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NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12th Physics, Chapter 6

Chapter 6: Electromagnetic Induction

About this Chapter

Electromagnetic induction is the production of an **e.m.f.** in a circuit by a changing magnetic flux. The chapter is built on **Faraday's law** $\varepsilon = -d\Phi_B/dt$ and **Lenz's law**, leading on to motional e.m.f., self and mutual inductance, eddy currents and AC generation. By the end you should be able to compute induced e.m.f.s and currents for moving conductors, time-varying fields and coupled coils, and reason about the direction of induced effects.

Topics covered: Magnetic flux • Faraday & Lenz laws • Motional e.m.f. • Self & mutual inductance • Eddy currents • Energy in a magnetic field

Quick Formula Sheet

Magnetic flux:
$\Phi_B = \int \vec{B}\cdot d\vec{A}$

Faraday's law:
$\varepsilon = -\frac{d\Phi_B}{dt}$

Motional e.m.f.:
$\varepsilon = Bvl$

Self inductance:
$L = \frac{\mu_0 N^2 A}{l}$

Mutual inductance:
$M_{12} = M_{21}$

NCERT Exemplar Problems

MCQ I — Multiple Choice (Single Correct)

Q 6.1 **A square of side L metres lies in the x–y plane in a region, where the magnetic field is given by $\vec{B} = B_0(2\hat{i} + 3\hat{j} + 4\hat{k})$ T, where B_0 is constant. The magnitude of flux passing through the square is**

(A) $2B_0L^2$ **Wb.**
(B) $3B_0L^2$ **Wb.**
(C) $4B_0L^2$ **Wb.**
(D) $\sqrt{29}\, B_0L^2$ **Wb.**

SOLUTION

Correct option: (C) $4B_0L^2$ Wb.

Concept used. The magnetic flux through a flat surface of area $\vec{A}$ in a uniform field $\vec{B}$ is the scalar product $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$, where θ is the angle between $\vec{B}$ and the area vector $\vec{A}$. The area vector is along the outward normal of the surface, and its magnitude equals the area.

Step 1. The square of side L lies in the x–y plane, so its normal points along $\hat{k}$. Hence

$$\vec{A} = L^2\,\hat{k}.$$

Step 2. Compute the dot product term by term:

$$\begin{aligned}\Phi_B &= \vec{B}\cdot\vec{A} = B_0(2\hat{i} + 3\hat{j} + 4\hat{k})\cdot(L^2\hat{k}) \\ &= B_0L^2\big[2(\hat{i}\cdot\hat{k}) + 3(\hat{j}\cdot\hat{k}) + 4(\hat{k}\cdot\hat{k})\big] \\ &= B_0L^2\big[0 + 0 + 4\big] = 4B_0L^2\,\text{Wb}.\end{aligned}$$

Step 3. Only the $\hat{k}$ component of $\vec{B}$ pierces the loop; the $\hat{i}$ and $\hat{j}$ components lie *in* the plane of the square and contribute zero flux. So (A) (which would give the $\hat{i}$ coefficient), (B) (the $\hat{j}$ coefficient) and (D) (which uses $|\vec{B}|$ as if the area vector were parallel to $\vec{B}$) are all wrong.

> **Final Answer:** Option **(C)**: $\Phi_B = 4B_0L^2$ Wb.

EXPERT'S SOLUTION *: Aarav Mehta, M.Sc. Physics, IIT Bombay*

Strategic angle. A loop only "sees" the field component along its normal. Project the field onto $\hat{n}$ first; everything else is arithmetic.

Step 1. Loop normal: $\hat{n} = \hat{k}$. Component of $\vec{B}$ along $\hat{n}$ is $B_z = 4B_0$.

Step 2. Flux $= B_z\cdot(\text{area}) = 4B_0\cdot L^2 = 4B_0L^2$ Wb.

Why this matters. The same trick handles every constant-field flux problem: identify $\hat{n}$, read off the parallel component, and multiply by area.

Alternative – via $|\vec{B}|\cos\theta$. The angle between $\vec{B}$ and $\hat{k}$ satisfies $\cos\theta = \dfrac{\vec{B}\cdot\hat{k}}{|\vec{B}|} = \dfrac{4}{\sqrt{29}}$, so $\Phi_B = |\vec{B}|A\cos\theta = B_0\sqrt{29}\cdot L^2\cdot\dfrac{4}{\sqrt{29}} = 4B_0L^2$ Wb – exactly the same answer, just a longer route. Choice (D) is the trap that drops the $\cos\theta$ factor.

Unit cross-check. $[B][A] =$T·m^2 $=$ Wb. Coefficient is dimensionless, so $4B_0L^2$ carries units of Wb as required.

Concept linkage. The very same projection trick reappears in Chapter 1 (electric-flux through a tilted area) and in Chapter 4 (force on a current-carrying loop – only the perpendicular component of $\vec{B}$ contributes to the torque). Flux is fundamentally a "component-along-normal $\times$ area" object.

Final Answer: $\Phi_B = 4B_0L^2$ Wb.

☞ Useful aside

For *any* planar loop in a uniform field, flux equals (normal-component of $\vec{B}$)$\times$(area). The in-plane components of $\vec{B}$ are "parallel passengers" – they thread along the surface but never pierce it.

Q 6.2 A loop, made of straight edges, has six corners at $A(0,0,0)$, $B(L,0,0)$, $C(L,L,0)$, $D(0,L,0)$, $E(0,L,L)$ and $F(0,0,L)$. A magnetic field $\vec{B} = B_0(\hat{i}+\hat{k})$ T is present in the region. The flux passing through the loop $ABCDEFA$ (in that order) is

(A) B_0L^2 **Wb.**

(B) $2B_0L^2$ **Wb.**

(C) $\sqrt{2}\,B_0L^2$ **Wb.**

(D) $4B_0L^2$ **Wb.**

SOLUTION

Correct option: (B) $2B_0L^2$ Wb.

Concept used. The loop is non-planar but still bounds a surface. We split it into two planar squares and add their fluxes $\Phi = \vec{B}\cdot\vec{A}_1 + \vec{B}\cdot\vec{A}_2$. The normals are fixed by the order in which the corners are traversed, using the right-hand rule (curl fingers along $ABCD\ldots$, thumb gives $\hat{n}$).

Step 1. *Sub-square 1:* $A(0,0,0)\to B(L,0,0)\to C(L,L,0)\to D(0,L,0)$ is a square in the x–y plane, traversed counter-clockwise as seen from $+z$. So $\vec{A}_1 = L^2\hat{k}$.

Step 2. *Sub-square 2:* $D(0,L,0)\to E(0,L,L)\to F(0,0,L)\to A(0,0,0)$ is a square in the y–z plane (i.e. the $x=0$ plane). Traversed in this order it is counter-clockwise as seen from $+x$. So $\vec{A}_2 = L^2\hat{i}$.

Step 3. Compute each flux:

$$\vec{B}\cdot\vec{A}_1 = B_0(\hat{i}+\hat{k})\cdot(L^2\hat{k}) = B_0L^2,$$
$$\vec{B}\cdot\vec{A}_2 = B_0(\hat{i}+\hat{k})\cdot(L^2\hat{i}) = B_0L^2.$$

Step 4. Total flux

$$\Phi_B = B_0L^2 + B_0L^2 = 2B_0L^2\,\text{Wb}.$$

Final Answer: Option **(B)**: $\Phi_B = 2B_0L^2$ Wb.

EXPERT'S SOLUTION : *Priya Sharma, M.Sc. Physics, IISc Bengaluru*

Strategic angle. Use the fact that flux depends only on the *boundary*, not on the chosen surface. Pick the easiest surface – two squares meeting along AD.

Step 1. Square in xy-plane bounded by $ABCD$: normal $\hat{k}$, area L^2, gives B_0L^2.

Step 2. Square in yz-plane bounded by $ADEF$: normal $\hat{i}$, area L^2, gives B_0L^2.

Step 3. Sum: $2B_0L^2$ Wb.

Why this matters. Flux through any cap with the same boundary is the same – a direct consequence of $\nabla\cdot\vec{B} = 0$.

Alternative – shrink the boundary. You can replace the L-shaped loop with the flat triangle that has the same edge sequence projected onto each face. Because $\vec{B} = B_0(\hat{i} + \hat{k})$ has *equal* x- and z-components, the two square-projections each carry B_0L^2, giving the same total $2B_0L^2$ – a nice consistency check.

Sign discipline. The corner order $A \to B \to C \to D \to E \to F \to A$ fixes *both* normals to point outward along $+\hat{k}$ and $+\hat{i}$ respectively. Had the problem said $FEDCBA$, both normals would flip and the answer would be $-2B_0L^2$. Marks are often lost on this.

Concept linkage. The independence of flux on the chosen cap is exactly why we can apply Faraday's law to any convenient surface – critical when handling solenoids and toroids later in this chapter.

Final Answer: $\Phi_B = 2B_0L^2$ Wb.

☞ **Recall**

$\nabla\cdot\vec{B} = 0$ (Gauss's law for magnetism) means magnetic field lines have *no sources*: they close on themselves. So flux *into* any closed surface = flux *out*, which is why splitting an open surface into pieces and adding their fluxes works no matter how the pieces are oriented.

Q 6.3 **A cylindrical bar magnet is rotated about its axis (Fig. 6.1). A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then**

(A) a direct current flows in the ammeter A.

(B) no current flows through the ammeter A.

(C) an alternating sinusoidal current flows through the ammeter A with a time period $T = 2\pi/\omega$.

(D) a time-varying non-sinusoidal current flows through the ammeter A.

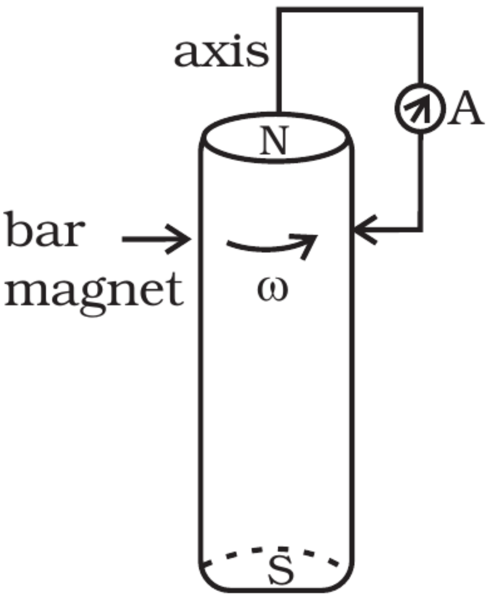


Fig. 6.1

Fig. 6.1, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Correct option: (B) No current flows through the ammeter.

Concept used. Faraday's law says $\varepsilon = -d\Phi_B/dt$. A current can flow only if the magnetic flux linked with the closed circuit *changes* with time.

Step 1. Identify the closed circuit: axis wire $\rightarrow$ ammeter $\rightarrow$ cylindrical surface contact, with the bar magnet itself acting as one "arm".

Step 2. As the magnet rotates about its own axis, its geometry and its magnetic field configuration are unchanged at every instant – the field has cylindrical symmetry about the rotation axis. Therefore the flux Φ_B linked with the circuit is *constant* in time:

$$\frac{d\Phi_B}{dt} = 0 \quad \Longrightarrow \quad \varepsilon = 0.$$

Step 3. With $\varepsilon = 0$, Ohm's law gives $I = \varepsilon/R = 0$. Hence no current flows, ruling out (A), (C) and (D).

Final Answer: Option **(B)**: No current; rotating an axially symmetric magnet about its own symmetry axis does not change the flux.

EXPERT'S SOLUTION : *Rohit Verma, M.Sc. Physics, University of Delhi*

Strategic angle. Spin-symmetry test – if rotating the source leaves it looking identical, the field pattern is time-independent and no e.m.f. can appear.

Step 1. The bar magnet is cylindrically symmetric about its axis. Rotation about that axis is a symmetry of the system.

Step 2. A symmetry of the source cannot change the flux through any fixed circuit – so $d\Phi/dt = 0$ and $\varepsilon = 0$.

Why this matters. A real homopolar generator needs the wire itself to move through the field, not just the magnet to spin; this question is its negative counterpart.
Alternative – motional emf check. Try to compute a motional e.m.f. $\int(\vec{v} \times \vec{B})\cdot d\vec{\ell}$ along the axis wire: $\vec{v} = 0$ for points on the axis (no translation), and along the radial brush contact the magnet's surface also moves with velocity tangential, but the closed loop is stationary in the lab. The conductor (wire + ammeter) does not move at all, so the line integral vanishes element-by-element.
Common pitfall. Students often confuse this setup with the *Faraday disc* (rotating conducting disc in a fixed field), where a real DC e.m.f. does appear. The difference: in the Faraday disc the *conductor* rotates through the field. Here the conductor is fixed and only the field source spins – but the field pattern itself is rotation-invariant, so nothing changes.
Concept linkage. "A field that is invariant under a continuous symmetry of its source cannot induce an e.m.f." generalises the "$d\Phi/dt = 0$" criterion to arbitrary geometries.

Final Answer: No current; symmetry forbids any change in flux.

☞ Exam Tip

Whenever a question mentions "rotating a magnet about its own axis" or "a uniformly rotating axially-symmetric source", the answer to "induced current?" is almost always *zero*. The trick word is **symmetry**: if the rotation maps the source onto itself, the field configuration is time-independent in the lab frame.

Q 6.4 There are two coils A and B as shown in Fig. 6.2. A current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counter-clockwise. B is kept stationary when A moves. We can infer that

(A) there is a constant current in the clockwise direction in A.

(B) there is a varying current in A.

(C) there is no current in A.

(D) there is a constant current in the counter-clockwise direction in A.

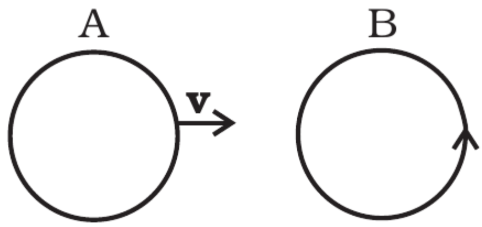


Fig. 6.2

Fig. 6.2, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Correct option: (D) A constant current in the counter-clockwise direction in A.

Concept used. An e.m.f. is induced in coil B only when the flux through B changes. The flux through B depends on (i) the current in A and (ii) the separation between A and B.

Step 1. The current in B exists *only while* A *is moving* and stops the instant A stops. Therefore the source of the changing flux is the motion of A, not the current variation in A itself. This rules out (B): if the current in A were varying, B would show a current even with A at rest.

Step 2. Since B does carry a current while A is moving, the flux through B must change with separation, which requires the current in A to be *non-zero*. This rules out (C).

Step 3. The question explicitly tells us the current in A is counter-clockwise (as seen in the figure). Hence (A), which specifies clockwise, is wrong.

Step 4. Combining: the current in A is steady (so motion alone causes the flux change) and counter-clockwise.

> **Final Answer:** Option **(D)**: constant counter-clockwise current in coil A.

EXPERT'S SOLUTION *: Sneha Patel, B.Tech Electrical Engineering, IIT Gandhinagar*

Strategic angle. Decouple the two causes of flux change – current variation vs. geometric motion – and test each against the data.

Step 1. "B has current iff A moves" $\Rightarrow$ flux change is purely geometric, not current-driven $\Rightarrow$ I_A is constant.

Step 2. "Current in A is counter-clockwise (given)". So I_A is constant and counter-clockwise.

Why this matters. This is exactly the principle of a transformer's primary at DC – moving a DC-energised coil mimics an AC primary for the secondary.
Alternative – mutual-inductance view. If $\Phi_B = M(t)\, I_A$, then $\varepsilon_B = -\dfrac{d\Phi_B}{dt} = -\dfrac{dM}{dt} I_A - M\dfrac{dI_A}{dt}$. The data ($\varepsilon_B$ exists $\Leftrightarrow$ A moves) tells us only the first term contributes: $dM/dt \neq 0$ (because separation is changing) but $dI_A/dt = 0$ (because I_A is steady). This is exactly the algebraic statement of step 1.
Common pitfall – mistaking ε_B for I_A. Some students see "current in B is in some direction" and conclude that I_A must be in that same direction. The current in B is the *induced* one and its sign is set by Lenz's law – it has no direct correspondence with I_A. The question states I_A's direction outright; that's what fixes (A) vs. (D).
Concept linkage. The same logic underlies wireless-charging pads – a DC-energised coil produces no e.m.f. in the receiver *unless* the geometry (separation, alignment, or current) changes.

> **Final Answer:** Constant counter-clockwise current in A.

✗ Common Mistake

"Current flows in B only while A moves" does *not* imply I_A is zero – it means I_A is *steady*. Only a changing *flux* (not a changing source-current alone) is required; here the geometry change does the job while I_A holds constant.

Q 6.5 **Same as problem 6.4 except the coil A is made to rotate about a vertical axis (Fig. 6.3). No current flows in B if A is at rest. The current in coil A, when the current in B (at $t = 0$) is counter-clockwise and the coil A is as shown at this instant ($t = 0$), is**
(A) constant current clockwise.
(B) varying current clockwise.
(C) varying current counter-clockwise.
(D) constant current counter-clockwise.

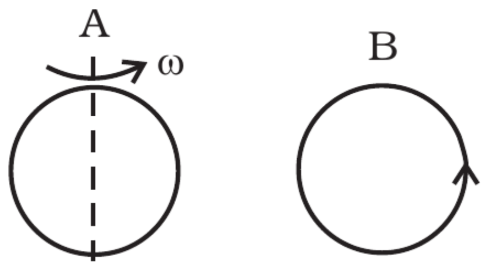


Fig. 6.3

Fig. 6.3, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Correct option: (A) Constant current in the clockwise direction in A.

