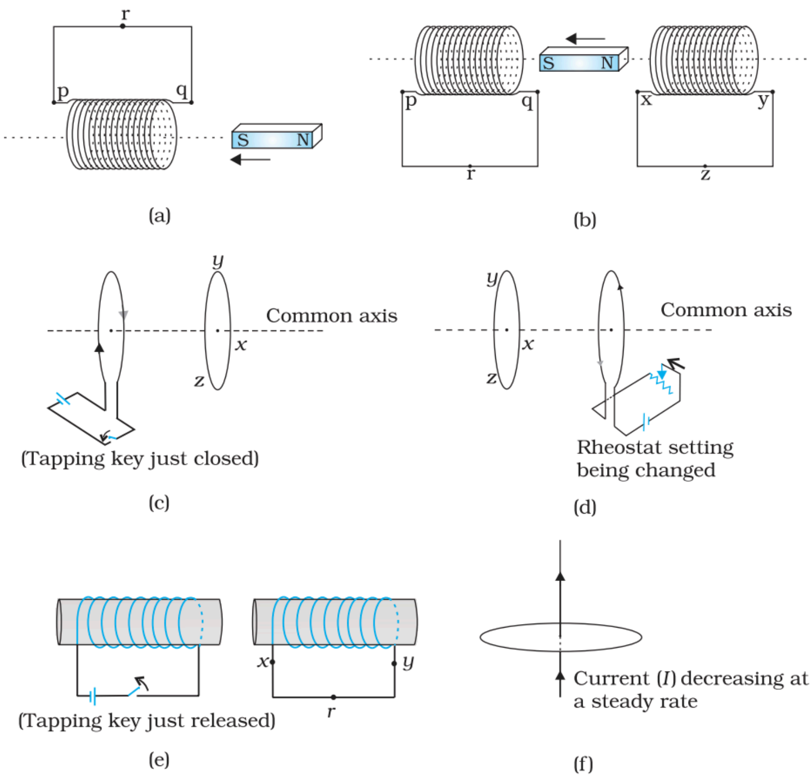
NCERT SOLUTIONS

Class 12 Physics

Chapter 6 - Electromagnetic Induction

Detailed Step-by-Step Exercise Solutions

Q1 **Predict the direction of the induced current in the situations described by the following Figs. 6.18(a) to (f).**



Solution

Concept: Lenz's Law and Direction of Induced Current

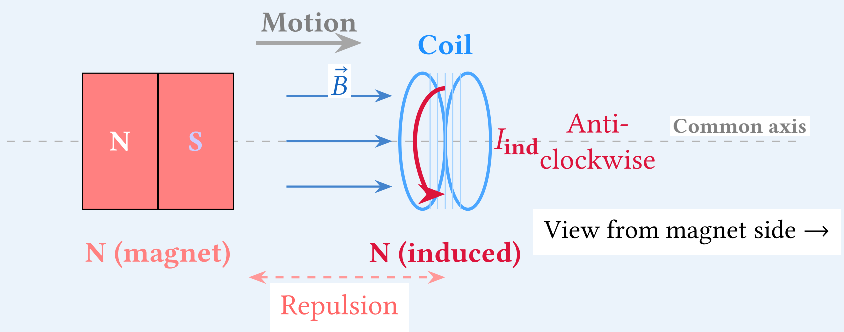
The direction of induced current is determined by **Lenz's Law**, which states that the induced current flows in such a direction that it **opposes the change** in magnetic flux that produced it. Mathematically, this is embodied in the negative sign in Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Steps to determine the direction of induced current:

1. Identify the direction of the external magnetic field ($\vec{B}$) through the coil.
2. Determine whether the magnetic flux (Φ_B) is **increasing** or **decreasing**.
3. The induced current creates its own magnetic field that **opposes** this change.
4. Use the Right-Hand Thumb Rule to find current direction from the induced field.

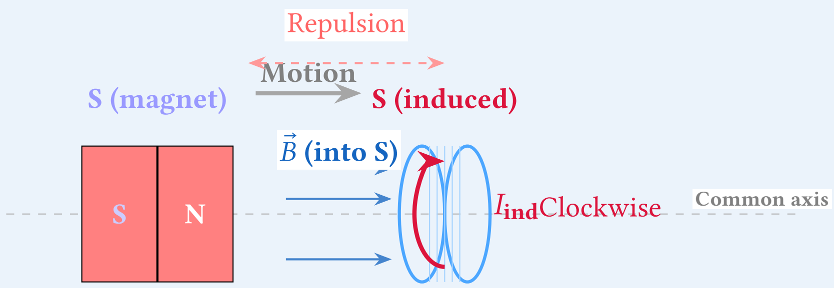
Part (a): North pole of a magnet moving towards the coil



- The North pole produces $\vec{B}$ directed **leftwards** (towards the coil).
- As the magnet moves closer, flux through the coil **increases**.
- By Lenz's law, the coil develops a **North pole** on its left face to repel the approaching magnet.
- Using the Right-Hand Thumb Rule: thumb points left (N-pole), fingers curl **anti-clockwise** as seen from the magnet side.

Answer (a): Anti-clockwise direction (when seen from the magnet side)

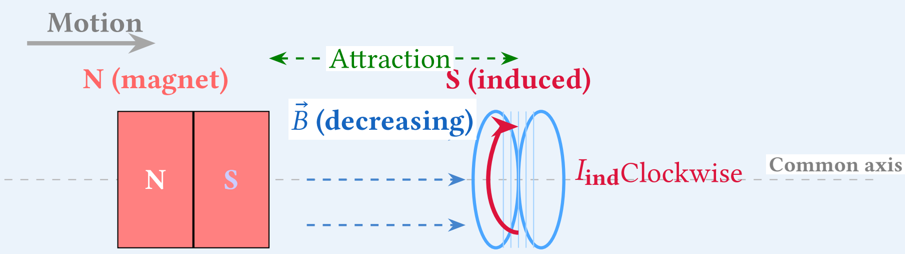
Part (b): South pole of a magnet moving towards the coil



- The South pole draws field lines **into** it, so $\vec{B}$ points **rightwards** (through the coil towards the magnet).
- As the magnet approaches, flux **increases**.
- The coil develops a **South pole** on its left face to repel the approaching South pole.
- For a South pole facing the magnet, current must be **clockwise** as seen from the magnet.

Answer (b): Clockwise direction (when seen from the magnet side)

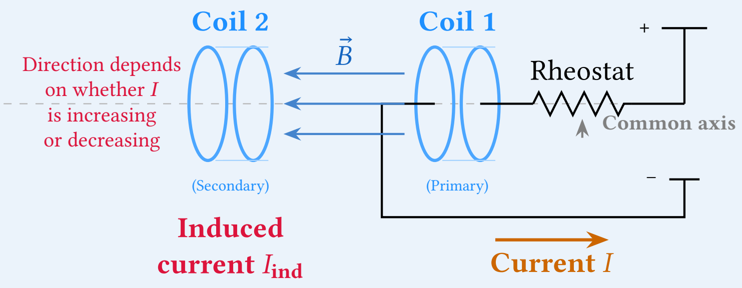
Part (c): North pole of a magnet moving away from the coil



- $\vec{B}$ from the North pole points **leftwards** through the coil.
- As magnet recedes, flux **decreases**.
- By Lenz's law, coil must **support** the field — it develops a **South pole** on its left face to attract the receding North pole.
- For a South pole facing the magnet, current must be **clockwise** as seen from the magnet.

