The Collegedunia NCERT Notes

The Ultimate NCERT Guide for Class 12 Physics

Chapter 6: Electromagnetic Induction

1 Introduction

The phenomenon of electromagnetic induction is the production of an electromotive force (emf) across an electrical conductor in a changing magnetic field. This discovery by Michael Faraday and Joseph Henry is one of the most fundamental principles of physics and forms the basis of technologies like electric generators, transformers, and inductors.

Electromagnetic Induction

Electromagnetic Induction is the process by which a changing magnetic field induces an electromotive force (emf) and consequently a current in a closed circuit. The induced emf is directly related to the rate of change of magnetic flux.

2 The Experiments of Faraday and Henry

The discovery of electromagnetic induction was made independently by Michael Faraday in England and Joseph Henry in the USA around 1831. Their experiments demonstrated that a changing magnetic field could produce an electric current.

2.1 Magnetic Flux (Φ_B)

Before discussing the experiments, it is crucial to define the concept of magnetic flux, which is central to the understanding of electromagnetic induction.

Magnetic Flux

The **magnetic flux** (Φ_B) through a surface of area **A** placed in a uniform magnetic field **B** is defined as the scalar product of the magnetic field vector and the area vector.

$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$$

- θ is the angle between the magnetic field vector **B** and the normal vector to the surface $\hat{\mathbf{n}}$.

- The area vector **A** has a magnitude equal to the area A and a direction normal to the surface.
- SI unit of magnetic flux is the **Weber (Wb)**. 1 Wb = 1 T m^2.
- It is a scalar quantity. The sign of flux depends on the choice of normal direction, but the *change* in flux is what induces emf.

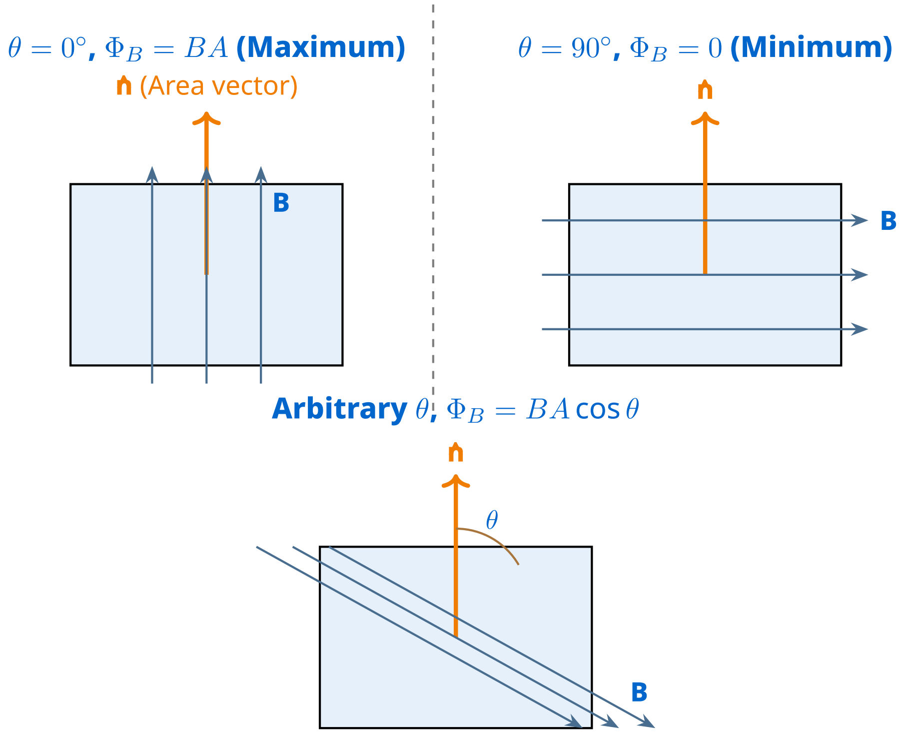


Figure 1: Illustration of magnetic flux $\Phi_B = \mathbf{B} \cdot \mathbf{A}$ for different orientations of the surface relative to the magnetic field.

2.2 Faraday and Henry's Experiments

Faraday and Henry performed a series of experiments. We can group them under six major demonstrations that reveal how a changing magnetic field induces an emf.

The Six Foundational Experiments

Experiment 1: Moving a magnet towards a coil.

- A bar magnet is pushed towards a coil connected to a galvanometer.
- The galvanometer needle deflects, indicating an induced current, only while the magnet is in motion. It returns to zero when the motion stops.
- Pulling the magnet away causes a deflection in the opposite direction.

Experiment 2: Moving a coil towards a stationary magnet.

- The roles are reversed. The coil is moved towards or away from a stationary bar magnet.
- The effect is identical. Deflection is observed only during relative motion between the coil and the magnet.

Experiment 3: Changing the current in a nearby coil (Mutual Induction).

- Two coils, C_1 and C_2, are placed close to each other. C_1 is connected to a battery and a tapping key, and C_2 to a galvanometer.
- On pressing the key (making current in C_1), a momentary deflection is seen in C_2.
- On releasing the key (breaking current in C_1), a momentary deflection in the opposite direction is seen.
- A steady current in C_1 produces no effect.

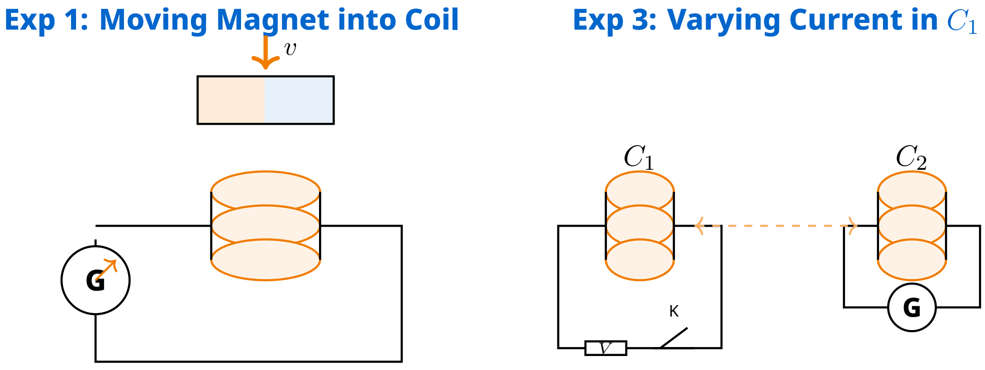


Figure 2: Schematic of Faraday's key experiments: (Left) A moving magnet induces current. (Right) A changing current in C_1 induces an emf in C_2.

The Six Foundational Experiments (continued)

Experiment 4: Relative motion between a coil and a steady current source.

- If the current-carrying coil C_1 is kept steady and coil C_2 is moved relative to it, a deflection is observed in C_2. This again proves that it is the relative motion and the change in magnetic flux that matters.

Experiment 5: Changing the current in a single coil (Self-Induction).

- A single coil is connected to a galvanometer and a battery through a tapping key.
- When the key is pressed, the current in the circuit rises from zero to its steady value. **During this rise time, the changing magnetic flux through the coil itself induces an emf in it.** The galvanometer shows a momentary kick.

- A similar kick in the opposite direction is observed when the key is released and the current falls.

Experiment 6: Coil rotating in a uniform magnetic field.

- A coil is rotated in a region of a uniform magnetic field (e.g., between the poles of a large magnet).
- The flux linking the coil changes continuously as it rotates, producing a continuously varying induced emf. This is the principle of the AC generator.

Unifying Conclusion of All Experiments

An induced emf is produced in a coil whenever the magnetic flux linked with it changes.
The relative motion between a magnet and a coil is just one way to change this flux.

