Electromagnetic Induction

The Core Idea

A changing magnetic field through a circuit produces an EMF and (if closed) a current.
Magnetism produces electricity !

Historical note

Faraday (UK, 1831) and ~~Hertz~~ Henry (USA) independently discovered EMI in the same year.

Faraday gave the famous experiments ; Henry studied self - induction in coils.

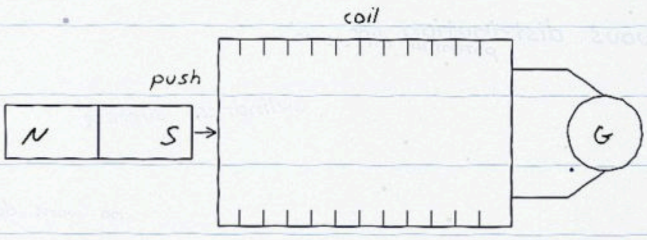


Fig. moving magnet -> current in coil -> G deflects

Topics ahead

Flux ; Faraday's law ; Lenz's law ; motional EMF ;

Faraday's Experiment - I

Setup : magnet + coil + G

Bar magnet moves towards or away from a coil connected to a galvanometer.

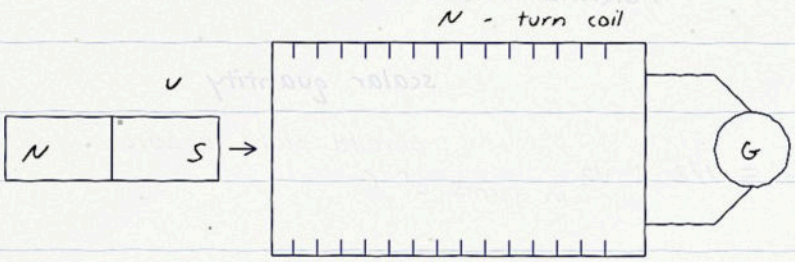


Fig. magnet pushed in -> G shows deflection

Observations

① Magnet still : G shows zero (no I).

② Push in : G deflects (say) right.

③ Pull out : G deflects (say) left.

Faster motion -> larger deflection.

Faraday's Experiment - II

Setup : primary & secondary coils

Primary coil P : carries current from a cell.

Secondary coil S : connected to a galvanometer.

Move P toward / away from S - just like the magnet in expt - I.

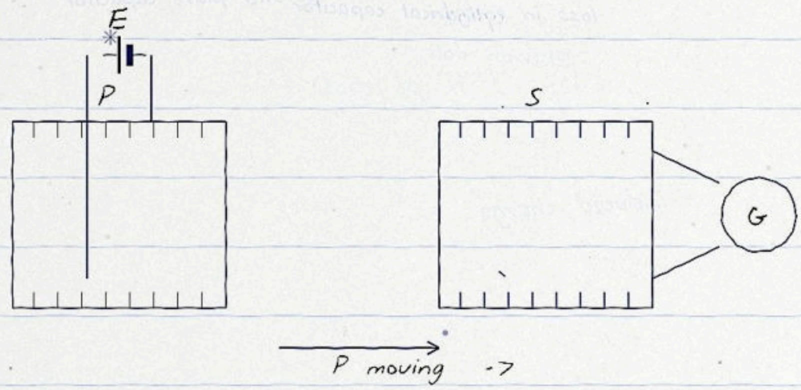


Fig. relative motion of P and S induces EMF in S

Observation

Same effect as expt - I : G deflects whenever P moves relative to S.

No motion -> steady I in P -> G reads zero.

Inference : what matters is the CHANGE in magnetic flux through S, not the cause.

Faraday's Experiment - III

Setup : stationary coils + key

Now both coils are stationary, but the current in P is changed by opening / closing a key K, or by inserting a resistor.