Answer (c): Clockwise direction (when seen from the magnet side)

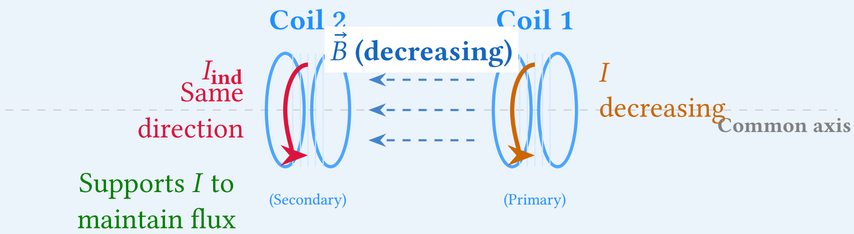
Part (d): Rheostat setting being changed in a two-coil system



- Current I in primary coil creates magnetic flux linked with secondary coil.
- Changing rheostat setting changes I, which changes the flux through Coil 2.
- **If I increases:** flux increases → induced current opposes (opposite direction to I).
- **If I decreases:** flux decreases → induced current supports (same direction as I).

Answer (d): Direction depends on whether the current in the primary circuit is increasing or decreasing (depends on how the rheostat setting is changed).

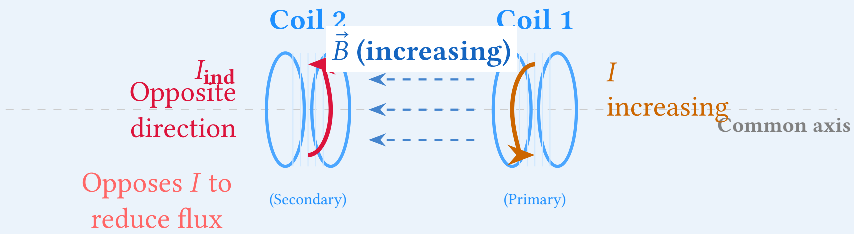
Part (e): Current in the primary coil decreasing at a steady rate



- Primary current I is **decreasing**, so magnetic flux through secondary is **decreasing**.
- By Lenz's law, induced current must **oppose** this decrease by **supporting** the existing flux.
- Hence, induced current flows in the **same direction** as the primary current to reinforce the magnetic field.

Answer (e): Induced current flows in the **same direction** as the current in the primary coil (to oppose the decrease in flux).

Part (f): Current in the primary coil increasing at a steady rate



- Primary current I is **increasing**, so magnetic flux through secondary is **increasing**.
- By Lenz's law, induced current must **oppose** this increase by creating a field in the opposite direction.
- Hence, induced current flows in the **opposite direction** to the primary current.

Answer (f): Induced current flows in the **opposite direction** to the current in the primary coil (to oppose the increase in flux).

Final Summary:

Fig.	Change in Flux	Coil's Response	Induced Current
(a)	Increasing (N approaches)	Develops N-pole (repels)	Anti-clockwise
(b)	Increasing (S approaches)	Develops S-pole (repels)	Clockwise
(c)	Decreasing (N recedes)	Develops S-pole (attracts)	Clockwise
(d)	Changing (rheostat change)	Opposes the change	Depends on ΔI
(e)	Decreasing ($I \downarrow$)	Supports primary flux	Same as I
(f)	Increasing ($I \uparrow$)	Opposes primary flux	Opposite to I

Expert's Solution — Rohit Sharma, B.Tech Electrical Engineering, IIT Bombay

Mastering Lenz's Law — The Conceptual Approach: Lenz's law is nature's version of "inertia" for magnetic flux. Just as mechanical inertia opposes changes in motion, the induced current opposes changes in magnetic flux.

The "Lazy Coil" Analogy:

- A conducting coil is "lazy" — it likes its current magnetic environment and resists any change.
- If you try to increase flux, the coil says "No!" and creates opposing flux.
- If you try to decrease flux, the coil says "Wait!" and tries to maintain the flux.

Quick Decision Algorithm:

1. **Identify $\vec{B}_{ext}$ direction**
2. **Is flux $\uparrow$ or $\downarrow$?**
3. **$\vec{B}_{ind}$ opposes the change**

4. **Right-hand rule:** Thumb → $\vec{B}_{\text{ind}}$, fingers → current

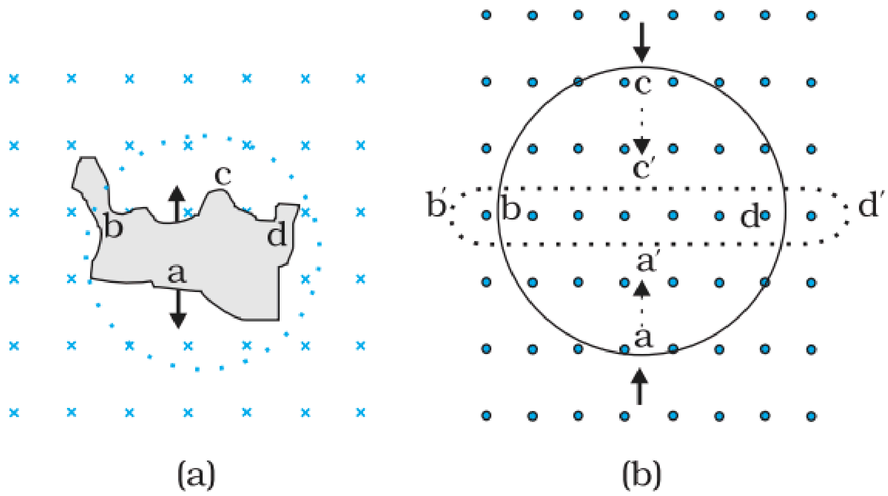
★ **Did You Know?**

Remember the key distinction:
For **parts (a)-(c)** — relative motion between magnet and coil.
For **parts (d)-(f)** — no physical motion, but changing current in one coil induces EMF in another. This is **mutual induction**, the principle behind transformers!
The common theme: **Changing magnetic flux → Induced EMF → Induced current that opposes the change.**

Q2 **Use Lenz's law to determine the direction of induced current in the situations described by Fig. 6.19:**

(a) A wire of irregular shape turning into a circular shape;

(b) A circular loop being deformed into a narrow straight wire.



Solution

Concept: Lenz's Law and Change in Magnetic Flux due to Area Change

According to **Faraday's law**, the induced emf is:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

When a magnetic field $\vec{B}$ is uniform and perpendicular to the plane of the loop, the flux is:

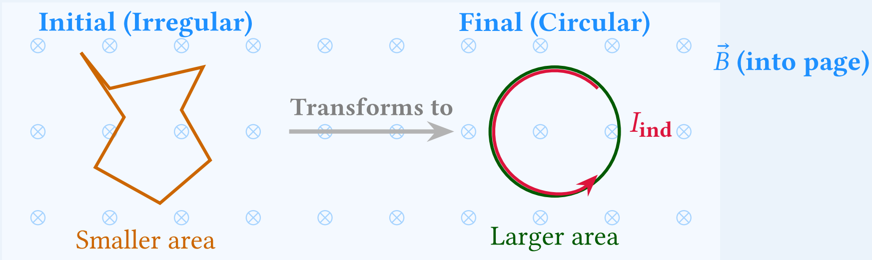
$$\Phi_B = B \cdot A$$

The negative sign embodies **Lenz's law**: the induced current flows in a direction that **opposes the change** in magnetic flux.