- **Experiment 1 & 2:** Flux changes due to the motion of a permanent magnet.
- **Experiment 3 & 4:** Flux changes due to a changing current in a neighboring circuit.
- **Experiment 5:** Flux changes in a single coil due to its own changing current.
- **Experiment 6:** Flux changes due to the rotation of the coil area vector in a steady field.

3 Faraday's Law of Induction and Lenz's Law

Based on his extensive experiments, Michael Faraday formulated the quantitative law that describes electromagnetic induction. Later, Heinrich Lenz gave a rule to determine the direction of the induced current, which is incorporated into the law.

3.1 Faraday's Law of Induction

Faraday's law establishes the precise relationship between the induced emf in a circuit and the rate of change of magnetic flux through it.

Faraday's Law of Electromagnetic Induction

The magnitude of the induced electromotive force (emf) in a closed circuit is directly proportional to the time rate of change of the magnetic flux passing through the circuit.

$$\boxed{|\mathcal{E}| = \frac{d\Phi_B}{dt}}$$

For a coil of N turns, the total induced emf is the sum of emfs induced in each turn. If the flux through each turn is the same:

$$\boxed{|\mathcal{E}| = N\frac{d\Phi_B}{dt}}$$

Key Concepts:

- $\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$. The flux can change by changing B, A, or θ.
- The emf exists only while the flux is *changing*.
- A larger rate of change produces a larger induced emf.

3.2 Lenz's Law

The direction of the induced emf and the induced current is given by Lenz's law, which is a direct consequence of the law of conservation of energy.

Lenz's Law

Statement: The polarity of the induced emf is such that the current it drives tends to **oppose the change in magnetic flux** that produced it.

- If the flux through a loop is **increasing**, the induced current will flow in a direction such that its own magnetic field **opposes** the increase (i.e., it creates a field in the opposite direction).
- If the flux is **decreasing**, the induced current will create a magnetic field in the **same direction** to oppose the decrease.

Conservation of Energy: If the induced current aided the change in flux, it would accelerate the process, leading to a creation of energy from nothing. Lenz's law ensures the induced current draws energy from the mechanical work done to change the flux.

3.3 Combined Faraday-Lenz Law

Lenz's law is mathematically incorporated into Faraday's law by introducing a minus sign. This is the complete statement of the law of induction.

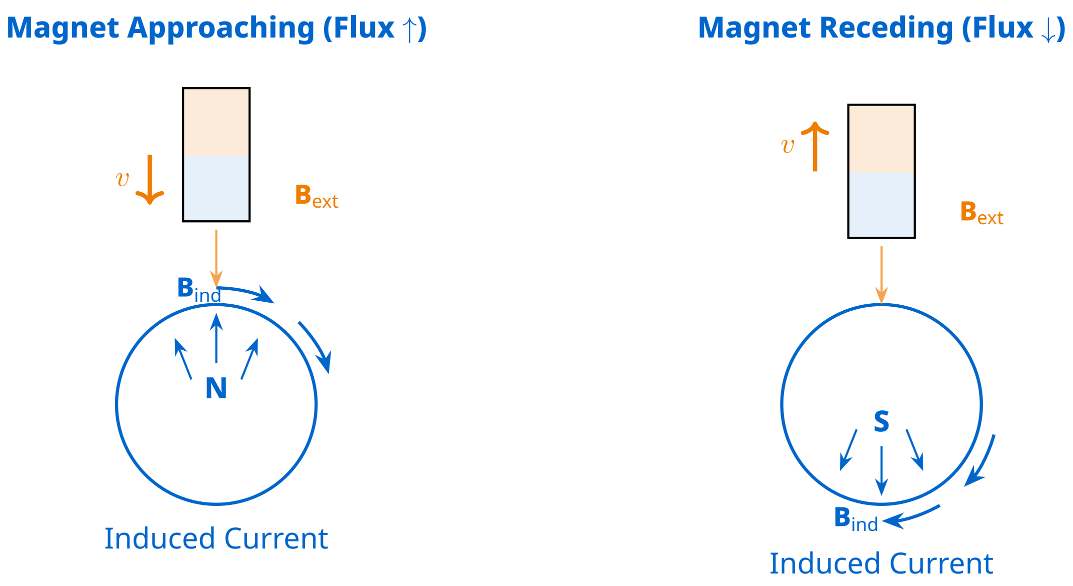


Figure 3: Illustration of Lenz's Law showing induced magnetic fields opposing the change in magnetic flux.

Faraday-Lenz Law

The induced emf in a circuit is equal to the negative of the time rate of change of magnetic flux through the circuit.

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

And for a coil of N turns:

$$\boxed{\mathcal{E} = -N\frac{d\Phi_B}{dt}}$$

Understanding the Negative Sign:

- The negative sign is the mathematical representation of Lenz's law.
- It indicates that the induced emf and the change in flux have opposite signs, i.e., they oppose each other.
- When calculating magnitude, we often use $|\mathcal{E}| = N|d\Phi_B/dt|$. The direction is then found separately using Lenz's law.

3.4 Methods of Producing Induced EMF

From the definition of flux $\Phi_B = BA\cos\theta$, we see that an emf can be induced by changing any of the three quantities.

Three Ways to Change Flux and Induce EMF

1. **By changing the magnetic field B with time:**
   - Keeping the area and orientation fixed. This is the most common method, e.g., in transformers.
   - $\mathcal{E} = -NA\cos\theta\left(\frac{dB}{dt}\right)$
2. **By changing the area A of the loop:**
   - For a loop in a uniform field, changing its shape, size, or moving a sliding conductor across a rail changes the effective area.
   - $\mathcal{E} = -NB\cos\theta\left(\frac{dA}{dt}\right)$
3. **By changing the orientation angle θ:**
   - Rotating a coil in a uniform magnetic field changes $\cos\theta$ periodically. This is the principle of the AC generator.
   - $\mathcal{E} = -NBA\left(\frac{d(\cos\theta)}{dt}\right) = NBA\omega\sin(\omega t)$

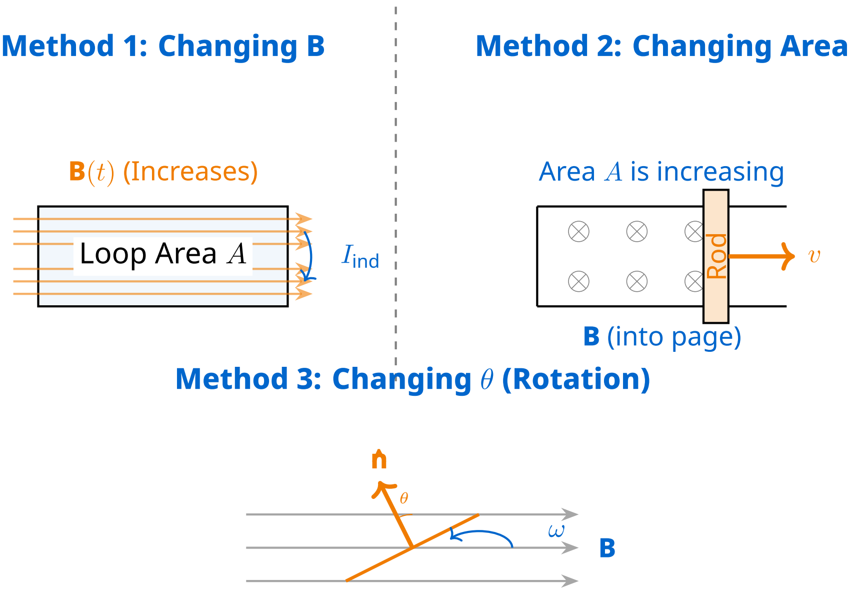


Figure 4: Three methods to change magnetic flux and induce an emf in a closed circuit.

