NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12th Physics, Chapter 7

Chapter 7: Alternating Current

About this Chapter

Chapter 7 of NCERT Class 12 Physics introduces **alternating current (AC)**, where the supply voltage varies sinusoidally with time. You will learn how resistors, inductors and capacitors respond differently to AC, the concept of **reactance** and **impedance** in series **LCR** circuits, **resonance** and **quality factor**, power dissipation with **power factor**, **LC oscillations**, and the working of **transformers** for transmitting electrical energy over long distances.

Topics covered: AC voltage & rms values • Reactance of L and C • Series LCR circuits • Resonance & Q-factor • Power in AC • LC oscillations • Transformers

Quick Formula Sheet

rms values:
$I_{\rm rms} = \frac{I_m}{\sqrt{2}}, \quad V_{\rm rms} = \frac{V_m}{\sqrt{2}}$

Reactance:
$X_L = \omega L, \quad X_C = \frac{1}{\omega C}$

Impedance (series LCR):
$Z = \sqrt{R^2 + (X_L - X_C)^2}$

Resonance frequency:
$\omega_0 = \frac{1}{\sqrt{LC}}$

Average power:
$\bar{P} = V_{\rm rms} I_{\rm rms} \cos\varphi$

Transformer:
$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$

MCQ I

Q 7.1 **If the rms current in a 50 Hz ac circuit is 5 A, the value of the current $\frac{1}{300}$ seconds after its value becomes zero is**

(a) $5\sqrt{2}$ A
(b) $5\sqrt{3/2}$ A
(c) $5/6$ A
(d) $5/\sqrt{2}$ A

SOLUTION

Correct option: (b) $5\sqrt{3/2}$ A.

Concept used. A sinusoidal alternating current can be written as $i(t) = I_m \sin(\omega t)$, where I_m is the peak (maximum) value of the current and $\omega = 2\pi f$ is the angular frequency. The peak value is related to the rms (root-mean-square) value by

$$I_m = \sqrt{2}\, I_{\text{rms}}.$$

If the current is zero at $t = 0$ and is rising, then at any later time t the instantaneous current is $i(t) = I_m \sin(\omega t)$.

Step 1. Compute the angular frequency:

$$\omega = 2\pi f = 2\pi \times 50 = 100\pi \text{ rad/s}.$$

Step 2. Compute the peak current from the rms value:

$$I_m = \sqrt{2}\, I_{\text{rms}} = \sqrt{2} \times 5 = 5\sqrt{2} \text{ A}.$$

Step 3. Find the phase at $t = 1/300$ s after the current was zero:

$$\omega t = 100\pi \times \frac{1}{300} = \frac{\pi}{3} \text{ rad}.$$

Step 4. Substitute into $i(t) = I_m \sin(\omega t)$:

$$i = 5\sqrt{2}\, \sin\left(\frac{\pi}{3}\right) = 5\sqrt{2} \cdot \frac{\sqrt{3}}{2} = \frac{5\sqrt{6}}{2} = 5\sqrt{\tfrac{3}{2}} \text{ A}.$$

Step 5. Numerical check: $5\sqrt{1.5} \approx 5 \times 1.2247 \approx 6.12$ A, which lies between 0 and the peak $I_m \approx 7.07$ A, as expected one-sixth of a cycle in.

> **Final Answer:** Option **(b)**: $i = 5\sqrt{3/2}$ A ≈ 6.12 A.

☞ Common trap

Many students confuse rms with peak: do not plug I_{rms} into $i(t) = I_m \sin \omega t$. Always convert to $I_m = \sqrt{2}\, I_{\text{rms}}$ first.

EXPERT'S SOLUTION *: Aarav Iyer, M.Sc Physics, IIT Madras*

Quick reading. The cue "$1/300$ s" is engineered to give $\omega t = \pi/3$, a clean reference angle. Recognise this once and the answer falls out.

Step 1. Write the standard sinusoidal form: $i(t) = I_m \sin(\omega t)$, with $I_m = \sqrt{2}\, I_{\text{rms}} = 5\sqrt{2}$ A.

Step 2. Plug in $\omega = 2\pi f = 100\pi$ rad/s and $t = 1/300$ s:

$$\omega t = \frac{100\pi}{300} = \frac{\pi}{3}.$$

Step 3. Use $\sin(\pi/3) = \sqrt{3}/2$:

$$i = 5\sqrt{2} \cdot \frac{\sqrt{3}}{2} = \frac{5\sqrt{6}}{2} = 5\sqrt{3/2}\text{ A}.$$

Step 4. **Phasor cross-check.** The rotating phasor $\vec{I}_m$ has length $5\sqrt{2}$ and sweeps ωt rad in time t. After $t = T/6 = 1/300$ s of the $T = 1/50$ s period, the phasor has turned through $360°/6 = 60°$. Its projection on the vertical axis (which represents the instantaneous current when starting from zero) is $5\sqrt{2}\sin 60° = 5\sqrt{2}(\sqrt{3}/2)$, matching the answer.

Step 5. **Period consistency.** $T = 1/f = 1/50 = 0.02$ s, so $1/300$ s $= T/6$. One sixth of a sinusoidal cycle from the rising zero-crossing is exactly $60° = \pi/3$ along the phase axis.

Why this matters. Knowing that $\omega t = \pi/3$ when $t = T/6$ saves trigonometric guesswork in exam-hall pressure.

> **Final Answer:** Option **(b)**: $5\sqrt{3/2}$ A.

☞ **Sinusoid angle↔time**

For a sinusoid of period T: $T/12 \leftrightarrow 30°$, $T/8 \leftrightarrow 45°$, $T/6 \leftrightarrow 60°$, $T/4 \leftrightarrow 90°$. Memorising these saves a multiplication every numerical AC problem.

Q 7.2 **An alternating current generator has an internal resistance R_g and an internal reactance X_g. It is used to supply power to a passive load consisting of a resistance R_g and a reactance X_L. For maximum power to be delivered from the generator to the load, the value of X_L is equal to**

(a) zero.

(b) X_g.

(c) $-X_g$.

(d) R_g.

SOLUTION

Correct option: (c) $-X_g$.

Concept used. Maximum power transfer theorem for AC sources. A source of internal impedance $Z_g = R_g + jX_g$ delivers maximum average power to a load impedance $Z_L = R_L + jX_L$ when Z_L is the *complex conjugate* of Z_g:

$$R_L = R_g, \qquad X_L = -X_g.$$

The reason: the reactive parts cancel in the series loop, so the total impedance becomes purely resistive ($R_g + R_L = 2R_g$), and all the energy that flows around the loop is dissipated only in the resistors.

Step 1. Total impedance of the loop:

$$Z = (R_g + R_L) + j(X_g + X_L).$$

Step 2. Magnitude squared:

$$|Z|^2 = (R_g + R_L)^2 + (X_g + X_L)^2.$$

Step 3. Average power delivered to the load resistor R_L:

$$P = I_{\text{rms}}^2 R_L = \frac{V_{\text{rms}}^2 R_L}{(R_g + R_L)^2 + (X_g + X_L)^2}.$$

Step 4. For a given source V_{rms} and given R_L, P is largest when the denominator is smallest. The reactance term $(X_g + X_L)^2 \geq 0$, so it is minimised when $X_g + X_L = 0$, i.e. $X_L = -X_g$.

Step 5. Why $-X_g$ and not $+X_g$: X_g might be inductive ($+\omega L$); to cancel it the load must contribute an equal capacitive reactance ($-1/\omega C$). The minus sign is the cancellation.

Final Answer: Option **(c)**: $X_L = -X_g$.

♥ Where this is used

Impedance matching with the conjugate condition is the basis of antenna matching networks, audio amplifier output stages and RF transmission-line design.

EXPERT'S SOLUTION : *Sneha Kapoor, Ph.D Physics, IISc Bangalore*

Structural angle. Treat the loop as $Z_g + Z_L$ in series; cancel reactances first, then balance resistances.

Step 1. The total loop impedance is $Z = (R_g + R_L) + j(X_g + X_L)$ and $P = V_{\text{rms}}^2 R_L/|Z|^2$.

Step 2. Minimising $|Z|^2$ over X_L alone: take $\partial(|Z|^2)/\partial X_L = 2(X_g + X_L) = 0 \Rightarrow X_L = -X_g$.

Step 3. With this choice the impedance becomes purely real, $|Z| = R_g + R_L$, and the further condition $R_L = R_g$ then maximises $P = V_{\text{rms}}^2 R_g/(2R_g)^2 = V_{\text{rms}}^2/(4R_g)$.

Step 4. **Phasor reasoning.** The voltage phasor $\vec{V}$ across the loop must be split between Z_g and Z_L. The reactive parts of Z_g and Z_L produce phasors that point in opposite directions if $X_L = -X_g$, so they cancel head-to-tail. What is left is purely resistive, and the current phasor aligns with $\vec{V}$ – the maximum-energy-transfer configuration.

Step 5. **Why "conjugate" is the right word.** In the complex plane, $Z_g = R_g + jX_g$ and $Z_L = R_g - jX_g$ are mirror images about the real axis. Their sum $2R_g$ lies on the real axis, which is exactly the condition for "no wasted reactive circulation".

Step 6. **Sanity scale.** With $V_{\text{rms}} = 240$ V and $R_g = 4\ \Omega$, the maximum deliverable power is $V^2/(4R_g) = 57600/16 = 3600$ W. A mismatched purely resistive $R_L = 8\ \Omega$ instead gives $V^2 R_L/(R_g + R_L)^2 = 240^2 \cdot 8/144 = 3200$ W, confirming the matched case is optimal.

Why this matters. The conjugate-matching condition $Z_L = Z_g^*$ generalises the DC rule $R_L = R_g$ to complex loads.

> **Final Answer:** Option **(c)**: $X_L = -X_g$.

☞ Sign convention for X

By convention, inductive reactance is taken positive $(+\omega L)$ and capacitive reactance negative $(-1/\omega C)$. The "$-X_g$" in the answer means: if the source is net-inductive, the matched load must be net-capacitive.

Q 7.3 When a voltage measuring device is connected to AC mains, the meter shows the steady input voltage of 220 V. This means

(a) input voltage cannot be AC voltage, but a DC voltage.

(b) maximum input voltage is 220 V.

(c) the meter reads not v but $\langle v^2 \rangle$ and is calibrated to read $\sqrt{\langle v^2 \rangle}$.

(d) the pointer of the meter is stuck by some mechanical defect.

SOLUTION

Correct option: (c).

Concept used. An AC voltmeter cannot show the instantaneous voltage (which oscillates 100 times per second for 50 Hz mains); its needle has too much inertia. Instead, the meter responds to a thermal or rectified effect that averages v^2 over time. The reading is then converted to the **root-mean-square voltage**

$$V_{\text{rms}} = \sqrt{\langle v^2 \rangle},$$

which equals $V_m/\sqrt{2}$ for a sinusoidal supply.

Step 1. For sinusoidal mains, $v(t) = V_m \sin(\omega t)$. Its time-average over one cycle is zero, so a true "average voltmeter" would read 0.

Step 2. The squared voltage $v^2(t) = V_m^2 \sin^2(\omega t)$ has a non-zero average $\langle v^2 \rangle = V_m^2/2$.

Step 3. Taking the square root: $\sqrt{\langle v^2 \rangle} = V_m/\sqrt{2} \equiv V_{\text{rms}}$. The meter is internally calibrated so that the needle position labelled "220 V" corresponds to this rms value.