Key Principle:

- The magnetic field $\vec{B}$ is directed **into the page** (represented by ⊗).
- Flux $\Phi_B = BA$ (with B constant).
- A change in **area** of the loop causes a change in flux.
- Induced current opposes this change in flux.

Part (a): A wire of irregular shape turning into a circular shape

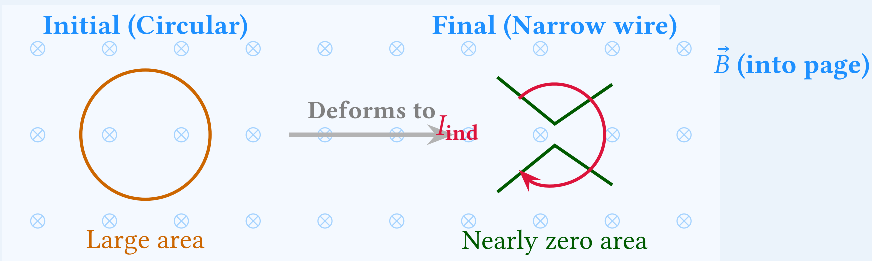


Analysis:

- For a given perimeter, a **circle encloses the maximum area**.
- Irregular shape ⟶ Circle: enclosed area **increases**.
- Since $\Phi_B = BA$, flux **increases** (more ⊗ field lines through the loop).
- By Lenz's law, induced current must **oppose** this increase.
- To oppose increasing ⊗ flux, induced current creates ⊙ (out of page) field.
- Right-Hand Thumb Rule: thumb out of page ⟶ fingers curl **anti-clockwise**.

Answer (a): Induced current flows in the **anti-clockwise** direction.

Part (b): A circular loop being deformed into a narrow straight wire



Analysis:

- Circular loop → Narrow wire: enclosed area **decreases** significantly.
- Since $\Phi_B = BA$, flux **decreases** (fewer ⊗ field lines through the loop).
- By Lenz's law, induced current must **oppose** this decrease.
- To oppose decreasing ⊗ flux, induced current creates ⊗ (into page) field to maintain the flux.
- Right-Hand Thumb Rule: thumb into page → fingers curl **clockwise**.

Answer (b): Induced current flows in the **clockwise** direction.

Conceptual Summary:

Case	Area	Flux	Induced $\vec{B}$	Current
Irregular → Circle	Increases	Increases	⊙ (opposes)	Anti-clockwise
Circle → Narrow wire	Decreases	Decreases	⊗ (supports)	Clockwise

Key Takeaway:

- For a given perimeter, the **circle has maximum area** (isoperimetric property).
- When area **increases** → induced current creates opposing field.
- When area **decreases** → induced current creates supporting field.

Expert's Solution — Arjun Mehta, B.Tech Electrical Engineering, IIT Roorkee

Understanding Flux Change Through Area Variation: This problem illustrates how **geometric deformation** of a conductor in a magnetic field generates induced EMF.

The Isoperimetric Property:

- Among all closed planar curves of a given perimeter, the **circle encloses the maximum area**.
- Irregular shape → Circle: Area increases ⇒ Flux increases ⇒ Anti-clockwise current.
- Circle → Narrow wire: Area decreases ⇒ Flux decreases ⇒ Clockwise current.

Real-World Connection: This principle is used in **flexible sensors** where mechanical deformation in a magnetic field induces measurable EMF for strain sensing.

★ **Did You Know?**

Quick Mnemonic (when $\vec{B}$ is into page):
AI → AC: Area **I**ncrease → **A**nti-**C**lockwise
AD → C: Area **D**ecrease → **C**lockwise

If $\vec{B}$ were out of page, the directions would reverse!

Q3 **A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?**

Solution

Given Data:

- Number of turns per unit length of solenoid, $n = 15$ turns per cm $= 15 \times 100 = 1500$ turns/m
- Area of the small loop, $A = 2.0\ \text{cm}^2 = 2.0 \times 10^{-4}\ \text{m}^2$
- Initial current, $I_1 = 2.0$ A
- Final current, $I_2 = 4.0$ A
- Time interval, $\Delta t = 0.1$ s
- The loop is placed **inside** the solenoid, **normal** to its axis (loop plane is perpendicular to the solenoid axis)

Concept: Magnetic Field Inside a Solenoid and Faraday's Law
A long solenoid produces a uniform magnetic field inside, given by:

$$B = \mu_0 n I$$

where $\mu_0 = 4\pi \times 10^{-7}\ \text{T m A}^{-1}$ is the permeability of free space.
Since the small loop is placed normal to the solenoid axis, the magnetic field lines pass perpendicularly through the area of the loop. Therefore, the magnetic flux through the loop is:

$$\Phi_B = B \cdot A = \mu_0 n I A$$

By **Faraday's law of electromagnetic induction**, the magnitude of induced emf is:

$$|\varepsilon| = \frac{d\Phi_B}{dt} = \frac{d}{dt}(\mu_0 n I A)$$

Since μ_0, n, and A are constants, and only the current I changes with time:

$$|\varepsilon| = \mu_0 n A \cdot \frac{dI}{dt}$$

For a steady (linear) change of current:

$$\frac{dI}{dt} = \frac{\Delta I}{\Delta t} = \frac{I_2 - I_1}{\Delta t}$$

Step 1: Calculate the rate of change of current

$$\frac{dI}{dt} = \frac{4.0 - 2.0}{0.1} = \frac{2.0}{0.1} = 20 \text{ A/s}$$

Step 2: Calculate the induced emf

$$|\varepsilon| = \mu_0 n A \cdot \frac{dI}{dt}$$

Substituting the values:

$$\begin{aligned}
|\varepsilon| &= (4\pi \times 10^{-7}) \times (1500) \times (2.0 \times 10^{-4}) \times (20) \\
&= 4\pi \times 10^{-7} \times 1500 \times 2.0 \times 10^{-4} \times 20 \\
&= 4\pi \times 10^{-7} \times 3000 \times 10^{-4} \times 20 \\
&= 4\pi \times 10^{-7} \times 60000 \times 10^{-4} \\
&= 4\pi \times 10^{-7} \times 6.0 \\
&= 24\pi \times 10^{-7} \\
&= 75.36 \times 10^{-7} \\
&= 7.536 \times 10^{-6} \text{ V}
\end{aligned}$$