Concept used. As in 6.4, the e.m.f. in B is driven by the change in flux through B. Here the change is produced by A *rotating*, not translating.

Step 1. "No current in B if A is at rest" $\Rightarrow$ the current in A does not vary on its own. So I_A is constant – rules out (B) and (C).

Step 2. As A rotates, the flux it sends through B oscillates, producing an alternating e.m.f. in B. The given direction in B at $t = 0$ is counter-clockwise.

Step 3. By Lenz's law, the induced current in B opposes the change that produced it. For B's current to be counter-clockwise at this instant, the magnetic flux from A through B must be *decreasing* in the direction pointing from A into B. Working backwards through the right-hand rule, this requires the steady current in A to circulate in the *clockwise* sense (as viewed from B). So (A) is correct, (D) is wrong.

> **Final Answer:** Option **(A)**: constant clockwise current in A.

EXPERT'S SOLUTION *: Aarav Mehta, M.Sc. Physics, IIT Bombay*

Strategic angle. Two pieces of evidence: (i) only motion drives B's current, so I_A is steady; (ii) Lenz's law converts the given direction in B at $t = 0$ into a direction in A.

Step 1. "At rest $\Rightarrow$ no current in B" fixes $I_A = \text{const}$.

Step 2. Sign analysis via Lenz's law on the instantaneous geometry of Fig. 6.3 reverses the sense from B to $A \Rightarrow$ clockwise in A.

Why this matters. A rotating DC-energised coil is the simplest AC generator – the same

mechanism powers commercial alternators.
Alternative – flux-sign book-keeping. At $t = 0$ let $\hat{n}_B$ point out of the page. If I_B is counter-clockwise (right-hand rule) then the *induced* field at B's centre is along $+\hat{n}_B$. By Lenz's law the induced field opposes the change, so the flux through B from A must be *decreasing* along $+\hat{n}_B$. The only way this happens with I_A steady is that A's axis is rotating away from coincidence with $\hat{n}_B$, with I_A producing field along $-\hat{n}_B$ at the instant shown – which (right-hand rule applied to A) means I_A runs clockwise.
Numerical cross-check. If $\Phi_B(t) = M_0 I_A \cos(\omega t)$ with $M_0 > 0$ and $I_A > 0$, then $\varepsilon_B = -\dot{\Phi}_B = +M_0 I_A \omega \sin(\omega t)$. At $t = 0^+$ this is positive in our convention, so the current in B flows in the positive sense (counter-clockwise as drawn) – matching the data with I_A steady.
Concept linkage. This is literally how a household ceiling fan generates back-EMF when momentarily disconnected – a DC-energised rotor in a stator coil acts as an instantaneous AC source.

Final Answer: Constant clockwise current in A.

☞ Exam Tip

For "what direction is I_A?" questions where A rotates and you are told I_B's direction at $t = 0$: (i) thumb-rule on I_B gives the induced field $\vec{B}_{\text{ind}}$ at B; (ii) Lenz says $\vec{B}_{\text{ind}}$ opposes $\frac{d\vec{\Phi}_A}{dt}$; (iii) reading the sign through the rotation reveals I_A's direction. Always do $I_B \to \vec{B}_{\text{ind}} \to \dot{\vec{\Phi}}_A \to I_A$ in that order.

Q 6.6 The self inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N, increases as

(A) l and A increase.
(B) l decreases and A increases.
(C) l increases and A decreases.
(D) both l and A decrease.

SOLUTION

Correct option: (B) l decreases and A increases.
Concept used. The self-inductance of a long air-core solenoid of length l, cross-sectional area A and N total turns is

$$L = \frac{\mu_0 N^2 A}{l},$$

derived from $\Phi = N\,BA$ with $B = \mu_0 (N/l) I$.

Step 1. Read off the dependence: with N fixed and μ_0 a constant, $L \propto A/l$.

Step 2. Therefore L increases iff A increases and/or l *decreases*.

Step 3. Check each option against $L \propto A/l$:

- (A): both up $\Rightarrow$ depends on which dominates; not guaranteed to increase.
- (B): $A\uparrow, l\downarrow \Rightarrow L\uparrow$. ✓
- (C): $A\downarrow, l\uparrow \Rightarrow L\downarrow$.
- (D): both down $\Rightarrow$ indeterminate.

> **Final Answer:** Option **(B)**: L increases when l decreases and A increases.

EXPERT'S SOLUTION : *Priya Sharma, M.Sc. Physics, IISc Bengaluru*

Strategic angle. Memorise the ratio A/l for a solenoid; the question is then pure proportionality.

Step 1. $L = \mu_0 N^2 A/l$, so the "increase" directions are $A\uparrow$ or $l\downarrow$.

Step 2. Only option (B) does both unambiguously.

Why this matters. Real inductors increase L by packing more turns into a shorter, fatter coil – exactly the (B) recipe.

Alternative – per-unit-length form. With $n = N/l$ (turns per metre), $B = \mu_0 n I$ and the flux linkage is $N\Phi = N \cdot BA = \mu_0 \dfrac{N^2}{l} AI$, so $L = \dfrac{\mu_0 N^2 A}{l}$. The same formula, derived from $\Phi = LI$, makes the A/l dependence obvious.

Numerical cross-check. For $N = 1000$, $A = 10^{-3}\,\text{m}^2$, $l = 0.5\,\text{m}$:
$L = \dfrac{4\pi \times 10^{-7} \cdot 10^6 \cdot 10^{-3}}{0.5} \approx 2.5\,\text{mH}$. Halving l to $0.25\,\text{m}$ doubles it to $5.0\,\text{mH}$; doubling A to $2 \times 10^{-3}\,\text{m}^2$ doubles it again to $10\,\text{mH}$ – consistent with (B).

Common pitfall. Many students try option (A) thinking "bigger means more inductance". But a longer coil with the same N reduces $n = N/l$ and so weakens B, dropping Φ *per turn* faster than the extra length helps. The dependence is on A/l, not on volume Al.

Concept linkage. The energy stored is $U = \frac{1}{2}LI^2 = \frac{1}{2\mu_0}B^2 \cdot Al$, so the energy density $u = B^2/(2\mu_0)$ is set by B alone (Section 6.10 of NCERT). L and volume together encode the same physics.

> **Final Answer:** $l\downarrow, A\uparrow \Rightarrow L\uparrow$.

♥ Why This Matters

The formula $L = \mu_0 N^2 A/l$ is the recipe for every choke, transformer primary and tank-circuit inductor: maximise N first (it appears *squared*), then maximise A/l. Iron cores multiply L by the relative permeability μ_r, often by $\sim 10^3$, which is why power transformers use laminated iron rather than air cores.

MCQ II — Multiple Choice (More Than One Correct)

Q 6.7 **A metal plate is getting heated. It can be because**

(A) a direct current is passing through the plate.

(B) it is placed in a time-varying magnetic field.

(C) it is placed in a space-varying magnetic field, but does not vary with time.

(D) a current (either direct or alternating) is passing through the plate.

SOLUTION

Correct options: (A), (B), (D).

Concept used. A metal plate dissipates energy (I^2R Joule heating, or eddy-current heating) whenever there is a current in it. The current may be supplied externally or induced via Faraday's law $\varepsilon = -d\Phi_B/dt$.

Step 1. **(A)** A direct current dissipates $P = I^2R$. ✓

Step 2. **(B)** A time-varying $\vec{B}$ produces $d\Phi/dt \neq 0$ and hence eddy currents in the plate – these dissipate as heat. ✓

Step 3. **(C)** A space-varying but *static* field has $\partial\vec{B}/\partial t = 0$ everywhere, so no e.m.f. is induced and (for a stationary plate) no eddy currents flow – no Joule heat. ✗

Step 4. **(D)** Any current, DC or AC, dissipates I^2R (or $I^2_{\text{rms}}R$). ✓

Final Answer: (A), (B), (D).

EXPERT'S SOLUTION *: Rohit Verma, M.Sc. Physics, University of Delhi*

Strategic angle. Pin down the two energy sources – an externally driven current ((A) and (D)) and induced eddy currents ((B)). Static spatial variation alone does no work, ruling out (C).

Step 1. External current $\Rightarrow$ I^2R heating: (A), (D).

Step 2. Time-varying $\vec{B}$ $\Rightarrow$ eddy currents $\Rightarrow$ heat: (B).

Step 3. Static field (even if non-uniform in space) $\Rightarrow$ no e.m.f. $\Rightarrow$ (C) is false.

Why this matters. Eddy-current heating is the principle of induction cooktops – a time-varying field heats the pan without contact.
Alternative – energy bookkeeping. Joule heat per unit volume is j^2/σ where j is the current density and σ the conductivity. So heating requires $j \neq 0$ somewhere inside the plate. The only ways to make $j \neq 0$ are: connect a battery (DC) or supply oscillation (AC) – options (A) and (D); or drive eddy currents with $\partial\vec{B}/\partial t \neq 0$ – option (B). A purely spatial $\vec{B}(r)$ that does not change in time leaves $\partial\vec{B}/\partial t = 0$ everywhere, hence $\vec{E} = 0$ in the plate's rest frame and $\vec{j} = \sigma\vec{E} = 0$.
Why (C) is wrong, in detail. "Space-varying but constant in time" is the field of a permanent magnet held still next to a stationary plate. No part of the plate sees a changing flux, so $\oint \vec{E}\cdot d\vec{\ell} = 0$ around every small loop. No e.m.f., no eddy currents, no heating.
Concept linkage. Eddy currents are not just heaters – they're also brakes. In electromagnetic braking of trains a copper plate moves through a static field; the motion of the conductor turns the static field into a time-varying one in the conductor's frame, dissipating kinetic energy as heat. (C) becomes a heating mechanism the moment the plate is allowed to move.

> **Final Answer:** (A), (B), (D).

✗ Common Mistake

"Non-uniform field heats the plate" is wrong if the plate is *at rest*. Spatial gradients of $\vec{B}$ don't create e.m.f. by themselves – it's $\partial\vec{B}/\partial t$ that does. The moment the plate or the field source starts moving relative to each other, option (C) becomes equivalent to (B) and heating returns.

Q 6.8 An e.m.f. is produced in a coil, which is not connected to an external voltage source. This can be due to

(A) the coil being in a time-varying magnetic field.

(B) the coil moving in a time-varying magnetic field.

(C) the coil moving in a constant magnetic field.

(D) the coil is stationary in an external spatially varying magnetic field, which does not change with time.

SOLUTION

Correct options: (A), (B), (C).
Concept used. Faraday's law $\varepsilon = -d\Phi_B/dt$. The flux $\Phi_B = \int \vec{B}\cdot d\vec{A}$ changes if either $\vec{B}$

changes with time, or the loop moves/changes shape so that the surface integral changes.

Step 1. **(A)** Stationary coil, $\vec{B}$ varies in time $\Rightarrow d\Phi/dt \neq 0 \Rightarrow \varepsilon \neq 0$. ✓

Step 2. **(B)** Coil moves *and* $\vec{B}$ varies in time – both contributions are present, $\varepsilon \neq 0$. ✓

Step 3. **(C)** Coil moves through a static $\vec{B}$ that is non-uniform across the coil (or the coil is reoriented) – motional e.m.f. $\varepsilon = \oint(\vec{v}\times\vec{B})\cdot d\vec{l} \neq 0$. ✓

Step 4. **(D)** Coil stationary in a static (but spatially varying) field $\Rightarrow$ no time-change of $\Phi \Rightarrow \varepsilon = 0$. ✗

Final Answer: (A), (B), (C).

EXPERT'S SOLUTION : *Sneha Patel, B.Tech Electrical Engineering, IIT Gandhinagar*

Strategic angle. Total $d\Phi/dt$ has two pieces – "$\vec{B}$ changes" and "loop moves". As long as at least one is non-zero, ε is non-zero.

Step 1. (A) $\vec{B}$ changes only: yes.

Step 2. (B) both change: yes.

Step 3. (C) loop moves only: yes (motional e.m.f.).

Step 4. (D) nothing changes: no.

Why this matters. A generator (case C) and a transformer (case A) are two sides of the same Faraday-law coin.

Alternative – the Leibniz decomposition. The total time-derivative of $\Phi = \int_S \vec{B}\cdot d\vec{A}$ for a moving surface $S(t)$ obeys

$$\frac{d\Phi}{dt} = \int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A} \; - \; \oint_{\partial S} (\vec{v}\times\vec{B})\cdot d\vec{\ell}.$$

The first term is the "transformer" piece (option A); the second is the "motional" piece (option C); option B has both. Option D zeros both terms.

Why (D) really fails. "Spatially varying but time-independent" $\vec{B}$ has $\partial\vec{B}/\partial t = 0$. A stationary loop has $\vec{v} = 0$, so both terms vanish identically. Even if $\vec{B}$ varies wildly in space, no induced e.m.f. appears – Φ is a fixed number set by the spatial integral.

Concept linkage. The two terms map onto two physical pictures we learn separately: transformers (term 1) and AC generators / linear generators / electromagnetic guns (term 2). Faraday's law unifies them.

Final Answer: (A), (B), (C).

☞ **Recall**
$\varepsilon = -\dfrac{d\Phi}{dt} = -\int_S \dfrac{\partial \vec{B}}{\partial t}\cdot d\vec{A} + \oint(\vec{v}\times\vec{B})\cdot d\vec{\ell}$. The first integral captures "$\vec{B}$ changes" and the second captures "loop moves". Either one suffices to generate an e.m.f.

Q 6.9 The mutual inductance M_{12} of coil 1 with respect to coil 2
(A) increases when they are brought nearer.
(B) depends on the current passing through the coils.
(C) increases when one of them is rotated about an axis.
(D) is the same as M_{21} of coil 2 with respect to coil 1.

SOLUTION

Correct options: (A), (D).
Concept used. Mutual inductance is a purely *geometric* quantity defined by

$$\Phi_{12} = M_{12}\, I_2,$$

where Φ_{12} is the flux through coil 1 due to current I_2 in coil 2. Reciprocity theorem: $M_{12} = M_{21}$.

Step 1. **(A)** Bringing coils nearer increases the flux of one through the other for the same current, so M increases. ✓

Step 2. **(B)** M is defined as Φ/I – the ratio is independent of I. Doubling the current doubles Φ, leaving M unchanged. So (B) is false. ✗

Step 3. **(C)** Rotation could either increase or decrease the overlap of flux lines – e.g. rotating one coil through 90° relative to the other reduces M. The statement "increases when rotated" is not generally true. ✗

Step 4. **(D)** The reciprocity theorem $M_{12} = M_{21}$ follows from the symmetry of Maxwell's equations in source-free regions. ✓

Final Answer: (A), (D).

EXPERT'S SOLUTION *: Aarav Mehta, M.Sc. Physics, IIT Bombay*

Strategic angle. Test each option against the defining relation $M_{12} = \Phi_{12}/I_2$ and the reciprocity $M_{12} = M_{21}$.

Step 1. Closer coils $\Rightarrow$ more flux linkage $\Rightarrow$ (A) true.

Step 2. M independent of current $\Rightarrow$ (B) false.

Step 3. Rotation can decrease coupling $\Rightarrow$ (C) not universally true.

Step 4. Reciprocity ⇒ (D) true.

Why this matters. Reciprocity (D) lets us choose the easier of the two flux integrals when computing M.

Alternative – vector-potential view. M_{12} can be written as Neumann's double-line integral

$$M_{12} = \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\vec{\ell}_1 \cdot d\vec{\ell}_2}{|\vec{r}_1 - \vec{r}_2|}.$$

The integrand is *manifestly symmetric* under $1 \leftrightarrow 2$, which proves $M_{12} = M_{21}$ on the spot (option D). And distance $|\vec{r}_1 - \vec{r}_2|$ appears in the denominator, so reducing it (bringing coils closer) increases M (option A).

Why (B) is firmly false. M is a *geometric coupling coefficient*, not a function of I. This is the same statement as "capacitance does not depend on the voltage stored".

Why (C) is conditional. Starting from $90°$ misalignment ($M \approx 0$) and rotating toward coaxial alignment *does* increase M. The blanket statement "rotation increases M" is too broad – it depends on the starting orientation. So (C) is not universally true and is counted wrong.

Numerical cross-check. For two coaxial circular coils of radii a and b ($b \ll a$) separated by z: $M = \dfrac{\mu_0 \pi a^2 b^2}{2(a^2 + z^2)^{3/2}}$. Independent of I. Decreases as z grows – consistent with (A).

Concept linkage. Reciprocity (D) is the same principle that gives equal mutual capacitance in a network – a deep result from linearity of Maxwell's equations.

Final Answer: (A), (D).

☞ Exam Tip

For any "does M depend on X?" MCQ, the only physical quantities M depends on are: number of turns, coil geometry (radii, lengths), relative position/orientation, and surrounding medium (μ_r). It *never* depends on the currents themselves – that's what makes M a "self-property of the geometry".

Q 6.10 **A circular coil expands radially in a region of magnetic field and no electromotive force is produced in the coil. This can be because**

(A) the magnetic field is constant.

(B) the magnetic field is in the same plane as the circular coil and it may or may not vary.

(C) the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably.

(D) there is a constant magnetic field in the perpendicular (to the plane of the coil)

direction.

SOLUTION

Correct options: (B), (C).

Concept used. For zero induced e.m.f., $d\Phi/dt = 0$, where $\Phi = BA\cos\theta$ uses only the field component perpendicular to the loop.

