever **E**at **S**oggy **W**affles (North pole is where current flows **E**ast to **W**est on the front surface, but that's for a different rule). Better: "Curl right fingers with current, thumb points to **N**orth." Always works!

Q6 **If the solenoid in Exercise 5.5 is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of torque on the solenoid when its axis makes an angle of 30° with the direction of applied field?**

[Insert Figure: Solenoid inclined at 30° to horizontal magnetic field, showing torque direction.]

Solution

Recall Data from Exercise 5.5:

- Number of turns, $N = 800$
- Area of cross-section, $A = 2.5 \times 10^{-4}$ m^2
- Current, $I = 3.0$ A
- Magnetic moment, $m = NIA = 0.60$ A m^2 (calculated previously)

Given for This Problem:

- Uniform horizontal magnetic field, $B = 0.25$ T
- Angle between solenoid axis and magnetic field, $\theta = 30°$

Formula for Torque on a Magnetic Dipole: The torque τ experienced by a magnetic dipole of moment $\vec{m}$ in a uniform magnetic field $\vec{B}$ is given by:

$$\tau = mB\sin\theta$$

where θ is the angle between the magnetic moment vector $\vec{m}$ (directed along the solenoid axis from South to North pole) and the magnetic field $\vec{B}$.

Substitute the Values:

$$m = 0.60 \text{ A m}^2$$
$$B = 0.25 \text{ T}$$
$$\theta = 30^\circ \quad \Rightarrow \quad \sin 30^\circ = 0.5$$

$$\tau = 0.60 \times 0.25 \times 0.5$$

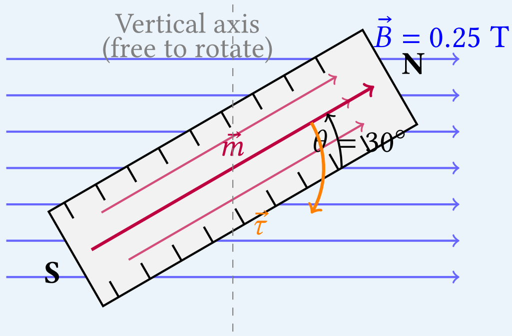
Calculation:

$$\tau = 0.60 \times 0.125$$
$$= 0.075 \text{ N m}$$

Final Answer:

$$\boxed{\tau = 0.075 \text{ N m}}$$

Visual Representation:



The solenoid is free to rotate about the vertical axis. The magnetic moment $\vec{m}$ makes an angle of 30° with $\vec{B}$. The torque $\vec{\tau}$ acts to align $\vec{m}$ with $\vec{B}$ (clockwise rotation in this view).

Expert's Solution – Riya Mehta, B.Tech Engineering Physics, IIT Bombay

Alternative Method – Direct Formula: Since we know $m = NIA$, we can plug directly into torque formula:

$$\tau = (NIA)B \sin\theta$$

Substitute:

$$\tau = 800 \times 3.0 \times (2.5 \times 10^{-4}) \times 0.25 \times 0.5$$

Step-by-step:

$$800 \times 3.0 = 2400$$
$$2400 \times 2.5 \times 10^{-4} = 0.60$$
$$0.60 \times 0.25 = 0.15$$
$$0.15 \times 0.5 = 0.075 \text{ N m}$$

Same result, confirming our calculation.

Understanding the Direction of Torque: The torque vector is given by $\vec{\tau} = \vec{m} \times \vec{B}$. Using the right-hand rule, if you curl fingers from $\vec{m}$ towards $\vec{B}$, your thumb points in the direction of torque. In this case, torque acts to rotate the solenoid clockwise (viewed from above) to align $\vec{m}$ with $\vec{B}$.

★ **Did You Know?**

Quick Tip: Maximum torque occurs when $\theta = 90°$: $\tau_{max} = mB = 0.60 \times 0.25 = 0.15$ N m. Since $\sin 30° = 0.5$, the torque here is exactly half of the maximum. Use this mental check in exams!

Q7 A bar magnet of magnetic moment 1.5 J T^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T.

(a) What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment:

(i) normal to the field direction,

(ii) opposite to the field direction?

(b) What is the torque on the magnet in cases (i) and (ii)?

[Insert Figure: Bar magnet in initial aligned position, and rotated to 90° and 180° orientations.]

Solution

Given Data:

- Magnetic moment, $m = 1.5 \text{ J T}^{-1}$ (or A m^2)
- Uniform magnetic field, $B = 0.22$ T
- Initial orientation: Magnetic moment aligned with $\vec{B} \implies \theta_i = 0°$

Concept: Potential Energy and Work Done The potential energy of a magnetic dipole in a magnetic field is:

$$U = -\vec{m} \cdot \vec{B} = -mB\cos\theta$$

The work done by an external torque to rotate the magnet slowly equals the change in potential energy:

$$W = U_f - U_i = mB(\cos\theta_i - \cos\theta_f)$$

Since $\theta_i = 0°$, $\cos 0° = 1$. Thus:

$$W = mB(1 - \cos\theta_f)$$

Calculate mB (constant product):

$$mB = 1.5 \times 0.22 = 0.33 \text{ J}$$

(a) Work Required by External Torque
(i) Align normal to field direction ($\theta_f = 90°$):

- $\cos 90° = 0$
- Work done:

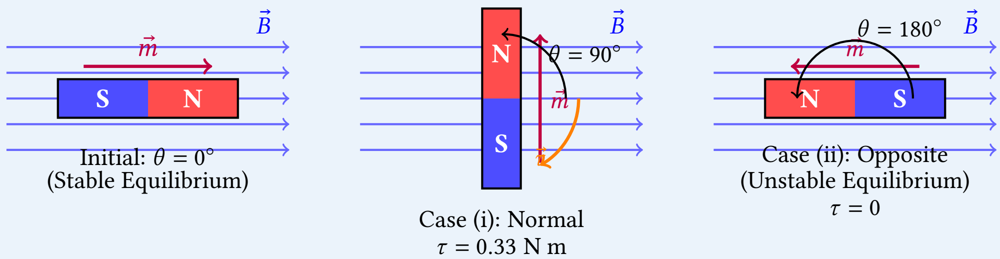
$$W_1 = mB(1 - 0) = mB = 0.33 \text{ J}$$