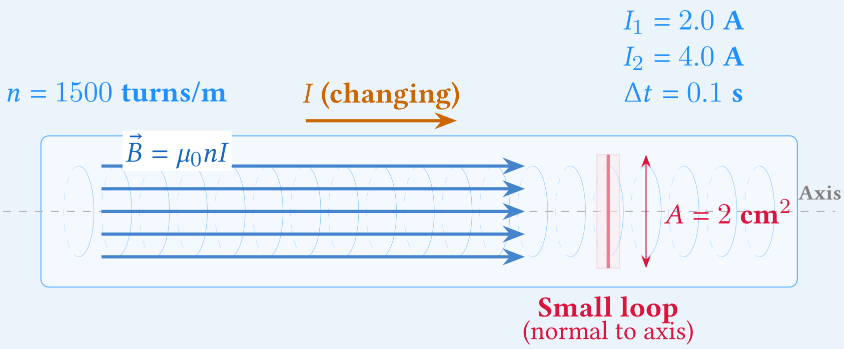
Step 3: Express in convenient units

$$|\varepsilon| = 7.54 \times 10^{-6} \text{ V} = 7.54\ \mu\text{V}$$

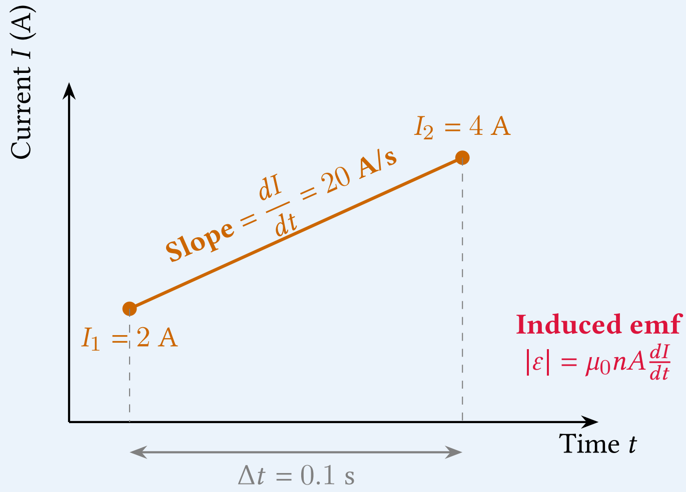
Final Answer:

$$\boxed{|\varepsilon| = 7.5 \times 10^{-6} \text{ V} \quad (\text{or } 7.5\ \mu\text{V})}$$

Visual Representation: Solenoid with Internal Loop



Circuit / Flux Change Visualization:



Expert's Solution — Priya Sharma, B.Tech Electrical Engineering, NIT Trichy

Understanding the Physics: This problem combines two fundamental concepts:

1. **Magnetic field of a solenoid:** $B = \mu_0 n I$ — uniform inside, independent of position.
2. **Faraday's law:** $\varepsilon = -d\Phi_B/dt$ — changing flux induces emf.

Why is the loop orientation important?

- The loop is **normal to the solenoid axis**, meaning $\vec{B}$ is perpendicular to the loop area.
- This gives maximum flux: $\Phi_B = BA\cos 0^\circ = BA$.
- If the loop were parallel to the axis, flux would be zero and no emf would be induced.

Step-by-step verification:

$$B_1 = \mu_0 n I_1 = 4\pi \times 10^{-7} \times 1500 \times 2.0 = 3.77 \times 10^{-3}\ \text{T}$$

$$B_2 = \mu_0 n I_2 = 4\pi \times 10^{-7} \times 1500 \times 4.0 = 7.54 \times 10^{-3}\ \text{T}$$

$$\Delta\Phi_B = (B_2 - B_1)A = (3.77 \times 10^{-3}) \times (2.0 \times 10^{-4}) = 7.54 \times 10^{-7}\ \text{Wb}$$

$$|\varepsilon| = \frac{\Delta\Phi_B}{\Delta t} = \frac{7.54 \times 10^{-7}}{0.1} = 7.54 \times 10^{-6}\ \text{V}$$

★ **Did You Know?**

Quick Tip:
For problems involving solenoids and internal loops, remember:
$\varepsilon = \mu_0 n A \cdot \frac{dI}{dt}$

All factors multiply directly — the emf is proportional to each of them.
If any of n, A, or dI/dt doubles, the induced emf doubles too!

Q4 A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s^{-1} in a direction normal to the (a) longer side, (b) shorter side of the loop? For how long does the induced voltage last in each case?

Solution

Given Data:

- Length of the loop, $l = 8$ cm $= 8 \times 10^{-2}$ m
- Width of the loop, $w = 2$ cm $= 2 \times 10^{-2}$ m
- Magnetic field, $B = 0.3$ T (directed normal to the loop)
- Velocity of the loop, $v = 1$ cm s^{-1} $= 1 \times 10^{-2}$ m s^{-1}
- The loop has a **small cut** (so it is an open loop)

Concept: Motional EMF

When a conductor moves in a magnetic field, charges inside the conductor experience a magnetic force:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

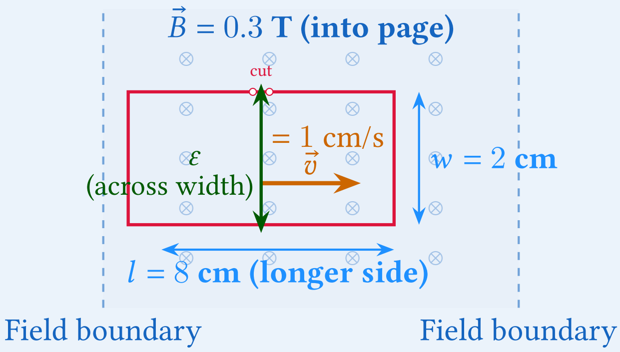
This leads to an induced emf across the ends of the conductor. For a straight conductor of length L moving with velocity v perpendicular to both its length and a uniform magnetic field B, the **motional emf** is:

$$\varepsilon = BLv$$

Understanding the Problem:

- The rectangular loop has a **small cut**, meaning it is not a closed circuit — no induced current flows.
- However, an emf is still developed across the cut due to charge separation.
- As the loop moves out of the magnetic field region, only the **portion inside the field** contributes to the emf.
- The side of the loop that is **cutting the magnetic field lines** is responsible for the motional emf.
- The emf is developed **across the width** (or the side perpendicular to the velocity).

Part (a): Velocity normal to the longer side (8 cm side)



Analysis:

- Velocity is **normal to the longer side** (8 cm), meaning the loop moves **along its length**.
- The side that cuts the magnetic field lines is the **shorter side** ($w = 2$ cm).
- The effective length for motional emf is $L = w = 2 \times 10^{-2}$ m.
- The emf is developed across the cut (across the width of the loop).