Step 4. Eliminating the other options:

- (a) is wrong: 220 V is a steady reading because rms is itself steady, not because the input is DC.
- (b) is wrong: 220 V is the rms value; the peak voltage is $V_m = 220\sqrt{2} \approx 311$ V.
- (d) is wrong: a steady reading is expected behaviour for an AC voltmeter, not a fault.

Final Answer: Option **(c)**: the meter reads $\sqrt{\langle v^2 \rangle}$, i.e. the rms voltage.

EXPERT'S SOLUTION *: Vivaan Rao, M.Sc Physics, IIT Madras*

Picture-first. Think of how a moving-iron meter works: deflection is proportional to current squared, not to instantaneous current. Averaging the squared signal naturally yields rms.

Step 1. In a moving-iron AC meter, the torque is proportional to i^2 (or v^2). Mechanical inertia averages this to $\langle i^2 \rangle$.

Step 2. The scale is engraved so that the deflection labelled V corresponds to $\sqrt{\langle v^2 \rangle}$, i.e. the rms voltage.

Step 3. For sinusoidal mains, $V_{\text{rms}} = V_m/\sqrt{2}$; the 220 V label hides a peak of ~ 311 V.

Step 4. **Why "true rms" digital meters exist.** A simple rectifier-type meter is calibrated assuming the input is a pure sine; on non-sinusoidal waveforms (e.g. chopped AC from a triac dimmer or the heavily distorted current drawn by a switched-mode power supply) the reading is wrong because $\langle|v|\rangle \neq \sqrt{\langle v^2\rangle}$ for non-sinusoids. A "true rms" meter actually computes $\sqrt{\langle v^2\rangle}$ digitally and gives the correct rms for any waveform.

Step 5. **Insulation note.** The 220 V label hides the design constraint: wall insulation and capacitor voltage ratings must withstand the peak ~ 311 V, not the rms 220 V. A 250 V-rated capacitor used on Indian mains will fail.

Why this matters. Quoting AC voltages and currents as rms makes Ohm's-law-like expressions ($P = V_{\text{rms}}\, I_{\text{rms}} \cos\varphi$) work the same way as in DC.

> **Final Answer:** Option **(c)**: rms reading.

☞ **Default convention**

Unless explicitly stated as "peak" or "maximum", every AC voltage or current value quoted in problems and on nameplates is the rms value.

Q 7.4 To reduce the resonant frequency in an LCR series circuit with a generator

(a) the generator frequency should be reduced.

(b) another capacitor should be added in parallel to the first.

(c) the iron core of the inductor should be removed.

(d) dielectric in the capacitor should be removed.

SOLUTION

Correct option: (b).

Concept used. In a series LCR circuit the **resonant angular frequency** is

$$\omega_0 = \frac{1}{\sqrt{LC}}, \qquad f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

The resonant frequency is a property of the circuit elements L and C alone; it does *not* depend on the generator's frequency. To *reduce* f_0, we must *increase* either L or C (or both).

Step 1. Examine each option and ask whether it changes L or C, and in which direction:

- (a) Changing the generator frequency changes the *driving* frequency, not the resonant frequency of the circuit. Wrong.
- (b) Adding a capacitor in parallel to the existing one: parallel capacitors add, so $C_{\text{new}} = C_1 + C_2 > C_1$. This *increases* C, which *decreases* $\omega_0 = 1/\sqrt{LC}$.

Correct.

- (c) Removing the iron core of the inductor reduces the relative permeability and hence reduces L. That *increases* ω_0. Wrong direction.
- (d) Removing the dielectric reduces the permittivity and hence reduces C. That also *increases* ω_0. Wrong direction.

Step 2. Conclusion: only (b) increases C, which lowers the resonant frequency.

> **Final Answer:** Option **(b)**: adding a capacitor in parallel increases C and so lowers f_0.

✗ Driving frequency vs natural frequency

The generator's frequency is a knob you turn; the resonant frequency is fixed by the circuit. Changing one does not change the other.

EXPERT'S SOLUTION *: Priya Mehta, M.Sc Physics, IIT Madras*

Strategic angle. Memorise $\omega_0 = 1/\sqrt{LC}$ and read every option through it.

Step 1. "Reduce f_0" ⇔ "increase LC".

Step 2. Among (a)–(d), only (b) (adding a parallel capacitor) achieves $C \to C_1 + C_2$, which makes LC bigger.

Step 3. Options (c) and (d) shrink L and C respectively; both *raise* f_0. Option (a) is about the source, not the circuit.

Step 4. **Quantitative feel.** If the original $C_1 = 10\ \mu$F and we add an equal $C_2 = 10\ \mu$F in parallel, $C_{\text{new}} = 20\ \mu$F. The resonant frequency drops by a factor $\sqrt{C_1/C_{\text{new}}} = 1/\sqrt{2} \approx 0.707$, a 29% reduction.

Step 5. **Topology check.** "In parallel" with the existing capacitor adds: $C_{\text{eff}} = C_1 + C_2$. If the question had said "in series", $1/C_{\text{eff}} = 1/C_1 + 1/C_2$ would *decrease* C_{eff} and *raise* f_0 – the opposite effect.

Why this matters. The same logic is used in radio tuners: a variable capacitor with a fixed inductor lets you slide the resonant frequency to pick stations.

> **Final Answer:** Option **(b)**.

Q 7.5 Which of the following combinations should be selected for better tuning of an LCR circuit used for communication?

(a) $R = 20\ \Omega$, $L = 1.5$ **H,** $C = 35\ \mu$**F.**
(b) $R = 25\ \Omega$, $L = 2.5$ **H,** $C = 45\ \mu$**F.**
(c) $R = 15\ \Omega$, $L = 3.5$ **H,** $C = 30\ \mu$**F.**
(d) $R = 25\ \Omega$, $L = 1.5$ **H,** $C = 45\ \mu$**F.**

SOLUTION

Correct option: (c) $R = 15\ \Omega$, $L = 3.5$ H, $C = 30\ \mu$F.
Concept used. "Better tuning" means the resonance curve must be *sharper*: only frequencies very close to ω_0 should produce a large current. The sharpness is measured by the **quality factor**

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

The higher the Q, the narrower the resonance peak. So we should pick the option that maximises Q.

Step 1. Compute Q for each option. Use $Q = (1/R)\sqrt{L/C}$ with C in farads.
(a) $\sqrt{L/C} = \sqrt{1.5/(35 \times 10^{-6})} = \sqrt{4.286 \times 10^4} \approx 207.0$. Then $Q = 207.0/20 \approx 10.4$.
(b) $\sqrt{L/C} = \sqrt{2.5/(45 \times 10^{-6})} = \sqrt{5.556 \times 10^4} \approx 235.7$. Then $Q = 235.7/25 \approx 9.4$.
(c) $\sqrt{L/C} = \sqrt{3.5/(30 \times 10^{-6})} = \sqrt{1.167 \times 10^5} \approx 341.6$. Then $Q = 341.6/15 \approx 22.8$.
(d) $\sqrt{L/C} = \sqrt{1.5/(45 \times 10^{-6})} = \sqrt{3.333 \times 10^4} \approx 182.6$. Then $Q = 182.6/25 \approx 7.3$.

Step 2. Comparing: $Q_c \approx 22.8 > Q_a \approx 10.4 > Q_b \approx 9.4 > Q_d \approx 7.3$. Option (c) wins by a wide margin.

Step 3. Physical intuition: option (c) has the smallest R and the largest L/C ratio, so dissipation is lowest and reactive "energy reservoir" is greatest.

Final Answer: Option **(c)**: $Q \approx 22.8$, the sharpest tuning.

☞ **Shortcut**
In any LCR-tuning MCQ, eyeball $L/(RC)$ proportionality. The option with both *small* R *and* large L relative to C wins.

EXPERT'S SOLUTION *: Arjun Verma, M.Sc Physics, IIT Madras*

Strategic angle. You don't need exact numbers, only the ranking of $L/(R^2C)$, since $Q^2 = L/(R^2C)$.

Step 1. Compare $L/(R^2C)$ across options:
(a) $1.5/(400 \cdot 35 \times 10^{-6}) = 107.1 \Rightarrow Q \approx 10.4$.
(b) $2.5/(625 \cdot 45 \times 10^{-6}) = 88.9 \Rightarrow Q \approx 9.4$.
(c) $3.5/(225 \cdot 30 \times 10^{-6}) = 518.5 \Rightarrow Q \approx 22.8$.
(d) $1.5/(625 \cdot 45 \times 10^{-6}) = 53.3 \Rightarrow Q \approx 7.3$.

Step 2. (c) is by far the largest.

Step 3. **Bandwidth corollary.** Each option's resonant angular frequency $\omega_0 = 1/\sqrt{LC}$ takes a different value, but the bandwidth is $\Delta\omega = \omega_0/Q = R/L$. For (c): $R/L = 15/3.5 \approx 4.29$ rad/s, the narrowest of the four. So (c) selects the smallest frequency window around resonance.

Step 4. **Energy interpretation.**
$Q = 2\pi \times$ (energy stored)/(energy dissipated per cycle). A high Q means the LCR circuit stores 22 times more energy in its reactive elements than it dissipates each cycle. That "ringing" is exactly what gives a sharp resonance.

Why this matters. A radio receiver must isolate a narrow band around a station's carrier frequency; a high-Q tank circuit is what makes that selectivity possible.

Final Answer: Option **(c)**.

☞ **Three equivalent Q expressions**

$Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{\omega_0 L}{R} = \dfrac{1}{\omega_0 RC}$. All three are equal at the resonant frequency $\omega_0 = 1/\sqrt{LC}$. Pick whichever has the cleanest given data.

Q 7.6 An inductor of reactance 1 Ω and a resistor of 2 Ω are connected in series to the terminals of a 6 V (rms) a.c. source. The power dissipated in the circuit is

(a) 8 W.
(b) 12 W.
(c) 14.4 W.
(d) 18 W.

SOLUTION

Correct option: (c) 14.4 W.

Concept used. Average power dissipated in an AC circuit equals the power dissipated in the resistive part only (inductors and capacitors do not consume average power). Thus

$$\bar{P} = I_{\text{rms}}^2\, R,$$

where $I_{\text{rms}} = V_{\text{rms}}/Z$ and $Z = \sqrt{R^2 + X_L^2}$ for an R–L series circuit.

Step 1. Compute the impedance:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{2^2 + 1^2} = \sqrt{5}\,\Omega.$$

Step 2. Compute the rms current:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{6}{\sqrt{5}}\text{ A}.$$

Step 3. Compute the average power:

$$\bar{P} = I_{\text{rms}}^2 R = \left(\frac{6}{\sqrt{5}}\right)^2 \times 2 = \frac{36}{5} \times 2 = \frac{72}{5} = 14.4\text{ W}.$$

Step 4. Sanity check: if there were no inductor, the power would have been $V_{\text{rms}}^2/R = 36/2 = 18$ W. Adding any reactance must *reduce* the current and hence the power; $14.4 < 18$ is consistent.

> **Final Answer:** Option **(c)**: $\bar{P} = 14.4$ W.

EXPERT'S SOLUTION *: Riya Banerjee, Ph.D Physics, IISc Bangalore*

Quick reading. For a series R–L, the only power-dissipating element is R.

Step 1. Total impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{5}\,\Omega$.