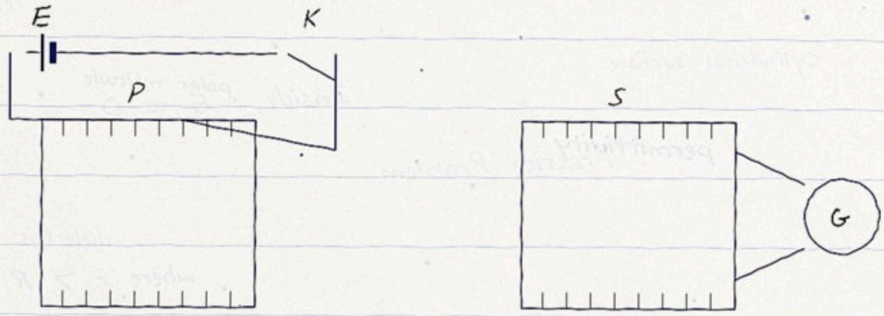


Fig. closing or opening K alters I in P

Observation

G deflects only at the instants K is *closed or opened. Steady I in P -> G reads zero.

Inference : flux through S changes only when I in P changes. -> EMF induced in S.

This is mutual induction (covered later).

Magnetic Flux

Number of magnetic field lines passing through a given area A is called magnetic flux.

$$\boxed{d\,\phi_B = B\,.\,d\,A = B\,dA\,\cos\,\theta}$$

<- theta : an
<- between

$$\boxed{\phi_B = \text{sum}\;B\,.\,d\,A}$$

<- SI : wéber (Wb)
<- 1 Wb = 1 T m 2

Dim. : $[\,M\,L^2\,T^{-2}\,A^{-1}\,]$

Sign and special cases

theta = 0 (B perp surface plane) : phi = BA

theta = 90 (B in plane of area) : phi = 0

phi is scalar but signed (sign of cos theta).

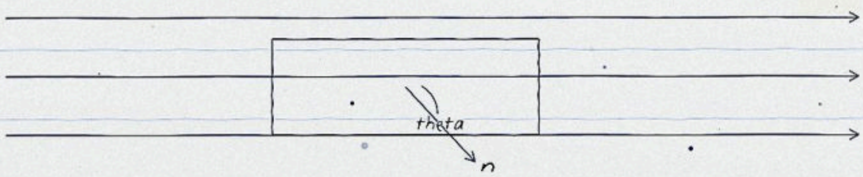


Fig. flux through a coil in uniform B

Faraday's Law of Induction

Statement

Whenever magnetic flux linked with a circuit changes, an EMF is induced; magnitude equals rate of change of flux.

$$\boxed{E = - d \phi_B / dt}$$

<- sign = Lenz's law

$$\boxed{E = - N d \phi_B / dt}$$

<- for N - turn coil

Ways to change flux

1. Change B (vary current in primary)
2. Change A (squeeze or open loop)
3. Change theta (rotate the coil)

Induced current and charge

$I_{ind} = E / R = (1/R)\, d\phi / dt$

$$\boxed{q = \phi_B / R \quad \text{(total induced charge)}}$$

(independent of how fast flux changes)

Lenz's Law

Statement

Direction of the induced EMF and current is always such that it OPPOSES the change in flux that produced it.

(That's where the minus sign in Faraday's law comes from.)

Why ?

Conservation of energy. If the induced current AIDED the change, the system would gain energy for free (perpetual motion violation!).

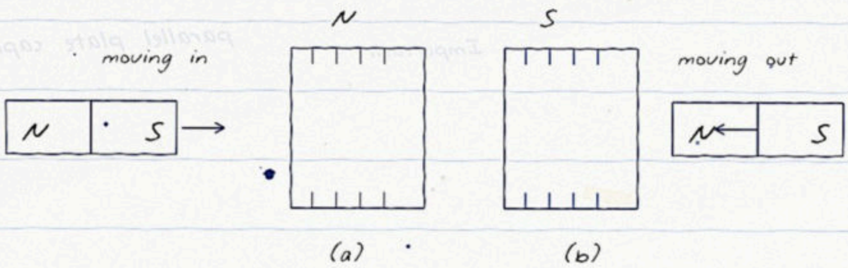


Fig. induced pole opposes the magnet's motion

Consequence

Lenz's Law - Applications

(1) Falling magnet through a copper pipe falls slower than a non - magnet : induced eddy currents oppose its motion.