Step 1. **(A)** "Field constant" alone is not enough; if it is also perpendicular to the coil, the expanding area changes Φ and an e.m.f. appears. So (A) is incomplete. ✗

Step 2. **(B)** If $\vec{B}$ lies in the plane of the coil, then $\vec{B}\cdot d\vec{A} = 0$, so $\Phi = 0$ regardless of how the area changes or how $\vec{B}$ varies in time. Hence $\varepsilon = 0$. ✓

Step 3. **(C)** If $B_\perp$ decreases at exactly the rate at which the area increases (so that $B_\perp A = \text{const}$), then $d\Phi/dt = d(B_\perp A)/dt = 0$ and $\varepsilon = 0$. ✓

Step 4. **(D)** Constant perpendicular field with expanding area gives $\Phi = B_\perp \pi r^2(t)$ which is increasing, hence $\varepsilon \neq 0$. ✗

> **Final Answer:** (B), (C).

EXPERT'S SOLUTION *: Priya Sharma, M.Sc. Physics, IISc Bengaluru*

Strategic angle. Write $\Phi = B_\perp A$ and force the *product* to be constant (or zero).

Step 1. $B_\perp = 0$ identically $\Rightarrow \Phi = 0 \Rightarrow$ (B).

Step 2. $B_\perp(t)A(t) = \text{const} \Rightarrow d\Phi/dt = 0 \Rightarrow$ (C).

Why this matters. A vanishing time-derivative of a product is not the same as either factor being constant – a subtlety that eliminates (A) and (D).

Alternative – product-rule cross-check. $\frac{d\Phi}{dt} = \dot{B}_\perp A + B_\perp \dot{A}$. For this to vanish in case (C), the two terms must cancel: $\dot{B}_\perp/B_\perp = -\dot{A}/A$, i.e. $B_\perp$ must decrease at the *same logarithmic rate* the area grows. Calling $r(t)$ the loop radius, $\frac{\dot{B}_\perp}{B_\perp} = -2\frac{\dot{r}}{r}$. So $B_\perp \propto 1/r^2$ keeps Φ constant – a very specific tuning.

Why (D) really fails. With $\vec{B}$ constant and perpendicular, $\Phi = B(\pi r^2)$. As r grows, $\frac{d\Phi}{dt} = 2\pi B r\dot{r} \neq 0$ unless $\dot{r} = 0$ (loop is not expanding). The problem states it *is* expanding, so (D) generates an e.m.f. – it cannot be one of the zero-emf cases.

Concept linkage. This setup is the inverse of a homopolar generator: instead of the e.m.f. driving current, here we force a special $B(r,t)$ to keep the e.m.f. at zero. The same idea is used in superconducting flux pumps to maintain constant flux against a changing area.

> **Final Answer:** (B), (C).

♥ **Why This Matters**

"Zero induced emf" has *two* distinct origins: (i) the loop intercepts no perpendicular flux at all (option B), or (ii) the flux *rate of change* from area and from $\vec{B}$ cancel out (option C). Real flux-locked loops in superconducting magnets exploit exactly mechanism (ii) to hold Φ at a target value.

VSA — Very Short Answer

Q 6.11 Consider a magnet surrounded by a wire with an on/off switch S (Fig. 6.4). If the switch is thrown from the off position (open circuit) to the on position (closed circuit), will a current flow in the circuit? Explain.

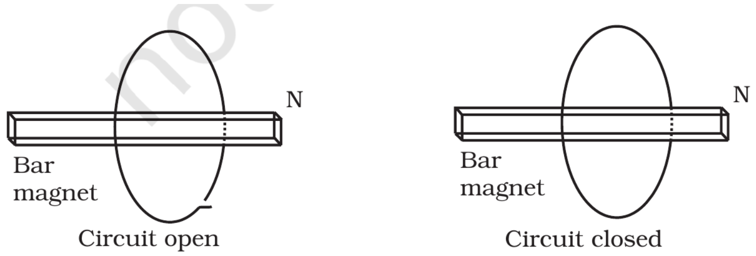


Fig. 6.4

Fig. 6.4, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. Faraday's law: an e.m.f. is induced in a loop only when the magnetic flux *through* that loop changes with time. Closing a switch makes the circuit conducting; it does *not* change the field of the (stationary) magnet.

Step 1. The bar magnet is at rest and its field is therefore static at every point. The wire loop is also at rest.

Step 2. Before the switch is closed, the loop is open – but the magnet's flux through the surface bounded by the wire still has some definite value Φ_0. After the switch is closed, the same field configuration gives the same Φ_0. Hence

$$\frac{d\Phi_B}{dt} = 0 \quad \Longrightarrow \quad \varepsilon = 0.$$

Step 3. With $\varepsilon = 0$, no current flows even though the loop is now closed and conducting.

> **Final Answer:** No – merely closing the switch does not change the flux, so no current flows. A current would appear only if the magnet (or the loop) starts moving, or the field is altered after the switch is closed.

EXPERT'S SOLUTION *: Rohit Verma, M.Sc. Physics, University of Delhi*

Strategic angle. Separate the topology of the circuit (open vs. closed) from the time-derivative of the flux. Only the latter drives an e.m.f.

Step 1. Closing the switch changes resistance from ∞ to a finite value but not Φ_B.

Step 2. $d\Phi/dt = 0 \Rightarrow \varepsilon = 0 \Rightarrow$ no current.

Alternative – circuit analogy. Think of the magnet's flux Φ_0 as a fixed reservoir. Faraday's law acts like a generator that needs Φ to *change*; if the reservoir level is constant, the generator outputs zero volts no matter whether you've connected a load (closed circuit) or left it open. Closing S only attaches the load.

Common pitfall. "Closing the switch should send a current into the loop because flux now passes through the closed loop." Wrong on two counts: (i) flux through the wire-bounded surface existed before too (Faraday's law cares about $d\Phi/dt$, not about loop "closedness"); (ii) the magnet has not moved, so Φ never changes. Currents need a *time-varying* flux, not just any flux.

Concept linkage. If immediately after closing S you started pulling the magnet away (or pushed it in), then $d\Phi/dt \neq 0$ and a current would appear. This is exactly Faraday's original 1831 experiment.

> **Final Answer:** No current flows on closing the switch.

☞ Useful aside

"Closing a switch" is a one-time topological event with no time extent. Faraday's law lives entirely on the *rate of change of flux*, so a one-shot reconfiguration of the circuit (with $\vec{B}$ unchanged) cannot drive a current.

Q 6.12 **A wire in the form of a tightly wound solenoid is connected to a DC source, and carries a current. If the coil is stretched so that there are gaps between successive elements of the spiral coil, will the current increase or decrease? Explain.**

SOLUTION

Concept used. For a tightly wound solenoid, the flux linked with the coil is $\Phi = LI$ with $L = \mu_0 n^2 V_{\text{sol}}$ and $n = N/l$ is the turn density. When the geometry is changed, Lenz's law dictates the direction of the induced response.

Step 1. Stretching the coil creates gaps between successive turns. Magnetic field lines now *leak out* through these gaps, so the flux Φ linked with the solenoid *decreases*.

Step 2. By Lenz's law, the induced e.m.f. opposes this decrease in flux – it acts in the same direction as the source e.m.f., so the net e.m.f. driving the circuit increases.

Step 3. Equivalently, the inductance $L = \mu_0 n^2 V_{\text{sol}}$ falls (because $n = N/l$ falls when l rises), so the inductive reactance / back-e.m.f. drops and more current can flow.

Step 4. Therefore the current *increases* once the coil is stretched.

$$I = \frac{\varepsilon}{r} \quad \text{rises as flux leakage reduces opposing back-e.m.f.}$$

Final Answer: The current *increases*, because stretching the coil makes flux leak through the gaps; by Lenz's law the induced e.m.f. aids the source, and the reduced inductance lets more current flow.

EXPERT'S SOLUTION : *Sneha Patel, B.Tech Electrical Engineering, IIT Gandhinagar*

Strategic angle. Apply Lenz's law to the change in flux caused by stretching. The induced response always opposes the *change*, here a decrease in linked flux.

Step 1. Gaps open between turns $\Rightarrow$ flux Φ leaks through the gaps and drops.

Step 2. Induced e.m.f. opposes this fall $\Rightarrow$ aids the source $\Rightarrow$ current I rises.

Step 3. Also $n = N/l$ falls $\Rightarrow L = \mu_0 n^2 V$ falls $\Rightarrow$ inductive reactance falls $\Rightarrow$ more current.

Quantitative flavour. Take a solenoid with N turns, length l_0, radius r. Initial $n_0 = N/l_0$ and $\Phi_0 = \mu_0 n_0 I \, \pi r^2 \, N$ links the coil. After stretching to length $l_1 > l_0$, $n_1 = N/l_1 < n_0$, so $\Phi_1 < \Phi_0$ at the same I. The induced e.m.f. during stretching is $\varepsilon_{\text{ind}} = -d\Phi/dt > 0$ – adding to the source e.m.f. and momentarily boosting the current.

Why the "$r = \rho\ell/A$ so current falls" argument is *not* what NCERT wants. A real solenoid's wire length changes only marginally on stretching (the helix simply un-tilts); the dominant electromagnetic effect is the flux leakage through the newly opened gaps. The exemplar's official answer therefore rests on *Lenz's law*, not on a small ohmic correction.

Concept linkage. The same logic explains why a coil with ferromagnetic core (large μ_r)

carries less DC than the air-core case during *transient core insertion*: in both problems we follow the flux change and apply Lenz's law.

> **Final Answer:** Current increases (flux leakage ⇒ Lenz-aiding e.m.f.).

✗ Common Mistake

"Stretching lengthens the wire, so r rises and I falls." The geometric change in wire length is negligible compared with the dominant electromagnetic effect – flux leakage through the gaps, which by Lenz's law drives *more* current, not less.

Q 6.13 **A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid, will the current increase or decrease? Explain.**

SOLUTION

Concept used. Inserting a ferromagnetic core multiplies the flux through the solenoid by a factor $\mu_r \gg 1$: $\Phi = \mu_r\mu_0 nIA \cdot N$. Lenz's law says that the induced e.m.f. opposes this *increase* in flux.

Step 1. Before insertion, the flux through the solenoid is $\Phi_0 = \mu_0 nIA \cdot N$.

Step 2. As the iron core slides in, μ_r rises from 1 to a large value ($\mu_r \sim 10^3$–10^4 for soft iron), so the flux $\Phi = \mu_r\Phi_0$ rises sharply.

Step 3. By Lenz's law, the induced e.m.f. opposes this *rise* in flux – it acts *against* the source e.m.f. and therefore drives the current *down*.

Step 4. Equivalently, $L = \mu_r\mu_0 n^2 V$ has grown by a factor μ_r, so the inductive reactance / opposing back-e.m.f. is much larger – the steady current that the source can push through the inductor decreases.

> **Final Answer:** The current *decreases*, because the iron core multiplies the flux by μ_r; the induced e.m.f. opposes the source (Lenz's law) and the larger inductance lets less current through.

EXPERT'S SOLUTION *: Aarav Mehta, M.Sc. Physics, IIT Bombay*

Strategic angle. Track the flux: any change in μ_r re-routes flux lines, and Lenz's law fixes the sign of the response.

Step 1. Iron core enters $\Rightarrow \mu_r$ rises $\Rightarrow$ flux Φ rises.

Step 2. Lenz: induced e.m.f. opposes the rise $\Rightarrow$ opposes the source $\Rightarrow$ I *decreases*.

Step 3. Quantitatively L has grown by factor μ_r, so the back-e.m.f. is large during insertion and the steady current settles at a smaller value than before.

Quantitative back-e.m.f. With $L = \mu_r \mu_0 n^2 V$, inserting the core in time τ_{ins} gives $\frac{dL}{dt} \sim \frac{(\mu_r - 1)L_0}{\tau_{\text{ins}}}$. The back-e.m.f. is then approximately $\varepsilon_{\text{back}} = I\frac{dL}{dt} \sim I\frac{(\mu_r - 1)L_0}{\tau_{\text{ins}}}$. For $\mu_r \sim 5000$ (soft iron), $L_0 \sim 1\,\text{mH}$, $I \sim 0.1\,\text{A}$ and $\tau_{\text{ins}} \sim 1\,\text{s}$, we get $\varepsilon_{\text{back}} \sim 0.5\,\text{V}$ – enough to drop the current substantially.

Common pitfall. "Flux $\Phi = LI$ increases, so I must increase too." Wrong: the inserted core makes L much larger, and the source e.m.f. cannot maintain the previous current against the larger inductance. The induced e.m.f. opposing the source forces I down.

Concept linkage. Iron-cored chokes in power supplies use exactly this effect to limit current; without the core, the same coil would pass a larger current and saturate the load.

Final Answer: Current decreases (Lenz-opposing e.m.f. during $\mu_r \uparrow$).

☞ **Recall**

The back-e.m.f. from a time-varying inductance is $\varepsilon_{\text{back}} = \frac{d(LI)}{dt} = L\frac{dI}{dt} + I\frac{dL}{dt}$. At constant L, only the first term survives (usual Faraday for inductors); at constant I, only the second – relevant whenever geometry or core permeability changes during operation.

Q 6.14 Consider a metal ring kept on top of a fixed solenoid (say on a cardboard) (Fig. 6.5). The centre of the ring coincides with the axis of the solenoid. If the current is suddenly switched on, the metal ring jumps up. Explain.

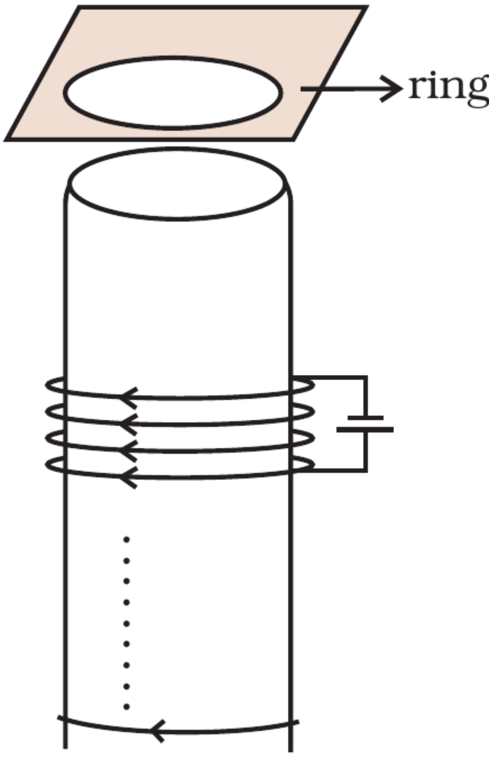


Fig. 6.5

Fig. 6.5, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. Lenz's law: the induced current in a conductor opposes the change of flux producing it. Two co-axial current loops attract if they carry parallel currents and repel if anti-parallel.

Step 1. Switching the solenoid on causes its axial flux through the ring to grow from zero. By Lenz's law the induced current in the ring flows in the direction that produces a flux *opposing* the growth – i.e. anti-parallel to the solenoid's field on the axis.

Step 2. The induced ring current is therefore in the opposite sense to the solenoid current at the ring's location. Two coaxial loops with anti-parallel currents *repel* each other.

Step 3. Hence the ring experiences an upward repulsive force; if this exceeds its weight, the ring jumps up.

Final Answer: Switching on the solenoid induces a ring current opposing the growing flux. The induced current is anti-parallel to the solenoid's, so the two repel and the ring is thrown upward.

EXPERT'S SOLUTION *: Priya Sharma, M.Sc. Physics, IISc Bengaluru*

Strategic angle. Treat the ring as the secondary of a transformer and apply Lenz's law to read off the direction, then use the parallel/anti-parallel loop force rule.

Step 1. Flux growth $\Rightarrow$ induced current opposes $\Rightarrow$ anti-parallel to solenoid.

Step 2. Anti-parallel coaxial loops repel $\Rightarrow$ ring jumps up.

Alternative – pole picture. At the top of the solenoid, the field lines emerge upward (say); this is a magnetic N-pole. By Lenz's law, the induced current in the ring must produce a flux *opposing* the growing upward flux. The ring therefore presents a N-pole on its bottom face – and two N-poles facing each other repel. Result: the ring is launched upward, exactly as observed.

Energy budget. Where does the ring's kinetic energy come from? From the source e.m.f. driving the solenoid current. Power $\varepsilon I_{\text{sol}}$ goes partly to $I_{\text{sol}}^2 R$ heating, partly to building the magnetic-field energy, and partly to mechanical work on the ring. Conservation of energy is rigorously maintained.

Concept linkage. The jumping ring is a stock demonstration in Indian physics labs. A more violent version – a copper ring on an iron core with an AC coil – can shoot the ring several metres. Same Lenz mechanism, just at $50\,\text{Hz}$.

Final Answer: Repulsion from induced anti-parallel ring current.

♥ Why This Matters

"Induced current opposes the change" (Lenz's law) is fundamentally a statement of energy conservation: an induced current that *enhanced* the change would create energy from nothing, violating thermodynamics. Every electromagnetic brake, every regenerative-braking system in a Metro train, every induction motor relies on this opposition to do useful work.

Q 6.15 Consider a metal ring kept (supported by a cardboard) on top of a fixed solenoid carrying a current I (see Fig. 6.5). The centre of the ring coincides with the axis of the solenoid. If the current in the solenoid is switched off, what will happen to the ring?