Step 2. rms current $I_{\text{rms}} = 6/\sqrt{5}$ A.

Step 3. Average power dissipated $= I_{\text{rms}}^2 R = (36/5)(2) = 14.4$ W.

Step 4. **Power-factor route.** $\cos\varphi = R/Z = 2/\sqrt{5}$. Then $\bar{P} = V_{\text{rms}} I_{\text{rms}} \cos\varphi = 6 \cdot (6/\sqrt{5}) \cdot (2/\sqrt{5}) = 72/5 = 14.4$ W. Same answer, different bookkeeping.

Step 5. **Phase angle.** $\tan\varphi = X_L/R = 1/2$, so $\varphi \approx 26.57°$. The current lags the voltage by about a quarter of a radian – modest, because the resistor still dominates.

Step 6. **Apparent vs. real power.** The apparent power is $S = V_{\text{rms}} I_{\text{rms}} = 6 \cdot 6/\sqrt{5} = 36/\sqrt{5} \approx 16.1$ VA, while the real (dissipated) power is 14.4 W. The 1.7 W gap circulates in and out of the inductor and is not consumed.

Why this matters. Equivalently, $\bar{P} = V_{\text{rms}} I_{\text{rms}} \cos\varphi$ with $\cos\varphi = R/Z = 2/\sqrt{5}$. Either route gives the same 14.4 W.

> **Final Answer:** Option **(c)**: 14.4 W.

✗ Don't use V^2/R blindly

Plugging $\bar{P} = V_{\text{rms}}^2/R = 36/2 = 18$ W gives the wrong answer because it ignores the inductor's contribution to the impedance. Always use Z first, find I_{rms}, then dissipate only in R.

Q 7.7 The output of a step-down transformer is measured to be 24 V when connected to a 12 watt light bulb. The value of the peak current is

(a) $1/\sqrt{2}$ **A.**

(b) $\sqrt{2}$ **A.**

(c) 2 A.

(d) $2\sqrt{2}$ **A.**

SOLUTION

Correct option: (a) $1/\sqrt{2}$ A.

Concept used. For a resistive load (an incandescent bulb is essentially resistive) at rms voltage V_{rms} drawing rms current I_{rms}, the average power is

$$\bar{P} = V_{\text{rms}} I_{\text{rms}}.$$

The **peak current** is related to the rms current by

$$I_m = \sqrt{2}\, I_{\text{rms}}.$$

Step 1. Find rms current from the power rating and rms voltage:

$$I_{\text{rms}} = \frac{\bar{P}}{V_{\text{rms}}} = \frac{12}{24} = 0.5 \text{ A} = \frac{1}{2} \text{ A}.$$

Step 2. Convert to peak current:

$$I_m = \sqrt{2}\, I_{\text{rms}} = \sqrt{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \text{ A}.$$

Step 3. Numerical: $1/\sqrt{2} \approx 0.707$ A.

Final Answer: Option **(a)**: $I_m = 1/\sqrt{2}$ A ≈ 0.707 A.

EXPERT'S SOLUTION : *Aanya Gupta, Ph.D Physics, IISc Bangalore*

Quick reading. Bulb is resistive, so power factor $= 1$. Two lines do it.

Step 1. $I_{\text{rms}} = P/V = 12/24 = 0.5$ A.

Step 2. Peak current $I_m = \sqrt{2} \cdot 0.5 = 1/\sqrt{2}$ A.

Step 3. **Unit check.** $[P]/[V] = \text{W}/\text{V} = (\text{V} \cdot \text{A})/\text{V} = \text{A}$. So $12\text{W}/24\text{V}$ is correctly 0.5 A. Then I_m inherits the same units.

Step 4. **Filament resistance.** For this bulb, $R = V_{\text{rms}}^2/P = 576/12 = 48\ \Omega$ (hot resistance at operating temperature). Cold resistance is typically 1/10 of this and is the reason filaments draw a brief surge current at switch-on.

Step 5. **Transformer turns ratio.** If the primary is at 240 V mains, the step-down ratio is $N_p/N_s = 240/24 = 10$. The primary current is then $I_p = I_s/10 = 0.05$ A (rms), peak $I_{p,m} = 0.05\sqrt{2} \approx 0.071$ A.

Why this matters. For a non-resistive load you would need $\cos\varphi$; here it is absent only because a filament bulb is purely resistive.

> **Final Answer:** Option **(a)**: $I_m = 1/\sqrt{2}$ A.

☞ **Power rating is real power**

The "12 W" on the bulb is the time-averaged power $\bar{P}$ – which for a resistive load equals $V_{\text{rms}} I_{\text{rms}}$. Never confuse it with peak power, which is twice this number.

MCQ II

Q 7.8 **As the frequency of an ac circuit increases, the current first increases and then decreases. What combination of circuit elements is most likely to comprise the circuit?**

(a) Inductor and capacitor.

(b) Resistor and inductor.

(c) Resistor and capacitor.

(d) Resistor, inductor and capacitor.

SOLUTION

Correct option: (d).

Concept used. In a series circuit driven by an AC source, the rms current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}, \qquad Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

The behaviour with frequency depends on what elements are present:

- Pure R: $Z = R$ is constant, so I does not change with ω.
- Series R–L: $Z = \sqrt{R^2 + \omega^2 L^2}$ grows monotonically with ω, so I *only decreases*.
- Series R–C: $Z = \sqrt{R^2 + 1/(\omega C)^2}$ shrinks monotonically as ω grows, so I *only increases*.
- Series R–L–C: Z first decreases (capacitor dominates), reaches a minimum $Z = R$ at **resonance** $\omega_0 = 1/\sqrt{LC}$, then increases (inductor dominates). Hence I *first increases, then decreases*.

Step 1. Match the observed pattern (rise then fall) with the four candidates. Only the series R–L–C combination has this peak-and-drop shape.

Step 2. Option (a), inductor and capacitor (no resistor), would show $|Z| = |\omega L - 1/(\omega C)|$. This drops to zero exactly at ω_0, where the current becomes infinite, then rises again; a singular sharp spike, not a smooth rise-and-fall.

Step 3. Option (b): R–L gives a monotonically decreasing I with ω. Wrong shape.

Step 4. Option (c): R–C gives a monotonically increasing I with ω. Wrong shape.

Step 5. Only option (d) reproduces the qualitative "rise to a peak, then fall" behaviour observed.

Final Answer: Option **(d)**: resistor, inductor and capacitor.

EXPERT'S SOLUTION *: Ananya Sharma, Ph.D Physics, IISc Bangalore*

Picture-first. The classic "rise-peak-fall" shape is the signature of a resonance.

Step 1. Resonance requires both an inductive reactance (ωL, growing with ω) and a capacitive reactance ($1/\omega C$, decreasing with ω).

Step 2. The peak occurs where the two reactances cancel, $\omega L = 1/\omega C$, i.e. $\omega_0 = 1/\sqrt{LC}$.

Step 3. A finite peak (no divergence) requires a non-zero resistance R, which sets the peak value to V_{rms}/R and limits the sharpness.

Step 4. **Eliminating distractors.** Option (a) is a pure LC tank without R; its current would diverge at ω_0, not rise smoothly. Options (b) and (c) lack one of the two reactive elements, so the impedance is monotonic in ω – no peak.

Step 5. **Symmetry of the curve.** On a log-frequency axis, $I(\omega)$ is symmetric about ω_0 (because $\omega_0/\omega \leftrightarrow \omega/\omega_0$ swaps X_L and X_C). On a linear axis the rise is steeper than the fall.

Why this matters. The same logic identifies any system with a single resonance: physical pendulums, LC oscillators, mass–spring–dashpot mechanical systems.

Final Answer: Option **(d)**.

☞ **Reactance directionality**

$X_L \propto \omega$ (grows linearly), $X_C \propto 1/\omega$ (decays as inverse). A peak in the current can only happen when both kinds of reactance fight each other, which forces the circuit to contain both L and C.

Q 7.9 In an alternating current circuit consisting of elements in series, the current increases on increasing the frequency of supply. Which of the following elements are likely to constitute the circuit?

(a) Only resistor.

(b) Resistor and an inductor.

(c) Resistor and a capacitor.

(d) Only a capacitor.

SOLUTION

Correct options: (c) and (d).

Concept used. The current rises with frequency only when the impedance *falls* with frequency. Among the basic elements:

- $X_R = R$ does not depend on ω.
- $X_L = \omega L$ *increases* with ω.
- $X_C = 1/(\omega C)$ *decreases* with ω.

So any circuit whose impedance is dominated by capacitive reactance will show I growing with ω.

Step 1. (a) Pure R: $I = V/R$ is independent of ω. Wrong.

Step 2. (b) Series R–L: $Z = \sqrt{R^2 + (\omega L)^2}$ grows with ω, so I falls. Wrong.

Step 3. (c) Series R–C: $Z = \sqrt{R^2 + 1/(\omega C)^2}$ shrinks as ω grows, so I rises. **Correct.**

Step 4. (d) Pure C: $Z = 1/(\omega C)$ shrinks with ω, so $I = V\omega C$ grows linearly. **Correct.**

Final Answer: Options **(c)** and **(d)**.

EXPERT'S SOLUTION : *Ishaan Pillai, Ph.D Physics, IISc Bangalore*

Quick reading. Only capacitive reactance falls with ω, so the circuit must contain C but no significant L.

Step 1. Exclude (a): a pure resistor's response is flat in frequency.

Step 2. Exclude (b): an inductor in series with R makes Z grow with ω.

Step 3. Keep (c): R–C gives $Z \to R$ as $\omega \to \infty$ from above, so I rises and asymptotes to V/R.

Step 4. Keep (d): pure C gives $I = V\omega C$, a linear rise.

Asymptote contrast.

- For (c), $I(\omega) = V_{\text{rms}}/\sqrt{R^2 + 1/(\omega C)^2}$. As $\omega \to \infty$, $I \to V_{\text{rms}}/R$, a finite ceiling.
- For (d), $I(\omega) = V_{\text{rms}}\omega C$. This grows without bound – a pure capacitor draws unlimited current at infinite frequency.

The shape of the curve thus distinguishes the two: (c) saturates, (d) keeps climbing.

Why this matters. Coupling capacitors in audio circuits "pass" high frequencies (low X_C) and block DC; the same principle is at work here.

Final Answer: Options **(c)**, **(d)**.

☞ **MCQ-II strategy**

For "which is/are correct" multi-answer questions, test each option against the same physical principle in sequence. Don't stop at the first correct option – multiple choices may be right.

Q 7.10 Electrical energy is transmitted over large distances at high alternating voltages. Which of the following statements is (are) correct?

(a) For a given power level, there is a lower current.

(b) Lower current implies less power loss.

(c) Transmission lines can be made thinner.

(d) It is easy to reduce the voltage at the receiving end using step-down transformers.

SOLUTION

Correct options: (a), (b) and (d).

Concept used. For a fixed power P to be delivered, the line current is $I = P/V$. So raising V lowers I. The power dissipated in the line resistance is $P_{\text{loss}} = I^2 R_{\text{line}}$. Therefore a low current means much smaller $I^2 R$ losses. Voltages are routinely stepped up at the generator and stepped down at the consumer using transformers.