(2) Electromagnetic brakes in trains : rotating wheel -> flux change -> eddy I opposes motion -> slows the train.

(3) Energy meter , induction motor.

(4) Strike -through : if Lenz's law were ~~along~~ opposite , energy conservation would be violated.

Direction recipe

1. Find the direction of B inside the coil.
2. Decide if phi is increasing or decreasing.
3. Induced B must oppose the change.
4. Right - hand grip rule -> current sense.

Sign convention check

If $d\phi / dt > 0$, then $E < 0$ (and vice versa).

i.e. EMF tries to drive current that reduces flux.

Lenz's Law & Energy Conservation

Push a bar magnet into a coil :

1. Flux through coil increases (B into face).
2. Induced I creates a field opposing the increase.
3. Coil's near face becomes a N - pole.
4. N - N pair -> repulsion on the magnet.
5. You DO work against this force -> it appears as electrical energy in the coil ($I^2 R$ losses).

$$\boxed{W_{you} = (\text{heat in } R)}$$

<- energy balance
<- no free energy !

If Lenz's law were reversed (impossible)

Then induced pole would be S , attracting the incoming magnet. Magnet accelerates , energy of the system grows - contradicting conservation.

Sources of induced EMF

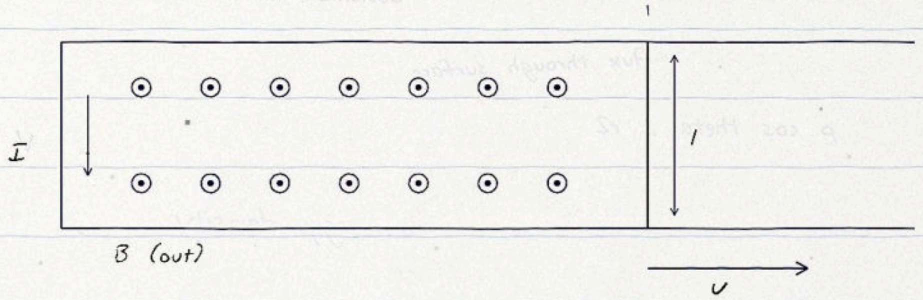
(1) Time - varying B (transformers).

(2) Conductor moving in steady B
(motional EMF , generators).

Both unified by Faraday's law on flux through the loop.

Motional EMF

Consider a straight conducting rod of length l moving with velocity v perpendicular to a uniform B (rod, B, v mutually perpendicular).



Lorentz - force derivation

Free electron's in the rod feel a force

$$F = -e(v \times B) \text{ along the rod.}$$

They drift -> charge separation -> E inside rod ; in steady state $eE = evB$:

$$\boxed{E = Blv}$$ <- motional EMF <- across rod

Direction : by right - hand rule on $v \times B$.

Motional EMF - Flux View

Same setup as before. As the rod moves right, the area enclosed by the loop increases :

$$A(t) = l \cdot x(t)$$
$$\phi_B(t) = B\,l\,x$$

Apply Faraday's law :

$$\mathcal{E} = -d\phi/dt = -B\,l\,(dx/dt) = -B\,l$$

Magnitude $= B\,l\,v$ (matches Lorentz view).

Induced current and power

Loop resistance R :

$$I = \mathcal{E}/R = B\,l\,v/R$$

Force on the rod : $F = B\,I\,l = B^2\,l^2\,v/R$

Mechanical power input : $P = F\,v = B^2\,l^2\,v^2/R$

$$\boxed{P_{mech} = P_{dissipated} = \mathcal{E}\,I}$$

<- energy
<- conservation

(work done against magnetic force -> Joule heat.)

Power, Force & Equilibrium

Force needed to keep v constant

$F_{ext} = B I l = B^2 l^2 v / R$

Without F_ext, magnetic force decelerates rod.

Energy book - keeping

$P_{ext} = F_{ext} v = B^2 l^2 v^2 / R$

$P_{dissipated} = I^2 R = (Blv)^2 / R$

P_ext = P_dissipated -> 100 % conversion.