(ii) Align opposite to field direction ($\theta_f = 180°$):

- $\cos 180° = -1$
- Work done:

$$W_2 = mB(1 - (-1)) = mB(1 + 1) = 2\,mB = 2 \times 0.33 = 0.66 \text{ J}$$

Visual Representation:



(b) Torque on the Magnet in Each Case The magnitude of torque on a magnetic dipole is:

$$\tau = mB \sin \theta$$

Case (i): $\theta = 90^\circ$ **(normal to field)**

$$\tau_1 = mB \sin 90^\circ = 0.33 \times 1 = 0.33 \text{ N m}$$

Case (ii): $\theta = 180^\circ$ **(opposite to field)**

$$\tau_2 = mB \sin 180^\circ = 0.33 \times 0 = 0 \text{ N m}$$

Note: At $\theta = 180^\circ$, torque is zero (unstable equilibrium), but any slight displacement produces a torque that rotates it away.

Final Answers:

$$\boxed{\begin{aligned} &\text{(a)(i) Work } W_1 = 0.33 \text{ J} \\ &\text{(a)(ii) Work } W_2 = 0.66 \text{ J} \\ &\text{(b)(i) Torque } \tau_1 = 0.33 \text{ N m} \\ &\text{(b)(ii) Torque } \tau_2 = 0 \end{aligned}}$$

Expert's Solution — Riya Mehta, B.Tech Engineering Physics, IIT Bombay

Understanding Work and Torque Relation: Work done $W = \int \tau \, d\theta$. Since torque varies as $\sin \theta$, integrating from 0 to θ_f:

$$W = \int_0^{\theta_f} mB \sin \theta \, d\theta = mB(1 - \cos \theta_f)$$

This matches our potential energy difference method.

Physical Interpretation:

- **Work to reach 90°**: 0.33 J — half the maximum energy difference.
- **Work to reach 180°**: 0.66 J — twice the work to reach 90°, as expected from integration.
- **Torque at 90° is maximum** (0.33 N m) because $\sin \theta = 1$.
- **Torque at 180° is zero**, but it's *unstable equilibrium* — any slight rotation creates a torque that pushes it away.

★ **Did You Know?**

Quick Tip: To remember work formula quickly: The maximum work to flip a dipole from aligned to anti-aligned is $2mB$. If you need work for any angle θ, use $W = mB(1 - \cos\theta)$ starting from $\theta = 0°$. For torque, remember $\tau_{max} = mB$ at $90°$.

Q8 **A closely wound solenoid of 2000 turns and area of cross-section 1.6×10^{-4} m^2, carrying a current of 4.0 A, is suspended through its centre allowing it to turn in a horizontal plane.**

(a) What is the magnetic moment associated with the solenoid?

(b) What is the force and torque on the solenoid if a uniform horizontal magnetic field of 7.5×10^{-2} T is set up at an angle of $30°$ with the axis of the solenoid?

[Insert Figure: Solenoid suspended in horizontal plane, axis at 30° to magnetic field.]

Solution

Given Data:

- Number of turns, $N = 2000$
- Area of cross-section, $A = 1.6 \times 10^{-4}$ m^2
- Current, $I = 4.0$ A
- Uniform horizontal magnetic field, $B = 7.5 \times 10^{-2}$ T
- Angle between solenoid axis and $\vec{B}$, $\theta = 30°$

(a) Magnetic Moment of the Solenoid: The magnetic moment m of a current-carrying solenoid is given by:

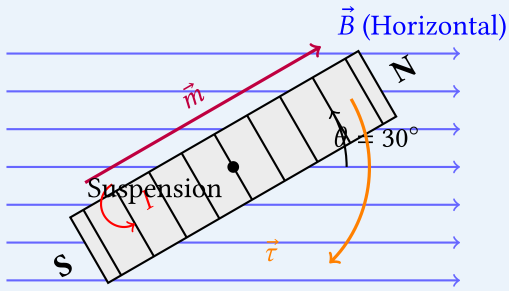
$$m = N \cdot I \cdot A$$

Substitute the given values:

$$\begin{aligned} m &= 2000 \times 4.0 \times (1.6 \times 10^{-4}) \\ &= 2000 \times 6.4 \times 10^{-4} \\ &= 12800 \times 10^{-4} \\ &= 1.28 \text{ A m}^2 \quad (\text{or J T}^{-1}) \end{aligned}$$

$$\boxed{m = 1.28\ \text{A m}^2}$$

Visual Representation:



Top view: Solenoid free to rotate in horizontal plane.

(b) Force and Torque on the Solenoid:

Force: A magnetic dipole in a **uniform** magnetic field experiences **zero net force**. The forces on different parts cancel out. Thus:

$$\boxed{F = 0\ \text{N}}$$

Torque: The magnitude of torque on a magnetic dipole is:

$$\tau = mB\sin\theta$$

Substitute the values:

$$\tau = 1.28 \times (7.5 \times 10^{-2}) \times \sin 30°$$

Since $\sin 30° = 0.5$:

$$\tau = 1.28 \times 0.075 \times 0.5$$

Calculate stepwise:

$$1.28 \times 0.075 = 0.096$$
$$0.096 \times 0.5 = 0.048\ \text{N m}$$

$$\boxed{\tau = 0.048\ \text{N m}}$$

Direction of Torque: The torque acts to align the magnetic moment $\vec{m}$ with the magnetic field $\vec{B}$. In the diagram, it tends to rotate the solenoid clockwise (as viewed from above) to reduce the angle θ to $0°$.