Step 1. (a): With P fixed, $I = P/V$. Increasing V from say 220 V to 11 kV reduces I by a factor of 50. **Correct.**

Step 2. (b): The line dissipation is $P_{\text{loss}} = I^2 R_{\text{line}}$. A 50× smaller I gives $2500\times$ smaller losses. **Correct.**

Step 3. (c): Thinner wires have *larger* R_{line}, so naively one would lose more power. The real reason transmission wires are not made arbitrarily thin is mechanical strength, voltage breakdown and skin-effect resistance. So (c) is not the main reason high-V transmission is used. **Wrong.**

Step 4. (d): Transformers transform AC voltages efficiently. At the load end, a step-down transformer brings 11 kV (or 220 kV) back to the 220 V that homes use. This is a feature of AC, not of DC. **Correct.**

> **Final Answer:** Options **(a)**, **(b)**, **(d)**.

♥ Why we choose AC

The headline reason AC won over DC for long-distance power transmission is precisely point (d): efficient voltage stepping with transformers, which DC cannot match without electronic converters.

EXPERT'S SOLUTION *: Karan Reddy, M.Sc Physics, IIT Madras*

Strategic angle. Walk through each option and ask: "does this follow from $P = VI$ and $P_{\text{loss}} = I^2 R$?"

Step 1. (a) $I = P/V$. Higher $V \Rightarrow$ lower I. True.

Step 2. (b) $P_{\text{loss}} \propto I^2$. Lower $I \Rightarrow$ much lower P_{loss}. True.

Step 3. (c) Conductor thickness is set by mechanical and electrical constraints (sag, span, skin depth), not by "thinner is fine because current is low". False.

Step 4. (d) Step-down transformers are the easy, lossless way (in principle) to recover usable voltage from a high-tension line. True.

Step 5. **Quantitative scale.** A 50 MW industrial feeder at 11 kV draws 4.5 kA, dropping 4.5 kV across a 1 Ω line and dissipating ~ 20 MW in heat. The same feeder at 220 kV draws 227 A, drops 227 V across the line, and dissipates only ~ 51 kW. A factor of 20 in voltage buys a factor of 400 in efficiency.

Step 6. **Why not arbitrarily high V?** Practical limits are insulator breakdown (air gaps must hold off the peak voltage), tower height (clearance to ground rises with V) and corona losses (above ~ 400 kV in air). India's HVDC links push 800 kV; transmission HV-AC sits at 220–765 kV.

Why this matters. The current–squared dependence is why I^2R losses scale so dramatically with V. A factor of 10 in V buys a factor of 100 in efficiency.

Final Answer: Options **(a)**, **(b)**, **(d)**.

Q 7.11 For an LCR circuit, the power transferred from the driving source to the driven oscillator is $P = I^2 Z \cos\varphi$.

(a) Here, the power factor $\cos\varphi \geq 0$, $P \geq 0$.

(b) The driving force can give no energy to the oscillator ($P = 0$) in some cases.

(c) The driving force cannot syphon out ($P < 0$) the energy out of oscillator.

(d) The driving force can take away energy out of the oscillator.

SOLUTION

Correct options: (a), (b) and (c).

Concept used. In a steady-state, sinusoidally driven LCR series circuit, the phase angle φ between the source voltage and the resulting current is defined by

$$\tan\varphi = \frac{X_L - X_C}{R} = \frac{\omega L - 1/(\omega C)}{R}.$$

Since $R > 0$ and $Z \geq R > 0$, we have $\cos\varphi = R/Z \in (0, 1]$. Therefore the average power delivered by the source to the circuit,

$$\bar{P} = V_{\text{rms}}\, I_{\text{rms}}\, \cos\varphi = I^2_{\text{rms}}\, Z \cos\varphi,$$

is always ≥ 0.

Step 1. Analyse (a): Because $\cos\varphi = R/Z$ and both $R, Z > 0$, $\cos\varphi$ is strictly positive. Hence $P \geq 0$. **Correct.**

Step 2. Analyse (b): $\cos\varphi = 0$ only when $R = 0$, i.e. for a pure LC circuit. In that ideal limit no energy is transferred from source to oscillator on average; the energy just sloshes between L and C. **Correct.** (More carefully: for finite but very small R the steady-state average power is positive but tiny.)

Step 3. Analyse (c): Since $\cos\varphi \geq 0$, P cannot be negative. The source therefore cannot remove energy on average from the oscillator. **Correct.**

Step 4. Analyse (d): This contradicts (c). If $P < 0$ were possible, the driving force would drain the oscillator; but this never happens in steady state. **Wrong.**

Final Answer: Options **(a)**, **(b)**, **(c)**.

EXPERT'S SOLUTION *: Krishna Joshi, Ph.D Physics, IISc Bangalore*

Structural angle. The sign of $\cos\varphi$ is fixed by R alone, and $R \geq 0$ always.

Step 1. Write $\cos\varphi = R/Z \in [0, 1]$. So $\bar{P} = I^2R \geq 0$ always.

Step 2. Equality holds only when $R = 0$ (a pure LC oscillator), where the source does no net work in steady state, confirming (b).

Step 3. Because $\bar{P}$ never goes negative in steady state, (c) is right and (d) is wrong.

Step 4. **Why the formula already encodes this.** $P = I^2Z\cos\varphi = I^2 \cdot Z \cdot (R/Z) = I^2R$. So the power formula collapses to "current-squared times resistance", a manifestly non-negative quantity.

Step 5. **When could a power-extractor exist?** Only if the load were *active* (an EMF of its own, like a battery being charged backwards). Passive elements – R, L, C alone – can never give back more than they take.

Why this matters. The fact that an AC source can only *do* work, not extract it, in steady state is what makes power-station billing possible: the meter only adds up.

Final Answer: Options **(a)**, **(b)**, **(c)**.

☞ Active vs. passive load

A passive load (made only of R, L, C) always has $\cos\varphi \geq 0$. An active load (with its own EMF, like a back-driving motor) can produce $\cos\varphi < 0$, meaning it pumps power back into the line.

Q 7.12 **When an AC voltage of 220 V is applied to the capacitor C**

(a) the maximum voltage between plates is 220 V.

(b) the current is in phase with the applied voltage.

(c) the charge on the plates is in phase with the applied voltage.

(d) power delivered to the capacitor is zero.

SOLUTION

Correct options: (c) and (d).

Concept used. For an ideal capacitor across a sinusoidal supply $v(t) = V_m \sin \omega t$:

- The charge on the plates is $q(t) = C\,v(t) = CV_m \sin \omega t$, so q is exactly in phase with v.
- The current is $i(t) = dq/dt = CV_m\omega \cos \omega t = I_m \sin(\omega t + \pi/2)$, so i *leads* v by $90°$.
- Average power $\bar{P} = V_{\rm rms} I_{\rm rms} \cos \varphi$ with $\varphi = \pi/2$, so $\cos \varphi = 0$ and $\bar{P} = 0$.

The quoted "220 V" is the rms value, not the maximum.

Step 1. (a): The maximum voltage between plates is $V_m = \sqrt{2} \cdot 220 \approx 311$ V, not 220 V. **Wrong.**

Step 2. (b): i leads v by $\pi/2$ in a pure capacitor; they are not in phase. **Wrong.**

Step 3. (c): From $q = Cv$, charge and voltage are in phase at every instant. **Correct.**

Step 4. (d): A pure capacitor stores energy on one quarter-cycle and returns it on the next. The time-average of vi is zero. **Correct.**

> **Final Answer:** Options **(c)**, **(d)**.

EXPERT'S SOLUTION : *Dev Nair, M.Sc Physics, IIT Madras*

Quick reading. "220 V" on AC is always rms; "in phase" rules out current but lets through charge.

Step 1. Charge follows voltage: $q(t) = Cv(t)$, so (c) is correct.

Step 2. Current is the derivative of charge: $i = C\,dv/dt$, hence i leads v by $\pi/2$, so (b) is wrong.

Step 3. Average power $\propto \cos \varphi = \cos(\pi/2) = 0$, so (d) is correct.

Step 4. "Maximum voltage" is $V_m = V_{\rm rms}\sqrt{2} \approx 311$ V, so (a) is wrong.

Step 5. **Phasor diagram.** Draw $\vec{V}$ along the $+x$ axis. The charge phasor $\vec{Q}$ also lies along $+x$ (in phase). The current phasor $\vec{I}$, being $j\omega C\vec{V}$, points along $+y$ – exactly $90°$ ahead of $\vec{V}$. The instantaneous power $p = vi$ then oscillates as $\sin \omega t \cos \omega t = \frac{1}{2} \sin 2\omega t$, averaging to zero.

Step 6. **Energy ledger.** During the first quarter-cycle the capacitor absorbs energy $\frac{1}{2}CV_m^2$ from the source; during the next quarter, it returns the same energy. The energy "sloshes" twice per cycle, but no net work is done.

Why this matters. A capacitor is a *wattless* element under AC: it shuttles energy back and forth without consuming any.

> **Final Answer:** Options **(c)**, **(d)**.

☞ **ICE and ELI rule**

For a pure capacitor: **I** leads **C** leads **E** (ICE). For a pure inductor: **E** leads **L** leads **I** (ELI). Memorise "ELI the ICE man" and never mix up which lead which.

Q 7.13 The line that draws power supply to your house from street has

(a) zero average current.

(b) 220 V average voltage.

(c) voltage and current out of phase by $90°$.

(d) voltage and current possibly differing in phase φ such that $|\varphi| < \frac{\pi}{2}$.

SOLUTION

Correct options: (a) and (d).

Concept used. Domestic mains is sinusoidal AC: $v(t) = V_m \sin \omega t$, $i(t) = I_m \sin(\omega t - \varphi)$. The phase φ is set by the load. The household load is a **mixed** load (resistive bulbs, inductive motors, capacitive electronics), so φ lies strictly between $-\pi/2$ and $+\pi/2$. The quoted "220 V" is the rms voltage, not the average voltage.

Step 1. (a): The time-average of $\sin \omega t$ over one cycle is zero. So the average current is exactly 0. **Correct.**

Step 2. (b): The time-average of $v(t)$ is also 0 (not 220 V). The number 220 V is V_{rms}. **Wrong.**

Step 3. (c): $\varphi = 90°$ would require a purely reactive load (no resistance). Real household loads always include some resistance, so $|\varphi| < \pi/2$. **Wrong.**

Step 4. (d): For any combination of R, L, C with $R > 0$, $\cos \varphi = R/Z > 0$ implies $|\varphi| < \pi/2$. **Correct.**

Final Answer: Options **(a)**, **(d)**.

EXPERT'S SOLUTION : *Tara Desai, M.Sc Physics, IIT Madras*

Quick reading. "Average voltage" and "average current" over a complete sinusoid are both zero. "220 V" is rms, not average.

Step 1. Mean of $\sin \omega t$ over a full period is 0, so (a) is right and (b) is wrong.

Step 2. A real domestic load always includes some pure resistance (filaments, heaters); the phase φ cannot reach $\pm\pi/2$. So (c) is wrong, (d) is right.

Step 3. **Half-wave average.** The mean of $|\sin \omega t|$ over a full cycle is $2/\pi \approx 0.637$. So if a problem asks for "average voltage of the rectified mains", the answer is

$0.637\,V_m = 0.9\,V_{\text{rms}}$, not zero. Read the question carefully.

Step 4. **Typical Indian household $\cos\varphi$.** Mixed lighting plus motors gives a typical domestic power factor of 0.7–0.9 lagging. Utilities penalise large industrial consumers whose $\cos\varphi$ falls below 0.85.