If F_ext removed

$m \, dv / dt = - B^2 l^2 v / R$

Velocity decays exponentially :

$v(t) = v_0 e^{-t / tau}$, tau $= m R / (B^2 l^2)$

Total charge

$q =$ sum $I \, dt = (1/R)$ sum $d\phi = \phi / R$

(depends only on net flux change, not its rate.)

Total mechanical energy spent = KE lost = heat.

Worked Example : Motional EMF

Problem : A rod of length 0.5 m slides on smooth rails in a uniform B = 0.4 T (perpendicular to plane) with v = 5 m/s. Circuit resistance R = 2 ohm. Find (a) EMF , (b) I , (c) F_ext , (d) P.

(a) EMF

$E = B\,l\,v = (0.4)\,(0.5)\,(5)$

$= 1.0\ V$

(b) Current

$I = E / R = 1.0 / 2 = 0.5\ A$

(c) External force

$F_{ext} = B\,I\,l = (0.4)(0.5)(0.5)$

$= 0.1\ N$

(d) Power

$P = F v = 0.1 \cdot 5 = 0.5\ W$

check : $I^2 R = 0.25 \cdot 2 = 0.5\ W$ OK

Quick sanity

q for delta x = 1 m : $\phi = B\,l\,x = 0.2\ Wb$

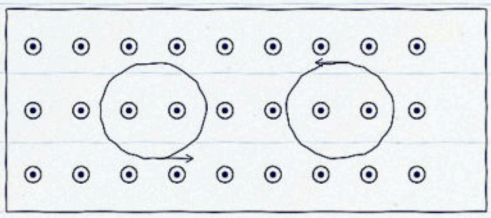
q $= \phi / R = 0.2 / 2 = \cancel{0.2}\ 0.1\ C$

q flux change ; does not depend on speed.

Eddy Currents

Closed loops of induced current that swirl inside a SOLID metal slab when the flux through it changes (Faraday + Lenz applied locally).

Fig. eddy currents in a metal slab



Joule heating

Eddy currents dissipate energy as heat :

$P = I^2 R$ in the metal.

To reduce eddy losses : laminate the metal into thin sheets insulated from each other.

(Each thin sheet has high R for swirling I.)

Used in transformer cores , motor armatures.

Eddy Currents - Applications

① Electromagnetic brakes (trains, dynamometers)
Magnetic flux changes -> eddy I in rotating wheel -> retarding force.

② Induction furnace
High-frequency B in a coil generates eddy currents in metal placed in it -> metal melts.

③ Energy meters
Aluminium disc in B from current coils ; eddy I -> torque -> rotation count.

④ Induction motors
Rotating B sets up eddy currents in the rotor -> torque due to interaction.

⑤ Metal detectors : coil drives an AC field, metal nearby alters its inductance.

⑥ Speedometers (eddy in metal disc)

Disadvantages

Heating losses in transformer cores, motors, etc.
Counter : laminations, high-resistivity steels.

Inductance - Intro

When a coil carries current, it produces its own magnetic flux -> if I changes -> flux changes -> EMF is induced.

Flux linkage N phi is proportional to I :

$$\boxed{N \phi = L I}$$

<- L = self
<- inductance

If two coils are nearby, flux from one threads the other :

$$\boxed{N_2 \phi_2 = M I_1}$$

<- M = mutual
<- inductance

Units & dimensions

SI unit : henry (H) = V . s / A = Wb / A
Dim. : $[M L^2 T^{-2} A^{-2}]$

Geometry only

L and M depend on the geometry (size, shape, number of turns, core material) but NOT on I.*

Compare with C = Q / V for capacitors - geometry only.

Self - Inductance

Definition

$$\boxed{L = N \phi / I}$$

<- flux linkage
<- per ampere

Self - induced EMF (back EMF)

If I changes :

$$\boxed{E = - L \, dI / dt}$$

<- minus : Lenz

Inductor opposes any change in current.
Acts like electric inertia for circuits.

Effects

1. Switch on : I rises slowly , not instantly.

2. Switch off : sudden break -> big back EMF -> sparking at contacts.

3. Steady DC : I constant -> no induced EMF , inductor acts like an ordinary wire.

In AC , inductor offers reactance X_L = omega L.