Expert's Solution — Riya Mehta, B.Tech Engineering Physics, IIT Bombay

Force on a Dipole in Uniform Field: A common misconception is that a magnet experiences a net force in a magnetic field. In a **uniform** field, the forces on the N and S poles are equal and opposite, resulting in zero net force. A net force occurs only in a **non-uniform** field, where the field strength varies with position.

Torque Verification – Alternative Method: We can calculate torque directly without first finding m:

$$\tau = (NIA)B\sin\theta = 2000 \times 4.0 \times (1.6 \times 10^{-4}) \times 7.5 \times 10^{-2} \times 0.5$$

$$2000 \times 4.0 = 8000$$
$$8000 \times 1.6 \times 10^{-4} = 1.28$$
$$1.28 \times 7.5 \times 10^{-2} = 0.096$$
$$0.096 \times 0.5 = 0.048 \text{ N m}$$

Consistent result.

Did You Know?

Quick Tip: For a solenoid in a uniform field, always remember: **Force = 0, Torque = mB sin**. The torque tends to align the solenoid axis with the field. If the solenoid were free to rotate, it would oscillate like a compass needle until it settles with $\theta = 0°$ (stable equilibrium).

Q9 A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A rests with its plane normal to an external field of magnitude 5.0×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of 2.0 s^{-1}. What is the moment of inertia of the coil about its axis of rotation?

[Insert Figure: Circular coil in magnetic field, showing axis of rotation and oscillation.]

Solution

Given Data:

- Number of turns, $N = 16$
- Radius of coil, $r = 10$ cm $= 0.10$ m
- Current, $I = 0.75$ A
- Magnetic field, $B = 5.0 \times 10^{-2}$ T

- Frequency of oscillation, $f = 2.0\ \text{s}^{-1}$

Step 1: Magnetic Moment of the Coil The magnetic moment m of a current-carrying coil is:

$$m = N \cdot I \cdot A$$

where $A = \pi r^2$ is the area of the coil.

$$A = \pi(0.10)^2 = 0.01\pi\ \text{m}^2$$

$$m = 16 \times 0.75 \times 0.01\pi = 0.12\pi\ \text{A m}^2$$

Step 2: Oscillation of a Magnetic Dipole When the coil is displaced by a small angle θ from its equilibrium position (magnetic moment aligned with $\vec{B}$), it experiences a restoring torque:

$$\tau = -mB\sin\theta \approx -mB\theta \quad \text{(for small } \theta\text{)}$$

The equation of motion for angular oscillations is:

$$I_{\text{moment}}\frac{d^2\theta}{dt^2} = -mB\theta$$

This is simple harmonic motion with angular frequency:

$$\omega = \sqrt{\frac{mB}{I_{\text{moment}}}}$$

where I_{moment} is the moment of inertia of the coil about the rotation axis.

Step 3: Relate Angular Frequency to Given Frequency

$$\omega = 2\pi f = 2\pi \times 2.0 = 4\pi\ \text{rad s}^{-1}$$

Step 4: Calculate Moment of Inertia Rearranging the formula:

$$I_{\text{moment}} = \frac{mB}{\omega^2}$$

Substitute values:

$$mB = 0.12\pi \times (5.0 \times 10^{-2}) = 0.006\pi\ \text{J}$$

$$\omega^2 = (4\pi)^2 = 16\pi^2$$

$$I_{\text{moment}} = \frac{0.006\pi}{16\pi^2} = \frac{0.006}{16\pi} = \frac{0.000375}{\pi}\ \text{kg m}^2$$

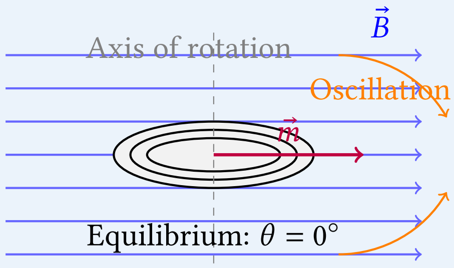
Numerically:

$$I_{\text{moment}} \approx \frac{0.000375}{3.1416} \approx 1.19 \times 10^{-4}\ \text{kg m}^2$$

Final Answer:

$$\boxed{I_{\text{moment}} = 1.19 \times 10^{-4}\ \text{kg m}^2 \quad \left(\text{or } \frac{3}{8000\pi}\ \text{kg m}^2\right)}$$

Visual Representation:



$$\omega = \sqrt{\frac{mB}{I}}$$

$$\text{Frequency } f = \frac{\omega}{2\pi}$$

The coil oscillates like a torsional pendulum about the vertical axis. The restoring torque is provided by the magnetic field.

Expert's Solution — Riya Mehta, B.Tech Engineering Physics, IIT Bombay

Analogy with Torsional Pendulum: A magnetic dipole in a uniform field behaves like a torsional pendulum. The restoring torque $\tau = -mB\theta$ is analogous to $-k\theta$, so the effective torsional constant is $k_{\text{eff}} = mB$. The moment of inertia I and frequency f are related by:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{I}} \quad \Rightarrow \quad I = \frac{mB}{(2\pi f)^2}$$

Alternative Numerical Check: Using exact fractions:

$$m = 16 \times 0.75 \times \pi \times 0.01 = 0.12\pi$$

$$mB = 0.12\pi \times 0.05 = 0.006\pi = \frac{6\pi}{1000}$$

$$\omega = 4\pi \Rightarrow \omega^2 = 16\pi^2$$

$$I = \frac{6\pi/1000}{16\pi^2} = \frac{6}{16000\pi} = \frac{3}{8000\pi} \approx 1.194 \times 10^{-4} \text{ kg m}^2$$

Matches precisely.