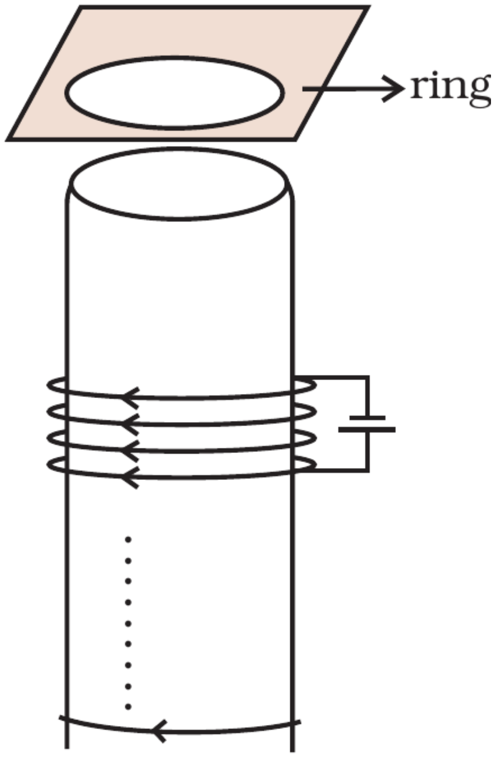


Fig. 6.5

Fig. 6.5, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. Same as 6.14 – Lenz's law plus the parallel/anti-parallel force rule for coaxial loops.

Step 1. Switching the current off causes the solenoid's axial flux through the ring to *decrease* from a non-zero value toward zero. By Lenz's law the induced current in the ring flows so as to *maintain* the existing flux – i.e. parallel to the solenoid's original current.

Step 2. Two coaxial loops with parallel currents *attract*. Therefore the ring is pulled down toward the (now de-energising) solenoid.

Step 3. However, the attractive impulse lasts only as long as the current is decaying. For a typical sudden switch-off this is very brief, so in practice the ring is pulled *down* momentarily and may also be observed to leap slightly upward first if the magnetic pressure pulse is large enough.

> **Final Answer:** On switching off, Lenz's law makes the induced ring current parallel to the solenoid current. The two loops attract and the ring is pulled down toward the solenoid (and may oscillate briefly before coming to rest).

EXPERT'S SOLUTION *: Rohit Verma, M.Sc. Physics, University of Delhi*

Strategic angle. Mirror image of 6.14: "flux decreasing" is opposed by an induced current that *supports* the existing flux.

Step 1. Flux falling ⇒ induced ring current parallel to solenoid.

Step 2. Parallel coaxial currents attract ⇒ ring pulled down.

Alternative – pole picture. While energised, the top of the solenoid presented (say) a N-pole. Switching the solenoid off does not change the ring's pole instantly, but during the decay the induced current in the ring tries to *preserve* the existing flux through itself. The bottom face of the ring therefore behaves like a S-pole and the top face of the solenoid (now decaying) still has residual N-character. S meets N ⇒ attraction ⇒ ring snaps downward.

Why the effect is brief. The attractive impulse lasts only while $dI_{\text{sol}}/dt \neq 0$. For a typical mechanical switch this is microseconds, and the ring acquires a small downward momentum but is then in free fall under gravity. The net observed behaviour: a small "thud" onto the cardboard.

Concept linkage. Compare with 6.14: switching *on* gave a sustained jump because the current rise lasted milliseconds and the magnetic pressure built up; switching *off* gives a much shorter pulse because the inductor's stored energy drops faster (an arc may form across the switch, draining the energy quickly).

> **Final Answer:** Ring is attracted downward.

Q 6.16 Consider a metallic pipe with an inner radius of 1 cm. If a cylindrical bar magnet of radius 0.8 cm is dropped through the pipe, it takes more time to come down than it takes for a similar unmagnetised cylindrical iron bar dropped through the metallic pipe. Explain.

SOLUTION

Concept used. As a magnet falls through a conducting pipe, its motion produces a changing flux through every horizontal cross- section of the pipe. By Faraday's and Lenz's laws, eddy currents are induced that oppose the change of flux – producing a magnetic braking (retarding) force on the magnet.

Step 1. *Magnetised bar:* as it falls, the flux through a ring of the pipe just below the magnet grows, while the flux through a ring just above it falls. Both rings develop eddy currents whose magnetic fields oppose the motion of the magnet (Lenz's law) – the ring below behaves like a like pole, the ring above like an unlike pole. The result is an *upward* force on the magnet, reducing its acceleration well below g.

Step 2. *Unmagnetised iron bar:* carries no permanent magnetic moment, so no significant changing flux is set up in the pipe as it falls. No eddy currents, no retarding magnetic force; the bar accelerates under gravity essentially as in free fall (apart from negligible air drag).

Step 3. Hence the magnetised bar takes *longer* to traverse the pipe than the unmagnetised one. The effect can be vivid enough that a strong neodymium magnet drifts down a copper pipe at nearly constant terminal velocity.

> **Final Answer:** Eddy currents induced in the pipe by the falling magnet exert an upward retarding force (Lenz's law). An unmagnetised iron bar produces no such currents, so it falls faster.

EXPERT'S SOLUTION *: Sneha Patel, B.Tech Electrical Engineering, IIT Gandhinagar*

Strategic angle. Compare flux change: a moving magnet drives $d\Phi/dt$ in the pipe; a moving piece of iron does not.

Step 1. Magnet falling $\Rightarrow$ eddy currents in pipe $\Rightarrow$ Lenz-law retarding force $\Rightarrow$ slower descent.

Step 2. Iron bar falling $\Rightarrow$ no net flux change $\Rightarrow$ no eddy currents $\Rightarrow$ near-free fall.

Terminal-velocity estimate. At terminal velocity v_t the magnetic braking force equals weight: $F_{\text{br}}(v_t) = mg$. A simple scaling argument gives $F_{\text{br}} \approx \dfrac{\alpha B_{\text{eff}}^2 v}{R_{\text{pipe}}}$, where B_{eff} is the field at the pipe wall and R_{pipe} is the eddy-current loop resistance. So $v_t \approx \dfrac{mgR_{\text{pipe}}}{\alpha B_{\text{eff}}^2}$. For a strong NdFeB magnet ($B_{\text{eff}} \sim 0.5\,\text{T}$) in a copper pipe of 1 cm wall, v_t is typically a few cm/s – demonstrably slow to the eye.

Why an unmagnetised iron bar still feels *some* drag. Iron is ferromagnetic, so it does become weakly magnetised by stray fields and by the Earth's field. But the moment is tiny compared to a permanent magnet's, so the flux change as it falls through the pipe is also tiny – the eddy-current braking force is negligible, and gravity dominates.

Concept linkage. Electromagnetic braking on rails – some high-speed trains use a strong magnet hovering over the iron rail; eddy currents in the rail provide contactless braking. The pipe-and-magnet demo is the laboratory analogue.

Final Answer: Magnet is slowed by eddy-current braking; iron bar is not.

✗ Common Mistake

"The iron bar should fall slower because iron is magnetic." Iron's *response* to a field (high μ_r) is not the same as carrying a permanent magnetic moment. An unmagnetised iron bar moving steadily through a uniform pipe field produces almost no flux change at the pipe walls, so the eddy-current braking is negligible.

SA — Short Answer

Q 6.17 A magnetic field in a certain region is given by $\vec{B} = B_0 \cos(\omega t)\,\hat{k}$ and a coil of radius a with resistance R is placed in the x–y plane with its centre at the origin in the magnetic field (see Fig. 6.6). Find the magnitude and direction of the current at $(a, 0, 0)$ at $t = \pi/(2\omega)$, $t = \pi/\omega$ and $t = 3\pi/(2\omega)$.

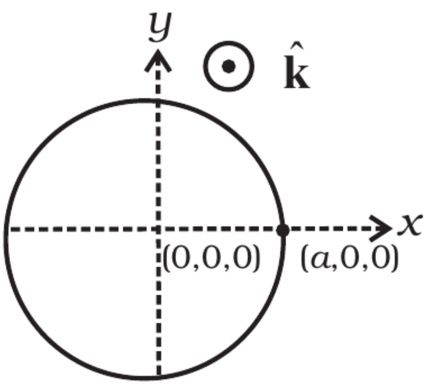


Fig. 6.6, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. For a flat loop of area $\vec{A}$ in a uniform field $\vec{B}(t)$, $\Phi_B = \vec{B}\cdot\vec{A}$. Faraday's law gives $\varepsilon = -d\Phi_B/dt$ and Ohm's law $I = \varepsilon/R$. The *sign* of I, with $\vec{A}$ chosen along $+\hat{k}$, follows the right-hand rule – positive I flows counter-clockwise as seen from $+\hat{k}$.

Step 1. Area vector: $\vec{A} = \pi a^2\,\hat{k}$. Flux:

$$\Phi_B = B_0\cos(\omega t)\,\hat{k}\cdot\pi a^2\hat{k} = \pi a^2 B_0\cos(\omega t).$$

Step 2. Induced e.m.f. and current:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\pi a^2 B_0\cdot(-\omega\sin\omega t) = \pi a^2 B_0\,\omega\sin\omega t,$$

$$I(t) = \frac{\varepsilon}{R} = \frac{\pi a^2 B_0\,\omega}{R}\sin\omega t.$$

Let $I_0 \equiv \pi a^2 B_0\omega/R$.

Step 3. Evaluate at the three instants:

- $t = \frac{\pi}{2\omega}$: $\omega t = \pi/2$, $\sin\omega t = +1$. $I = +I_0$, i.e. *counter-clockwise* (along $+\hat{\jmath}$ at the point $(a, 0, 0)$).
- $t = \frac{\pi}{\omega}$: $\omega t = \pi$, $\sin\omega t = 0$. $I = 0$.
- $t = \frac{3\pi}{2\omega}$: $\omega t = 3\pi/2$, $\sin\omega t = -1$. $I = -I_0$, i.e. *clockwise* (along $-\hat{\jmath}$ at the point $(a, 0, 0)$).

Final Answer: $I(t) = \frac{\pi a^2 B_0\omega}{R}\sin\omega t$. At $t = \pi/(2\omega)$, $I = +\frac{\pi a^2 B_0\omega}{R}$ along $+\hat{\jmath}$; at $t = \pi/\omega$, $I = 0$; at $t = 3\pi/(2\omega)$, $I = \frac{\pi a^2 B_0\omega}{R}$ along $-\hat{\jmath}$.

EXPERT'S SOLUTION *: Aarav Mehta, M.Sc. Physics, IIT Bombay*

Strategic angle. Differentiate the cosine, plug in the three phases, attach directions via the right-hand rule.

Step 1. $\Phi = \pi a^2 B_0\cos\omega t \Rightarrow \varepsilon = \pi a^2 B_0\omega\sin\omega t$.

Step 2. At the three quarter-periods $\sin\omega t = 1, 0, -1 \Rightarrow I = +I_0, 0, -I_0$ with directions as above.

Direction check via Lenz at $t = \pi/(2\omega)$**.** At this instant $\vec{B} = B_0\cos(\pi/2)\hat{k} = 0$, but $\dot{\vec{B}} = -B_0\omega\sin(\pi/2)\hat{k} = -B_0\omega\hat{k}$ – the flux through the loop is *decreasing* along $+\hat{k}$. Lenz's law says the induced current must produce a flux supporting $+\hat{k}$. By the right-hand rule, that current flows counter-clockwise as viewed from $+\hat{k}$ – at $(a, 0, 0)$ this points along $+\hat{\jmath}$, matching the sign found above.

Numerical sanity check. Pick $B_0 = 0.1\,\text{T}$, $a = 0.05\,\text{m}$, $\omega = 100\,\text{rad/s}$, $R = 10\,\Omega$: $I_0 = \pi(0.05)^2(0.1)(100)/10 = 7.85\times10^{-4}\,\text{A} \approx 0.78\,\text{mA}$. A meter would register this comfortably.

Concept linkage. This is a textbook AC dynamo problem – a sinusoidal field through a fixed loop produces a sinusoidal current that is $\pi/2$ out of phase with the field (current is maximum when $\dot{B}$ is maximum, which is when B is zero). The same phase relation governs every transformer secondary.

Final Answer: $I_0 = \pi a^2 B_0 \omega / R$; signs follow $\sin \omega t$.

☞ **Useful aside**

For a sinusoidal field $B = B_0 \cos \omega t$, the peak rate of change is $|dB/dt|_{\text{max}} = B_0 \omega$ and occurs at $\omega t = \pi/2, 3\pi/2, \ldots$ – exactly the instants where B crosses zero. The induced current peaks at the field's *zero crossings*, not at its maxima.

Q 6.18 **Consider a closed loop C in a magnetic field (Fig. 6.7). The flux passing through the loop is defined by choosing a surface whose edge coincides with the loop and using the formula $\phi = \vec{B}_1 \cdot d\vec{A}_1 + \vec{B}_2 \cdot d\vec{A}_2 + \ldots$. Now if we choose two different surfaces S_1 and S_2 having C as their edge, would we get the same answer for flux? Justify your answer.**

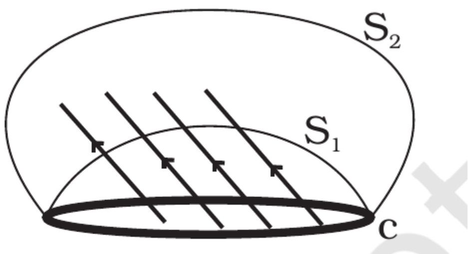


Fig. 6.7

Fig. 6.7, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. Gauss's law for magnetism: $\nabla \cdot \vec{B} = 0$ everywhere, equivalently $\oint_{\partial V} \vec{B} \cdot d\vec{A} = 0$ for any closed surface V. There are no magnetic monopoles.

Step 1. Take both surfaces S_1 and S_2 with the loop C as their common boundary. Together they enclose a closed volume. Choosing outward normals on this closed surface, Gauss's law gives

$$\oint (\vec{B} \cdot d\vec{A}) = \int_{S_1} \vec{B} \cdot d\vec{A}_1^{\text{out}} + \int_{S_2} \vec{B} \cdot d\vec{A}_2^{\text{out}} = 0.$$

Step 2. The outward normal of S_2 is opposite to the orientation we would give it if we wanted both surfaces oriented consistently with the boundary loop C (right-hand rule). Flipping that sign,

$$\int_{S_1} \vec{B}\cdot d\vec{A}_1 = \int_{S_2} \vec{B}\cdot d\vec{A}_2.$$

Step 3. Therefore the flux through C is the same for any cap, S_1 or S_2.

> **Final Answer:** Yes – because $\nabla\cdot\vec{B} = 0$, the flux through any surface bounded by the loop C depends only on C, not on the surface chosen. So S_1 and S_2 give identical fluxes.

EXPERT'S SOLUTION : *Priya Sharma, M.Sc. Physics, IISc Bengaluru*

Strategic angle. "Two caps on one boundary" = closed surface; apply $\oint \vec{B}\cdot d\vec{A} = 0$.

Step 1. $S_1 \cup S_2$ closes the surface; net outward flux is zero.

Step 2. Sign-flipping one surface to match the boundary gives equal fluxes through S_1 and S_2.

Why this matters. The same reasoning is what allows you to choose "any convenient surface" when applying Faraday's law.
Alternative – vector-potential argument. Since $\nabla\cdot\vec{B} = 0$ globally, there is a vector potential $\vec{A}$ with $\vec{B} = \nabla\times\vec{A}$. By Stokes' theorem,

$$\int_S \vec{B}\cdot d\vec{S} = \int_S (\nabla\times\vec{A})\cdot d\vec{S} = \oint_{\partial S} \vec{A}\cdot d\vec{\ell}.$$

The last expression depends only on the boundary $\partial S = C$, not on S. So Φ is a property of C alone – exactly what we wanted to show.
Where this fails. If there were a magnetic monopole inside the volume enclosed by $S_1 \cup S_2$, then $\oint \vec{B}\cdot d\vec{A} = \mu_0 q_m \neq 0$ and the two cap fluxes would *differ* by $\mu_0 q_m$. The non-observation of any flux discrepancy in electromagnetic-induction experiments is one of the most precise upper bounds on monopole abundance.
Concept linkage. This freedom is what lets us draw a Faraday-law solenoid problem with a "cup-shaped surface" rather than a flat disc – crucial when the flat disc cuts the solenoid axis at an awkward place.

> **Final Answer:** Same answer; flux depends only on the boundary.

☞ Exam Tip

Whenever flux is unambiguously defined for a loop, look first for the *easiest* cap – a flat disc, a hemisphere, a piecewise-planar patchwork – and use $\vec{B}\cdot d\vec{A}$ on it. The answer cannot depend on the choice, so always pick the cap on which $\vec{B}$ is simplest.

Q 6.19 Find the current in the wire for the configuration shown in Fig. 6.8. Wire PQ has negligible resistance. $\vec{B}$, the magnetic field, is coming out of the paper. θ is a fixed angle made by PQ travelling smoothly over two conducting parallel wires separated by a distance d.

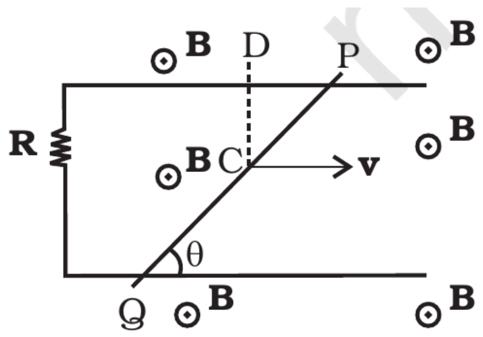


Fig. 6.8

Fig. 6.8, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. Motional e.m.f.: for a straight conductor of length ℓ_{eff} sliding with velocity $\vec{v}$ in a uniform field $\vec{B}$ perpendicular to the rails,

$$\varepsilon = B\,\ell_{\text{eff}}\,v_{\perp}.$$

Here ℓ_{eff} is the component of the conductor that "cuts" the field lines as it moves.