Example 6.1: NCERT Based

Problem: A circular coil of mean radius 0.1 m and having 100 turns is placed with its plane perpendicular to a uniform magnetic field of 0.1 T. The coil is flipped by $180°$ about a diameter in 0.5 s. Find the average emf induced.
Solution: Initially, $\Phi_1 = NBA\cos 0° = 100 \times 0.1 \times (\pi \times 0.1^2) \times 1 = 0.1\pi$ Wb Finally, $\Phi_2 = NBA\cos 180° = 0.1\pi \times (-1) = -0.1\pi$ Wb Change in flux, $\Delta\Phi =$

$\Phi_2 - \Phi_1 = -0.2\pi$ Wb Magnitude of average induced emf:

$$|\mathcal{E}| = \left|\frac{\Delta\Phi}{\Delta t}\right| = \frac{0.2\pi}{0.5} = 0.4\pi \approx 1.26 \text{ V}$$

Key Points on Lenz's Law and Faraday's Law

- **Faraday's Law** quantifies *how much* emf is induced: $\mathcal{E} = -N d\Phi_B/dt$.
- **Lenz's Law** determines the *direction* (polarity) of the induced emf and current. It is a consequence of the **conservation of energy**.
- The induced emf is in **Volts (V)** if flux is in Webers (Wb) and time is in seconds (s).
- A constant flux, no matter how large, produces zero induced emf.
- The term $d\Phi_B/dt$ represents the **slope** of the Φ_B vs. t graph.

4 Motional Electromotive Force

When a conductor moves through a uniform magnetic field, the free charges inside the conductor experience a magnetic Lorentz force. This force acts as a non-electrostatic source that can drive a current. The emf generated this way is called a **motional emf**.

4.1 Origin of Motional EMF

Consider a straight conductor of length l moving with a velocity **v** perpendicular to a uniform magnetic field **B**.

Magnetic Force on Charges in a Moving Conductor

- A charge q inside the conductor moving with velocity **v** in field **B** experiences the Lorentz force: $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$.
- This magnetic force pushes the free electrons towards one end, say P_2.
- Consequently, end P_1 becomes positively charged and end P_2 negatively charged.
- This charge separation creates an internal **electrostatic field E** inside the conductor.
- The charge buildup continues until the electrostatic force $q\mathbf{E}$ exactly balances the magnetic force $q(\mathbf{v} \times \mathbf{B})$.

$$\textbf{At equilibrium:} \quad \mathbf{E} = -(\mathbf{v} \times \mathbf{B})$$

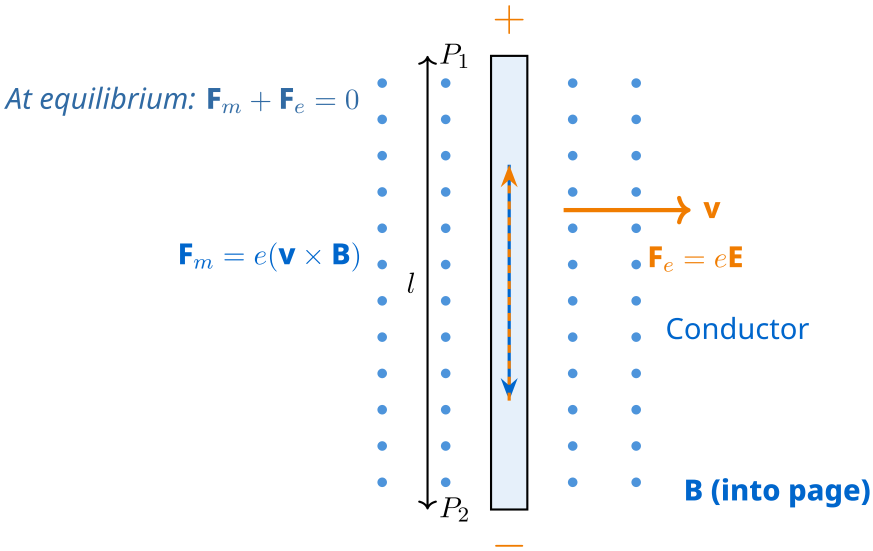


Figure 5: A straight conductor of length l moving with velocity **v** in a uniform magnetic field **B** (into the page). The magnetic Lorentz force separates charges, creating an internal electrostatic field **E**. Equilibrium is reached when the electrostatic force $\mathbf{F}_e$ balances the magnetic force $\mathbf{F}_m$.

4.2 Expression for Motional EMF

The potential difference between the ends P_1 and P_2 due to the induced electric field is the motional emf.

Motional EMF in a Straight Conductor

The magnitude of the motional emf induced across a straight conductor of length l moving with velocity v perpendicular to a uniform field B is:

$$\boxed{\mathcal{E} = Blv}$$

General Vector Form:

$$\mathcal{E} = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

For a straight rod, if the rod, its velocity, and the field are mutually perpendicular, the magnitude simplifies to Blv. If they are not mutually perpendicular, the component of v perpendicular to both B and the rod is used.

4.3 Motional EMF in a Sliding Conductor on Rails

This is a classic example where the motional emf drives a current in a complete circuit, converting mechanical energy into electrical energy.

The Sliding Rod Problem

Consider a U-shaped conductor (rails) placed in a uniform magnetic field B perpendicular to its plane. A movable conducting rod of length l slides along the rails with velocity v.

- The area enclosed by the loop changes as the rod moves.
- Area swept by rod per unit time: $dA/dt = l \times v$.
- The change in magnetic flux: $d\Phi_B/dt = B \times l \times v$.
- By Faraday's law, the induced emf: $\mathcal{E} = Blv$.
- If the circuit has resistance R, the induced current is $I = \mathcal{E}/R = Blv/R$.

Power Dissipation: The mechanical power Fv supplied to move the rod equals the electrical power I^2R dissipated as heat.

$$F = IlB = \frac{B^2l^2v}{R} \quad \Rightarrow \quad P = Fv = \frac{B^2l^2v^2}{R} = I^2R$$

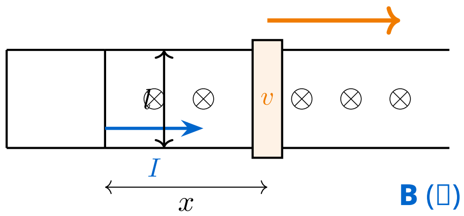


Figure 6: A sliding conductor on frictionless rails in a uniform magnetic field. The induced current is determined by the right-hand rule or Lenz's law.

4.4 Concept of Induced Electric Field

A changing magnetic flux induces an emf, which implies that an **electric field** is created even in the absence of a physical conductor.

Relation Between Induced EMF and Electric Field

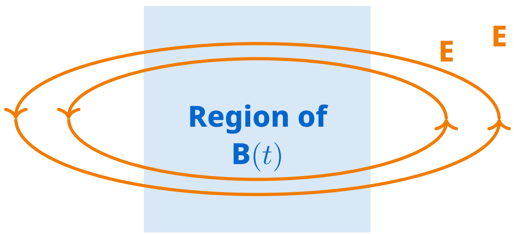
Faraday's law can be expressed in terms of the non-conservative electric field induced by a changing magnetic field:

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

- This is one of Maxwell's equations of electromagnetism.
- The induced electric field lines form closed loops, unlike the electrostatic field lines produced by static charges which start and end on charges.

- A time-varying magnetic field produces an electric field that is **non-conservative** in nature.

Differential form: $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$



Changing **B** induces circular **E**-field lines.

Figure 7: A time-varying magnetic field (directed into the page) produces circulating induced electric field lines. The non-conservative electric field is the physical origin of the induced emf.

Example 6.4: NCERT Based

Problem: A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.
Solution: We use the formula for the emf induced in a rotating rod of length R:

$$\mathcal{E} = \frac{1}{2} B\omega R^2$$

$$\mathcal{E} = \frac{1}{2} \times 0.5 \times 400 \times (1.0)^2$$
$$= 0.5 \times 200 \times 1 = 100 \text{ V}$$

The emf developed between the centre and the metallic ring is 100 V.