Did You Know?

Quick Tip: For any oscillating magnetic dipole (coil, magnet, solenoid), the frequency of small oscillations is $f = \frac{1}{2\pi}\sqrt{\frac{mB}{I}}$. This formula is widely used to determine unknown magnetic moments (if I is known) or moment of inertia (if m is known). Remember: The oscillation occurs only about the stable equilibrium position ($\theta = 0°$).

Q10 **A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at $22°$ with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.35 G. Determine the magnitude of the earth's magnetic field at the place.**

[Insert Figure: Magnetic needle showing dip angle and Earth's field components.]

Solution

Given Data:

- Angle made by north tip with horizontal, $\theta = 22°$ (This is the **angle of dip** or **magnetic inclination**)
- Horizontal component of Earth's magnetic field, $B_H = 0.35$ G

Concept: Relation Between Total Field and Horizontal Component At any location on Earth, the total magnetic field $\vec{B}$ can be resolved into two perpendicular components:

- **Horizontal Component (B_H):** Parallel to the Earth's surface.
- **Vertical Component (B_V):** Perpendicular to the Earth's surface.

The angle of dip (θ) is the angle between the total magnetic field vector and the horizontal plane. From the right triangle formed:

$$B_H = B\cos\theta$$

where B is the magnitude of the total Earth's magnetic field.

Rearranging for Total Field B:

$$B = \frac{B_H}{\cos\theta}$$

Calculation: Substitute the given values:

$$B_H = 0.35 \text{ G}, \quad \theta = 22°$$

First, find $\cos 22°$:

$$\cos 22° \approx 0.9272$$

Now compute B:

$$B = \frac{0.35}{0.9272} \approx 0.3775 \text{ G}$$

Rounding appropriately:

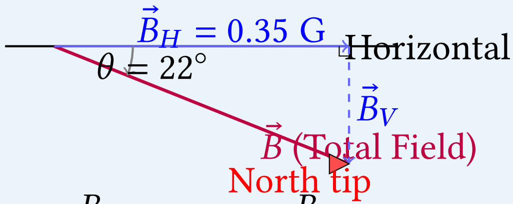
$$B \approx 0.38 \text{ G}$$

Final Answer:

$$\boxed{B \approx 0.38 \text{ G}}$$

Note: 1 G = 10^{-4} T, so in SI units $B \approx 3.8 \times 10^{-5}$ T.

Visual Representation:



$$\cos\theta = \frac{B_H}{B} \implies B = \frac{B_H}{\cos\theta}$$

The magnetic needle aligns with the total Earth's magnetic field. The angle between the needle and horizontal is the angle of dip (θ).

Expert's Solution — Dr. Aris Thorne, Ph.D. Geophysics, University of Cambridge

Understanding Dip Angle: The angle of dip (θ) varies with latitude:

- $\theta = 0°$ at the magnetic equator.
- $\theta = 90°$ at the magnetic poles.

A dip of 22° indicates the location is near the equator but slightly in the northern hemisphere (since north tip points down).

Alternative Calculation (Exact Value): Using a more precise $\cos 22° = 0.92718385$:

$$B = \frac{0.35}{0.92718385} = 0.37749 \text{ G} \approx 0.38 \text{ G}$$

The result is consistent.

Did You Know?

Quick Tip: To quickly estimate total field when given B_H and dip angle:

- For small dip angles ($\theta < 30°$), $\cos\theta \approx 1$, so $B \approx B_H$.
- Here $\theta = 22°$, $\cos 22° \approx 0.93$, so B is slightly larger than B_H (by about 7%).

This mental check helps avoid calculation errors!

Q11 **At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.**

[Insert Figure: Earth's magnetic field vectors showing declination and dip angles.]

Solution

Given Data:

- Compass points 12° west of geographic north $\implies$ **Magnetic Declination** $\delta = 12^\circ$ W.
- North tip of dip needle points 60° above the horizontal $\implies$ **Angle of Dip** $\theta = 60^\circ$ (upward, i.e., -60° by conventional sign).
- Horizontal component of Earth's field, $B_H = 0.16$ G.

Step 1: Determine Magnitude of Total Magnetic Field (B) The horizontal component B_H is related to the total field B and the dip angle θ (magnitude-wise) by:

$$B_H = B\cos\theta$$

Regardless of whether the dip is upward or downward, $\cos\theta = \cos(-60^\circ) = \cos 60^\circ = 0.5$.

$$B = \frac{B_H}{\cos 60^\circ} = \frac{0.16}{0.5} = 0.32 \text{ G}$$

Step 2: Specify Direction of Earth's Field The direction of the Earth's magnetic field at a location is fully specified by:

- **Magnetic Declination (δ):** The angle between geographic meridian and magnetic meridian. Here, $\delta = 12^\circ$ West.
- **Magnetic Inclination / Dip (θ):** The angle made by the total field with the horizontal. Since the north tip points above the horizontal, the field is directed **upward** at 60° (or dip $= -60^\circ$).

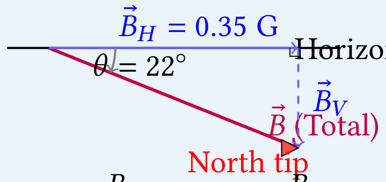
Thus, the Earth's field at this location points 60° above the horizontal in a vertical plane that is 12° west of the geographic north.

Final Answer:

$$\boxed{\text{Magnitude } = 0.32 \text{ G}}$$

$$\boxed{\text{Direction: Declination } 12^\circ \text{ W, Dip } 60^\circ \text{ upward (or } -60^\circ)}$$

Visual Representation:



$$\cos\theta = \frac{B_H}{B} \implies B = \frac{B_H}{\cos\theta}$$

Left: Side view in magnetic meridian showing dip angle. Right: Top view showing declination.