Step 5. **Symmetry check.** For φ to be *exactly* $\pm\pi/2$ requires $R = 0$. Since real wires and bulbs have non-zero resistance, $|\varphi| < \pi/2$ always.

Why this matters. "Average current" vs. "rms current" is the most common AC vocabulary trap. Always pause and identify which one a question is asking for.

> **Final Answer:** Options **(a)**, **(d)**.

✗ "Average" ≠ "rms"

A common trap: assume that "220 V average" is the same as "220 V rms". They are not. The true time-average over a full sinusoidal cycle is zero; the rms is $V_m/\sqrt{2}$. Mains specifications always quote rms.

VSA

Q 7.14 **If an LC circuit is considered analogous to a harmonically oscillating spring–block system, which energy of the LC circuit would be analogous to potential energy and which one analogous to kinetic energy?**

SOLUTION

Concept used. In a mass–spring oscillator, the kinetic energy $\frac{1}{2}m\dot{x}^2$ is associated with motion (the velocity $\dot{x}$), and the potential energy $\frac{1}{2}kx^2$ is associated with displacement (the position x). In an LC circuit the analogous quantities are

$$q \longleftrightarrow x, \qquad i = \dot{q} \longleftrightarrow \dot{x}, \qquad \frac{1}{C} \longleftrightarrow k, \qquad L \longleftrightarrow m.$$

So inductive energy plays the role of kinetic energy and capacitive energy plays the role of potential energy.

Step 1. Write the two energies for the LC circuit:

$$U_C = \frac{q^2}{2C} \quad \text{(stored in the capacitor)}, \qquad U_L = \frac{1}{2}Li^2 \quad \text{(stored in the inductor)}.$$

Step 2. Compare term by term with $\frac{1}{2}kx^2$ and $\frac{1}{2}m\dot{x}^2$:

$$U_C = \frac{1}{2}\left(\frac{1}{C}\right)q^2 \leftrightarrow \frac{1}{2}kx^2 \text{ (PE)}, \qquad U_L = \frac{1}{2}L\,i^2 \leftrightarrow \frac{1}{2}m\dot{x}^2 \text{ (KE)}.$$

Step 3. Therefore the capacitor's electric-field energy is analogous to spring potential energy, and the inductor's magnetic-field energy is analogous to the block's kinetic energy.

> **Final Answer:** $U_C = q^2/2C$ acts like PE, $U_L = Li^2/2$ acts like KE.

EXPERT'S SOLUTION *: Aditi Bhat, M.Sc Physics, IIT Madras*

Quick reading. Charge mirrors position, current mirrors velocity. Energies follow automatically.

Step 1. Spring PE ($\propto x^2$) lines up with capacitor energy ($\propto q^2$).

Step 2. Mass KE ($\propto \dot{x}^2$) lines up with inductor energy ($\propto i^2$).

Step 3. **Period check.** Spring–mass oscillates at $\omega = \sqrt{k/m}$; LC circuit oscillates at $\omega_0 = \sqrt{(1/C)/L} = 1/\sqrt{LC}$. The form is identical: stiffness divided by inertia.

Step 4. **Maximum–zero crossing.** When the spring is fully stretched ($x = x_{\text{max}}$, KE $= 0$, PE is maximum), the analogous LC state has the capacitor fully charged ($q = q_{\text{max}}$, $i = 0$, U_C is maximum). A quarter-cycle later, the block races past equilibrium ($x = 0$, KE maximum) just as i peaks in the inductor.

Step 5. **Energy exchange.** The total $U_C + U_L =$ constant in an ideal LC, mirroring the total mechanical KE + PE being constant in a frictionless spring–mass.

Why this matters. This analogy is why the LC oscillator equation $L\ddot{q} + q/C = 0$ has the same sinusoidal solutions as $m\ddot{x} + kx = 0$.

> **Final Answer:** Capacitor energy $\leftrightarrow$ PE; inductor energy $\leftrightarrow$ KE.

☞ **Mechanical $\leftrightarrow$ Electrical map**

$m \leftrightarrow L$, $k \leftrightarrow 1/C$, $x \leftrightarrow q$, $\dot{x} \leftrightarrow i$, $b \leftrightarrow R$ (damping). Every formula from SHM transfers directly to LCR.

Q 7.15 **Draw the effective equivalent circuit of the circuit shown in Fig 7.1, at very high frequencies and find the effective impedance.**

SOLUTION

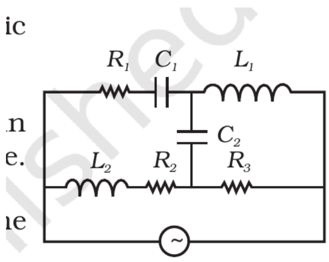


Fig. 7.1

Fig. 7.1, NCERT Exemplar Class 12 Physics, Chapter 7.

Concept used. For an inductor $X_L = \omega L$ and for a capacitor $X_C = 1/(\omega C)$. In the high-frequency limit ($\omega \to \infty$):

- Every inductor behaves like an *open circuit* (its reactance is huge).
- Every capacitor behaves like a *short circuit* (its reactance is negligible).

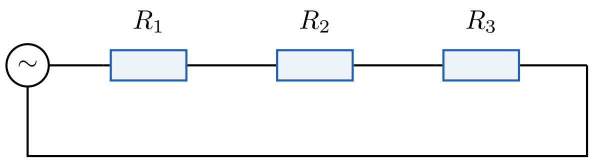
Replacing L_1, L_2 with open breaks, and C_1, C_2 with short pieces of wire, simplifies the network drastically.

Step 1. Top arm $R_1 – C_1 – L_1$: the L_1 is an open, so no current flows through the top arm.

Step 2. Bottom arm $L_2 – R_2 – R_3$: the L_2 is an open, so the left end of the bottom arm is disconnected from the source's left terminal.

Step 3. The remaining conducting path from the source's left terminal to its right terminal is: through R_1, through the shorted C_1 (zero impedance), to the central node, through the shorted C_2 (zero impedance) to the bottom arm, through R_2, then R_3, back to the source.

Step 4. Equivalent circuit therefore reduces to R_1, R_2 and R_3 in series across the source. Inductors are removed (open) and capacitors collapse to wires.



High-frequency equivalent

Final Answer: $Z_{\text{eff}} = R_1 + R_2 + R_3$.

EXPERT'S SOLUTION *: Rohit Singh, M.Sc Physics, IIT Madras*

Quick reading. "Very high frequency" is a one-line filter: $L \to$ open, $C \to$ short.

Step 1. Open every L, short every C on the schematic.

Step 2. Trace the only surviving path from one source terminal to the other.

Step 3. Read off the series sum of resistances along that path.

Step 4. **Numerical sense.** If $L = 10$ mH and $\omega = 10^7$ rad/s, $X_L = \omega L = 10^5\ \Omega$, vastly larger than any resistor in the circuit. If $C = 1\ \mu$F at the same ω, $X_C = 1/(\omega C) = 0.1\ \Omega$, vastly smaller. The asymptotic limits are reached fast.

Step 5. **Mirror trick for low frequencies.** The opposite limit $\omega \to 0$ converts capacitors to opens and inductors to shorts. For *this* circuit it would isolate the top arm completely (because C_1 blocks DC), leaving only the bottom arm L_2–R_2–R_3 in circuit. Hence $Z_{\text{low}} = R_2 + R_3$.

Why this matters. The same trick lets you sketch the low-frequency limit too ($L \to$ short, $C \to$ open) without doing any algebra.

> **Final Answer:** $Z_{\text{eff}} = R_1 + R_2 + R_3$.

☞ Mnemonic for asymptotic limits

"Capacitors block DC, pass AC; inductors pass DC, block AC." At $\omega \to \infty$: C is a wire and L is a break. At $\omega \to 0$: C is a break and L is a wire.

Q 7.16 **Study the circuits (a) and (b) shown in Fig 7.2 and answer the following questions.**

(a) Under which conditions would the rms currents in the two circuits be the same?

(b) Can the rms current in circuit (b) be larger than that in (a)?

SOLUTION

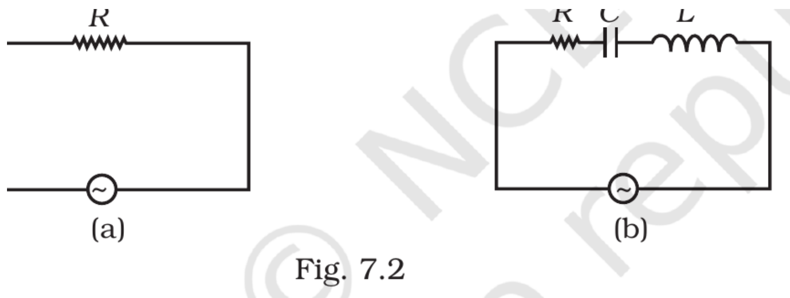


Fig. 7.2, NCERT Exemplar Class 12 Physics, Chapter 7.

Concept used. In circuit (a) only a resistor is present, so the impedance is $Z_a = R$ and the rms current is

$$I_a = \frac{V_{\text{rms}}}{R}.$$

In circuit (b) we have R, C and L in series, so

$$Z_b = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}, \qquad I_b = \frac{V_{\text{rms}}}{Z_b}.$$

Since $Z_b^2 = R^2 + (X_L - X_C)^2 \geq R^2$, always $Z_b \geq R$ and so $I_b \leq I_a$.

Step 1. Part (a), when are the currents equal? $I_a = I_b$ requires $Z_b = R$, which forces

$$\omega L - \frac{1}{\omega C} = 0 \quad \Rightarrow \quad \omega = \omega_0 = \frac{1}{\sqrt{LC}}.$$

That is, the source must drive the circuit at its **resonance** frequency.

Step 2. Part (b), can $I_b > I_a$? Since $Z_b \geq R = Z_a$ for every ω, the current in (b) is at most equal to that in (a). It can never exceed it.

> **Final Answer:** (a) At resonance, $\omega = 1/\sqrt{LC}$. (b) No, I_b can never exceed I_a.

EXPERT'S SOLUTION : *Pooja Chatterjee, M.Sc Physics, IIT Madras*

Structural angle. R is a lower bound on impedance for any series LCR.

Step 1. $Z_b^2 = R^2 + (X_L - X_C)^2 \geq R^2$ with equality at resonance.

Step 2. Hence $I_b \leq I_a$, and the equality holds at $\omega = \omega_0 = 1/\sqrt{LC}$.

Step 3. **What happens off-resonance.** For $\omega \neq \omega_0$, the reactive term $(X_L - X_C)^2$ is strictly positive; $Z_b > R$ and $I_b < I_a$. The further ω is from ω_0, the larger the gap.

Step 4. **Phasor picture.** In circuit (b), the resistor voltage $V_R = IR$ is in phase with I, while the net reactive voltage $V_X = I(X_L - X_C)$ is at $90°$ to I. The source voltage is the hypotenuse: $V^2 = V_R^2 + V_X^2 \geq V_R^2$, so the same V delivers less V_R and hence less I in (b) than in (a).

Why this matters. The resonance condition is the only way to "hide" L and C from the current.

> **Final Answer:** (a) $\omega = 1/\sqrt{LC}$; (b) no.

☞ **Three resonance signatures**

At series resonance: (i) $X_L = X_C$, (ii) Z reaches its minimum $Z = R$, (iii) I reaches its maximum

V/R, and (iv) voltage and current are in phase ($\varphi = 0$).