Motional EMF: Key Takeaways

- **Motional EMF:** $\mathcal{E} = Blv$ (for mutually perpendicular B, v, l). The direction can be found using the right-hand rule for $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$.
- **Mechanical $\rightarrow$ Electrical Energy:** The sliding rod problem perfectly illustrates the conversion of mechanical work (done against the magnetic force IlB) into electrical energy dissipated in the resistor.
- **Induced Electric Field:** Even in vacuum, a changing B-field creates an

E-field. This E-field is non-conservative ($\oint \mathbf{E} \cdot d\mathbf{l} \neq 0$) and is responsible for the emf.

- The expression for a **rotating rod** is $\mathcal{E} = \frac{1}{2}B\omega l^2$, which can be derived by integrating Blv over the length.

5 Eddy Currents

So far, we have discussed induced currents in well-defined conducting paths like wires and loops. However, when a bulk piece of a conductor is subjected to a changing magnetic flux, the induced currents circulate within the volume of the conductor itself. These are called eddy currents.

5.1 What are Eddy Currents?

Eddy currents (also called Foucault currents) are loops of electrical current induced within conductors by a changing magnetic field in the conductor, due to Faraday's law of induction.

Eddy Currents

Eddy currents are induced circulating currents that flow within the volume of a bulk conductor when the magnetic flux linked with the conductor changes. They flow in closed loops, in planes perpendicular to the magnetic field, according to Lenz's law.

- The term "eddy" refers to the whirlpool-like pattern of their flow.
- They were discovered by French physicist Léon Foucault in 1851.
- Eddy currents dissipate energy as heat (I^2R losses) because the bulk conductor has a finite, non-zero resistance.

5.2 Undesirable Effects of Eddy Currents

In many electrical devices, eddy currents are a nuisance because they lead to energy losses and heating.

Disadvantages of Eddy Currents

1. **Energy Loss (Heat):** The I^2R losses produce unwanted heat. In transformers and AC motors, this reduces efficiency.
2. **Magnetic Braking / Damping:** As the eddy currents oppose the change in flux, they produce a drag force on the moving conductor. While useful in some cases, this can cause unwanted damping in sensitive instruments like galvanometers.

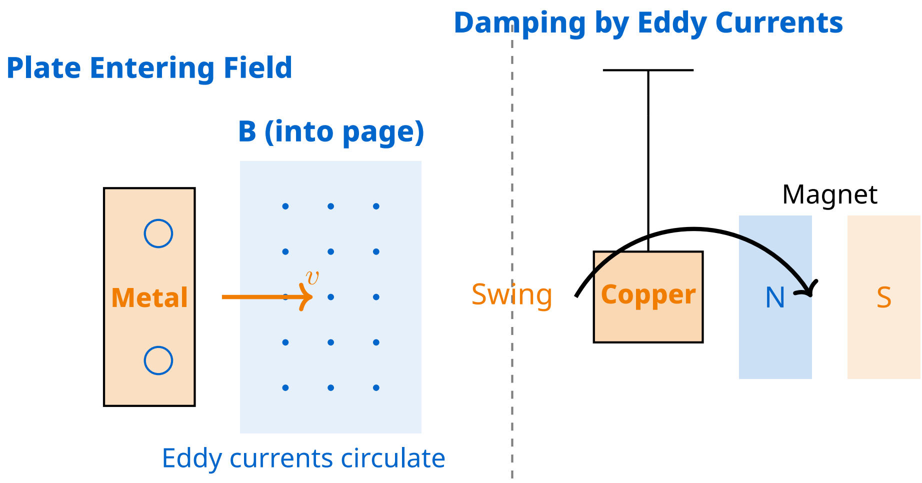


Figure 8: (Left) Eddy currents are induced in a metallic plate as it enters a magnetic field, opposing its motion. (Right) A solid copper pendulum bob swinging between the poles of a strong magnet is heavily damped and comes to rest quickly due to eddy currents.

5.3 Practical Applications of Eddy Currents

Despite their drawbacks, the properties of eddy currents are cleverly exploited in numerous applications.

Applications of Eddy Currents

1. **Magnetic Braking:** Used in high-speed trains and roller coasters. A strong magnetic field is applied to the rails, inducing eddy currents in the train's metallic wheels, creating a smooth, non-contact braking force.

2. **Induction Furnace:** A metallic object is placed in a rapidly changing magnetic field. The induced eddy currents heat it up so much that it melts. This is used for producing alloys and in metallurgy.

3. **Speedometers:** In an automobile's speedometer, a magnet rotates at a speed proportional to the vehicle's speed. It induces eddy currents in an aluminium drum. The torque from the magnetic interaction deflects a pointer.

4. **Electromagnetic Damping:** In galvanometers, a coil is wound on a metallic frame. When deflected, the motion induces eddy currents that provide damping, bringing the needle to rest quickly without oscillation.

5. **Metal Detectors and Security Scanners:** A changing magnetic field generates eddy currents in buried metallic objects or concealed weapons, which in turn alter the impedance of the detector coil, triggering an alarm.

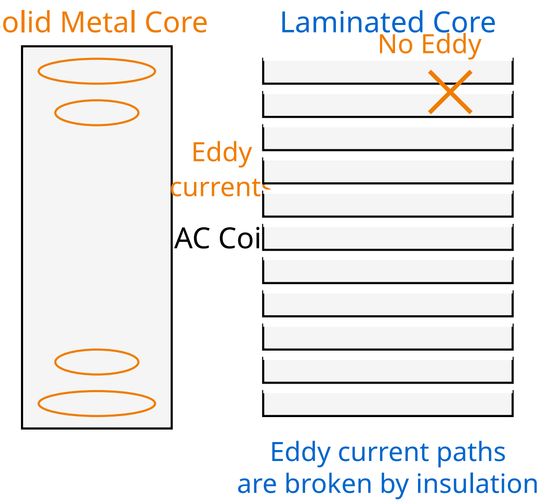


Figure 9: (Left) A solid metal core exposed to a changing magnetic field experiences large eddy currents, generating significant heat loss. (Right) To minimize this, transformer cores are **laminated**—made of thin, insulated sheets—which restrict the paths of the eddy currents and reduce I^2R losses.

5.4 Minimization of Eddy Currents

To reduce the undesirable energy losses, eddy currents are minimized in practical applications.

Reducing Eddy Current Losses

- **Laminating the Core:** For AC applications, the iron core is not a solid block but is built up of thin, flat sheets (laminations) stacked together. Each lamination is electrically insulated from the next (by a varnish layer).
- **Why it works:** The thin plates increase the resistance of the eddy current paths and confine the currents to smaller areas, drastically reducing the magnitude of the eddy currents ($I \propto 1/R$) and the associated heat loss.
- **Material Choice:** Soft iron or silicon steel is used for transformer cores as they have high electrical resistivity and high magnetic permeability.

Eddy Currents: Advantages vs. Disadvantages

Useful Applications	Disadvantages and Mitigation
1. Magnetic braking in trains	1. Undesirable heat loss in transformer/motor cores
2. Induction furnaces for melting metals	2. Undesirable damping in moving-coil instruments
3. Speedometers	3. Reduced efficiency in electrical machines
4. Electromagnetic damping in galvanometers	4. **Solution:** Use **laminated cores** to break eddy current paths.
5. Metal detectors and security scanners	

6 Inductance

In Faraday's experiments, we saw that a changing current in a coil induces an emf not just in a neighboring coil (mutual induction), but also in itself (self-induction). The property of a circuit or a coil that quantifies its ability to produce this induced emf is called **inductance**.

6.1 Self-Inductance

When the current through a coil changes, the magnetic flux linked with the coil also changes. This changing flux induces an emf in the same coil that opposes the change in current.