Calculation of EMF:

$$\varepsilon = BLv = B \cdot w \cdot v$$

$$\varepsilon = 0.3 \times (2 \times 10^{-2}) \times (1 \times 10^{-2})$$

$$\varepsilon = 0.3 \times 2 \times 10^{-4} = 6 \times 10^{-5} \text{ V}$$

$$\varepsilon = 0.06 \text{ mV} = 60 \ \mu\text{V}$$

Time for which induced voltage lasts:

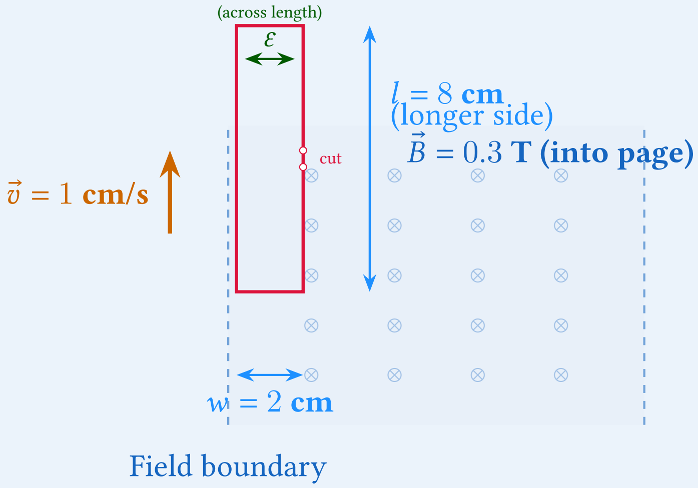
- The loop moves normal to the longer side, so it travels a distance equal to the **longer side length** (8 cm) to completely exit the field.
- Time $= \dfrac{\text{Distance}}{\text{Velocity}} = \dfrac{l}{v}$

$$t = \frac{8 \times 10^{-2}}{1 \times 10^{-2}} = 8 \text{ s}$$

- The induced voltage lasts as long as the loop is partially inside the field (one short side cutting flux while the other is outside).

Answer (a): $\varepsilon = 6 \times 10^{-5}$ V (or 60 μV), lasting for $t = 8$ s.

Part (b): Velocity normal to the shorter side (2 cm side)



Analysis:

- Velocity is **normal to the shorter side** (2 cm), meaning the loop moves **along its width** (perpendicular to the longer side).
- The side that cuts the magnetic field lines is the **longer side** ($l = 8$ cm).
- The effective length for motional emf is $L = l = 8 \times 10^{-2}$ m.
- The emf is developed across the cut (across the length of the loop).

Calculation of EMF:

$$\varepsilon = BLv = B \cdot l \cdot v$$

$$\varepsilon = 0.3 \times (8 \times 10^{-2}) \times (1 \times 10^{-2})$$
$$\varepsilon = 0.3 \times 8 \times 10^{-4} = 24 \times 10^{-5} \text{ V}$$
$$\varepsilon = 2.4 \times 10^{-4} \text{ V} = 0.24 \text{ mV} = 240\ \mu\text{V}$$

Time for which induced voltage lasts:

- The loop moves normal to the shorter side, so it travels a distance equal to the **shorter side length** (2 cm) to completely exit the field.
- Time $= \dfrac{\text{Distance}}{\text{Velocity}} = \dfrac{w}{v}$

$$t = \frac{2 \times 10^{-2}}{1 \times 10^{-2}} = 2 \text{ s}$$

Answer (b): $\varepsilon = 2.4 \times 10^{-4}$ V (or 240 μV), lasting for $t = 2$ s.

Final Summary:

Case	Effective Length L	Induced EMF	Duration
(a) $v \perp$ longer side	$w = 2$ cm (shorter side)	6×10^{-5} V	8 s
(b) $v \perp$ shorter side	$l = 8$ cm (longer side)	2.4×10^{-4} V	2 s

Key Insight:

- The induced emf depends on the length of the side that is **cutting the magnetic flux** (i.e., the side perpendicular to velocity).
- In case (a), the shorter side cuts flux $\Rightarrow$ smaller emf, longer duration (travels 8 cm).
- In case (b), the longer side cuts flux $\Rightarrow$ larger emf, shorter duration (travels 2 cm).
- Product of emf and time: $\varepsilon \cdot t = BL \cdot \dfrac{L_{\text{travel}}}{v} = \dfrac{B \cdot \text{Area}}{v}$ — same for both cases.

Expert's Solution — Vikram Rathore, B.Tech Engineering Physics, NIT Surathkal

Understanding Motional EMF: The emf in this problem arises from the **magnetic force** on free electrons in the conductor:

$$\vec{F}_B = -e(\vec{v} \times \vec{B})$$

Visualizing Charge Separation:

- As the loop moves, the magnetic force pushes electrons to one end of the cutting side.
- This creates an electric field that opposes further charge movement.
- At equilibrium: $eE = evB$, giving the emf: $\varepsilon = EL = BLv$.
- The small cut prevents current flow, so the emf appears as a potential difference across the cut.

Why different emf values?

- $\varepsilon \propto L$ (the length of the side cutting the field).
- Case (a): $L = 2$ cm, giving $\varepsilon = 60\ \mu$V.
- Case (b): $L = 8$ cm, giving $\varepsilon = 240\ \mu$V.
- The ratio is $4 : 1$, exactly matching the side length ratio $8 : 2$.

★ **Did You Know?**

Quick Mnemonic:
For motional emf: $\varepsilon = BLv$
L = **L**ength of the side that is **c**utting the field (perpendicular to $\vec{v}$).

Duration = **D**istance the loop must travel divided by velocity:

$$t = \frac{\text{length of side parallel to } \vec{v}}{v}$$

Key: Longer cutting side → larger emf but shorter time, and vice versa!

Q5 A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Solution

Given Data:

- Length of the metallic rod, $l = 1.0$ m
- Angular frequency, $\omega = 400$ rad s^{-1}
- Magnetic field, $B = 0.5$ T (parallel to the axis of rotation)

- The rod rotates about an axis through one end
- The other end is in contact with a circular metallic ring

Concept: Motional EMF in a Rotating Rod

When a conducting rod rotates in a uniform magnetic field perpendicular to the plane of rotation, an emf is induced between its ends. This is because different segments of the rod move with different linear velocities in the magnetic field.

Method 1: Using Motional EMF of a Small Element

Consider a small element of length dx at a distance x from the axis of rotation. The linear velocity of this element is:

$$v = \omega x$$

The motional emf induced across this small element is:

$$d\varepsilon = B \cdot v \cdot dx = B \cdot \omega x \cdot dx$$

The total emf between the centre (axis) and the ring (other end) is obtained by integrating from $x = 0$ to $x = l$:

$$\varepsilon = \int_0^l B\,\omega\,x\,dx = B\,\omega \int_0^l x\,dx$$

$$\varepsilon = B\,\omega \left[\frac{x^2}{2}\right]_0^l = \frac{1}{2}B\,\omega\,l^2$$

Method 2: Using Faraday's Law (Area Swept)

In one complete revolution, the rod sweeps out a circular area πl^2. The time period for one revolution is $T = 2\pi/\omega$. The average emf is:

$$\varepsilon = \frac{\Delta\Phi_B}{\Delta t} = \frac{B \cdot \pi l^2}{2\pi/\omega} = \frac{1}{2}B\,\omega\,l^2$$

Both methods give the same result.