Q 7.17 **Can the instantaneous power output of an ac source ever be negative? Can the average power output be negative?**

SOLUTION

Concept used. The **instantaneous power** delivered by an AC source is $p(t) = v(t)\, i(t)$. With $v = V_m \sin \omega t$ and $i = I_m \sin(\omega t - \varphi)$,

$$p(t) = V_m I_m \sin \omega t \, \sin(\omega t - \varphi) = \tfrac{1}{2} V_m I_m [\cos \varphi - \cos(2\omega t - \varphi)]\,.$$

The **average power** over a full cycle is

$$\bar{P} = \frac{1}{T}\int_0^T p(t)\, dt = \tfrac{1}{2} V_m I_m \cos \varphi = V_{\text{rms}} I_{\text{rms}} \cos \varphi.$$

Step 1. Instantaneous: $p(t)$ has an oscillating term $\cos(2\omega t - \varphi)$ whose amplitude is $\frac{1}{2} V_m I_m$. During parts of the cycle this term exceeds the constant $\frac{1}{2} V_m I_m \cos \varphi$, so $p(t) < 0$ during those intervals. Physically, this is the time when the inductor/capacitor returns energy to the source. So **yes**, instantaneous power can be negative.

Step 2. Average: For a passive load the phase φ satisfies $\cos \varphi = R/Z \geq 0$, so $\bar{P} \geq 0$. The source delivers (or at worst, breaks even with) the load over a full cycle. So **no**, average power cannot be negative for any passive AC circuit.

> **Final Answer:** Instantaneous power: **yes**, can be negative; average power: **no**, cannot be negative.

EXPERT'S SOLUTION *: Yash Gupta, Ph.D Physics, IISc Bangalore*

Quick reading. Instantaneous sign tracks vi; the average is locked to the power factor.

Step 1. Half a cycle out of every two, v and i have opposite signs in any non-resistive AC circuit, so $p(t) = vi < 0$ there.

Step 2. The full-cycle average reduces to $V_{\text{rms}} I_{\text{rms}} \cos \varphi$. For passive loads $\cos \varphi \geq 0$, so $\bar{P} \geq 0$.

Step 3. **Worked decomposition.** Take a pure inductor: $v = V_m \sin \omega t$, $i = (V_m/\omega L) \sin(\omega t - \pi/2) = -(V_m/\omega L) \cos \omega t$. Then $p(t) = -(V_m^2/\omega L) \sin \omega t \cos \omega t = -(V_m^2/2\omega L) \sin 2\omega t$. This is positive over half of each half-cycle and negative over the other half; the integral over T is zero.

Step 4. **Time fraction with $p < 0$.** For a general φ, $p(t) = \frac{1}{2}V_m I_m[\cos\varphi - \cos(2\omega t - \varphi)]$. The negative excursions take a smaller fraction of the cycle as $\cos\varphi$ grows. For a pure resistor ($\varphi = 0$), p is always ≥ 0. For a pure reactance ($\varphi = \pm\pi/2$), half of every cycle has $p < 0$.

Why this matters. Negative $p(t)$ is the moment a capacitor or inductor returns energy to the source; but it can never give back more than it took.

Final Answer: Instantaneous: yes; average: no.

☞ Sign cue

"Instantaneous" invites a yes-it-can-be-negative answer; "average" invites no-it-cannot. Read which word appears in the question stem before answering.

Q 7.18 **In series LCR circuit, the plot of I_{max} vs ω is shown in Fig 7.3. Find the bandwidth and mark in the figure.**

SOLUTION

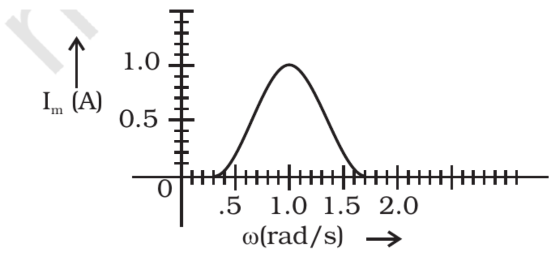


Fig. 7.3

Fig. 7.3, NCERT Exemplar Class 12 Physics, Chapter 7.

Concept used. The **bandwidth** $\Delta\omega$ of a resonance curve is defined as the difference between the two frequencies $\omega_1 < \omega_0 < \omega_2$ at which the current drops to $I_{max}/\sqrt{2}$ (the **half-power** points, since power $\propto I^2$ drops to $\frac{1}{2}$ at those frequencies). Then

$$\Delta\omega = \omega_2 - \omega_1.$$

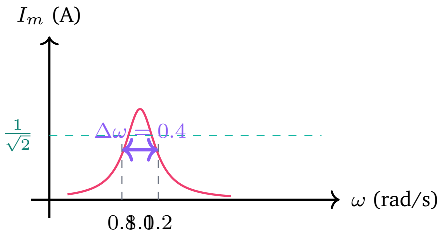
Step 1. Read the peak current from Fig. 7.3: $I_{max} = 1.0$ A at $\omega_0 = 1.0$ rad/s.

Step 2. Half-power level: $I_{max}/\sqrt{2} = 1.0/\sqrt{2} \approx 0.707$ A.

Step 3. Drop horizontally from this I value onto the curve and read off the two intersection points: $\omega_1 \approx 0.8$ rad/s on the left and $\omega_2 \approx 1.2$ rad/s on the right.

Step 4. Bandwidth:

$$\Delta\omega = \omega_2 - \omega_1 = 1.2 - 0.8 = 0.4 \text{ rad/s}.$$



Final Answer: $\Delta\omega \approx 0.4$ rad/s (the band between $\omega_1 \approx 0.8$ and $\omega_2 \approx 1.2$ rad/s).

EXPERT'S SOLUTION *: Aditya Verma, M.Sc Physics, IIT Madras*

Picture-first. The bandwidth is the width of the resonance peak at $I_{\text{max}}/\sqrt{2}$.

Step 1. Identify peak: $I_{\text{max}} = 1.0$ A at $\omega_0 = 1.0$ rad/s.

Step 2. Cut horizontally at $1.0/\sqrt{2} \approx 0.707$ A.

Step 3. The intersections lie at $\omega_1 \approx 0.8$ and $\omega_2 \approx 1.2$.

Step 4. Bandwidth $\Delta\omega = \omega_2 - \omega_1 = 0.4$ rad/s.

Step 5. **Q-factor extraction.** $Q = \omega_0/\Delta\omega = 1.0/0.4 = 2.5$. With $Q = (\omega_0 L)/R$, if $L = 1$ H and $\omega_0 = 1$ rad/s then $R = \omega_0 L/Q = 1/2.5 = 0.4\ \Omega$.

Step 6. **Why $1/\sqrt{2}$.** The power dissipated is $P = I^2 R$, so the half-power points sit where P drops to $\frac{1}{2}P_{\text{max}}$, i.e. $I^2 = \frac{1}{2}I_{\text{max}}^2$, i.e. $I = I_{\text{max}}/\sqrt{2}$. The convention is set by power, not amplitude.

Why this matters. Bandwidth $\Delta\omega$, the quality factor Q and the resonance frequency ω_0 are tied by $Q = \omega_0/\Delta\omega$. Here $Q \approx 2.5$.

Final Answer: $\Delta\omega \approx 0.4$ rad/s.

♥ Tuning a radio

$Q \approx 2.5$ would be far too poor for an FM receiver, where stations sit 200 kHz apart at ~ 100 MHz: you need $Q \sim 500$ to separate them. AC theory and electromagnetic design meet at the antenna stage.

Q 7.19 **The alternating current in a circuit is described by the graph shown in Fig 7.4. Show rms current in this graph.**

SOLUTION

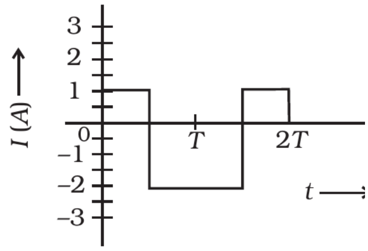


Fig. 7.4

Fig. 7.4, NCERT Exemplar Class 12 Physics, Chapter 7.

Concept used. The **rms current** is defined by

$$I_{\text{rms}} = \sqrt{\langle i^2 \rangle} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}.$$

For a piecewise-constant waveform that takes value i_k for a fraction $f_k = t_k/T$ of the period (with $\sum f_k = 1$), this simplifies to

$$I_{\text{rms}} = \sqrt{\sum_k f_k\, i_k^2}.$$

Step 1. Read Fig. 7.4: the current is $+1$ A for half the period and -2 A for the other half. So $i_1 = +1$ A with $f_1 = 1/2$, and $i_2 = -2$ A with $f_2 = 1/2$.

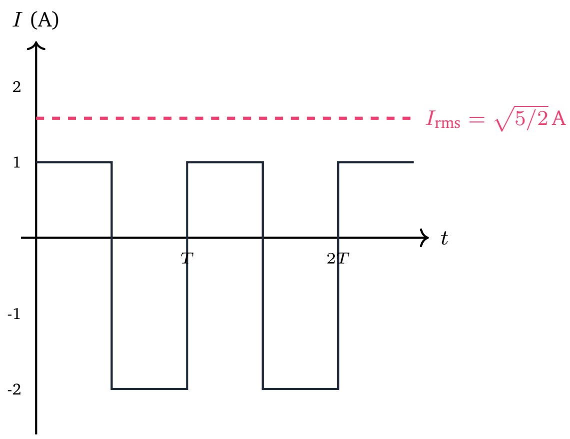
Step 2. Mean of i^2:

$$\langle i^2 \rangle = \tfrac{1}{2}(1)^2 + \tfrac{1}{2}(-2)^2 = \tfrac{1}{2} + \tfrac{1}{2}\cdot 4 = \tfrac{1}{2} + 2 = \tfrac{5}{2}.$$

Step 3. Take the square root:

$$I_{\text{rms}} = \sqrt{\tfrac{5}{2}} = \sqrt{2.5} \approx 1.58 \text{ A}.$$

Step 4. Marking the result: draw a horizontal line at $I = +1.58$ A across the entire t-axis on Fig. 7.4 (and a symmetric one at -1.58 A would also represent the same rms value, since rms is sign-blind).



Final Answer: $I_{\text{rms}} = \sqrt{5/2}$ A ≈ 1.58 A, marked as a horizontal dashed line on Fig. 7.4.

EXPERT'S SOLUTION : *Aditi Joshi, M.Sc Physics, IIT Madras*

Strategic angle. For any piecewise-constant current, take the time-weighted average of i^2, then take the square root.

Step 1. Weighted mean of i^2: $\frac{1}{2}(1)^2 + \frac{1}{2}(2)^2 = 5/2$.

Step 2. $I_{\text{rms}} = \sqrt{5/2} \approx 1.58$ A.

Step 3. **Average vs. rms.** Average current: $\bar{i} = \frac{1}{2}(+1) + \frac{1}{2}(-2) = -0.5$ A. Rms: 1.58 A. Average tells you net charge transferred; rms tells you heating effect.

Step 4. **Sign-blind heating.** Because i^2 ignores the sign of i, the -2 A interval and a $+2$ A interval contribute equally to rms. So the same load would dissipate $1.58^2R = 2.5R$ watts whether the negative excursion were -2 A or $+2$ A.