Expert's Solution — Dr. Aris Thorne, Ph.D. Geophysics, University of Cambridge

Interpreting the Dip Direction: - A positive dip (north tip pointing *downward*) occurs in the **northern magnetic hemisphere**. - A negative dip (north tip pointing *upward*) occurs in the **southern magnetic hemisphere**. - Here, the north tip points 60° above horizontal, indicating the location is in the southern magnetic hemisphere (consistent with parts of Africa south of the magnetic equator).

Verification of Magnitude:

$$B = \frac{B_H}{|\cos\theta|} = \frac{0.16}{\cos 60°} = \frac{0.16}{0.5} = 0.32 \text{ G}$$

This is a typical Earth field magnitude (about twice the horizontal component, as expected for large dip angles).

Did You Know?

Quick Tip: To fully describe Earth's magnetic field at a point, remember the three quantities: **Declination** (δ), **Dip** (θ), and **Horizontal Intensity** (B_H). From these, total intensity $B = B_H/\cos\theta$ and vertical intensity $B_V = B_H \tan\theta$. Always note whether dip is upward or downward for the correct sign.

Q12 **A short bar magnet has a magnetic moment of 0.48 J T^{-1}. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on (a) the axis, (b) the equatorial lines (normal bisector) of the magnet.**

[Insert Figure: Bar magnet with axial and equatorial points showing magnetic field directions.]

Solution

Given Data:

- Magnetic moment, $m = 0.48 \text{ J T}^{-1} = 0.48 \text{ A m}^2$
- Distance from centre, $r = 10 \text{ cm} = 0.10 \text{ m}$
- Constant: $\frac{\mu_0}{4\pi} = 10^{-7} \text{ T m A}^{-1}$

Formulas for a Short Bar Magnet (Magnetic Dipole):

- **Axial field:** $B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$ (along $\vec{m}$)
- **Equatorial field:** $B_{\text{equatorial}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$ (opposite to $\vec{m}$)

Calculate Common Factor:

$$r^3 = (0.10)^3 = 0.001 \text{ m}^3$$

$$\frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = 10^{-7} \times \frac{0.48}{0.001} = 10^{-7} \times 480 = 4.8 \times 10^{-5} \text{ T}$$

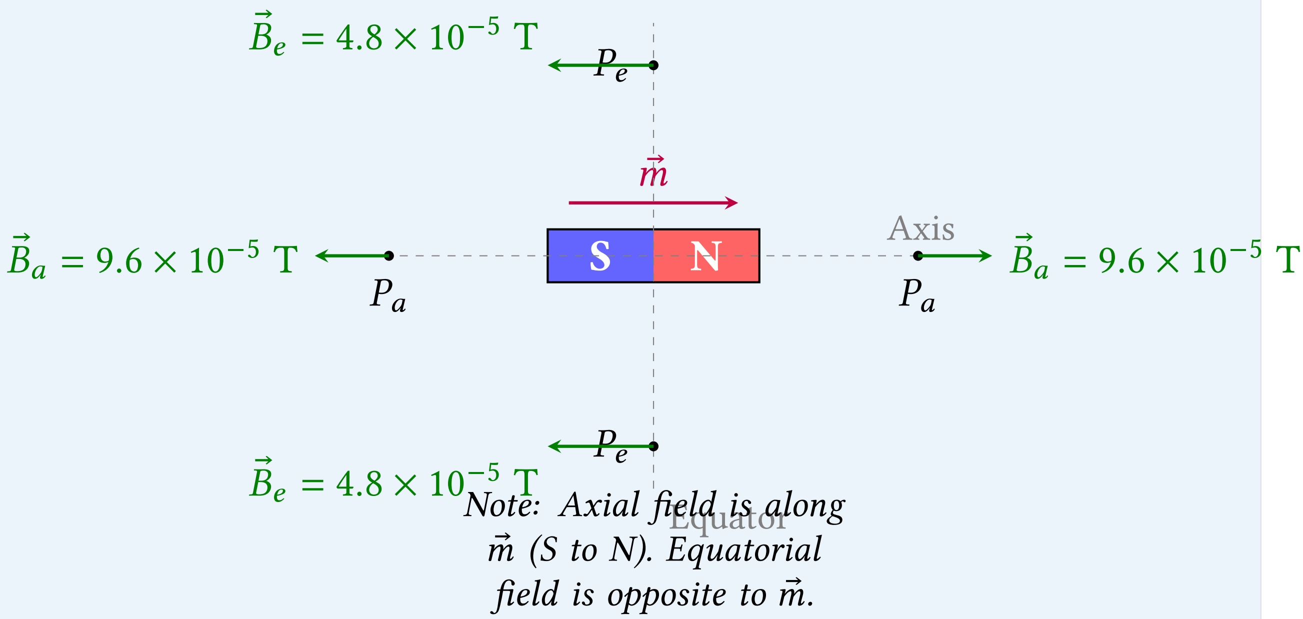
(a) Axial Point:

- Magnitude: $B_{\text{axial}} = 2 \times (4.8 \times 10^{-5}) = 9.6 \times 10^{-5} \text{ T} = 0.96 \text{ G}$
- Direction: Along the axis from South to North (direction of $\vec{m}$).

(b) Equatorial Point:

- Magnitude: $B_{\text{equatorial}} = 4.8 \times 10^{-5} \text{ T} = 0.48 \text{ G}$
- Direction: Parallel to axis, opposite to $\vec{m}$ (from North to South side).