Calculation:

Substituting the given values:

$$\varepsilon = \frac{1}{2}B\,\omega\,l^2$$

$$\varepsilon = \frac{1}{2} \times 0.5 \times 400 \times (1.0)^2$$

$$\varepsilon = \frac{1}{2} \times 0.5 \times 400 \times 1$$

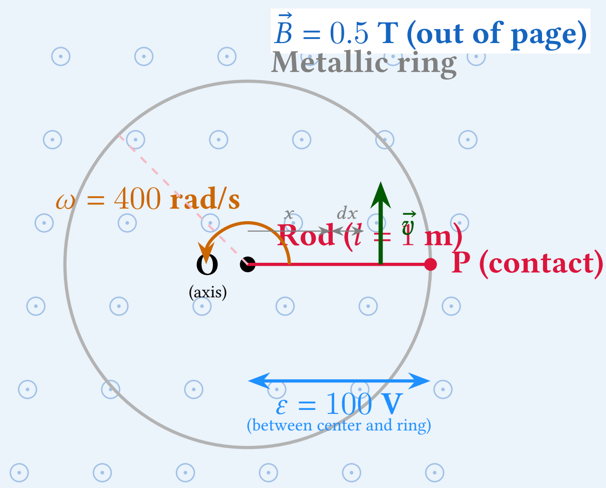
$$\varepsilon = \frac{1}{2} \times 200$$

$$\varepsilon = 100 \text{ V}$$

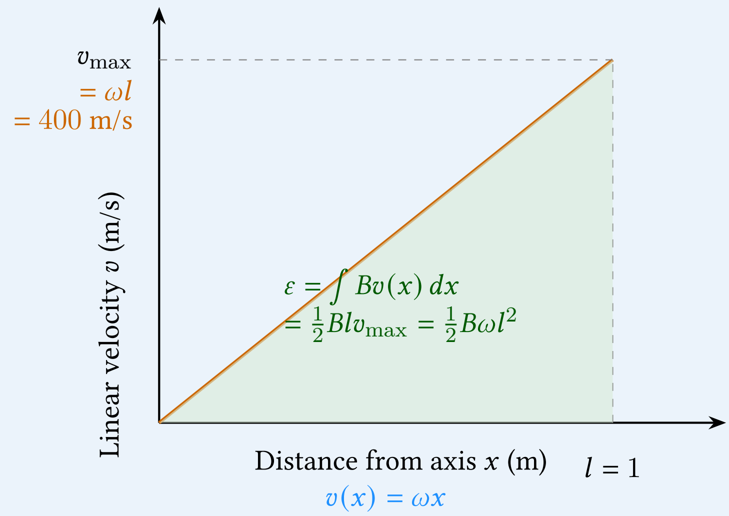
Final Answer:

$$\boxed{\varepsilon = 100 \text{ V}}$$

Visual Representation: Rotating Rod with Circular Ring



Velocity Variation Along the Rod:



Expert's Solution — Sneha Patel, B.Tech Electrical Engineering, IIT Gandhinagar

The Rotating Rod — A Classic Faraday's Dynamo: This setup is essentially the **Faraday disc generator** simplified to a single rotating rod, also known as a **homopolar generator**.

Why Integration is Necessary:

- Unlike linear motion where all points have the same velocity, in rotation **different points have different velocities**.

- Velocity increases linearly from 0 at the axis to ωl at the end.
- The **average velocity** of the rod is $\bar{v} = \frac{0+\omega l}{2} = \frac{\omega l}{2}$.
- Using this average: $\varepsilon = Bl\bar{v} = Bl \cdot \frac{\omega l}{2} = \frac{1}{2}B\omega l^2$.

Alternative Formula Using Frequency:

$$\varepsilon = \frac{1}{2}B\omega l^2 = \frac{1}{2}B(2\pi f)l^2 = \pi B f l^2$$

where f is the rotational frequency in Hz. Here $f = \frac{\omega}{2\pi} = \frac{400}{2\pi} \approx 63.66$ Hz.

Practical Application: This principle is used in **tachometers** — devices that measure rotational speed. By measuring the induced emf, the angular velocity can be determined since $\omega = \frac{2\varepsilon}{Bl^2}$. The linear relationship between ω and ε makes calibration straightforward.

Did You Know?

Quick Tip:
Don't confuse this with a coil rotating in a magnetic field (AC generator). Here:

- Flux through any fixed area is **not changing** — B is uniform.
- The emf arises from **motional emf** (Blv) at the microscopic level.
- The formula $\varepsilon = \frac{1}{2}B\omega l^2$ is specific to a rod rotating about one end.

If the rod rotated about its centre, the emf between the centre and either end would be $\frac{1}{8}B\omega l^2$.

Q6 **A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of magnitude 3.0×10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10 Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?**

Solution

Given Data:

- Radius of the coil, $r = 8.0$ cm $= 8.0 \times 10^{-2}$ m
- Number of turns, $N = 20$

- Angular speed, $\omega = 50$ rad s^{-1}
- Magnetic field, $B = 3.0 \times 10^{-2}$ T (horizontal)
- Rotation axis: vertical diameter
- Resistance of the closed loop, $R = 10\ \Omega$

Concept: AC Generator — EMF Induced in a Rotating Coil
When a coil of N turns and area A rotates with angular speed ω in a uniform magnetic field B, the magnetic flux through the coil at any instant t is:

$$\Phi_B = NBA\cos\theta = NBA\cos(\omega t)$$

where $\theta = \omega t$ is the angle between the magnetic field and the normal to the plane of the coil. By **Faraday's law**, the induced emf is:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}[NBA\cos(\omega t)] = NBA\omega\sin(\omega t)$$

The emf varies sinusoidally with time:

$$\varepsilon = \varepsilon_0 \sin(\omega t)$$

where $\varepsilon_0 = NBA\omega$ is the **maximum (peak) emf**.

Step 1: Calculate the area of the coil

$$A = \pi r^2 = \pi \times (8.0 \times 10^{-2})^2 = \pi \times 64 \times 10^{-4} = 64\pi \times 10^{-4}\ \text{m}^2$$

Step 2: Calculate the maximum emf (ε_0)

$$\begin{aligned}
\varepsilon_0 &= NBA\omega \\
&= 20 \times (3.0 \times 10^{-2}) \times (64\pi \times 10^{-4}) \times 50 \\
&= 20 \times 3.0 \times 10^{-2} \times 64\pi \times 10^{-4} \times 50 \\
&= 20 \times 3.0 \times 64\pi \times 50 \times 10^{-6} \\
&= 192000\pi \times 10^{-6} \\
&= 0.192\pi \\
&= 0.192 \times 3.1416 \\
&= 0.603\ \text{V}
\end{aligned}$$

Step 3: Calculate the average emf

The instantaneous emf is $\varepsilon = \varepsilon_0 \sin(\omega t)$. For a complete cycle (from 0 to T):

$$\varepsilon_{avg} = \frac{1}{T}\int_0^T \varepsilon_0 \sin(\omega t)\, dt = 0$$

Over a half-cycle, the average emf is:

$$\varepsilon_{avg\ (half\text{-}cycle)} = \frac{2\varepsilon_0}{\pi} = \frac{2 \times 0.603}{3.1416} \approx 0.384 \text{ V}$$

However, since the question asks for "average emf" without specification, the conventional interpretation is the average over a **full cycle**, which is **zero** for a sinusoidal AC emf.