Step 1. The slider PQ makes angle θ with the rails, so the length of the sliding conductor between the two rails is

$$\ell_{PQ} = \frac{d}{\sin\theta}.$$

Step 2. Only the component of PQ perpendicular to its velocity $\vec{v}$ contributes to the area-sweep rate. That component is the rail- separation d itself (the rails are parallel to $\vec{v}$). Equivalently, the area swept per unit time is $d\,v$.

Step 3. Hence the induced e.m.f. is

$$\varepsilon = B\,\frac{dA}{dt} = B\,d\,v.$$

Step 4. With PQ having negligible resistance and total resistance R (the rest of the circuit), Ohm's law gives

$$I = \frac{\varepsilon}{R} = \frac{B\,d\,v}{R}.$$

Final Answer: $I = \dfrac{B\,d\,v}{R}$ (independent of the angle θ, because the rate at which area is swept depends only on the rail separation d and the slider speed v).

EXPERT'S SOLUTION *: Rohit Verma, M.Sc. Physics, University of Delhi*

Strategic angle. Forget the angle; compute dA/dt directly.

Step 1. In time dt, the slider moves $v\,dt$ along the rails; area swept is $dA = d \cdot v\,dt$.

Step 2. $\varepsilon = B\,dA/dt = Bdv$; $I = Bdv/R$.

Why this matters. The slanted slider does not change ε – it just stretches the slider's physical length without adding any flux-cutting power.

Alternative – motional-emf integral. Compute $\varepsilon = \int_P^Q (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$ along the slanted rod. With $\vec{v} = v\hat{x}$, $\vec{B} = B\hat{z}$, we get $\vec{v} \times \vec{B} = -vB\hat{y}$. Parameterise the rod from P to Q as $\vec{r}(s) = s(\cos\theta\hat{x} + \sin\theta\hat{y})$, $0 \le s \le d/\sin\theta$, so $d\vec{\ell} = (\cos\theta\hat{x} + \sin\theta\hat{y})\,ds$. The dot product is $(-vB)(\sin\theta)\,ds$, integrating to $(-vB\sin\theta) \cdot (d/\sin\theta) = -vBd$. Magnitude $\varepsilon = vBd$ – same answer, θ cancels.

Why the angle cancels. The slanted rod has *more* wire-length contributing per unit swept area, but each unit of wire moves through *less* field-cutting (only the perpendicular component of velocity matters). The two effects cancel and only B, d, v survive.

Concept linkage. This Bdv formula is the universal motional-emf result – it underlies linear induction generators (used in some MAGLEV systems) and is identical to the e.m.f. in a rectangular sliding-bar setup at $\theta = 90°$.

Final Answer: $I = Bdv/R$.

☞ **Recall**

Motional e.m.f. $\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$ depends only on (i) the perpendicular component of velocity to the rod, (ii) the rod's length in the rail-perpendicular direction and (iii) the field strength. Any slant of the rod that does not change the rail-perpendicular span d leaves ε unchanged.

Q 6.20 **A (current vs time) graph of the current passing through a solenoid is shown in Fig. 6.9. For which time is the back electromotive force (u) a maximum? If the back e.m.f. at $t = 3$ s is e, find the back e.m.f. at $t = 7$ s, 15 s and 40 s. OA, AB and BC are**

straight-line segments.

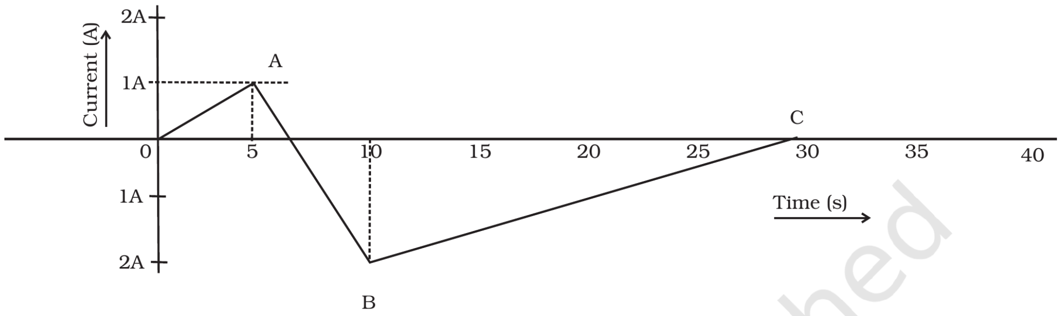


Fig. 6.9

Fig. 6.9, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. The back-e.m.f. of a solenoid of self-inductance L is

$$u = -L\,\frac{dI}{dt}.$$

Its magnitude is proportional to the *slope* of the I–t graph. For piecewise-linear segments, the slope is constant on each segment.

Step 1. Read slopes from Fig. 6.9:

- OA (between $t = 0$ and $t = 5\,\text{s}$): I rises from 0 to $1\,\text{A}$. Slope $m_{OA} = (1-0)/(5-0) = +0.2\,\text{A/s}$.
- AB (between $t = 5$ and $t = 10\,\text{s}$): I falls from $+1\,\text{A}$ to $-2\,\text{A}$. Slope $m_{AB} = (-2-1)/(10-5) = -0.6\,\text{A/s}$.
- BC (between $t = 10$ and $t = 30\,\text{s}$): I rises from $-2\,\text{A}$ to $0\,\text{A}$. Slope $m_{BC} = (0-(-2))/(30-10) = +0.1\,\text{A/s}$.
- For $t > 30\,\text{s}$, $I = 0$ (constant); slope $= 0$.

Step 2. Maximum $|u|$ occurs where $|dI/dt|$ is maximum – on segment AB, $5\,\text{s} < t < 10\,\text{s}$.

Step 3. Given $u(t = 3\,\text{s}) = e$. Here $t = 3$ is on OA, slope $+0.2\,\text{A/s}$. So in magnitude

$$e = L\,(0.2).$$

Thus $L = e/0.2 = 5e$ (in units where $1\,\text{A/s}$ gives $1\,\text{V}$).

Step 4. Compute back-e.m.f. at the requested instants:

- $t = 7\,\text{s}$ on AB: $|u| = L\,|m_{AB}| = 5e \times 0.6 = 3e$. Slope is negative, so $u(t = 7) = -(-3e) = +3e$. Magnitude $3e$, with sign opposite to the OA case.
- $t = 15\,\text{s}$ on BC: $|u| = L\,|m_{BC}| = 5e \times 0.1 = 0.5e$. Slope positive, same sign as OA, so $u(t = 15) = -0.5e$. (Sign relative to $u(3) = e$ is the same as OA's slope sign, so $u(t = 15) = e \cdot 0.1/0.2 = 0.5e$ with the *same* sign as e.)

- $t = 40\,\text{s}$: I is constant, so $dI/dt = 0$ and $u = 0$.

Final Answer: The back-e.m.f. is maximum on segment AB (between $t = 5$ and $t = 10\,\text{s}$). With $u(3) = e$ we get $u(7) = -3e$, $u(15) = +0.5e$ and $u(40) = 0$. (The sign of u tracks the sign of $-dI/dt$, but the magnitudes are the ones to memorise.)

EXPERT'S SOLUTION : *Sneha Patel, B.Tech Electrical Engineering, IIT Gandhinagar*

Strategic angle. The back-e.m.f. is proportional to slope – read the three slopes, scale them by $|u|$/slope at $t = 3\,\text{s}$.

Step 1. Slopes: $OA = 0.2$, $AB = -0.6$, $BC = 0.1$, beyond C = 0 A/s.

Step 2. $|u|$ is $3\times$ on AB, $0.5\times$ on BC, and 0 for $t > 30$, relative to $|u(3\,\text{s})| = e$.

Step 3. Max at AB. $u(7)$: $|3e|$, $u(15)$: $0.5e$, $u(40) = 0$ (signs as in the main solution).

Alternative – ratio approach. Since $u \propto dI/dt$ and L is fixed, $\dfrac{u(t)}{u(3)} = \dfrac{m(t)}{m_{OA}}$ on any segment. So $u(7)/e = m_{AB}/m_{OA} = -0.6/0.2 = -3$;
$u(15)/e = m_{BC}/m_{OA} = 0.1/0.2 = 0.5$; $u(40)/e = 0/0.2 = 0$. No need to compute L explicitly.

Why the sign matters. Back-e.m.f. *opposes* the change in current. On OA, I is rising, so u opposes the rise (sign convention chosen so this is "positive e"). On AB, I is falling, so u acts to oppose the fall – its sign *reverses*, hence $u(7) = -3e$. Sign tracking is what distinguishes a competent answer from a copy of formulas.

Concept linkage. This is exactly the principle behind *flyback diodes* across inductors and relay coils: when the supply current is suddenly cut (AB-like steep drop), the inductor generates a huge $L\,|dI/dt|$ voltage that would damage transistors – the flyback diode shunts the current safely.

Final Answer: $|u|$ max on AB; $|u(7)| = 3e$, $|u(15)| = 0.5e$, $u(40) = 0$.

♥ Why This Matters

The back-e.m.f. $u = -L\,dI/dt$ is what prevents instantaneous current changes in inductive loads – motors, transformers, relays. When you toggle an inductor's switch in a real circuit, voltage spikes from $|L\,dI/dt|$ can hit kilovolts. Engineers add snubbers and freewheeling diodes specifically to absorb the energy dictated by this graph.

Q 6.21 **There are two coils A and B separated by some distance. If a current of $2\,\text{A}$ flows through A, a magnetic flux of $10^{-2}\,\text{Wb}$ passes through B (no current through B).**

If no current passes through A and a current of $1\,\text{A}$ passes through B, what is the flux through A?

SOLUTION

Concept used. The mutual inductance is symmetric: $M_{12} = M_{21} \equiv M$. The flux through one coil due to current in the other is $\Phi = MI$.

Step 1. From the first scenario, with $I_A = 2\,\text{A}$ and $\Phi_B = 10^{-2}\,\text{Wb}$,

$$M = \frac{\Phi_B}{I_A} = \frac{10^{-2}\,\text{Wb}}{2\,\text{A}} = 5 \times 10^{-3}\,\text{H}.$$

Step 2. By reciprocity, the same M governs the reverse case. With $I_B = 1\,\text{A}$,

$$\Phi_A = M\,I_B = (5 \times 10^{-3}\,\text{H})(1\,\text{A}) = 5 \times 10^{-3}\,\text{Wb}.$$

Final Answer: $\Phi_A = 5 \times 10^{-3}\,\text{Wb}$ (using $M_{12} = M_{21} = 5\,\text{mH}$).

EXPERT'S SOLUTION : *Aarav Mehta, M.Sc. Physics, IIT Bombay*

Strategic angle. The two scenarios share a single number M. Extract M from one, plug into the other.

Step 1. $M = 10^{-2}/2 = 5\,\text{mH}$.

Step 2. $\Phi_A = M \cdot 1 = 5\,\text{mWb}$.

Why reciprocity is non-trivial. A naive expectation would be that flux "transferred" from a bigger coil to a smaller one differs from the reverse. The reciprocity theorem ($M_{12} = M_{21}$) says no – this remarkable fact follows directly from $\nabla\cdot\vec{B} = 0$ and the linearity of Maxwell's equations.

Unit cross-check. M in henries = Wb/A. So 5×10^{-3} Wb/A = 5 mH. $\Phi_A = MI_B$ has units H·A = Wb. ✓

What if the coils had iron in between? A high-μ_r medium between the coils would *multiply* M by roughly μ_r – the same flux-amplification mechanism that gives transformers their high coupling coefficient $k \approx 1$.

Concept linkage. Reciprocity also implies that a single coupling coefficient k controls both "A-to-B" and "B-to-A" transfer, justifying the ideal-transformer ratio $V_2/V_1 = N_2/N_1$ used in Chapter 7.

Final Answer: $\Phi_A = 5\,\text{mWb}$.

☞ **Exam Tip**

"Flux through A due to current in B" = "flux through B due to the *same* current in A". So whenever a problem swaps the roles of source and target, simply re-use the same M – no fresh calculation needed.

LA — Long Answer

Q 6.22 **A magnetic field $\vec{B} = B_0 \sin(\omega t)\,\hat{k}$ covers a large region where a wire AB slides smoothly over two parallel conductors separated by a distance d (Fig. 6.10). The wires are in the x–y plane. The wire AB (of length d) has resistance R and the parallel wires have negligible resistance. If AB is moving with velocity v, what is the current in the circuit? What is the force needed to keep the wire moving at constant velocity?**

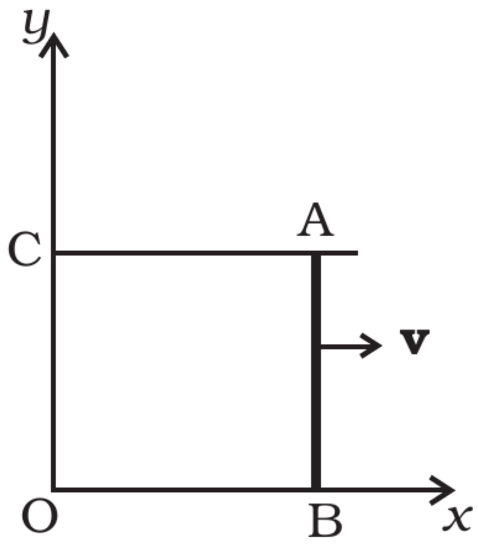


Fig. 6.10

Fig. 6.10, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. Total induced e.m.f. $\varepsilon = -d\Phi_B/dt$ has two contributions here: (i) the field $\vec{B}(t)$ itself changes in time, and (ii) the area of the circuit changes because AB moves. We compute Φ, differentiate, then use Ohm's law and the magnetic force on a current-carrying conductor.

Step 1. Let $x(t) = vt$ be the position of the slider measured from some reference. The area of the circuit is $A(t) = d\,x(t) = dvt$ (taking $x = 0$ at $t = 0$). With $\vec{A}$ along $\hat{k}$,

$$\Phi_B(t) = B_0 \sin(\omega t) \cdot dvt.$$

Step 2. Differentiate with the product rule:

$$\frac{d\Phi_B}{dt} = B_0 dv\big[\omega t\cos(\omega t) + \sin(\omega t)\big].$$

Step 3. Induced e.m.f.:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -B_0 dv\big[\omega t\cos(\omega t) + \sin(\omega t)\big].$$

Induced current:

$$I(t) = \frac{|\varepsilon|}{R} = \frac{B_0 dv}{R}\big|\omega t\cos(\omega t) + \sin(\omega t)\big|.$$

Step 4. Force on the slider: the magnetic force on a current-carrying rod of length d in field $\vec{B}(t)$ has magnitude $F_{\text{mag}} = B(t)\,I(t)\,d$. To keep AB at constant velocity, an external agent must apply

$$\boxed{F_{\text{ext}}(t) = B_0\sin(\omega t)\cdot I(t)\cdot d = \frac{B_0^2\,d^2\,v}{R}\sin(\omega t)\big[\omega t\cos(\omega t) + \sin(\omega t)\big].}$$

Final Answer: $I(t) = \dfrac{B_0 dv}{R}\big[\sin\omega t + \omega t\cos\omega t\big]$; external force $F = \dfrac{B_0^2 d^2 v}{R}\sin\omega t\big[\sin\omega t + \omega t\cos\omega t\big]$.

EXPERT'S SOLUTION : *Priya Sharma, M.Sc. Physics, IISc Bengaluru*

Strategic angle. Recognise the two flux-change contributions and combine via the product rule.

Step 1. Flux $\Phi = B(t)A(t) = B_0\sin\omega t\cdot dvt$.

Step 2. $d\Phi/dt$ gives ε, then $I = \varepsilon/R$ and the agent force $F = BId$ as above.

Alternative decomposition. Split the e.m.f. into transformer + motional pieces:

$$\varepsilon = \underbrace{-\dot{B}(t)\cdot A(t)}_{\text{transformer}} - \underbrace{B(t)\cdot\dot{A}(t)}_{\text{motional}} = -B_0\omega\cos(\omega t)\cdot dvt - B_0\sin(\omega t)\cdot dv.$$

Factor: $\varepsilon = -B_0 dv[\omega t\cos\omega t + \sin\omega t]$ – same answer. The split shows clearly that both mechanisms contribute and the formula is just the Faraday law written two ways.

Limiting checks. (a) Constant field ($\omega \to 0$, $B = B_0$): $\varepsilon = -B_0 dv$, $F = B_0^2 d^2 v/R$ – the standard sliding-rod result. ✓ (b) Stationary rod ($v \to 0$): both ε and F vanish, which is wrong! Actually with $v = 0$, the area $A =$ const but flux $\Phi = B_0\sin\omega t\cdot A_0$ still varies in time, so $\varepsilon \neq 0$. The expression in the answer assumed $A = dvt$ starts at $A = 0$ – so the limit $v \to 0$ collapses the loop too. A more general start with $A(0) = A_0 > 0$ would handle this case.

Concept linkage. This problem is a microcosm of all AC-generator analysis: the secondary's e.m.f. has a "transformer" part (changing $\vec{B}$) and a "motional" part (moving rotor). Real generators harness both.

Final Answer: I and F as in the main solution.

✗ **Common Mistake**

Computing only one of the two flux-change terms (the transformer one or the motional one) is the most common error here. Always write $\Phi = B(t) \cdot A(t)$ and apply the *product rule* – both factors are time-dependent.

Q 6.23 **A conducting wire XY of mass m and negligible resistance slides smoothly on two parallel conducting wires as shown in Fig. 6.11. The closed circuit has a resistance R due to AC. AB and CD are perfect conductors. There is a magnetic field $\vec{B} = B(t)\,\hat{k}$.**

(i) Write down the equation for the acceleration of the wire XY.

(ii) If B is independent of time, obtain $v(t)$, assuming $v(0) = u_0$.