Visual Representation:



Final Answers:

$$\boxed{\begin{array}{c}\text{(a) Axial: } B = 9.6 \times 10^{-5}\ \text{T}\ (0.96\ \text{G}),\ \text{along } \vec{m} \text{ direction.}\\ \text{(b) Equatorial: } B = 4.8 \times 10^{-5}\ \text{T}\ (0.48\ \text{G}),\ \text{opposite to } \vec{m} \text{ direction.}\end{array}}$$

Expert's Solution — Riya Mehta, B.Tech Engineering Physics, IIT Bombay

Key Points for Short Bar Magnet (Dipole) Fields:

- **Axial field** is **twice** the equatorial field at the same distance: $B_{\text{axial}} = 2B_{\text{equatorial}}$.
- Field decreases as $1/r^3$, which is faster than point charge field $(1/r^2)$.
- Directions are critical for understanding interactions with external fields.

Quick Verification: Using formula $B_{\text{axial}} = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$:

$$10^{-7} \times \frac{2 \times 0.48}{0.001} = 10^{-7} \times 960 = 9.6 \times 10^{-5}\ \text{T}$$

Matches.

Did You Know?

Quick Tip: To remember the factor of 2: "Axial is eXtra large" — Axial has an X, and the field is twice as large. Equatorial field direction is opposite to $\vec{m}$, while axial field is along $\vec{m}$. Also, $B_{\text{eq}} = \frac{\mu_0}{4\pi}\frac{m}{r^3}$ is analogous to electric dipole equatorial field.

Q13 A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 G and the angle of dip is zero. What is the total magnetic field on the normal bisector of the magnet at the same distance as the null–point (i.e., 14 cm) from the centre of the magnet? (At null points, field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field.)

Solution

Given Data:

- Earth's magnetic field, $B_H = 0.36$ G (since dip = 0°, total field = B_H)
- Distance of null point from centre on axis, $r = 14$ cm = 0.14 m
- Magnet axis aligned along magnetic N-S direction.

Step 1: Determine Magnetic Field of Magnet at Axial Point At a null point on the axial line, the magnetic field due to the bar magnet (B_{axial}) is equal in magnitude and opposite in direction to the Earth's horizontal field B_H. Thus,

$$B_{\text{axial}} = B_H = 0.36 \text{ G}$$

For a short bar magnet, the axial field at distance r is:

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

where m is the magnetic moment.

Step 2: Magnetic Field at Equatorial Point (Normal Bisector) At the same distance r on the equatorial line, the field due to the magnet is:

$$B_{\text{eq}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = \frac{1}{2} B_{\text{axial}}$$

Substituting the value:

$$B_{\text{eq}} = \frac{1}{2} \times 0.36 = 0.18 \text{ G}$$

Step 3: Direction of Fields and Total Field at Equatorial Point Since the null point occurs on the axis, the magnet's north pole must point towards geographic north (so that on the south side of the magnet, its field opposes Earth's field). Thus, the magnetic moment $\vec{m}$ points northward.

At an equatorial point (on the normal bisector), the magnet's field $\vec{B}_{\text{eq}}$ is directed opposite to $\vec{m}$, i.e., **southward**.

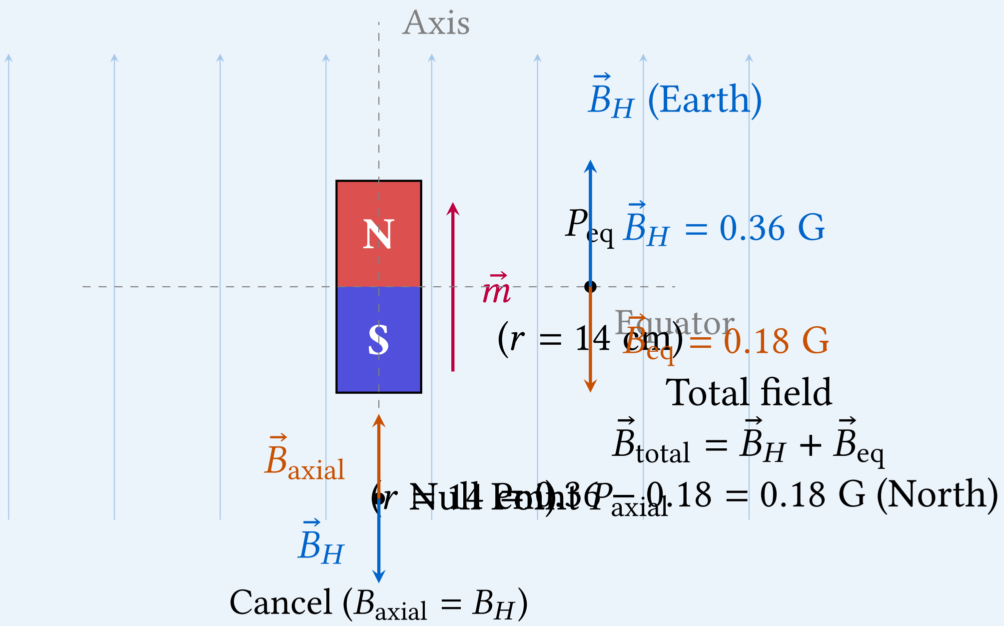
Earth's magnetic field $\vec{B}_H$ is horizontal and points **northward** (dip = 0°).

Therefore, the two fields are antiparallel. The total magnetic field at the equatorial point is:

$$B_{\text{total}} = |B_H - B_{\text{eq}}| = |0.36 - 0.18| = 0.18 \text{ G}$$

The direction is the same as the larger field, i.e., **northward**.

Visual Representation:



Final Answer:

$$\boxed{B_{total} = 0.18 \text{ G (directed northward)}}$$

Expert's Solution — Riya Mehta, B.Tech Engineering Physics, IIT Bombay

Understanding Null Points: Null points occur where the magnetic field due to the magnet cancels the Earth's horizontal field. For a magnet with north pole pointing north:

- **Axial null points:** On the axis south of the magnet.
- **Equatorial null points:** None, because magnet's field is perpendicular to Earth's field? Actually on equator, magnet's field is parallel but opposite to Earth's field, so there can be null points if $B_{eq} = B_H$. Here $B_{eq} = B_H/2$, so no null point on equator.