Self-Inductance (L)

Self-inductance of a coil is the property by virtue of which it opposes any change in the current flowing through it.

- The magnetic flux Φ_B linked with a coil is directly proportional to the current I flowing through it:

$$\Phi_B \propto I \quad \Rightarrow \quad N\Phi_B = LI$$

- The constant of proportionality L is called the coefficient of self-inductance or simply self-inductance.

- Using Faraday's law, the **self-induced emf** is:

$$\boxed{\mathcal{E}_L = -L\frac{dI}{dt}}$$

Unit: The SI unit of self-inductance is the **Henry (H)**.

- 1 Henry = 1 Weber per Ampere (1 H = 1 Wb/A) = 1 Volt-second per Ampere (1 H = 1 Vs/A).
- A circuit has an inductance of 1 Henry if a current changing at the rate of 1 A/s induces an emf of 1 Volt.

6.2 Self-Inductance of a Long Solenoid

A long solenoid is an ideal component to understand the physical factors affecting inductance.

Inductance of a Long Solenoid

For a long air-core solenoid of cross-sectional area A, length l, and total number of turns N:

$$\text{Magnetic field inside: } B = \mu_0 nI = \mu_0 \frac{N}{l} I$$

$$\text{Total flux linkage: } N\Phi_B = N(BA) = \mu_0 \frac{N^2}{l} AI$$

$$\text{Self-inductance: } \boxed{L = \mu_0 \frac{N^2}{l} A = \mu_0 n^2 Al}$$

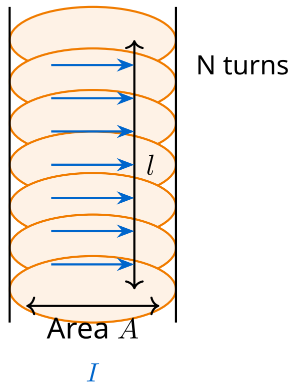
Where $n = N/l$ is the number of turns per unit length.
Key Observations:

- $L \propto N^2$ — Inductance is directly proportional to the square of the number of turns.
- $L \propto A$ — A larger cross-sectional area gives more flux, hence larger L.
- $L \propto 1/l$ — A shorter coil has higher inductance for the same number of turns (more concentrated flux).
- If the core is filled with a material of relative permeability μ_r, μ_0 is replaced by $\mu_r\mu_0$, drastically increasing L.

6.3 Mutual Inductance

When two coils are placed close together, a changing current in one coil induces an emf in the other. The effectiveness of this coupling is measured by mutual inductance.

Solenoid (Self-Inductance L)



LR Circuit Symbol

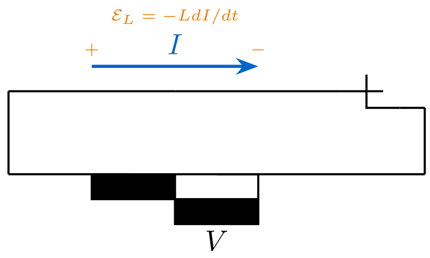


Figure 10: (Left) Physical parameters determining the self-inductance of a solenoid: area A, length l, and number of turns N. (Right) The symbol for an inductor and its behavior in a circuit—the self-induced emf opposes the change in current.

Mutual Inductance (M)

Consider two coaxial solenoids (or any two nearby coils). Let a current I_1 flow in coil 1, producing a flux Φ_2 through each turn of coil 2.

$$N_2\Phi_2 = M_{21}I_1$$
$$M_{21} = \frac{N_2\Phi_2}{I_1}$$

M_{21} is the mutual inductance of coil 2 with respect to coil 1. An emf is induced in coil 2 when I_1 changes:

$$\boxed{\mathcal{E}_2 = -M_{21}\frac{dI_1}{dt}}$$

It can be proven that $M_{21} = M_{12} = M$. **Mutual inductance is symmetrical.**
Unit: The SI unit is also the **Henry (H)**.

Mutual Inductance of Two Coaxial Solenoids

For two long coaxial solenoids of the same length l, with N_1 and N_2 turns and area A (assuming flux from inner solenoid fully links with the outer one):

Flux from inner solenoid (coil 1) through outer (coil 2): $\Phi_2 = B_1 A = \left(\mu_0 \frac{N_1}{l} I_1\right) A$

$$N_2\Phi_2 = \mu_0 \frac{N_1 N_2}{l} A I_1$$

$$\boxed{M = \mu_0 \frac{N_1 N_2}{l} A = \mu_0 n_1 n_2 A l}$$

Note that self-inductance $L_1 = \mu_0 N_1^2 A/l$ and $L_2 = \mu_0 N_2^2 A/l$. Thus, for perfectly coupled coils:

$$\boxed{M = \sqrt{L_1 L_2}}$$

Mutual Inductance (M)

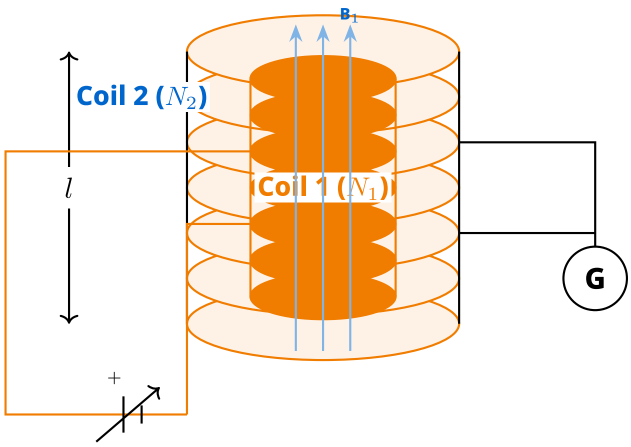


Figure 11: Mutual induction between two coaxial solenoids. The primary circuit (Coil 1) is shown on the left, and the secondary (Coil 2) on the right to maintain clarity.

6.4 Energy Stored in an Inductor

An inductor stores energy in its magnetic field when a current flows through it. This energy is analogous to the energy stored in a charged capacitor.

Energy Stored in an Inductor

The work done by an external source against the induced emf to establish a current I in an inductor L is:

$$dW = \mathcal{E}_L \, dq = L \left(\frac{dI}{dt} \right) (I \, dt) = LI \, dI$$

Total work done to establish current from 0 to I:

$$\boxed{U = \int_0^I LI \, dI = \frac{1}{2} LI^2}$$

This energy is stored in the magnetic field of the inductor. For a solenoid, expressing $L = \mu_0 n^2 Al$, $I = B/(\mu_0 n)$, we get the energy density:

$$u_B = \frac{U}{\text{Volume}} = \frac{1}{2}\frac{B^2}{\mu_0}$$

6.5 Inductors in Series and Parallel

Like resistors and capacitors, inductors can be combined.

Combination of Inductors

1. Inductors in Series: Assuming the mutual inductance is negligible (coils are far apart), the total induced emf is the sum of individual emfs.

$$L_{\text{eq}} = L_1 + L_2 + L_3 + \dots$$

2. Inductors in Parallel: For parallel combination (negligible mutual inductance):

$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots$$

These formulas are exactly analogous to resistors.