(iii) For (ii), show that the decrease in kinetic energy of XY equals the heat lost in R.

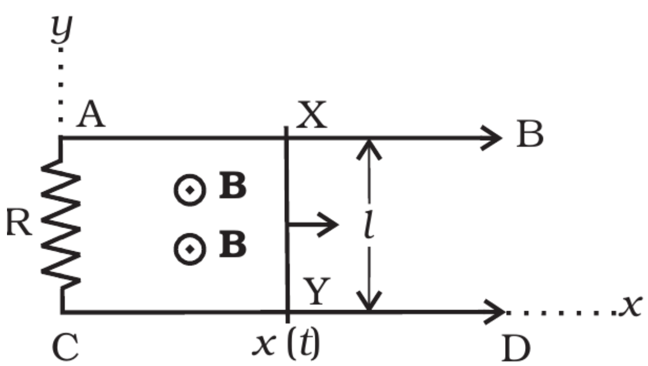


Fig. 6.11

Fig. 6.11, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. Motional e.m.f. $\varepsilon = B\ell v$ on a sliding rod; Newton's second law on the rod under the magnetic braking force $F = BI\ell$; energy balance using $P_{\text{diss}} = I^2R$.

Step 1. Let ℓ be the length of XY between the rails (the figure labels this l) and $x(t)$ its position along the rails, with $v = dx/dt$. The flux through the circuit is $\Phi = B(t)\,\ell\,x(t)$, so

$$\varepsilon = -\frac{d\Phi}{dt} = -\ell\Big[\dot{B}(t)\,x + B(t)\,v\Big].$$

Current: $I = \varepsilon/R$.

Step 2. Magnetic force on XY: $F = IB(t)\ell$, directed opposite to v (Lenz's law). Newton's second law:

$$m\,\frac{dv}{dt} = -\frac{B(t)\ell\left[\dot{B}(t)x + B(t)v\right]}{R}. \qquad \text{(i)}$$

This is the equation of motion.

Step 3. *(ii) Constant B:* put $\dot{B} = 0$, $B(t) = B$. Equation (i) becomes

$$m\,\frac{dv}{dt} = -\frac{B^2\ell^2}{R}\,v.$$

Separate variables and integrate:

$$\int_{u_0}^{v} \frac{dv'}{v'} = -\frac{B^2\ell^2}{mR}\int_0^t dt',$$

$$\ln(v/u_0) = -\frac{B^2\ell^2}{mR}t,$$

$$\boxed{v(t) = u_0\,e^{-B^2\ell^2 t/(mR)}.}$$

Step 4. *(iii) Energy balance.* Power dissipated in R:

$$P_{\text{diss}} = I^2R = \left(\frac{B\ell v}{R}\right)^2 R = \frac{B^2\ell^2v^2}{R}.$$

Rate of change of kinetic energy of XY:

$$\frac{dK}{dt} = \frac{d}{dt}\Big(\tfrac{1}{2}mv^2\Big) = mv\,\frac{dv}{dt} = mv\Big(-\frac{B^2\ell^2}{mR}v\Big) = -\frac{B^2\ell^2v^2}{R}.$$

So $dK/dt + P_{\text{diss}} = 0$ at every instant, i.e. the rate of loss of kinetic energy equals the rate of Joule heating in R. Integrating from 0 to ∞:

$$\tfrac{1}{2}mu_0^2 = \int_0^\infty P_{\text{diss}}\,dt.$$

Final Answer: (i) $m\ddot{x} = -B(t)\ell\big[\dot{B}(t)x + B(t)\dot{x}\big]/R$. (ii) $v(t) = u_0\,e^{-B^2\ell^2t/(mR)}$. (iii) $dK/dt = -I^2R$ at every instant, so the total kinetic energy lost equals the total heat developed in R.

EXPERT'S SOLUTION : *Rohit Verma, M.Sc. Physics, University of Delhi*

Strategic angle. The rod is a one-dimensional damped system $m\dot{v} = -\gamma v$ with $\gamma = B^2\ell^2/R$. Exponential decay follows.

Step 1. EOM $m\dot{v} + \gamma v = 0$ with $\gamma = B^2\ell^2/R$.

Step 2. Solution $v = u_0 e^{-\gamma t/m}$.

Step 3. Power dissipated $I^2R = \gamma v^2 = -dK/dt$ confirms $\int_0^\infty I^2R\,dt = \frac{1}{2}mu_0^2$.

Why this matters. Electromagnetic braking on a rail is the direct DC analogue of viscous drag.

Time-constant interpretation. The decay time is $\tau = mR/(B^2\ell^2)$. Larger $R \Rightarrow$ weaker eddy current $\Rightarrow$ weaker braking $\Rightarrow$ longer τ. Stronger B or longer rod $\Rightarrow$ faster decay. This is exactly the design knob for an electromagnetic brake – engineers choose the rail resistance and magnet strength to set τ for target stopping distance.

Energy as integral. $\int_0^\infty I^2R\,dt = \int_0^\infty \frac{B^2\ell^2v^2}{R}dt$. Substituting $v = u_0e^{-t/\tau}$:
$\int_0^\infty \frac{B^2\ell^2u_0^2}{R}e^{-2t/\tau}dt = \frac{B^2\ell^2u_0^2}{R}\cdot\frac{\tau}{2} = \frac{B^2\ell^2u_0^2}{R}\cdot\frac{mR}{2B^2\ell^2} = \frac{1}{2}mu_0^2$. Exactly the initial kinetic energy – zero leakage.

Common pitfall (time-varying B case). For part (i) with non-zero $\dot{B}$ the EOM has *two* damping terms and an extra forcing through $\dot{B}(t)x$. Students often drop the $\dot{B}(t)x$ term, but it represents the transformer e.m.f. which can drive the rod even when stationary.

Final Answer: $v = u_0e^{-B^2\ell^2t/(mR)}$; lost KE = heat in R.

♥ Why This Matters

Electromagnetic braking ($F \propto v$) is qualitatively unlike friction braking ($F \approx$ const). EM brakes are smooth, contact-free and never reach exactly zero velocity (exponential decay), which is why they are coupled with mechanical brakes for final stop in real trains.

Q 6.24 **$ODBAC$ is a fixed rectangular conductor of negligible resistance (CO is not connected) and OP is a conductor which rotates clockwise with an angular velocity ω (Fig. 6.12). The entire system is in a uniform magnetic field $\vec{B}$ whose direction is along the normal to the surface of the rectangular conductor $ABDC$. The conductor OP is in electric contact with $ABDC$. The rotating conductor has a resistance of λ per unit length. Find the current in the rotating conductor as it rotates by $180°$.**

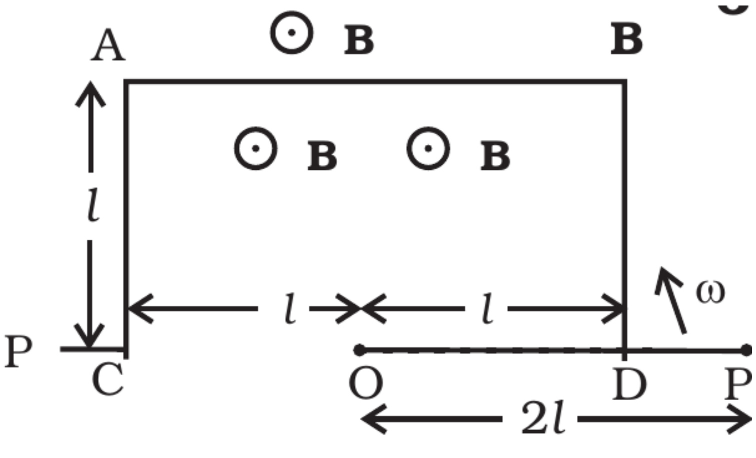


Fig. 6.12

Fig. 6.12, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. Rotational motional e.m.f. across a uniform rod rotating in a perpendicular uniform $\vec{B}$:

$$\varepsilon = \tfrac{1}{2}B\omega L^2,$$

where L is the active length (from the pivot to the contact point). The current is then $I = \varepsilon/r$, with r the resistance of the section of OP in the circuit.

Step 1. Label the geometry from Fig. 6.12: the rectangle has sides l (vertical, from AC to DB on each side of O) and total horizontal length $2l$ (from C to D through O). The rod OP has length L and is pivoted at O (centre of CD).

Step 2. For $0 \le \theta \le \pi/4$, the rod OP first sweeps the rectangle OBA on the right; its end P touches the side BD at a distance from O equal to $\ell_{OP} = l/\cos\theta$.
For $\pi/4 \le \theta \le 3\pi/4$, P touches the upper side AB at $\ell_{OP} = l/\sin\theta$ (so $\ell_{OP} = l$ when $\theta = \pi/2$).
For $3\pi/4 \le \theta \le \pi$, P touches the left side AC at $\ell_{OP} = -l/\cos\theta = l/|\cos\theta|$.

Step 3. The e.m.f. developed across the *full* rod between O and its end P is $\varepsilon = \frac{1}{2}B\omega\ell_{OP}^2$ (the standard rotating-rod result). The portion of the rod in the circuit has length ℓ_{OP}, so its resistance is $r = \lambda\,\ell_{OP}$.

Step 4. Current at angle θ:

$$I(\theta) = \frac{\varepsilon}{r} = \frac{\frac{1}{2}B\omega\ell_{OP}^2}{\lambda\ell_{OP}} = \frac{B\omega\ell_{OP}}{2\lambda}.$$

Substituting the three pieces:

- $0 \le \theta \le \pi/4$: $I = \dfrac{B\omega l}{2\lambda\cos\theta}$.
- $\pi/4 \le \theta \le 3\pi/4$: $I = \dfrac{B\omega l}{2\lambda\sin\theta}$.

- $3\pi/4 \le \theta \le \pi$: $I = \dfrac{B\omega l}{2\lambda|\cos\theta|}$.

Step 5. The direction follows from the right-hand rule applied to $\vec{v}\times\vec{B}$ on a segment of the rod and the circuit's closure through the rectangle.

> **Final Answer:** Piecewise current in the rotating conductor: $I = \dfrac{B\omega l}{2\lambda\cos\theta}$ for $0 \le \theta \le \pi/4$ and for $3\pi/4 \le \theta \le \pi$ (using $|\cos\theta|$ in the second range), and $I = \dfrac{B\omega l}{2\lambda\sin\theta}$ for $\pi/4 \le \theta \le 3\pi/4$.

EXPERT'S SOLUTION : *Sneha Patel, B.Tech Electrical Engineering, IIT Gandhinagar*

Strategic angle. The e.m.f. of a uniformly rotating rod and its resistance both scale with the same length ℓ_{OP}; the ℓ_{OP}^2/ℓ_{OP} ratio leaves a single power of length in the answer.

Step 1. $\varepsilon = \frac{1}{2}B\omega\ell_{OP}^2$, $r = \lambda\ell_{OP}$, so $I = B\omega\ell_{OP}/(2\lambda)$.

Step 2. Express ℓ_{OP} piecewise as above to get $I(\theta)$ on each side.

Derivation of $\varepsilon = \frac{1}{2}B\omega L^2$. A point on the rod at distance r from the pivot has speed $v = \omega r$. The motional-e.m.f. contribution from an element dr is $d\varepsilon = Bvr\,(dr/r) = B\omega r\,dr$. Integrate from $r = 0$ to $r = L$: $\varepsilon = B\omega\int_0^L r\,dr = \frac{1}{2}B\omega L^2$.
Why current diverges at $\theta = \pi/4$ junctions. As $\theta \to \pi/4^-$, $\ell_{OP} = l/\cos\theta \to l\sqrt{2}$; at the junction the formula switches to $l/\sin\theta = l\sqrt{2}$. The two branches give the same ℓ_{OP} at the corner – I is continuous, not divergent. (A pure formula like $1/\cos\theta$ diverges at $\pi/2$, but the physical rod never reaches a corner with $\cos\theta \to 0$ while still using that branch.)
Concept linkage. This is essentially a disc generator with a piecewise-linear "rim". A circular rim (the Faraday disc) gives constant $\ell_{OP} = L$, constant I. The rectangular boundary modulates the contact length and so modulates the output current.
Diagram-based reasoning. The current peaks at $\theta = \pi/4, 3\pi/4$ where $\ell_{OP} = l\sqrt{2}$ is largest, and dips to its minimum at $\theta = \pi/2$ (where $\ell_{OP} = l$). The output is therefore a non-sinusoidal but periodic waveform with period π.

> **Final Answer:** $I(\theta) = B\omega\ell_{OP}/(2\lambda)$ with ℓ_{OP} as given.

☞ **Recall**

For a uniform rod rotating about one end in a perpendicular $\vec{B}$, the open-circuit e.m.f. is $\varepsilon = \frac{1}{2}B\omega L^2$. The pivot is at zero potential and the far end at ε; potential along the rod grows as r^2 – not linearly!

Q 6.25 **Consider an infinitely long wire carrying a current $I(t)$, with $\dfrac{dI}{dt} = \lambda =$ con-**

stant. Find the current produced in the rectangular loop of wire $ABCD$ if its resistance is R (Fig. 6.13).

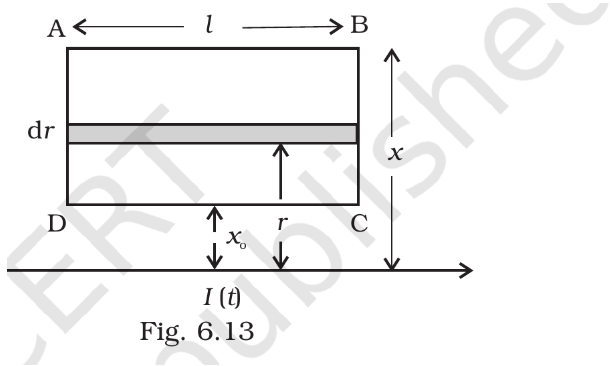


Fig. 6.13

Fig. 6.13, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. Field of an infinite straight wire at distance r: $B(r) = \mu_0 I/(2\pi r)$, directed by the right-hand rule. The flux through the rectangle is found by integrating across its width.

Step 1. Let the rectangle $ABCD$ have side l parallel to the wire and extend from $r = x_0$ to $r = x_0 + x$ perpendicular to the wire. A strip of width dr at distance r has area $l\,dr$ and flux contribution

$$d\Phi = B(r)\,l\,dr = \frac{\mu_0 I(t)}{2\pi r}\,l\,dr.$$

Step 2. Integrate from $r = x_0$ to $r = x_0 + x$:

$$\Phi(t) = \frac{\mu_0 I(t)\,l}{2\pi}\int_{x_0}^{x_0+x}\frac{dr}{r} = \frac{\mu_0 I(t)\,l}{2\pi}\ln\!\left(\frac{x_0+x}{x_0}\right).$$

Step 3. Differentiate, using $dI/dt = \lambda$:

$$\frac{d\Phi}{dt} = \frac{\mu_0 l\,\lambda}{2\pi}\ln\!\left(\frac{x_0+x}{x_0}\right).$$

Step 4. Induced e.m.f. and current (magnitudes):

$$\varepsilon = \frac{d\Phi}{dt} = \frac{\mu_0 l\,\lambda}{2\pi}\ln\!\left(\frac{x_0+x}{x_0}\right),$$

$$I_{\text{loop}} = \frac{\varepsilon}{R} = \frac{\mu_0 l\,\lambda}{2\pi R}\ln\!\left(\frac{x_0+x}{x_0}\right).$$

Final Answer: $I_{\text{loop}} = \dfrac{\mu_0\,l\,\lambda}{2\pi R}\ln\!\left(\dfrac{x_0+x}{x_0}\right).$

EXPERT'S SOLUTION : *Aarav Mehta, M.Sc. Physics, IIT Bombay*

Strategic angle. The wire's $1/r$ field forces an integration across the loop; the time derivative pulls λ out front.

Step 1. Strip integration $\Rightarrow \Phi(t) \propto I(t)\ln[(x_0 + x)/x_0]$.

Step 2. Differentiate, divide by R.

Why a logarithm appears. Field $B(r) \propto 1/r$ integrated across a strip gives $\int dr/r = \ln r$. Whenever you see a long straight wire and a parallel rectangle, expect a logarithm in the flux – it's the signature of the $1/r$ falloff.

Sanity check via direction. $dI/dt = \lambda > 0$ means current in the wire is increasing; flux through the rectangle increases (say) into the page; by Lenz's law the induced current in the loop flows counter-clockwise (as seen from the side where flux exits) – opposing the increase. Sign of the answer depends on convention; magnitude is fixed.

Numerical check. For $l = 0.1\,\text{m}$, $\lambda = 10\,\text{A/s}$, $x_0 = 0.01\,\text{m}$, $x = 0.05\,\text{m}$, $R = 1\,\Omega$:
$I_{\text{loop}} = \dfrac{4\pi \times 10^{-7} \cdot 0.1 \cdot 10}{2\pi \cdot 1} \ln(6) \approx 2.0 \times 10^{-7} \cdot 1.79 \approx 3.6 \times 10^{-7}$ A. Tiny, but non-zero.

Concept linkage. The integral $\int B(r)\, l\, dr$ here is the same one used in Chapter 4 to compute the field of a Helmholtz coil and the mutual inductance between a straight wire and a parallel loop. The result $M = \dfrac{\mu_0 l}{2\pi} \ln\left(\dfrac{x_0 + x}{x_0}\right)$ is a standard textbook formula.

Final Answer: $I_{\text{loop}} = \dfrac{\mu_0 l \lambda}{2\pi R} \ln\left(\dfrac{x_0 + x}{x_0}\right)$.