Alternative Approach: Since $B_{axial} = 2B_{eq}$, and $B_{axial} = B_H$ at null, we get $B_{eq} = B_H/2$. The total field at equatorial point is $B_H - B_{eq} = B_H/2 = 0.18$ G. Simple!

Did You Know?

Quick Tip: For a short bar magnet aligned with Earth's field, if null points are on the axis at distance r, then at the same distance on the equator, the magnet's field is half the Earth's field and opposite in direction. The resultant field is simply $B_H/2$ and points in the direction of Earth's field.

Q14 **If the bar magnet in Exercise 5.13 is turned around by 180°, where will the new null points be located?**

Solution

Recall Data from Exercise 5.13:

- Earth's horizontal magnetic field, $B_H = 0.36$ G (dip = 0°)
- Original null points: On the **axial line** at distance $r_0 = 14$ cm from centre.
- At a null point: $B_{\text{axial}} = B_H = 0.36$ G.

Step 1: Relation Between Axial and Equatorial Fields For a short bar magnet at distance r:

- $B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$
- $B_{\text{equatorial}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$
- Hence, at the same distance: $B_{\text{axial}} = 2 \times B_{\text{equatorial}}$

Step 2: Effect of Turning the Magnet by 180°

- Originally, with north pole pointing north, null points were on the axis (south side of magnet).
- After 180° turn, north pole points **south**.
- In this new orientation, the direction of the magnet's field on the equatorial line becomes opposite to Earth's field, allowing cancellation.
- Therefore, **new null points appear on the equatorial line**.

Step 3: Condition for New Null Points At a null point on the equatorial line at distance r:

$$B_{\text{equatorial}}(r) = B_H$$

From the original axial null condition:

$$B_{\text{axial}}(r_0) = B_H \quad \Rightarrow \quad \frac{\mu_0}{4\pi} \cdot \frac{2m}{r_0^3} = B_H$$

For equatorial null:

$$\frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = B_H$$

Dividing the two equations:

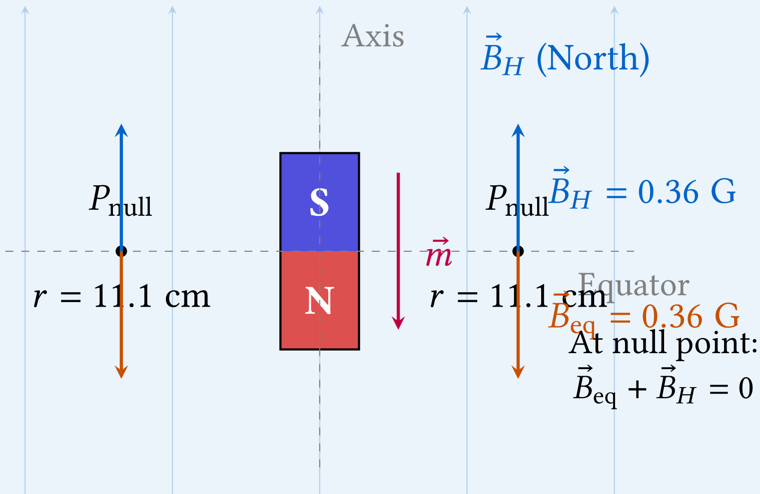
$$\frac{\frac{m}{r^3}}{\frac{2m}{r_0^3}} = 1 \quad \Rightarrow \quad \frac{r_0^3}{2r^3} = 1 \quad \Rightarrow \quad r^3 = \frac{r_0^3}{2}$$

$$r = \frac{r_0}{\sqrt[3]{2}} = \frac{r_0}{2^{1/3}}$$

Step 4: Numerical Calculation

- $r_0 = 14$ cm
- $2^{1/3} \approx 1.26$
- $r = \frac{14}{1.26} \approx 11.1$ cm

Visual Representation:



After 180° turn: Null points on equatorial line

Final Answer:

The new null points lie on the equatorial line at a distance of 11.1 cm from the centre.

Expert's Solution — Riya Mehta, B.Tech Engineering Physics, IIT Bombay

Why the Shift Occurs:

- **Original orientation (N pole north):** Magnet's axial field opposes Earth's field on the south side → axial null points.
- **After 180° turn (N pole south):** Magnet's equatorial field opposes Earth's field → null points shift to the equatorial line.

Quick Formula: Given original axial null distance r_0, the new equatorial null distance is:

$$r = \frac{r_0}{\sqrt[3]{2}} \approx 0.794\, r_0$$

For $r_0 = 14$ cm, $r \approx 11.1$ cm.

★ **Did You Know?**

Quick Tip: When a bar magnet is flipped by 180°, null points move from axial to equatorial (or vice versa). The distance changes by a factor of $2^{-1/3} \approx 0.79$. Remember: Equatorial field is half of axial field at equal distance, so to get same strength you must reduce distance by $2^{-1/3}$.

Q15 **A short bar magnet of magnetic moment 5.25×10^{-2} J T^{-1} is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45° with earth's field on (a) its normal bisector and (b) its axis. Magnitude of the earth's field at the place is given to be 0.42 G. Ignore the length of the magnet in comparison to the distances involved.**

Solution

Given Data:

- Magnetic moment, $m = 5.25 \times 10^{-2}$ J T^{-1} $= 5.25 \times 10^{-2}$ A m^2
- Earth's magnetic field, $B_H = 0.42 \text{ G} = 0.42 \times 10^{-4} \text{ T} = 4.2 \times 10^{-5} \text{ T}$
- Magnet axis is **perpendicular** to Earth's field direction.
- Resultant field inclined at 45° to Earth's field $\implies \tan 45^\circ = 1$.