Example 6.8: NCERT Based

Problem: A solenoid has a length of 50 cm and a radius of 2 cm. It has 500 turns. Calculate its self-inductance. If a current of 5 A is passed through it, what is the energy stored in its magnetic field? ($\mu_0 = 4\pi \times 10^{-7}$ SI units).
Solution: Area, $A = \pi r^2 = \pi \times (0.02)^2 = 4\pi \times 10^{-4}$ m² Self-inductance:

$$L = \mu_0 \frac{N^2}{l} A = \frac{(4\pi \times 10^{-7}) \times 500^2 \times (4\pi \times 10^{-4})}{0.5}$$

$$L = \frac{4\pi \times 10^{-7} \times 25 \times 10^4 \times 4\pi \times 10^{-4}}{0.5} = \frac{16\pi^2 \times 10^{-7}}{0.5} \approx 3.16 \times 10^{-4} \text{ H}$$

Energy stored:

$$U = \frac{1}{2}LI^2 = \frac{1}{2} \times (3.16 \times 10^{-4}) \times (5)^2 = 3.95 \times 10^{-3} \text{ J}$$

Key Formulas on Inductance

- **Self-Inductance Emf:** $\mathcal{E}_L = -L\frac{dI}{dt}$. The direction is given by Lenz's Law.
- **SI Unit: Henry (H).** 1 H $= 1$ Wb/A $= 1$ V s/A.
- **Solenoid:** $L = \mu_0 n^2 Al$. $L \propto N^2$.
- **Mutual Inductance Emf:** $\mathcal{E}_2 = -M\frac{dI_1}{dt}$. $M_{21} = M_{12} = M$.

- **Coaxial Solenoids:** $M = \mu_0 n_1 n_2 A l$. $M = \sqrt{L_1 L_2}$ for perfect coupling.
- **Energy Stored:** $U = \frac{1}{2}LI^2$. Energy density $u_B = \frac{1}{2}B^2/\mu_0$.

7 AC Generator

The AC generator (or alternator) is a device that converts mechanical energy into alternating electrical energy. It is the most important practical application of Faraday's law of electromagnetic induction and powers the entire modern electrical grid.

7.1 Principle of an AC Generator

The operation of an AC generator is based on the phenomenon of electromagnetic induction—specifically, inducing an emf by changing the orientation of a coil in a uniform magnetic field.

Principle

Principle: When a closed coil is rotated rapidly in a uniform magnetic field, the magnetic flux linked with the coil changes continuously. This change in flux induces an alternating emf and, consequently, an alternating current in the coil if the circuit is closed.

- The flux through the coil is $\Phi_B = NBA\cos\theta$, where $\theta = \omega t$ is the angle between the magnetic field and the area vector.
- By Faraday's law, $\mathcal{E} = -d\Phi_B/dt = NBA\omega\sin(\omega t)$.
- The induced emf varies sinusoidally with time, giving the name **alternating current**.

7.2 Construction and Main Components

A practical AC generator consists of the following essential parts:

Main Components of an AC Generator

1. **Field Magnet:** A strong permanent magnet or electromagnet that produces a uniform magnetic field. The poles (North and South) are shaped to direct the field radially in some designs.
2. **Armature (Coil):** A rectangular coil consisting of a large number of turns of insulated copper wire wound over a soft iron core (laminated to reduce eddy currents). This coil rotates in the magnetic field. The iron core increases the flux and hence the induced emf.

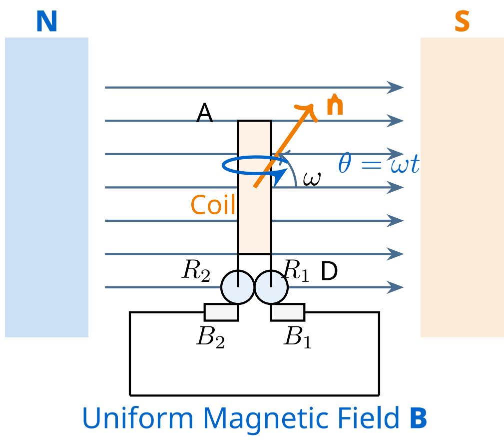


Figure 12: Schematic diagram of an AC generator. The coil (armature) rotates in a uniform magnetic field. The ends are connected to slip rings R_1 and R_2, which rotate with the coil, and the current is collected by the stationary brushes B_1 and B_2.

3. **Slip Rings:** Two complete, concentric metallic rings (R_1 and R_2) to which the two ends of the armature coil are connected. These rings rotate with the coil.
4. **Brushes:** Two stationary carbon brushes (B_1 and B_2) that press lightly against the slip rings. They maintain electrical contact between the rotating coil and the external stationary circuit to collect the induced current.
5. **Rotor and Stator:** In large generators, the magnet is often the rotor (rotating part) and the armature is the stator (stationary part). This avoids using brushes for large currents. In our basic model, the coil is the rotor.

7.3 Working and Expression for Induced EMF

Let the rectangular coil have N turns, area A, and rotate with constant angular velocity ω in a uniform magnetic field B.

Derivation of EMF for AC Generator

Step 1: Magnetic Flux at any instant t:

$$\Phi_B = N(\mathbf{B} \cdot \mathbf{A}) = NBA\cos\theta$$

Since $\theta = \omega t$:

$$\Phi_B = NBA\cos(\omega t)$$

Step 2: Induced EMF by Faraday's Law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}[NBA\cos(\omega t)]$$

$$\boxed{\mathcal{E} = NBA\omega \sin(\omega t)}$$

Step 3: Instantaneous Value:

$$\boxed{\mathcal{E} = \mathcal{E}_0 \sin(\omega t)}$$

Where $\mathcal{E}_0 = NBA\omega$ is the **peak emf** or amplitude of the alternating emf. If the circuit has resistance R, the induced current is:

$$I = \frac{\mathcal{E}_0}{R} \sin(\omega t) = I_0 \sin(\omega t)$$

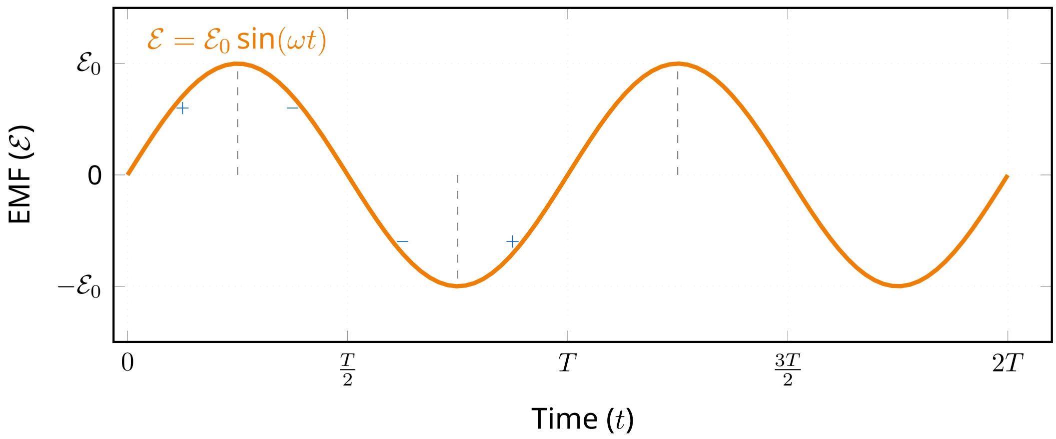


Figure 13: The output of an AC generator is a sinusoidal alternating emf. The emf changes polarity every half cycle ($T/2$). $\mathcal{E}_0$ is the maximum or peak value.

7.4 Frequency of AC

The frequency of the alternating emf depends on the speed of rotation of the coil.

Frequency of Alternating Current

If the coil makes n revolutions per second, the angular frequency is $\omega = 2\pi n$. The emf completes one full cycle when $\omega t = 2\pi$, so the time period $T = 2\pi/\omega$.

$$\boxed{f = \frac{\omega}{2\pi} = n}$$

- In India, the mains frequency is **50 Hz**, meaning the coil in the power plant rotates 50 times per second, and the current changes direction 100 times per second.
- In the USA, the standard is 60 Hz.