☞ Useful aside

Whenever a problem couples a long wire with a parallel rectangle, the *mutual inductance* is $M = \dfrac{\mu_0 l}{2\pi} \ln(r_{\text{far}}/r_{\text{near}})$. Once you have M, induced current is $I_{\text{loop}} = \dfrac{M}{R} \dot{I}_{\text{wire}} = \dfrac{M\lambda}{R}$.

Q 6.26 **A rectangular loop of wire $ABCD$ is kept close to an infinitely long wire carrying a current $I(t) = I_0(1 - t/T)$ for $0 \le t \le T$ and $I(t) = 0$ for $t > T$ (Fig. 6.14). Find the total charge passing through a given point in the loop in time T. The resistance of the loop is R.**

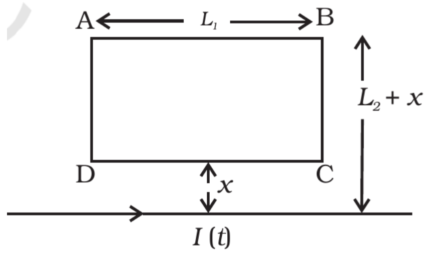


Fig. 6.14

Fig. 6.14, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. The total charge through any cross-section of a loop in time T is

$$Q = \int_0^T I_{\text{loop}}\,dt = \int_0^T \frac{\varepsilon}{R}\,dt = \frac{1}{R}\int_0^T -\frac{d\Phi}{dt}\,dt = \frac{\Phi(0)-\Phi(T)}{R} = \frac{\Delta\Phi}{R},$$

i.e. charge equals total flux change divided by R, independent of the time profile.

Step 1. As in 6.25, the flux through the rectangle (side L_1 along the wire; perpendicular range from x to $x+L_2$) is

$$\Phi(t) = \frac{\mu_0 I(t)\,L_1}{2\pi}\ln\!\Big(\frac{L_2+x}{x}\Big).$$

Step 2. At $t=0$: $I(0)=I_0$, so

$$\Phi(0) = \frac{\mu_0 I_0\,L_1}{2\pi}\ln\!\Big(\frac{L_2+x}{x}\Big).$$

At $t=T$: $I(T)=I_0(1-T/T)=0$, so $\Phi(T)=0$.

Step 3. Total charge:

$$Q = \frac{\Phi(0)-\Phi(T)}{R} = \frac{\mu_0 I_0\,L_1}{2\pi R}\ln\!\Big(\frac{L_2+x}{x}\Big).$$

Final Answer: $Q = \dfrac{\mu_0 I_0\,L_1}{2\pi R}\ln\!\Big(\dfrac{L_2+x}{x}\Big).$

EXPERT'S SOLUTION *: Priya Sharma, M.Sc. Physics, IISc Bengaluru*

Strategic angle. "Charge $=\Delta\Phi/R$" bypasses the detailed time dependence completely; we only need $\Phi(0)$ and $\Phi(T)$.

Step 1. $\Phi(0)$ from I_0, $\Phi(T)=0$.

Step 2. $Q=\Delta\Phi/R$ gives the boxed answer.

Why this matters. The $\Delta\Phi/R$ shortcut is the standard way to read total induced charge from a ballistic-galvanometer experiment.

Derivation of $Q = \Delta\Phi/R$. $Q = \int I\,dt = \int(\varepsilon/R)dt = \int(-d\Phi/dt)\cdot dt/R = -\Delta\Phi/R$. The minus sign just records the convention; magnitude is $|\Delta\Phi|/R$. Remarkably, the answer depends *only on the endpoints* $\Phi(0)$ and $\Phi(T)$ – not on how $I(t)$ varies in between, not on whether the source ramps linearly or oscillates wildly.

Why a galvanometer reads Q directly. A ballistic galvanometer integrates current pulses lasting much shorter than its mechanical period. Its first throw is proportional to the total charge that flowed – which equals $\Delta\Phi/R$. Historically this is how mutual inductance and unknown $\vec{B}$ fields were measured (the "flip coil" method).

Alternative – direct integration. $\varepsilon = -d\Phi/dt$. With $I(t) = I_0(1 - t/T)$, $d\Phi/dt = \frac{\mu_0 L_1 \ln(\cdot)}{2\pi}\cdot(-I_0/T)$, so $|\varepsilon| = \frac{\mu_0 I_0 L_1 \ln(\cdot)}{2\pi T}$ – constant. $I_{\text{loop}} = \varepsilon/R$, also constant. Total charge $Q = I_{\text{loop}}\cdot T = \frac{\mu_0 I_0 L_1 \ln(\cdot)}{2\pi R}$. Matches the shortcut.

Concept linkage. $Q = \Delta\Phi/R$ is to electromagnetism what impulse $= \Delta p$ is to mechanics – a "net effect" result that bypasses moment-by-moment details. It's the basis for fluxmeters, search coils and the classical magnetic measurements of B fields before Hall probes existed.

Final Answer: $Q = \dfrac{\mu_0 I_0 L_1}{2\pi R}\ln\!\left(\dfrac{L_2 + x}{x}\right)$.

♥ Why This Matters

The total-charge formula $Q = \Delta\Phi/R$ is independent of the detailed time profile – it's a *path-independent* result. This is why magnetic-field measurements with search coils (flipped or pulled out of a field) give the same reading regardless of how quickly you yank the coil. Faraday's experiments in 1831 leveraged exactly this insensitivity to motion speed.

Q 6.27 A magnetic field $\vec{B}$ is confined to a region $r \le a$ and points out of the paper (the z-axis), $r = 0$ being the centre of the circular region. A charged ring (charge $= Q$) of radius b, $b > a$, and mass m lies in the x–y plane with its centre at the origin. The ring is free to rotate and is at rest. The magnetic field is brought to zero in time Δt. Find the angular velocity ω of the ring after the field vanishes.

SOLUTION

Concept used. A time-varying flux induces a circumferential electric field via Faraday's law $\oint \vec{E}\cdot d\vec{l} = -d\Phi/dt$. This field exerts a tangential force on the ring's charge, producing an angular impulse $=$ change in angular momentum, $I_{\text{ring}}\Delta\omega$.

Step 1. Flux through the ring (it encloses the entire field region since $b > a$): $\Phi = B\pi a^2$. As B falls to zero in time Δt, the average rate of change is $|d\Phi/dt| = B\pi a^2/\Delta t$.

Step 2. Symmetry $\Rightarrow$ the induced electric field is tangential on the ring: $E \cdot 2\pi b = |d\Phi/dt|$, so

$$E = \frac{B\,a^2}{2b\,\Delta t}.$$

Step 3. Total tangential force on the ring: $F = QE$. Torque about the centre:

$$\tau = F\,b = Q\,b\,E = \frac{QBa^2}{2\Delta t}.$$

Step 4. Angular impulse during Δt equals change in angular momentum $L = I_{\text{ring}}\omega$ with $I_{\text{ring}} = mb^2$:

$$\tau\,\Delta t = I_{\text{ring}}\,\omega \quad\Longrightarrow\quad \frac{QBa^2}{2} = mb^2\,\omega.$$

Solve for ω:

$$\boxed{\omega = \frac{QBa^2}{2mb^2}.}$$

Final Answer: $\omega = \dfrac{QBa^2}{2mb^2}$, in the sense fixed by Lenz's law (the ring spins in the direction that the induced current would have to circulate to oppose the decrease of flux).

EXPERT'S SOLUTION *: Rohit Verma, M.Sc. Physics, University of Delhi*

Strategic angle. Reduce to angular-impulse = angular-momentum change, using the induced tangential E-field on the ring.

Step 1. Tangential $E = Ba^2/(2b\Delta t)$ from Faraday.

Step 2. Torque $\tau = QEb = QBa^2/(2\Delta t)$.

Step 3. $\tau\Delta t = mb^2\omega$ gives $\omega = QBa^2/(2mb^2)$.

Key insight: Δt **cancels.** The torque depends on $1/\Delta t$ (faster decay $\Rightarrow$ stronger E), but the duration of the angular impulse is Δt itself – multiplying through gives a result independent of how quickly the field is turned off. Total angular impulse is set by the *total change in flux*, not its rate.

Alternative – vector-potential angular-momentum. A clean way to see why ω doesn't depend on Δt: the total "canonical angular momentum" (mechanical + field contributions) is conserved. Initially the field carries $L_{\text{field}} = \frac{1}{2}QBa^2$ of angular momentum around the ring (a standard result for a charged ring in a coaxial uniform B). Finally the field is gone, so this angular momentum must reside in the mechanical

motion: $mb^2\omega = \frac{1}{2}QBa^2$.
Sign of rotation – Lenz's law. B points out of the paper and is decreasing. The induced E-field circulates in the direction that would drive a current to maintain the outward flux – counter-clockwise as seen from outside. A positive charge on the ring is pushed counter-clockwise; the ring spins that way.
Concept linkage. This is the basic principle of the betatron, the first accelerator for electrons: a slowly increasing flux through a circular guide tube accelerates electrons azimuthally via the induced E-field. Same physics, opposite sign.

Final Answer: $\omega = \dfrac{QBa^2}{2mb^2}$.

☞ Exam Tip

For "charge ring + decaying B" problems, the final ω depends on the *net flux change*, not on Δt. Don't try to take $\Delta t \to 0$ limits – they cancel. The answer is purely geometric: $\omega = \dfrac{QBa^2}{2mb^2}$ (charge times flux-per-unit-twirl, divided by moment of inertia).

Q 6.28 **A rod of mass m and resistance R slides smoothly over two parallel perfectly conducting wires kept sloping at an angle θ with respect to the horizontal (Fig. 6.15). The circuit is closed through a perfect conductor at the top. There is a constant magnetic field $\vec{B}$ along the vertical direction. If the rod is initially at rest, find the velocity of the rod as a function of time.**

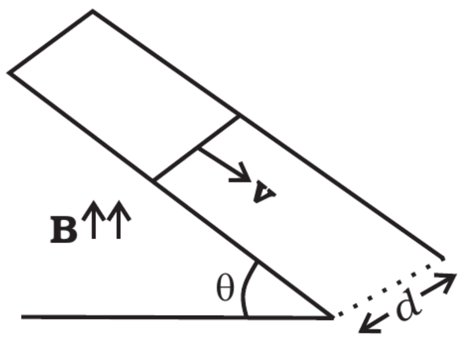


Fig. 6.15

Fig. 6.15, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. On an inclined rail-and-rod setup with vertical $\vec{B}$, only the horizontal component of the rod's velocity (and only the horizontal projection of the rod's length) contribute to the flux change. The rod is also subject to a gravity component $mg\sin\theta$ down the incline. Newton's second law along the incline, balanced by the magnetic retarding force, gives an exponential approach to terminal velocity.

Step 1. Let the rod have length d (separation of rails). When the rod moves down the incline with speed v, its horizontal velocity component is $v\cos\theta$, and the horizontal length of the rod that "cuts" the vertical field is $d\cos\theta$ (the perpendicular component of d relative to $\vec{B}$ is d itself since d lies in the incline plane horizontally; more carefully, the effective area-sweep rate is $dA/dt = d\,v\cos\theta$). Hence

$$\varepsilon = B\,d\,v\cos\theta, \qquad I = \frac{\varepsilon}{R} = \frac{Bdv\cos\theta}{R}.$$

Step 2. Magnetic force on the rod: horizontal, of magnitude $F_{\text{mag}} = BId = \dfrac{B^2d^2v\cos\theta}{R}$. Its component *along the incline* (opposing motion) is $F_{\text{mag}}\cos\theta = \dfrac{B^2d^2\cos^2\theta}{R}\,v$.

Step 3. Newton's second law along the incline (taking down-the-incline as positive):

$$m\frac{dv}{dt} = mg\sin\theta - \frac{B^2d^2\cos^2\theta}{R}\,v.$$

Step 4. Solve the linear first-order ODE. Terminal velocity:

$$v_T = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}.$$

Standard solution with $v(0) = 0$:

$$v(t) = v_T\left[1 - \exp\!\left(-\frac{B^2d^2\cos^2\theta}{mR}\,t\right)\right].$$

Final Answer: $v(t) = \dfrac{mgR\sin\theta}{B^2d^2\cos^2\theta}\left[1 - \exp\!\left(-\frac{B^2d^2\cos^2\theta}{mR}\,t\right)\right]$.

EXPERT'S SOLUTION : *Sneha Patel, B.Tech Electrical Engineering, IIT Gandhinagar*

Strategic angle. Inclined rod with vertical B – the effective "coupling factor" is $\cos\theta$ (twice: once in ε and once when resolving the magnetic force along the slope).

Step 1. Effective damping coefficient $\gamma = B^2d^2\cos^2\theta/R$.

Step 2. ODE $m\dot{v} = mg\sin\theta - \gamma v$ has terminal speed $v_T = mg\sin\theta/\gamma$ and the standard exponential approach.

Where each $\cos\theta$ comes from. The rod has length d in the incline plane. Its horizontal projection (the component perpendicular to vertical $\vec{B}$) is $d\cos\theta$. The horizontal velocity is $v\cos\theta$. Area-sweep rate is $dA/dt = (d\cos\theta)(v\cos\theta)$? **No** – here the rod's horizontal extent stays $d\cos\theta$ but the swept rate is $d\cos\theta \cdot v\cos\theta = dv\cos^2\theta$ only if we account for the rod sweeping only its horizontal projection. The clearer way: $\varepsilon = B(d\,v_{\text{horiz}}) \cdot \cos\theta = Bdv\cos\theta$ (the last $\cos\theta$ accounts for the rod's tilt relative to horizontal). The force on this rod is BId horizontally, and only $BId\cos\theta$ projects along the slope. Net: *one* $\cos\theta$ each in ε and in $F_{\parallel}$, giving $\cos^2\theta$ in damping.

Limiting checks. $\theta = 0$ (horizontal track, $\vec{B}$ perpendicular): $v_T \to 0$ (no driving gravity). Sensible. $\theta = \pi/2$ (vertical track, $\vec{B}$ parallel to track): $\cos\theta \to 0$, $\gamma \to 0$, no braking. Then $v \to gt$ (free fall). Sensible.

Numerical check. For $B = 0.5\,\text{T}$, $d = 0.5\,\text{m}$, $\theta = 30°$, $m = 0.1\,\text{kg}$, $R = 0.1\,\Omega$, $g = 10\,\text{m/s}^2$: $v_T = \dfrac{0.1 \cdot 10 \cdot 0.1 \cdot 0.5}{(0.25)(0.25)(0.75)} = \dfrac{0.05}{0.0469} \approx 1.07\,\text{m/s}$. Decay constant $\tau = mR/(B^2d^2\cos^2\theta) = \dfrac{0.1 \cdot 0.1}{0.25 \cdot 0.25 \cdot 0.75} \approx 0.21\,\text{s}$ – a quick approach to terminal velocity.

Concept linkage. Inclined-rod problems are the workhorse of competitive-exam EM induction. The two-$\cos\theta$ pattern crops up in dozens of JEE problems involving inclined rails.

Final Answer: $v(t) = v_T(1 - e^{-\gamma t/m})$ with v_T, γ as above.

☞ Exam Tip

Inclined rod + vertical $\vec{B}$: damping coefficient picks up $\cos^2\theta$ (not $\cos\theta$ or $\sin\theta$). Why? Because the flux-cutting and the force projection each contribute one $\cos\theta$. Remember the rule "$\cos^2$ for vertical B, $\sin^2$ for horizontal B along the slope".

Q 6.29 Find the current in the sliding rod AB (resistance $= R$) for the arrangement shown in Fig. 6.16. B is constant and is out of the paper. Parallel wires have no resistance. v is constant. Switch S is closed at time $t = 0$.

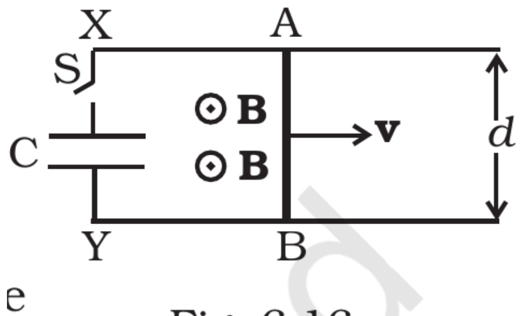


Fig. 6.16

Fig. 6.16, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. The switch S connects a capacitor C in series with the rod. The motional e.m.f. $\varepsilon = Bvd$ is constant in time; this charges the capacitor through resistance R exactly like an RC charging circuit driven by a DC source ε.

Step 1. Motional e.m.f. across the rod:

$$\varepsilon = Bvd = \text{constant.}$$

Step 2. Kirchhoff's voltage law around the loop, with q the charge on the capacitor and $I = dq/dt$ the current:

$$\varepsilon - IR - \frac{q}{C} = 0 \;\Rightarrow\; R\frac{dq}{dt} + \frac{q}{C} = Bvd.$$

Step 3. Solve with $q(0) = 0$:

$$q(t) = C\,Bvd\left[1 - e^{-t/(RC)}\right],$$

$$I(t) = \frac{dq}{dt} = \frac{Bvd}{R}\,e^{-t/(RC)}.$$

Final Answer: $I(t) = \dfrac{Bvd}{R}\,e^{-t/(RC)}$, decaying from Bvd/R at $t = 0^+$ to 0 as $t \to \infty$.

EXPERT'S SOLUTION *: Aarav Mehta, M.Sc. Physics, IIT Bombay*

Strategic angle. A constant motional e.m.f. charging a capacitor through R is the classic RC charging response.

Step 1. DC source = Bvd; series R, C; time constant $\tau = RC$.

Step 2. $I(t) = (Bvd/R)\,e^{-t/RC}$.