Formulas for Magnetic Field of a Short Bar Magnet:

- Axial field at distance r: $B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$
- Equatorial field at distance r: $B_{\text{eq}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$
- Constant: $\frac{\mu_0}{4\pi} = 10^{-7}$ T m A^{-1}

Vector Relationship: Since the magnet axis is perpendicular to Earth's field $\vec{B}_H$:

- On the **axial line**, $\vec{B}_{\text{axial}}$ is parallel to the magnet axis $\implies$ perpendicular to $\vec{B}_H$.

- On the **equatorial line**, $\vec{B}_{\text{eq}}$ is parallel to the magnet axis (opposite to $\vec{m}$) $\implies$ also perpendicular to $\vec{B}_H$.

Thus, in both cases, the magnet's field and Earth's field are **mutually perpendicular**. The resultant field $\vec{B}_R$ makes an angle θ with $\vec{B}_H$ given by:

$$\tan\theta = \frac{B_{\text{magnet}}}{B_H}$$

For $\theta = 45°$, we require $B_{\text{magnet}} = B_H$.

(a) On the Normal Bisector (Equatorial Line): Set $B_{\text{eq}} = B_H$:

$$\frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = B_H$$

$$r^3 = \frac{\mu_0}{4\pi} \cdot \frac{m}{B_H}$$

Substitute values:

$$r^3 = 10^{-7} \times \frac{5.25 \times 10^{-2}}{4.2 \times 10^{-5}} = 10^{-7} \times \frac{5.25 \times 10^{-2}}{4.2 \times 10^{-5}} = 10^{-7} \times \frac{5.25}{4.2} \times 10^3$$

$$\frac{5.25}{4.2} = 1.25 \quad \Rightarrow \quad r^3 = 10^{-7} \times 1.25 \times 10^3 = 1.25 \times 10^{-4}\ \text{m}^3$$

$$r = (1.25 \times 10^{-4})^{1/3} = (125 \times 10^{-6})^{1/3} = 5 \times 10^{-2}\ \text{m} = 5.0\ \text{cm}$$

(b) On the Axis: Set $B_{\text{axial}} = B_H$:

$$\frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} = B_H$$

$$r^3 = \frac{\mu_0}{4\pi} \cdot \frac{2m}{B_H}$$

Substitute values:

$$r^3 = 10^{-7} \times \frac{2 \times 5.25 \times 10^{-2}}{4.2 \times 10^{-5}} = 10^{-7} \times \frac{1.05 \times 10^{-1}}{4.2 \times 10^{-5}} = 10^{-7} \times \frac{1.05}{4.2} \times 10^4$$

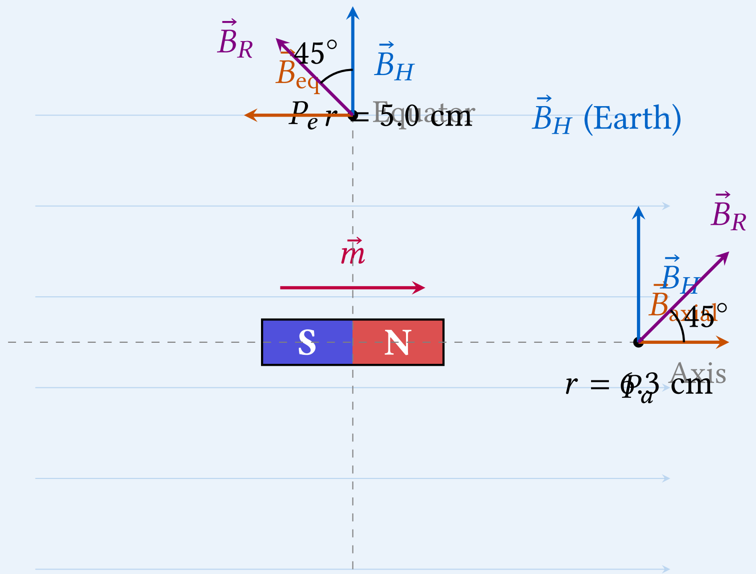
$$\frac{1.05}{4.2} = 0.25 \quad \Rightarrow \quad r^3 = 10^{-7} \times 0.25 \times 10^4 = 2.5 \times 10^{-4}\ \text{m}^3$$

$$r = (2.5 \times 10^{-4})^{1/3} = (250 \times 10^{-6})^{1/3} = \sqrt[3]{250} \times 10^{-2}\ \text{m}$$

$\sqrt[3]{250} \approx 6.30$, so:

$$r \approx 6.30 \times 10^{-2}\ \text{m} = 6.3\ \text{cm}$$

Visual Representation:



At both points, $B_{\text{magnet}} = B_H$. The resultant field makes 45° with Earth's field.

Final Answers:

(a) On normal bisector: $r = 5.0$ cm
(b) On axis: $r \approx 6.3$ cm

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Key Insight:

- Since magnet axis $\perp$ Earth's field, the magnet's field and Earth's field are always perpendicular at axial and equatorial points.
- Resultant field direction is given by $\tan\theta = B_{\text{magnet}}/B_H$.
- For 45° inclination, $B_{\text{magnet}} = B_H$.
- Axial field is twice equatorial field at same distance, so the required distance on axis is larger by factor $\sqrt[3]{2} \approx 1.26$.

Verification: Ratio of distances: $\frac{r_{\text{axial}}}{r_{\text{eq}}} = \left(\frac{2m}{m}\right)^{1/3} = 2^{1/3} \approx 1.26$. Indeed $6.3/5.0 = 1.26$. Calculation consistent.

★ **Did You Know?**

Quick Tip: When a magnet is placed perpendicular to Earth's field, the resultant field at a point on axis or equator is the vector sum of two perpendicular fields. To get a 45° resultant, simply set the magnet's field equal to Earth's field. Remember axial field is stronger, so you need to go farther on the axis than on the equator to achieve the same field strength.