Self - Inductance of a Solenoid

Long solenoid : length l , area A , n turns per unit length , total turns $N = n\,l$, current I.

Derivation

Inside : $B = \mu_0 \, n \, I$

Flux through one turn : $\phi = B\,A = \mu_0 \, n \, I \, A$

Linkage : $N\,\phi = (n\,l)(\mu_0 \, n \, I \, A)$

$$= \mu_0 \, n^2 \, A \, l \, . \, I$$

By definition L = N phi / I :

$$\boxed{L = \mu_0 \, n^2 \, A \, l}$$

<- for long solenoid

Equivalent forms

$L = \mu_0 \, N^2 \, A \, / \, l$ (use $N = n\,l$)

If a magnetic material of relative permeability u_r fills the core :

$$\boxed{L = \mu_0 \, \mu_r \, n^2 \, A \, l}$$

<- iron core
<- huge L boost

More turns -> L N 2 ; doubling N -> 4 x L.

Energy Stored in an Inductor

While current I builds up from 0 to its final value, the source does work against the back EMF : this work is stored in the field.

Derivation

Instantaneous power : $P = \mathcal{E} \; I = L \, I \, dI/dt$

Energy in small step : $dU = L \, I \cdot dI$

Total $U =$ sum $L \, I \, dI$ from 0 to I =

*

$$\boxed{U = (1/2) \, L \, I^2}$$

<- magnetic
<- energy of coil

Energy density (long solenoid)

$U = (1/2)(\mu_0 \, n^2 \, A \, l) \, I^2$; volume $= A \, l$

$$\boxed{u = B^2 / (2 \mu_0)}$$

<- magnetic energy
<- per unit volume

Compare with capacitor

Capacitor : $U = 1/2 \; C \; V^2$; $u = 1/2$ eps_0 E^2

Inductor : $U = 1/2 \; L \; I^2$; $u = B^2 / 2\mu_0$

Both store energy in their respective fields.

Combinations of Inductors

Series (no mutual coupling)

Same dI/dt through both ; back EMFs add :

$$\boxed{L_s = L_1 + L_2 + \dots}$$

<- L_s > any single L

Parallel (no mutual coupling)

Same EMF across each ; currents add :

$$\boxed{1 / L_p = 1 / L_1 + 1 / L_2 + \dots}$$

<- L_p < smalles

With mutual coupling (extra info)

If two coils are linked (coupling coeff = κ) :

$$\boxed{L_s = L_1 + L_2 +/- 2M}$$

<- + aiding,
<- - opposing

$$\boxed{M = \kappa \, \mathrm{sqrt}(L_1 L_2)}$$

<- $0 \le \kappa \le 1$

$\kappa = 1$ -> perfect coupling (all flux shared)

$\kappa = 0$ -> no coupling

Used in transformers (κ 0.99 with iron core).

Mutual Inductance

Definition

Flux through coil 2 due to current I_1 in coil 1 :

$$N_2 \Phi_{21} = M_{21} I_1$$

$$\boxed{M = N_2 \Phi_{21} / I_1}$$

<- SI : henry

Induced EMF in coil 2

$$\boxed{E_2 = - M \, dI_1 / dt}$$

<- induced by
<- primary's change

Reciprocity theorem

$$\boxed{M_{12} = M_{21} = M}$$

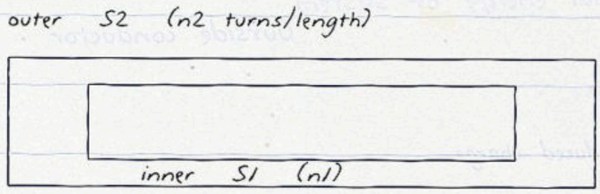
<- swap coils -
<- same M

Factors affecting M

1. Geometry of both coils (radius , length)
2. Relative orientation (max when coaxial)
3. Core : ferromagnetic core boosts M.

M for Coaxial Solenoids

Inner solenoid (length l, area A) with n_1 turns per unit length placed inside outer solenoid (same length l, n_2 turns/length).