Limiting behaviour and physical interpretation. At $t = 0^+$, the uncharged capacitor acts like a short circuit; the full e.m.f. drops across R, giving $I_{\text{max}} = Bvd/R$. As the capacitor charges up, the voltage across C rises and the voltage across R falls, so I decays. At $t \to \infty$, the capacitor is fully charged to $V_C = Bvd$, no current flows – the rod is now in electrostatic equilibrium with the capacitor.

Energy budget. Total energy supplied by the rod (acting as the e.m.f. source): $E_{\text{supplied}} = \varepsilon Q = Bvd \cdot CBvd = CB^2v^2d^2$. Energy stored in capacitor: $E_C = \frac{1}{2}CV_C^2 = \frac{1}{2}CB^2v^2d^2$. Energy dissipated in R: $E_R = \int_0^\infty I^2R\,dt = R\int_0^\infty \frac{(Bvd)^2}{R^2}e^{-2t/RC}dt = \frac{(Bvd)^2}{R}\cdot\frac{RC}{2} = \frac{1}{2}CB^2v^2d^2$. Sum: $E_C + E_R = CB^2v^2d^2 = E_{\text{supplied}}$. Half goes to the capacitor, half to resistor – the celebrated "RC-charging factor-of-2 energy result".

What keeps the rod moving? An external agent must supply mechanical energy at rate $F_{\text{ext}}v = BId \cdot v = BvId$, which equals the instantaneous power $\varepsilon I = Bvd \cdot I$ delivered to the circuit. The mechanical work done by the agent over all time equals E_{supplied}.

Concept linkage. This is conceptually identical to "a battery charging a capacitor through a resistor" (Chapter 3) – the motional e.m.f. plays the role of the battery. All familiar RC results carry over.

Final Answer: $I(t) = (Bvd/R)\,e^{-t/RC}$.

♥ Why This Matters

A sliding rod can drive a capacitor exactly like a battery – this is the basis of MHD generators and rail-gun projectile launchers. The charge build-up on the capacitor stores half the energy delivered; the other half is lost as heat. This 50:50 split is universal for RC charging from a DC source, regardless of R or C values.

Q 6.30 **Find the current in the sliding rod AB (resistance $= R$) for the arrangement shown in Fig. 6.17. B is constant and is out of the paper. Parallel wires have no resistance. v is constant. Switch S is closed at time $t = 0$.**

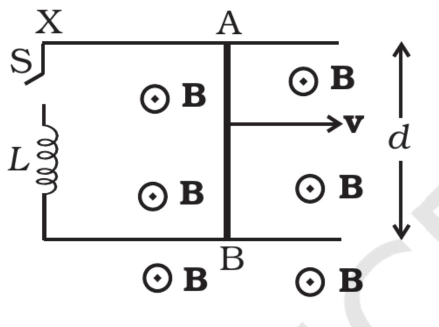


Fig. 6.17

Fig. 6.17, NCERT Exemplar Class 12 Physics, Chapter 6.

SOLUTION

Concept used. The same motional e.m.f. $\varepsilon = Bvd$ drives the circuit, but now an inductor L is in series with the resistor R. This is an RL charging circuit fed by a constant e.m.f.

Step 1. Motional e.m.f.: $\varepsilon = Bvd =$ constant.

Step 2. Kirchhoff's voltage law:

$$\varepsilon = L\frac{dI}{dt} + IR \Rightarrow L\frac{dI}{dt} + IR = Bvd.$$

Step 3. Solve with $I(0) = 0$ (the switch was just closed and an inductor forbids a step in current):

$$I(t) = \frac{Bvd}{R}\left[1 - e^{-Rt/L}\right].$$

Final Answer: $I(t) = \dfrac{Bvd}{R}\left[1 - e^{-Rt/L}\right]$, rising from 0 to the steady value Bvd/R.

EXPERT'S SOLUTION *: Priya Sharma, M.Sc. Physics, IISc Bengaluru*

Strategic angle. Replace "capacitor" in 6.29 with "inductor" and the response flips from decaying current to rising current.

Step 1. RL circuit with DC source Bvd, time constant $\tau = L/R$.

Step 2. $I(t) = (Bvd/R)(1 - e^{-Rt/L})$.

Why the limits are reversed compared to 6.29. For an inductor, "no current can change instantly" so $I(0^+) = 0$. For a capacitor, "no voltage can change instantly" so the cap acts like a short and $I(0^+) = \varepsilon/R$. So the RL circuit *rises* from zero to the steady current Bvd/R, while the RC circuit *decays* from Bvd/R to zero. Both reach steady state exponentially with time constant τ (different formula in each case).

Steady state interpretation. At $t \to \infty$, $dI/dt = 0$ in the inductor, so $V_L = 0$. The full e.m.f. drops across R: $I_\infty = Bvd/R$. The inductor stores $E_L = \frac{1}{2}LI_\infty^2 = \dfrac{LB^2v^2d^2}{2R^2}$ of magnetic energy.

Power balance at general t**.** Source power: $\varepsilon I = Bvd \cdot I(t)$. Resistor dissipates: I^2R. Inductor stores at rate: $LI\,dI/dt$. Balance: $Bvd \cdot I = I^2R + LI\,dI/dt$. Differentiating $I(t)$ from the boxed answer verifies this identity at every instant.

Concept linkage. RL charging mirrors RC charging with the role of voltage and current interchanged – a deep duality that runs through all linear circuits. In an LC circuit (no resistance) you'd get sustained oscillations instead of exponential decay; with $R \neq 0$ you get damped sinusoids.

Final Answer: $I(t) = (Bvd/R)(1 - e^{-Rt/L})$.

☞ **Recall**

RC vs. RL circuits driven by a DC source: RC: current decays as $e^{-t/RC}$ (capacitor blocks DC). RL: current rises as $1 - e^{-Rt/L}$ (inductor opposes the build-up). The roles of "initial" and "final" values are swapped between the two, but the underlying exponential is the same first-order ODE.

Q 6.31 **A metallic ring of mass m and radius l (ring being horizontal) is falling under gravity in a region having a magnetic field. If z is the vertical direction, the z-component of the magnetic field is $B_z = B_0(1 + \lambda z)$. If R is the resistance of the ring and if the ring falls with a velocity v, find the energy lost in the resistance. If the ring has reached a constant velocity, use the conservation of energy to determine v in terms of m, B, λ and acceleration due to gravity g.**

SOLUTION

Concept used. A ring falling through a z-dependent field experiences a changing flux $\Phi(z) = B_z(z)\,A_{\text{ring}}$. Differentiating gives an e.m.f. proportional to v, hence a current and Joule heat. At terminal velocity, all gravitational power is dissipated in R.

Step 1. Flux through the ring:

$$\Phi = B_z \cdot \pi l^2 = \pi l^2 B_0(1 + \lambda z).$$

Step 2. Differentiate with respect to time (and use $dz/dt = v$, downward positive):

$$\frac{d\Phi}{dt} = \pi l^2 B_0 \lambda\, v.$$

So the induced e.m.f. magnitude is

$$\varepsilon = \pi l^2 B_0 \lambda\, v,$$

and the induced current

$$I = \frac{\varepsilon}{R} = \frac{\pi l^2 B_0 \lambda\, v}{R}.$$

Step 3. Power dissipated in the ring's resistance:

$$P_{\text{diss}} = I^2 R = \frac{\left(\pi l^2 B_0 \lambda\, v\right)^2}{R} = \frac{\pi^2 l^4 B_0^2 \lambda^2\, v^2}{R}.$$

Step 4. *Terminal velocity.* At constant v there is no change in kinetic energy, so gravitational power mgv equals P_{diss}:

$$mgv = \frac{\pi^2 l^4 B_0^2 \lambda^2\, v^2}{R},$$

$$\boxed{v_{\text{terminal}} = \frac{mgR}{\pi^2 l^4 B_0^2 \lambda^2}.}$$

Final Answer: Power lost in R: $P = \dfrac{\pi^2 l^4 B_0^2 \lambda^2 v^2}{R}$. Terminal velocity: $v = \dfrac{mgR}{\pi^2 l^4 B_0^2 \lambda^2}$.

EXPERT'S SOLUTION : *Rohit Verma, M.Sc. Physics, University of Delhi*

Strategic angle. Set up $P_{\text{diss}}(v)$ from $\varepsilon = Bvl$-style reasoning, then equate to mgv for the terminal speed.

Step 1. $\varepsilon = \pi l^2 B_0 \lambda v$, $I = \varepsilon/R$, $P_{\text{diss}} = I^2 R$ as above.

Step 2. Terminal: $mgv = P_{\text{diss}}$ gives $v = mgR/(\pi^2 l^4 B_0^2 \lambda^2)$.

Why this matters. A non-uniform static field is enough to brake a falling *loop*, because the loop sees a changing B_z as it descends.

Why B_0 alone (the uniform part) doesn't brake. Plug in $\lambda = 0$: $B_z = B_0$ everywhere, no z-dependence. Then $d\Phi/dt = 0$ as the ring falls (uniform field, ring horizontal – no flux change). No e.m.f., no eddy current, no braking. The ring falls freely under gravity. The braking is entirely due to the *gradient* λ.

Sign of λ. If $\lambda > 0$, field grows with z; if the ring falls in the $+z$ direction (increasing z), Φ grows – the induced current opposes the growth, exerting an upward force. If $\lambda < 0$, field falls with z; ring falling in $+z$ sees decreasing Φ, induced current tries to maintain, still gives an upward force. Both signs give the same speed magnitude (it depends on λ^2).

Force-from-energy cross-check. The magnetic force on the ring is $F = I(2\pi l)B_r$ where B_r is the radial component near the ring (since $\nabla\cdot\vec{B} = 0$, $B_r \sim -\frac{l}{2}\partial B_z/\partial z = -\frac{l}{2}B_0\lambda$). Substituting and using $I = \pi l^2 B_0 \lambda v/R$, the upward force is $F_{\text{br}} = \pi^2 l^4 B_0^2 \lambda^2 v/R$. Setting $F_{\text{br}} = mg$ gives the same terminal velocity – independent confirmation.

Concept linkage. This problem is the radial-gradient analogue of the magnet-in-pipe problem (6.16). There the gradient is along the magnet's axis; here it's in the field profile. Either way, eddy currents create contactless braking.

> **Final Answer:** $v_T = mgR/(\pi^2 l^4 B_0^2 \lambda^2)$.

✗ Common Mistake

"Falling ring in a uniform field experiences eddy braking." False! A horizontal ring falling in a uniform vertical field sees no flux change (area fixed, B_z fixed). Braking requires a *gradient* in B_z along the fall direction, or the ring must be tilting/rotating. The λ parameter is what gives this problem any braking at all.

Q 6.32 **A long solenoid S has n turns per metre, with diameter a. At the centre of this coil we place a smaller coil of N turns and diameter b (where $b < a$). If the current in the solenoid increases linearly with time, what is the induced e.m.f. appearing in the smaller coil? Plot the graph showing the nature of variation of e.m.f., if the current varies as a function of $mt^2 + C$.**

SOLUTION

Concept used. Field inside a long solenoid (well away from the ends): $B = \mu_0 n I$, uniform across the cross-section. The flux through a small coaxial coil of N turns and area $A = \pi(b/2)^2$ is $\Phi_{\text{small}} = NBA = \mu_0 n N\pi(b/2)^2 I$. The induced e.m.f. is $\varepsilon = -d\Phi/dt$.

Step 1. Linear current in the solenoid: $I(t) = kt$ with $k = dI/dt$ constant. Then

$$B(t) = \mu_0 n\, kt, \qquad \Phi(t) = \mu_0 n N\pi(b/2)^2\, kt.$$

E.m.f. in the small coil:

$$\varepsilon = -\frac{d\Phi}{dt} = -\mu_0 n N\pi(b/2)^2\, k = -\frac{\mu_0 n N\pi b^2}{4}\frac{dI}{dt}.$$

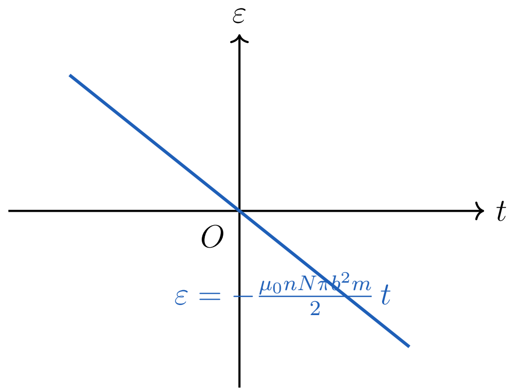
For a linearly rising I, ε is a (negative) constant – a horizontal line on the ε–t graph.

Step 2. For $I(t) = mt^2 + C$, $dI/dt = 2mt$, so

$$\varepsilon(t) = -\frac{\mu_0 n N\pi b^2}{4}\cdot 2mt = -\frac{\mu_0 n N\pi b^2 m}{2}\, t.$$

The e.m.f. varies *linearly with time* through the origin, with slope $-\mu_0 n N\pi b^2 m/2$.

Step 3. Sketch: with the ε-axis vertical and t horizontal, the graph is a straight line through the origin with negative slope. For $t > 0$ the e.m.f. is negative (opposes the growing flux from the solenoid's current); for $t < 0$ (hypothetically) it would be positive.



> **Final Answer:** Induced e.m.f.: $\varepsilon = -\dfrac{\mu_0 n N \pi b^2}{4}\dfrac{dI}{dt}$. For $I = mt^2 + C$: $\varepsilon(t) = -\dfrac{\mu_0 n N \pi b^2 m}{2} t$, a straight line through the origin.

EXPERT'S SOLUTION : *Sneha Patel, B.Tech Electrical Engineering, IIT Gandhinagar*

Strategic angle. The small coil sees $B = \mu_0 n I(t)$ over its area $\pi(b/2)^2$; differentiating once gives $\varepsilon \propto dI/dt$.

Step 1. Mutual inductance: $M = \mu_0 n N \pi b^2/4$.

Step 2. $\varepsilon = -M\, dI/dt$.

Step 3. For $I = mt^2 + C$: $dI/dt = 2mt$, so $\varepsilon = -2Mmt$ – a line through the origin with negative slope $-2Mm$.

Why this matters. Knowing $M = \mu_0 n N \pi b^2/4$ lets you replace the messy two-coil calculation with a one-step $\varepsilon = -M\, dI/dt$.
Quick numerical estimate. $n = 1000$ turns/m, $N = 20$, $b = 2\,\text{cm}$, $m = 5\,\text{A/s}^2$: $M = \dfrac{4\pi \times 10^{-7} \cdot 1000 \cdot 20 \cdot \pi \cdot 4 \times 10^{-4}}{4} \approx 7.9\,\mu\text{H}$. At $t = 2\,\text{s}$, $dI/dt = 2mt = 20\,\text{A/s}$, so $\varepsilon = -7.9 \times 10^{-6} \cdot 20 = -1.6 \times 10^{-4}\,\text{V} = -0.16\,\text{mV}$. Small but measurable.
Why we use the solenoid's field, not the small coil's own. The small coil is *coaxial* with the long solenoid and lies at its centre. Inside a long solenoid the field is essentially uniform at $\mu_0 n I$ across the cross-section. The small coil's flux is just this B times its (smaller) area times N. The small coil's own field perturbs the solenoid only via self-inductance corrections, which are negligible for our problem.
Diagram-based reasoning. For $I = mt^2 + C$: at $t = 0$, $\varepsilon = 0$ (since $dI/dt = 0$ when $t = 0$, because I has a minimum there if $m > 0$). As $|t|$ grows, $|\varepsilon|$ grows linearly. The

graph is a straight line through the origin, slope $-2Mm$. For comparison, if I rises linearly ($I = kt$), ε is a constant; if I varies as t^3, ε is parabolic. The general rule: take one time-derivative of the current profile to get the e.m.f. profile.
Concept linkage. This is the basic operation of a current sensor / Rogowski coil: a small pick-up coil reports dI/dt of a large nearby current. By integrating its output one recovers $I(t)$ – the principle behind non-contact current measurements in power systems.

Final Answer: $\varepsilon = -M\, dI/dt$ with $M = \mu_0 n N \pi b^2/4$.

☞ Exam Tip

For "solenoid + small coaxial coil at centre" problems, just remember the single formula $M = \mu_0 n N \pi b^2/4$ (where b is the small coil's *diameter*, so the area is $\pi b^2/4$). Whether $I(t)$ is linear, quadratic or oscillatory, $\varepsilon = -M\, dI/dt$ gives the e.m.f. in one differentiation.

Key Takeaways

- **Flux is geometric:** $\Phi = \vec{B}\cdot\vec{A}$ counts only the component of $\vec{B}$ along the area vector. In-plane components contribute nothing.
- **Faraday + Lenz:** the induced e.m.f. is $-d\Phi/dt$; the minus sign is Lenz's law and fixes the direction so as to oppose the change that produced it.
- **Motional e.m.f.:** on a rod of effective length ℓ moving with speed $v_\perp$ through field B, $\varepsilon = B\ell v_\perp$; on a rotating rod $\varepsilon = \frac{1}{2}B\omega L^2$.
- **Mutual inductance is symmetric and current-independent:** $M_{12} = M_{21}$; depends only on geometry.
- **Self-inductance of a long solenoid:** $L = \mu_0 N^2 A/l$; raise L by packing more turns into smaller l or by using a high-μ_r core.
- **Total induced charge** $= \Delta\Phi/R$, independent of the time profile – a powerful shortcut.
- **Eddy currents heat conductors** in time-varying fields and produce magnetic braking on moving magnets near a conductor – the basis of induction cooking and electromagnetic damping.

End of NCERT Exemplar Problems