Example 6.10: NCERT Based

Problem: A small AC generator has a rectangular coil of area 0.05 m^2 with 100 turns. The coil rotates at 50 revolutions per second in a magnetic field of 0.1 T. Calculate the maximum value of the induced emf. What is the instantaneous emf when the coil has rotated through $30°$ from its initial position?
Solution: Maximum emf:

$$\begin{aligned}\mathcal{E}_0 &= NBA\omega = NBA(2\pi f)\\ &= 100 \times 0.1 \times 0.05 \times (2\pi \times 50)\\ &= 100 \times 0.1 \times 0.05 \times 100\pi = 50\pi \approx 157 \text{ V}\end{aligned}$$

Instantaneous emf at $\theta = 30° = \pi/6$ rad:

$$\mathcal{E} = \mathcal{E}_0 \sin(\pi/6) = 157 \times 0.5 = 78.5 \text{ V}$$

The emf is halfway to its peak value.

AC Generator: Important Facts

- **Principle:** Faraday's Law of Electromagnetic Induction. Mechanical energy $\rightarrow$ Electrical energy.
- **Key Formula:** $\mathcal{E} = NBA\omega \sin(\omega t) = \mathcal{E}_0 \sin(\omega t)$.
- **Peak EMF:** $\mathcal{E}_0 = NBA\omega$. It depends on the number of turns, field strength, coil area, and angular speed.
- **Slip Rings:** Ensure that the connection to the external circuit does not twist, and allow the alternating emf to be delivered. (In contrast, a DC generator uses a **split-ring commutator** to produce a unidirectional pulsating current.)
- **Frequency:** $f = \omega/2\pi$. Indian grid frequency is 50 Hz.
- In actual power stations, the armature is stationary (stator) and the magnet (electromagnet) rotates as the rotor to avoid passing large currents through brushes.

8 Transformer

A transformer is a static (no moving parts) electrical device that transfers alternating current energy from one circuit to another by the principle of electromagnetic induction. It can increase (step up) or decrease (step down) the voltage of an AC supply.

8.1 Principle and Basic Construction

The operation of a transformer is based on the principle of **mutual induction** between two magnetically coupled coils.

Principle of a Transformer

Principle of Mutual Induction: An alternating voltage applied to one coil (the primary) sets up an alternating magnetic flux in a shared core. This changing flux links with the second coil (the secondary) and induces an alternating emf in it.

Basic Construction:

- **Core:** A rectangular frame made of a **laminated** soft iron (or silicon steel) to minimize eddy current losses and provide high magnetic permeability.
- **Primary Coil (P):** The coil connected to the input AC source. It has N_p turns.
- **Secondary Coil (S):** The coil connected to the output load. It has N_s turns.
- The coils are wound on the two opposite arms of the core (or one over the other) to ensure tight magnetic coupling.

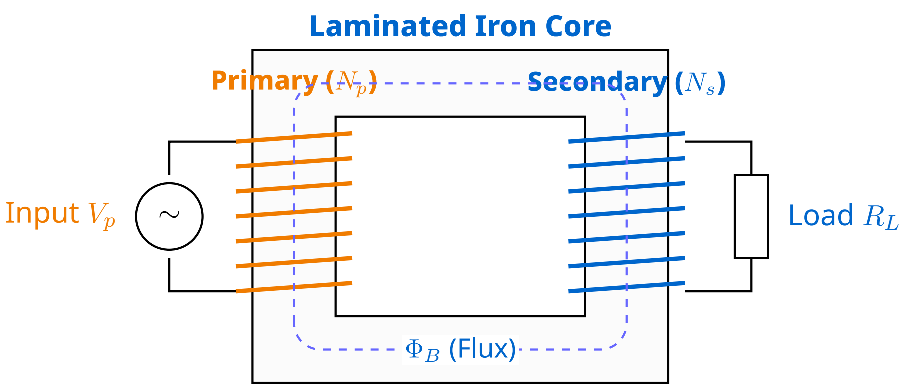


Figure 14: Schematic of a transformer. Alternating current in the primary coil generates a varying flux Φ_B in the laminated core, inducing a voltage in the secondary coil.

8.2 Theory and Working of an Ideal Transformer

An ideal transformer is one with no energy losses—all the flux generated by the primary links with the secondary, and there are no resistive or magnetic losses.

Transformation Ratio in an Ideal Transformer

1. Flux Linkage:

- The alternating current in the primary sets up an alternating flux Φ_B in the core, which links both coils.
- Induced emf in primary: $\mathcal{E}_p = -N_p \frac{d\Phi_B}{dt}$
- Induced emf in secondary: $\mathcal{E}_s = -N_s \frac{d\Phi_B}{dt}$

2. Voltage Ratio (Turns Ratio):

$$\boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p}}$$

For an ideal transformer, the terminal voltages equal the induced emfs ($V_p = |\mathcal{E}_p|$, $V_s = |\mathcal{E}_s|$).

3. Current Ratio (From Energy Conservation): Input power = Output power (for 100% efficiency)

$$V_p I_p = V_s I_s \quad \Rightarrow \quad \boxed{\frac{I_s}{I_p} = \frac{N_p}{N_s}}$$

Types Based on Turns Ratio:

- **Step-Up Transformer:** $N_s > N_p \Rightarrow V_s > V_p$, $I_s < I_p$. (Voltage increased, current decreased).
- **Step-Down Transformer:** $N_s < N_p \Rightarrow V_s < V_p$, $I_s > I_p$. (Voltage decreased, current increased).

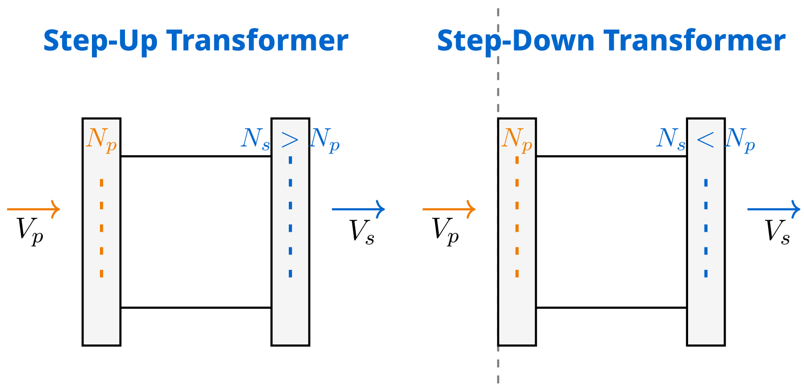


Figure 15: Comparison of a step-up transformer ($N_s > N_p$, voltage is raised) and a step-down transformer ($N_s < N_p$, voltage is lowered).

8.3 Energy Losses in a Transformer

Practical transformers are not ideal; they have several energy losses which reduce their efficiency.

Types of Losses in a Practical Transformer

1. **Copper Losses (I^2R Losses):**
   - Heat generated in the primary and secondary windings due to their electrical resistance.
   - Minimised by using thick copper wires of low resistivity.
2. **Flux Leakage:**
   - All the flux produced by the primary coil may not link with the secondary coil. Some flux may leak through the air.
   - Minimised by winding the primary and secondary coils one over the other or using a well-designed core.
3. **Eddy Current Losses:**
   - The alternating flux induces eddy currents in the iron core, producing heat.
   - Minimised by using a **laminated core**—made of thin sheets of iron insulated from each other (varnish/shellac).
4. **Hysteresis Losses:**
   - Energy lost due to the repeated magnetisation and demagnetisation of the core material.
   - The work done per cycle per unit volume equals the area of the hysteresis loop.
   - Minimised by choosing a core material with a **narrow hysteresis loop** (e.g., soft iron, silicon steel, mu-metal).
5. **Magnetostriction:**
   - The humming sound in a transformer is due to the slight expansion and contraction of the core under the alternating magnetic field. This also accounts for some minor energy loss.

8.4 Efficiency of a Transformer

The efficiency quantifies how well a transformer converts input power to output power.

Transformer Efficiency (η)

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} = \frac{V_s I_s}{V_p I_p}$$

- For an ideal transformer, $\eta = 1$ (or 100%).
- For well-designed practical large transformers, efficiency can be as high as **98–99%**.
- Small transformers (like in phone chargers) have lower efficiency (around 85–90%).