Step 5. **Sinusoid contrast.** A sine wave of amplitude A has rms $A/\sqrt{2} \approx 0.707\,A$. A square wave of amplitude A has rms exactly A. The waveform shape, not just the amplitude, decides rms.

Why this matters. The rms of a non-sinusoidal current is *not* just amplitude/$\sqrt{2}$; you must integrate the actual waveform.

Final Answer: $I_{\text{rms}} = \sqrt{5/2}$ A.

✗ Don't divide by $\sqrt{2}$ blindly

A common error: divide the larger peak (-2 A) by $\sqrt{2}$ to get 1.414 A. That formula is only valid for a *sinusoid*. For piecewise waveforms, integrate i^2 instead.

Q 7.20 **How does the sign of the phase angle φ, by which the supply voltage leads the current in an LCR series circuit, change as the supply frequency is gradually increased from very low to very high values?**

SOLUTION

Concept used. In a series LCR circuit driven by an AC source, the source voltage leads the current by an angle φ given by

$$\tan\varphi = \frac{X_L - X_C}{R} = \frac{\omega L - 1/(\omega C)}{R}.$$

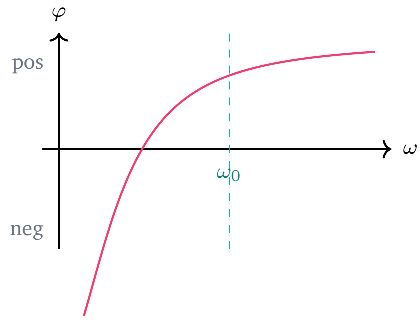
The sign of φ depends on the relative size of X_L and X_C, which themselves depend on ω.

Step 1. Very low ω: $X_L = \omega L$ is small, $X_C = 1/(\omega C)$ is large. So $X_L - X_C < 0$, giving $\tan\varphi < 0$, i.e. φ is **negative** (the voltage actually *lags* the current; equivalently the current leads the voltage).

Step 2. At resonance, $\omega = \omega_0 = 1/\sqrt{LC}$: $X_L = X_C$, so $\tan\varphi = 0$ and $\varphi = 0$. Voltage and current are in phase.

Step 3. Very high ω: X_L is large, X_C is small. So $X_L - X_C > 0$, $\tan\varphi > 0$, and φ is **positive** (voltage genuinely leads current).

Step 4. As ω increases from very low to very high, φ moves continuously from a negative value, through 0 at ω_0, to a positive value.



Final Answer: φ starts **negative** at low ω, passes through $\varphi = 0$ at the resonance frequency $\omega_0 = 1/\sqrt{LC}$, then becomes **positive** at high ω.

EXPERT'S SOLUTION : *Sanya Reddy, M.Sc Physics, IIT Madras*

Quick reading. The sign of φ tracks the sign of $X_L - X_C$.

Step 1. Low ω: X_C big, circuit looks capacitive, $\varphi < 0$.

Step 2. At ω_0: $X_L = X_C$, $\varphi = 0$.

Step 3. High ω: X_L big, circuit looks inductive, $\varphi > 0$.

Step 4. **Asymptotic limits.** As $\omega \to 0$, $X_C \to \infty$ so $\tan\varphi \to -\infty$ and $\varphi \to -\pi/2$. As $\omega \to \infty$, $X_L \to \infty$ so $\tan\varphi \to +\infty$ and $\varphi \to +\pi/2$. The full range of φ is the open interval $(-\pi/2, +\pi/2)$.

Step 5. **Power factor consequence.** $\cos\varphi$ peaks at 1 exactly at ω_0 (maximum power transfer), and drops symmetrically on both sides. Capacitive operation is signalled by leading current; inductive operation by lagging current. The names "leading" and "lagging" refer to the current relative to the voltage.

Why this matters. The same crossing tells you the circuit's character: capacitive below resonance, inductive above.

> **Final Answer:** Negative below ω_0, zero at ω_0, positive above ω_0.

☞ Sign convention

$\varphi > 0$: voltage leads current (inductive, ELI). $\varphi < 0$: voltage lags current (capacitive, ICE). $\varphi = 0$: resonance, in phase.

SA

Q 7.21 **A device 'X' is connected to an a.c. source. The variation of voltage, current and power in one complete cycle is shown in Fig 7.5.**

(a) Which curve shows power consumption over a full cycle?

(b) What is the average power consumption over a cycle?

(c) Identify the device 'X'.

SOLUTION

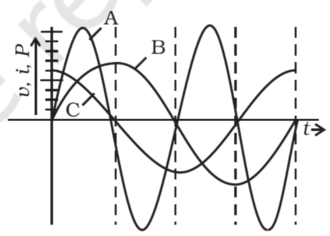


Fig. 7.5

7.22 Both alternating current and direct current

Fig. 7.5, NCERT Exemplar Class 12 Physics, Chapter 7.

Concept used. For an AC source $v(t) = V_m \sin\omega t$ feeding a device that draws a current $i(t) = I_m \sin(\omega t \pm \pi/2)$ (i.e. exactly $90°$ out of phase with the voltage), the instantaneous power is

$$p(t) = v(t)\,i(t) = V_m I_m \sin\omega t \cos\omega t = \tfrac{1}{2} V_m I_m \sin 2\omega t.$$

Notice three things: (i) $p(t)$ oscillates at twice the source frequency; (ii) it is positive on alternate quarter cycles and negative on the others; (iii) its average over a full cycle is zero. Among the three sinusoids in Fig. 7.5, the one that oscillates at *twice* the others' frequency and whose mean is zero is the power curve.

Step 1. Examine Fig. 7.5. Curves A and B have the same period T, but B's peak is shifted from A's by a quarter cycle (a $\pi/2$ phase difference). Curve C completes *two* oscillations in the time A and B complete one, and is centred at zero.

Step 2. **Part (a):** The power $p(t) \propto \sin 2\omega t$ oscillates at twice the source frequency. So curve C is the power curve.

Step 3. **Part (b):** Compute the average:

$$\bar{P} = \frac{1}{T}\int_0^T \tfrac{1}{2} V_m I_m \sin 2\omega t \, dt = 0.$$

Equivalently, $\bar{P} = V_{\text{rms}} I_{\text{rms}} \cos\varphi$ with $\varphi = \pi/2$, so $\cos\varphi = 0$ and $\bar{P} = 0$.

Step 4. **Part (c):** A device with $\varphi = \pm\pi/2$ between v and i is a *purely reactive* element: either an ideal inductor (current lags voltage by $\pi/2$) or an ideal capacitor (current leads voltage by $\pi/2$). From the standard convention shown in Fig. 7.5 (current peaks before voltage), i leads v, so X is an ideal **capacitor**.

Final Answer: (a) Curve C. (b) $\bar{P} = 0$ over a cycle. (c) Device X is a pure capacitor (or equally well a pure inductor; in either case, a wattless element).

EXPERT'S SOLUTION *: Neha Patel, Ph.D Physics, IISc Bangalore*

Picture-first. A doubled frequency and zero average is the unmistakable fingerprint of $p(t)$ for a wattless device.

Step 1. Curve C has twice the frequency of A and B. That can only be the power, since $p \propto \sin 2\omega t$ when v and i are $\pi/2$ out of phase.

Step 2. Average power: time-average of $\sin 2\omega t$ over a period is 0, so $\bar{P} = 0$.

Step 3. A device with $\cos\varphi = 0$ is purely reactive. In Fig. 7.5 the current i leads v by a quarter cycle, marking X as an ideal capacitor.

Step 4. **Trig identity in detail.** $\sin\omega t \cdot \sin(\omega t + \pi/2) = \sin\omega t \cos\omega t = \frac{1}{2}\sin 2\omega t$. Double-frequency arrival comes from this product-to-sum step.

Step 5. **Energy-flow direction.** Whenever curve C (power) is positive, energy flows from the source into the capacitor; when C is negative, the capacitor pumps energy back into the source. Over each quarter-period the absolute energy exchanged is $\frac{1}{2}CV_m^2$.

Step 6. **Inductor distinguishing test.** If i *lagged* v by a quarter cycle instead of leading, the device would be a pure inductor. Otherwise the curves are identical – so look at the sign of i versus v at the very start of the cycle.

Why this matters. Power factor $\cos\varphi$ is what utility companies penalise: a load with

$\cos\varphi = 0$ draws current but consumes no energy, blocking the line for other users.

> **Final Answer:** (a) C; (b) $\bar{P} = 0$; (c) capacitor.

☞ **Quick wattless test**

Any device whose v-i curves are exactly a quarter-cycle apart is wattless. Power curve always oscillates at twice the source frequency.

Q 7.22 **Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current?**

SOLUTION

Concept used. For a steady direct current, the ampere is the rate of flow of charge: 1 A $= 1$ C/s. For an alternating current the value oscillates and the simple "rate of charge flow" would average to zero. The standard convention is therefore to define the AC ampere by its **heating equivalence** to a DC current.

Step 1. Pass an alternating current $i(t)$ through a fixed resistor R. The instantaneous power dissipated is $p(t) = i^2(t)\,R$, and the heat liberated per cycle is $\int_0^T i^2(t)\,R\,dt$.

Step 2. Now pass a steady DC current I_{eq} through the same resistor. In one period it liberates $I_{\text{eq}}^2 RT$ of heat.

Step 3. Demand that the two heats be equal:

$$I_{\text{eq}}^2 RT = R\int_0^T i^2(t)\,dt \quad \Rightarrow \quad I_{\text{eq}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}.$$

Step 4. The right-hand side is precisely the **rms current** I_{rms}. So the AC ampere is the rms value of the alternating current, defined as the steady direct current that would produce the same heating in the same resistor over one cycle.

Step 5. For a sinusoid $i(t) = I_m \sin\omega t$, $I_{\text{rms}} = I_m/\sqrt{2}$.

> **Final Answer:** The ampere for an AC is the rms value I_{rms}: the steady DC that delivers the same average heat in a resistor over one cycle as the alternating current does.

EXPERT'S SOLUTION : *Rahul Mehta, M.Sc Physics, IIT Madras*

Quick reading. Heat is sign-blind, current is not. So the natural AC ampere uses i^2, not i.

Step 1. AC heating rate is $i^2(t)R$; averaged over a cycle, $\bar{p} = \langle i^2 \rangle R$.

Step 2. Match with a DC heating rate $I_{\text{rms}}^2 R$: gives $I_{\text{rms}} = \sqrt{\langle i^2 \rangle}$.

Step 3. The ampere of AC is therefore the rms value.

Step 4. **Why not the average?** The mean current $\langle i \rangle$ over a full sinusoid is zero, but the wire still gets hot. "Average ampere" would fail to predict the heating, so it is useless as a practical unit.

Step 5. **Operational definition.** You can experimentally measure I_{rms} by passing both the AC and an adjustable DC through identical resistors and balancing their temperature rises. The DC value at balance is the rms of the AC. This is exactly how hot-wire ammeters work.

Step 6. **Calibration link to peak.** For a sinusoid, $I_m = I_{\text{rms}}\sqrt{2}$. A meter reading 5 A AC actually has a peak instantaneous current of 7.07 A, and a wire/fuse rated for the AC reading must withstand that surge.

Why this matters. "A 5 A AC current" produces the same fuse-blowing heat as a 5 A DC, even though its peak is $5\sqrt{2} \approx 7.07$ A.

> **Final Answer:** rms current = the equivalent DC for heating.