Derivation

Take I_1 in inner S_1 : $B = \mu_0 n_1 I_1$

Flux per turn of S_2 : $\phi = BA = \mu_0 n_1 I_1 A$

Turns of S_2 in length l : $N_2 = n_2 l$

Total linkage $\therefore$ $N_2 \phi = \mu_0 n_1 n_2 A l I_1$

$$\boxed{M = \mu_0 n_1 n_2 A l}$$ <- coaxial pair

Same expression whether we drive S_1 or S_2 - consistent with reciprocity ($M_{12} = M_{21}$).

AC Generator - Principle

Idea

A coil rotating in a uniform B undergoes continuous flux change, hence continuous EMF. This is the basis of every power station !

Construction

1. Armature : N - turn coil of area A wound on a soft iron core.

2. Field magnet : strong permanent or electromagnet supplying B (constant).

3. Slip rings (2) + brushes : take output without twisting the coil leads.

4. Drive : turbine (steam, hydro, wind) rotates armature at constant omega.

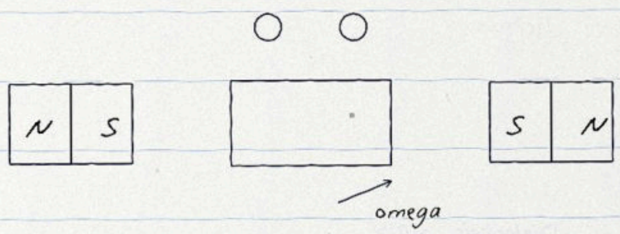


Fig. schematic of a single - coil AC generator

AC Generator - EMF Derivation

At time t, the normal of the coil makes angle theta = omega t with B.

Flux through one turn : $\phi = B A \cos(w t)$

Total linkage : $N\phi = N B A \cos(w t)$

Apply Faraday's law :

$$E = -d(N\phi)/dt$$
$$= N B A w \sin(w t)$$

$$\boxed{E = E_0 \sin(w t)}$$ <- sinusoidal

$$\boxed{E_0 = N B A w}$$ <- peak (maximum) <- EMF

Frequency

omega = 2 pi f

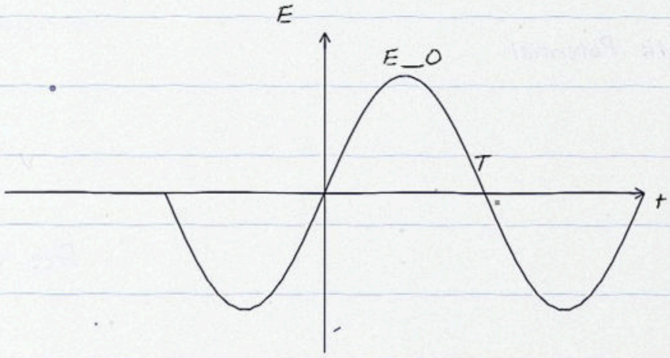
$f = 50$ Hz (India / Europe)

$f = 60$ Hz (USA)

Useful peak / RMS values :

$E_{rms} = E_0 / \mathrm{sqrt}(2)$; $I_{rms} = I_0 / \mathrm{sqrt}(2)$

AC Generator - Output Curve



Features

1. Sinusoidal : $E = E_0 \sin \omega t$.

2. Peak : $E_0 = N B A \omega$.

3. Direction reverses every half cycle.

4. Zero crossings when coil's normal B.

AC obtained directly ; DC needs a commutator.

Worked Example : Self Inductance

Problem : A solenoid 0.5 m long , 4 cm 2 area of cross - section , has 1000 turns. Find L , and energy stored when I = 2 A.

Inductance

$n = N / l = 1000 / 0.5 = 2000$ turns/m

$L = \mu_0 n^2 A l$

$= (4 \text{ pi} \times 10^{-7}) (2000)^2 (4 \times 10^{-4}) (0.5)$

$= 4 \text{ pi} \times 10^{-7} . 4 \times 10^6 . 2 \times 10^{-4}$

1.0×10^{-3} H $= 1.0$ mH .