Step 4: Calculate the maximum current Using Ohm's law:

$$I_0 = \frac{\varepsilon_0}{R} = \frac{0.603}{10} = 0.0603 \text{ A} = 60.3 \text{ mA}$$

Step 5: Calculate the average power loss (Joule heating)

The instantaneous power dissipated is:

$$P(t) = \frac{\varepsilon^2(t)}{R} = \frac{\varepsilon_0^2 \sin^2(\omega t)}{R}$$

The average power over a full cycle:

$$P_{avg} = \frac{\varepsilon_0^2}{R} \cdot \overline{\sin^2(\omega t)} = \frac{\varepsilon_0^2}{R} \cdot \frac{1}{2} = \frac{\varepsilon_0^2}{2R}$$

Substituting values:

$$P_{avg} = \frac{(0.603)^2}{2 \times 10} = \frac{0.3636}{20} = 0.01818 \text{ W} = 18.2 \text{ mW}$$

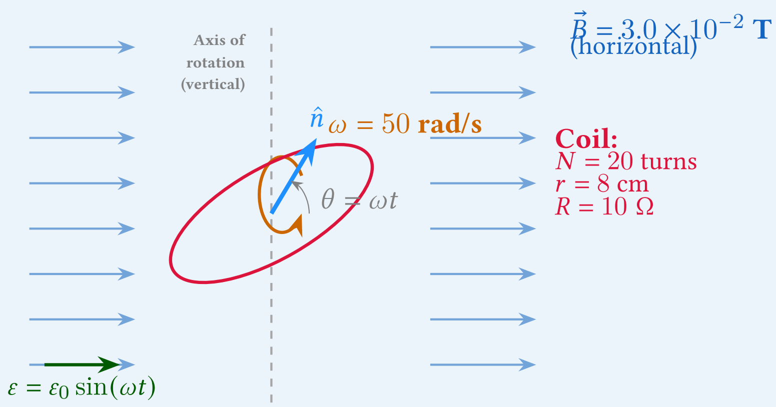
Step 6: Source of the power The electrical power dissipated as heat comes from the **mechanical work** done in rotating the coil against the magnetic torque. When current flows in the coil, it experiences a magnetic torque that opposes the rotation (Lenz's law). The external agent rotating the coil must do work against this opposing torque, converting mechanical energy into electrical energy, which is then dissipated as Joule heat.

Final Answers:

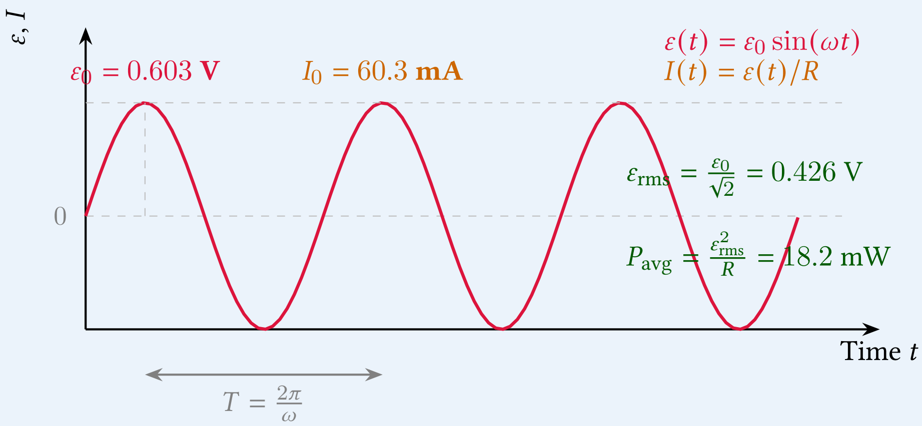
$$\begin{aligned} \text{Maximum emf: } & \varepsilon_0 = 0.60 \text{ V} \quad (\text{or } 0.603 \text{ V}) \\ \text{Average emf (full cycle): } & \varepsilon_{avg} = 0 \text{ V} \\ \text{Maximum current: } & I_0 = 6.03 \times 10^{-2} \text{ A} \quad (\text{or } 60.3 \text{ mA}) \\ \text{Average power loss: } & P_{avg} = 1.82 \times 10^{-2} \text{ W} \quad (\text{or } 18.2 \text{ mW}) \end{aligned}$$

Power source: The mechanical work done by the external agent rotating the coil against the opposing magnetic torque.

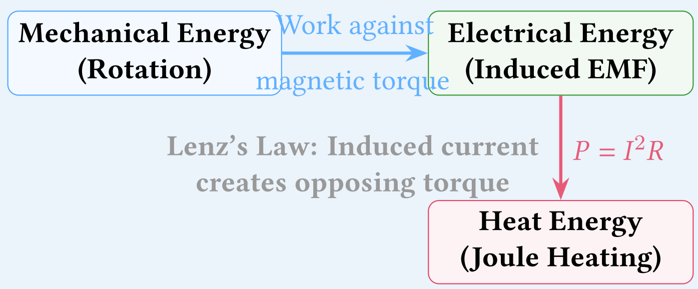
Visual Representation: Rotating Coil AC Generator



EMF and Current Waveforms:



Energy Conversion:



Expert's Solution — Rahul Verma, B.Tech Electrical Engineering, NIT Jalandhar

The AC Generator Principle: This problem describes the basic working principle of an **AC generator** (alternator). The sinusoidal variation of flux produces a sinusoidal emf:

$$\varepsilon(t) = NBA\omega \sin(\omega t) = \varepsilon_0 \sin(\omega t)$$

Key Results for Sinusoidal AC:

Quantity	Formula	Value
Maximum emf	$\varepsilon_0 = NBA\omega$	0.603 V
RMS emf	$\varepsilon_{rms} = \varepsilon_0/\sqrt{2}$	0.426 V
Average emf (full cycle)	$\varepsilon_{avg} = 0$	0
Average emf (half cycle)	$\varepsilon_{avg} = 2\varepsilon_0/\pi$	0.384 V
Maximum current	$I_0 = \varepsilon_0/R$	0.0603 A
Average power	$P_{avg} = \varepsilon_0^2/(2R)$	18.2 mW

Why Average EMF is Zero Over a Full Cycle:

- The emf is $\varepsilon = \varepsilon_0 \sin(\omega t)$, which is symmetric about zero.
- The positive half-cycle is exactly cancelled by the negative half-cycle.
- This is why we use RMS values for AC calculations.

★ **Did You Know?**

Quick Tip — The Power Source Question:
The power doesn't come "for free." The external agent (prime mover — like a turbine, engine, or hand crank) must supply mechanical power equal to the electrical power dissipated plus any losses:

$$P_{mechanical} = P_{electrical} + P_{losses}$$

This is a beautiful demonstration of **energy conservation** in electromagnetic systems. The coil resists being rotated (Lenz's law), and the work done against this resistance becomes the electrical energy output.

Q7 A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1}, at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb m^{-2}.

(a) What is the instantaneous value of the emf induced in the wire?

(b) What is the direction of the emf?

(c) Which end of the wire is at the higher electrical potential?