Example 7.8: NCERT Based

Problem: A transformer has 500 turns in its primary coil and 10000 turns in its secondary. The primary voltage is 220 V. Assuming no losses, calculate (a) the secondary voltage, and (b) the primary and secondary currents if the power in the secondary is 11 kW.
Solution: (a) For an ideal transformer:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \Rightarrow \quad V_s = V_p \frac{N_s}{N_p} = 220 \times \frac{10000}{500} = 220 \times 20 = 4400 \text{ V}$$

This is a step-up transformer.
(b) Power in secondary: $P_s = V_s I_s = 11 \times 10^3$ W.

$$I_s = \frac{P_s}{V_s} = \frac{11000}{4400} = 2.5 \text{ A}$$

For an ideal transformer, input power = output power: $V_p I_p = V_s I_s = 11$ kW.

$$I_p = \frac{11000}{220} = 50 \text{ A}$$

Even with high efficiency, switching to high voltage reduces current for long-distance transmission.

Transformer: Key Takeaways

- **Principle:** Mutual Induction. The core confines and guides the magnetic flux.
- **Key Equations:**
  - $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ (Voltage Ratio)
  - For an ideal transformer: $V_p I_p = V_s I_s$.
- **Step-Up:** $N_s > N_p$, $V_s > V_p$, $I_s < I_p$. Used at power stations to send power at high voltage/low current over long distances to reduce I^2R losses.

- **Step-Down:** $N_s < N_p$, $V_s < V_p$, $I_s > I_p$. Used in local substations and home appliances (like phone chargers) for safe low voltages.
- **A transformer works ONLY on AC.** A steady DC current produces a constant flux, which does not induce any emf in the secondary.
- **Losses:** Copper, eddy current (laminated core), hysteresis (soft iron core), flux leakage.

9 NCERT Solved Examples

Example 6.1: Faraday's Law and Lenz's Law

Problem: A rectangular loop of area $A = 0.01$ m^2 is placed in a magnetic field of strength $B = 0.2$ T, with its plane perpendicular to the field. The loop is withdrawn completely from the field in a time of 0.05 s.

(a) Calculate the magnitude of the average induced emf.

(b) If the loop has a resistance of $2\,\Omega$, determine the induced current and its direction (using Lenz's Law) as the loop leaves the field.

Solution: (a) Initial flux: $\Phi_1 = BA = 0.2 \times 0.01 = 2.0 \times 10^{-3}$ Wb. Final flux (outside field): $\Phi_2 = 0$. Change in flux: $\Delta\Phi = 0 - 2.0 \times 10^{-3} = -2.0 \times 10^{-3}$ Wb. Time taken: $\Delta t = 0.05$ s. Magnitude of average induced emf:

$$|\mathcal{E}| = \left|\frac{\Delta\Phi}{\Delta t}\right| = \frac{2.0 \times 10^{-3}}{0.05} = 0.04 \text{ V} = 40 \text{ mV}$$

(b) Induced current: $I = \frac{|\mathcal{E}|}{R} = \frac{0.04}{2} = 0.02$ A = 20 mA.
Direction: As the loop leaves the field, the magnetic flux through the loop **decreases** (directed into the page). According to Lenz's Law, the induced current will flow in a direction that **opposes** this decrease—i.e., it will try to create a magnetic field into the page. Using the right-hand thumb rule, the current must flow **clockwise**.

Example 6.2: Motional EMF in a Moving Rod

Problem: An aeroplane with a wingspan of 30 m flies horizontally at a speed of 900 km/h at a place where the vertical component of the Earth's magnetic field is 0.5×10^{-4} T. Find the potential difference developed between the tips of its wings.

Solution: The wings are a conducting rod moving through the Earth's magnetic field.

$$\text{Speed: } v = 900 \times \frac{5}{18} = 250 \text{ m/s}$$
$$\begin{aligned}\text{Motional EMF: } \mathcal{E} &= B_v l v \\ &= (0.5 \times 10^{-4}) \times 30 \times 250 \\ &= 0.5 \times 10^{-4} \times 7500 \\ &= 3750 \times 10^{-4} = 0.375 \text{ V}\end{aligned}$$

A potential difference of 0.375 V is developed. This tiny voltage is not hazardous.

Example 6.3: Self-Inductance of a Coil

Problem: A coil has a self-inductance of 1.5 H. If the current flowing through it changes from 0 A to 10 A in 0.5 s, find the magnitude of the induced emf. Also, find the energy stored in the coil when the current is steady at 10 A.
Solution: Magnitude of induced emf:

$$|\mathcal{E}| = L\left|\frac{dI}{dt}\right| = 1.5 \times \frac{10-0}{0.5} = 1.5 \times 20 = 30 \text{ V}$$

Energy stored in the magnetic field:

$$U = \frac{1}{2}LI^2 = \frac{1}{2} \times 1.5 \times (10)^2 = 0.75 \times 100 = 75 \text{ J}$$

Example 6.4: AC Generator

Problem: A simple AC generator consists of a 100-turn coil of area 0.1 m^2. It rotates at a frequency of 60 Hz in a uniform magnetic field of 0.05 T. Calculate the peak voltage output.
Solution:

$$\text{Angular frequency: } \omega = 2\pi f = 2\pi \times 60 = 120\pi \text{ rad/s}$$
$$\begin{aligned}\text{Peak EMF: } \mathcal{E}_0 &= NBA\omega \\ &= 100 \times 0.05 \times 0.1 \times 120\pi \\ &= 100 \times 0.005 \times 120\pi = 0.5 \times 120\pi = 60\pi \approx 188.5 \text{ V}\end{aligned}$$

Example 6.5: Transformer Application

Problem: A power transmission line feeds input power at 2300 V to a step-down transformer with its primary winding having 4000 turns. What should be the number of turns in the secondary to get output power at 230 V?

Solution: For a transformer: $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

$$\begin{aligned} N_s &= N_p \times \frac{V_s}{V_p} \\ &= 4000 \times \frac{230}{2300} \\ &= 4000 \times \frac{1}{10} = 400 \text{ turns} \end{aligned}$$

The secondary coil must have 400 turns.

10 Chapter Summary: Key Concepts to Master

Electromagnetic Induction: Final Recap

1. **Magnetic Flux:** $\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$. Its change is the central requirement for inducing emf.
2. **Faraday's Law (Magnitude):** $|\mathcal{E}| = N\frac{d\Phi_B}{dt}$.
3. **Lenz's Law (Direction):** The induced current opposes the change in flux; signified by the negative sign in $\mathcal{E} = -Nd\Phi_B/dt$. It proves the conservation of energy.
4. **Motional EMF:** $\mathcal{E} = Blv$ for a straight rod moving perpendicular to B. It's a manifestation of the Lorentz force acting on charges.
5. **Eddy Currents:** Induced circulating currents in bulk conductors. They can cause undesirable heat loss (minimized by lamination) but are useful in magnetic braking, induction furnaces, and speedometers.
6. **Self-Induction:** The coil opposes the change in its own current. $\mathcal{E}_L = -LdI/dt$. Energy stored: $U = \frac{1}{2}LI^2$.
7. **Mutual Induction:** A changing current in one coil induces emf in a neighboring coil. $\mathcal{E}_s = -MdI_p/dt$. The principle of a transformer.
8. **AC Generator:** Converts mechanical energy to electrical energy. $\mathcal{E} = \mathcal{E}_0\sin(\omega t)$, with $\mathcal{E}_0 = NBA\omega$.
9. **Transformer:** Converts AC voltage to a higher or lower value. $V_s/V_p = N_s/N_p = I_p/I_s$. Works only on AC. Uses a laminated soft iron core to minimize losses.