Energy stored

$U = (1/2) L I^2 = (1/2)(10^{-3})(2)^2$

$= (1/2)(10^{-3})(4) = 2 \times 10^{-3}$ J

$= 2$ mJ

Back EMF if I changes uniformly

If I drops from 2 A to 0 in 0.01 s :

$E = L \, dI/dt = (10^{-3})(2 / 0.01)$

$= 0.2$ V

(Small for a small inductor ; large for big L.)

Worked Example : Mutual Inductance

Problem : Two coaxial solenoids, 0.4 m long, area 8 cm 2. Inner has 1500 turns, outer 1200. Find M and induced EMF in outer if dI_1/dt in inner = 100 A/s.

Mutual inductance

$n_1 = 1500/0.4 = 3750$ /m
$n_2 = 1200/0.4 = 3000$ /m

$$M = \mu_0 n_1 n_2 A l$$
$$= (4 \text{ pi} \times 10^{-7})(3750)(3000)(8 \times 10^{-4})(0.4)$$

$$4.52 \times 10^{-3} \text{ H}$$
$$= 4.5 \text{ mH}$$

Induced EMF in outer coil

$$E_2 = M \cdot dI/dt$$
$$= (4.5 \times 10^{-3}) \cdot (100)$$
$$= 0.45 \text{ V}$$

Reciprocity check

If outer coil drives the current, same $M = 4.5$ mH
i.e. $M_{12} = M_{21}$ (theorem holds).
Add iron core -> M boosted by factor u_r.

Worked Example : AC Generator

Problem : An AC generator has a 200 - turn coil of area 0.05 m 2 in a uniform $B = 0.1$ T , rotating at omega = 100 pi rad/s.
Find (a) peak EMF , (b) RMS EMF , (c) frequency f , (d) EMF at $t = 5$ ms.

(a) Peak EMF

$E_0 = N B A$ omega
= (200)(0.1)(0.05)(100 pi)
= 100 pi 314.16 V

(b) RMS EMF

$E_{rms} = E_0$ / sqrt(2) = 314.16 / 1.414
222.2 V

(c) Frequency

f = omega / (2 pi) = 100 pi / (2 pi) = 50 Hz

(d) EMF at t = 5 ms

omega t = (100 pi)(5×10^{-3}) = pi / 2 = 90
$E = E_0$ sin 90 = E_0 314 V

Mains voltage : ~~110 V~~ 220 V (India , RMS)
Peak 311 V ; what your switch sees.

Common Pitfalls + Quick MCQ Tips

Conceptual checks

1. Induced EMF arises from flux CHANGE; constant B and constant area -> no EMF.

2. Faster change of B -> larger EMF, but TOTAL CHARGE q depends only on net flux change.

3. Lenz : induced effect always opposes the cause - never aids it.

4. L, M, C all depend on geometry only, not on instantaneous I or V.

5. Inductor resists change in current, not the current itself. Like inertia.

6. Energy in inductor = 1/2 L I 2; energy density = B 2 / (2 u_0).

Sign reminders

Use right - hand grip rule for B and Lenz's law to fix the direction of induced current.

If you get a negative answer, flip the sense.

Summary - Key Formulae

Flux & Faraday

$\phi = B A \cos \theta$ SI : weber (Wb)

$E = - d\phi/dt$; $E\_N = - N\, d\phi/dt$

$q = \phi / R$ (total induced charge)

Lenz

sign of E opposes the change in flux

Motional EMF

$E = B l v$; $I = Blv/R$

$P = B^2 l^2 v^2 / R$

$v(t) = v_0 \exp(-t / \tau)$; $\tau = mR / B^2 l^2$

Inductance

$L = N\phi/I$; $E = - L\, dI/dt$

$L\,(\text{solenoid}) = \mu_0 n^2 A l$

$U = 1/2\, L I^2$; $u = B^2 / (2\mu_0)$

Mutual induction

$M = N_2 \phi_{21} / I_1$; $E_2 = - M\, dI/dt$

$M\,(\text{coaxial}) = \mu_0 n_1 n_2 A l$; $M = k\, \mathrm{sqrt}(L1\, L2)$

AC generator

$E = N B A\, \omega \sin \omega t$; $E\_{rms} = E\_0 / \mathrm{sqrt}($