Solution

Given Data:

- Length of the wire, $l = 10$ m
- Speed of the falling wire, $v = 5.0$ m s^{-1} (vertically downward)
- Horizontal component of Earth's magnetic field, $B_H = 0.30 \times 10^{-4}$ Wb m^{-2} $= 0.30 \times 10^{-4}$ T
- Wire orientation: East to West
- Wire is **at right angles** to B_H

Concept: Motional EMF
When a straight conductor of length l moves with velocity $\vec{v}$ in a magnetic field $\vec{B}$, such that $\vec{v}$, $\vec{B}$, and $\vec{l}$ are mutually perpendicular, the motional emf induced across the ends of the conductor is:

$$\varepsilon = Blv$$

Part (a): Instantaneous value of the induced emf

$$\begin{aligned}
\varepsilon &= B_H \cdot l \cdot v \\
&= (0.30 \times 10^{-4}) \times 10 \times 5.0 \\
&= 0.30 \times 10^{-4} \times 50 \\
&= 15 \times 10^{-4} \\
&= 1.5 \times 10^{-3} \text{ V}
\end{aligned}$$

Answer (a): $\varepsilon = 1.5 \times 10^{-3}$ V $= 1.5$ mV

Part (b): Direction of the induced emf
To determine the direction of induced emf, we use the **Fleming's Right-Hand Rule** (for generators):

- **Thumb:** Motion of the conductor ($\vec{v}$) — **vertically downward**
- **Index finger:** Magnetic field ($\vec{B}$) — **Earth's horizontal component**
- **Middle finger:** Direction of induced emf/current — gives the direction from negative to positive

Understanding the geometry:

- The wire extends from **East to West**.

- Earth's horizontal magnetic field component (B_H) points from **South to North** (geographic).
- The wire falls **vertically downward**.
- $\vec{v}$ (downward), $\vec{B}$ (northward), and $\vec{l}$ (east-west) are **mutually perpendicular**.

Applying Fleming's Right-Hand Rule:

- Thumb $\longrightarrow$ Downward (motion)
- Index finger $\longrightarrow$ Northward (B_H)
- Middle finger $\longrightarrow$ **Westward**

The middle finger gives the direction of induced conventional current, which flows from **negative to positive** inside the source. Therefore, the induced emf is directed from **East to West**.

Answer (b): The induced emf is directed from **East to West** (or Westward).

Part (c): Which end is at higher potential?

The direction of induced emf gives the direction of increasing potential. Inside the source (the moving wire), conventional current flows from **negative (lower potential) to positive (higher potential)**.

Since the emf is directed from East to West:

- The **East end** is at **negative** (lower) potential.
- The **West end** is at **positive** (higher) potential.

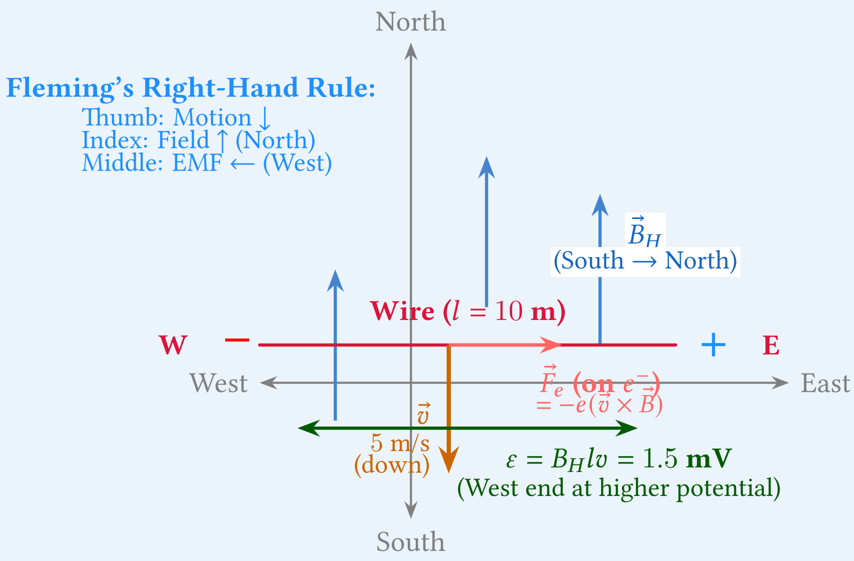
Alternatively, using the Lorentz force on free electrons:

$$\vec{F} = -e(\vec{v} \times \vec{B})$$

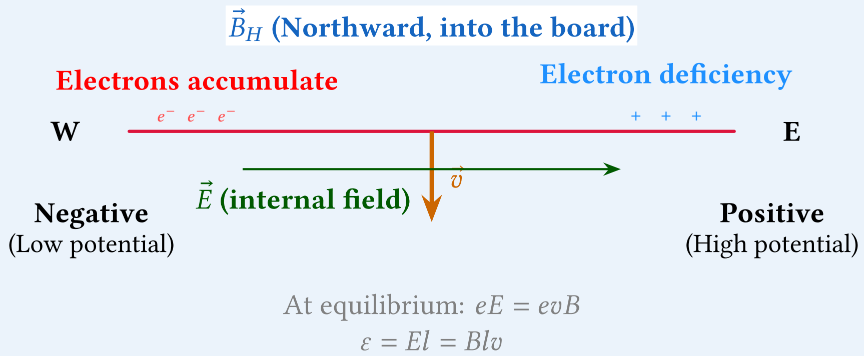
- $\vec{v} \times \vec{B}$ = downward $\times$ northward = westward
- Force on negative electrons is **eastward** (opposite to $\vec{v} \times \vec{B}$)
- Electrons accumulate at the **East end** $\longrightarrow$ East end is negative.
- **West end** is deficient in electrons $\longrightarrow$ West end is positive.

Answer (c): The **West end** of the wire is at the higher electrical potential.

Visual Representation: Motional EMF in a Falling Wire



Charge Separation and Electric Field:



Expert's Solution — Karan Joshi, B.Tech Engineering Physics, IIT Roorkee

Understanding Earth's Magnetic Field in EMI Problems: The Earth's magnetic field has both horizontal (B_H) and vertical (B_V) components. In India:

- $B_H \approx 0.30 \times 10^{-4}$ T (South to North)
- $B_V \approx 0.40 \times 10^{-4}$ T (downward in Northern hemisphere)

Why Only B_H Matters Here:

- The wire is horizontal (East-West) and moves vertically downward.
- B_H is perpendicular to both $\vec{v}$ (downward) and $\vec{l}$ (horizontal) — this gives the maximum motional emf.
- B_V is parallel to $\vec{v}$ (both vertical) — $\vec{v} \times \vec{B_V} = 0$, so it contributes nothing.

Vector Cross Product Verification:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\vec{v} = -v\,\hat{j}, \quad \vec{B}_H = B_H\,\hat{i} \ \ \text{(South to North)}$$

$$\vec{v} \times \vec{B}_H = (-v\hat{j}) \times (B_H\hat{i}) = vB_H\,\hat{k}$$

The force pushes positive charges toward **West** ($\hat{k}$ direction if East-West is along $\hat{k}$), confirming West end is at higher potential.

★ **Did You Know?**

Quick Mnemonic — Fleming's Right-Hand Rule:
Thumb — **Th**rust (Motion)
Forefinger — **F**ield
Middle — EMF (conventional current inside source)

Remember: "Generators Are Right" — use **Right** hand for generators (Fleming's **Right**-Hand Rule).
For motors, use **Left** hand (Fleming's **Left**-Hand Rule).