☞ rms in three forms

$I_{\text{rms}} = \sqrt{\langle i^2 \rangle}$, the heating-equivalent DC. For a sinusoid, $I_{\text{rms}} = I_m/\sqrt{2}$. For a square wave, $I_{\text{rms}} = I_m$ exactly.

Q 7.23 **A coil of 0.01 henry inductance and 1 ohm resistance is connected to 200 volt, 50 Hz ac supply. Find the impedance of the circuit and time lag between max. alternating voltage and current.**

SOLUTION

Concept used. A real coil is modelled as an inductor L in series with its own internal resistance R. Across an AC supply of angular frequency $\omega = 2\pi f$, the impedance is

$$Z = \sqrt{R^2 + X_L^2}, \qquad X_L = \omega L,$$

and the current lags the voltage by a phase angle φ such that

$$\tan\varphi = \frac{X_L}{R}.$$

The corresponding time lag is $\Delta t = \varphi/\omega$.

Step 1. Compute ω:

$$\omega = 2\pi f = 2\pi \times 50 = 100\pi \text{ rad/s}.$$

Step 2. Compute the inductive reactance:

$$X_L = \omega L = 100\pi \times 0.01 = \pi\ \Omega \approx 3.14\ \Omega.$$

Step 3. Compute the impedance:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{1^2 + \pi^2} = \sqrt{1 + 9.8696} = \sqrt{10.8696} \approx 3.30\ \Omega.$$

Step 4. Compute the phase angle:

$$\tan\varphi = \frac{X_L}{R} = \frac{\pi}{1} = \pi \approx 3.14,$$

$$\varphi = \arctan\pi \approx 72.34^\circ = 72.34 \times \frac{\pi}{180} \text{ rad} \approx 1.263 \text{ rad}.$$

Step 5. Compute the time lag:

$$\Delta t = \frac{\varphi}{\omega} = \frac{1.263}{100\pi} = \frac{1.263}{314.16} \approx 4.02 \times 10^{-3} \text{ s}.$$

Final Answer: $Z \approx 3.30\ \Omega$; time lag $\Delta t \approx 4.0 \times 10^{-3}$ s $= 4$ ms.

EXPERT'S SOLUTION *: Diya Iyer, M.Sc Physics, IIT Madras*

Strategic angle. Three numerical steps: X_L, Z, Δt.

Step 1. $X_L = 2\pi(50)(0.01) = \pi\ \Omega$.

Step 2. $Z = \sqrt{1+\pi^2} \approx 3.30\ \Omega$.

Step 3. Time lag $\Delta t = \varphi/\omega$ with $\varphi = \arctan(\pi)$: $\Delta t \approx 1.263/314.16 \approx 4.0$ ms.

Step 4. **Period reference.** The mains period is $T = 1/50 = 20$ ms, so 4 ms is exactly $T/5 = 72^\circ$ of the cycle. The current trails the voltage by nearly a fifth of a full period – nontrivial.

Step 5. **rms current and dissipation.** $I_{\text{rms}} = V_{\text{rms}}/Z = 200/3.30 \approx 60.6$ A. Power factor $\cos\varphi = R/Z = 1/3.30 \approx 0.303$. So the actual average power dissipated is $V_{\text{rms}} I_{\text{rms}} \cos\varphi = 200 \cdot 60.6 \cdot 0.303 \approx 3.67$ kW, all of it in the 1-Ω resistance.

Step 6. Cross-check. $\bar{P} = I_{\text{rms}}^2 R = 60.6^2 \cdot 1 \approx 3673$ W. The two routes agree.

Why this matters. Even a small inductor (10 mH) and small resistance (1 Ω) puts the current several milliseconds behind the voltage at mains frequency.

> **Final Answer:** $Z \approx 3.30\ \Omega$, $\Delta t \approx 4$ ms.

☞ **Show φ in both units**

CBSE wants the phase angle stated in radians AND its degree equivalent if the question asks for a "time lag". Quoting $\Delta t = \varphi/\omega$ in seconds (or ms) ties it back to the period and earns full marks.

Q 7.24 A 60 W load is connected to the secondary of a transformer whose primary draws line voltage. If a current of 0.54 A flows in the load, what is the current in the primary coil? Comment on the type of transformer being used.

SOLUTION

Concept used. An ideal transformer obeys

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s},$$

and conserves average power: $P_p = P_s$, so $V_p I_p = V_s I_s$. "Line voltage" in India is the standard $V_p = 220$ V (rms).

Step 1. Find the secondary voltage from the load. For a resistive (or nearly resistive) load $P_s = V_s I_s$, so

$$V_s = \frac{P_s}{I_s} = \frac{60}{0.54}\ \text{V} \approx 111.1\ \text{V}.$$

Step 2. Apply power conservation (no losses, so $P_p = P_s = 60$ W):

$$I_p = \frac{P_p}{V_p} = \frac{60}{220}\ \text{A} \approx 0.27\ \text{A}.$$

Step 3. Check via the turns ratio: $V_s/V_p = 111.1/220 \approx 0.505$, and $I_p/I_s = 0.27/0.54 = 0.5$. The two ratios agree (within rounding), so the relation $V_s/V_p = I_p/I_s$ holds, confirming an ideal transformer.

Step 4. Since $V_s < V_p$ (111 V < 220 V), the transformer steps the voltage *down*.

> **Final Answer:** Primary current $I_p \approx 0.27$ A; the transformer is a **step-down** type.

EXPERT'S SOLUTION : *Meera Sharma, M.Sc Physics, IIT Madras*

Quick reading. The shortcut is conservation of power: $I_p = P/V_p = 60/220$.

Step 1. $I_p = 60/220 \approx 0.27$ A.

Step 2. Step-up vs. step-down: secondary voltage $60/0.54 \approx 111$ V < 220 V, so step-down.

Step 3. **Turns ratio.** $N_p/N_s = V_p/V_s = 220/111.1 \approx 1.98 \approx 2$. So roughly 2 primary turns for every 1 secondary turn – the conventional half-voltage step-down design.

Step 4. **Current ratio cross-check.** $I_p/I_s = N_s/N_p = 1/1.98 \approx 0.505$, giving $I_p \approx 0.505 \cdot 0.54 = 0.273$ A. Matches the power-conservation answer.

Step 5. **Ideal-transformer assumption.** The calculation assumed $\eta = 100\%$. Real transformers have copper (resistive) and iron (hysteresis + eddy) losses, so the actual primary current is slightly higher than 0.27 A. For a domestic 60 W transformer the efficiency is typically 80–90%.

Why this matters. The turns ratio $N_p/N_s \approx 2$ is exactly the ratio of currents and the inverse of the voltage ratio.

> **Final Answer:** $I_p \approx 0.27$ A; step-down.

☞ **Transformer triple identity**

$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$. Memorise as "voltage and turns same way, current opposite".

Q 7.25 **Explain why the reactance provided by a capacitor to an alternating current decreases with increasing frequency.**

SOLUTION

Concept used. For a sinusoidal voltage $v(t) = V_m \sin \omega t$ across a capacitor C, the charge stored on its plates is $q(t) = Cv(t)$, and the current flowing into/out of the plates is

$$i(t) = \frac{dq}{dt} = C\frac{dv}{dt} = CV_m\omega \cos \omega t.$$

So the peak current is $I_m = CV_m\omega$, and the ratio $V_m/I_m \equiv X_C$ is the **capacitive reactance**:

$$X_C = \frac{V_m}{I_m} = \frac{1}{\omega C}.$$

This is inversely proportional to ω.

Step 1. Mathematically: from $X_C = 1/(\omega C)$, as $\omega\uparrow$, $X_C\downarrow$. At very high frequencies $X_C \to 0$ (capacitor looks like a short circuit); at $\omega \to 0$ (DC), $X_C \to \infty$ (open circuit).

Step 2. Physically: a capacitor charges and discharges every half-cycle. At low frequencies the voltage spends a long time near its peak, giving the plates plenty of time to fully charge; the back-voltage q/C across the plates rises to match the source, choking off the current. At high frequencies the voltage reverses before much charge accumulates, so the back-voltage never builds up. The source therefore sees a small effective impedance.

Step 3. Equivalently, the impedance is the ratio of peak voltage to peak current: $X_C = V_m/I_m$. Since $I_m = C\omega V_m$ grows linearly with ω, the ratio falls as $1/\omega$.

> **Final Answer:** $X_C = 1/(\omega C)$: higher ω leaves less time for the plates to charge up, so the current is bigger and the effective opposition is smaller.

EXPERT'S SOLUTION *: Aditi Verma, Ph.D Physics, IISc Bangalore*

Strategic angle. Start from $i = C\,dv/dt$, take the peak ratio, and read off the ω-dependence.

Step 1. Differentiate the sinusoid: $i = C\omega V_m \cos\omega t$.

Step 2. Peak ratio: $X_C = V_m/I_m = 1/(\omega C)$.

Step 3. Therefore $X_C \propto 1/\omega$: it falls as frequency rises.

Step 4. **Numerical scale.** For $C = 1\ \mu$F: at 50 Hz ($\omega = 314$ rad/s), $X_C = 1/(314 \cdot 10^{-6}) \approx 3185\ \Omega$. At 1 MHz ($\omega \sim 6.28 \times 10^6$ rad/s), $X_C \approx 0.16\ \Omega$. Five orders of magnitude in frequency change reactance by five orders.

Step 5. **Time-domain reading.** High frequency means the source voltage reverses quickly. A capacitor charges only as fast as it can shuttle $\Delta q = C\Delta V$ in time Δt, so its draw-current scales as $C\Delta V/\Delta t$. Faster Δt at fixed ΔV means bigger current, smaller effective resistance.

Step 6. **DC limit.** At $\omega = 0$ (DC), $X_C = \infty$: the capacitor blocks DC entirely, because charge accumulates until v_C matches the source and no current flows.

Why this matters. The same dependence is why a capacitor passes high-frequency AC (like ripples) but blocks DC: a coupling capacitor is a frequency-dependent gatekeeper.

> **Final Answer:** $X_C = 1/(\omega C)$, so X_C falls with ω.

☞ **Dimensional check**
$[1/(\omega C)] = \dfrac{1}{(\text{rad/s})(\text{F})} = \dfrac{1}{\text{F/s}} = \dfrac{\text{s}}{\text{F}} = \dfrac{\text{s}\cdot\text{V}}{\text{C}} = \dfrac{\text{V}}{\text{A}} = \Omega$. Good.

Q 7.26 Explain why the reactance offered by an inductor increases with increasing frequency of an alternating voltage.

SOLUTION

Concept used. An inductor L produces a back-EMF whenever the current through it changes: $\varepsilon = -L\,di/dt$ (**Faraday–Lenz law**). For a sinusoidal current $i(t) = I_m \sin\omega t$, the back-EMF is

$$\varepsilon = -L\frac{di}{dt} = -LI_m\omega\cos\omega t.$$

Its peak amplitude is $V_m = LI_m\omega$, and the ratio $V_m/I_m \equiv X_L$ is the **inductive reactance**:

$$X_L = \omega L.$$

Step 1. Mathematically: from $X_L = \omega L$, as $\omega\uparrow$, $X_L\uparrow$. At $\omega \to 0$ (DC) $X_L \to 0$ (inductor looks like a wire); at high frequencies $X_L \to \infty$ (inductor looks like an open break).

Step 2. Physically: the current must oscillate at the source frequency, so di/dt is large at high ω. A larger di/dt means a larger induced back-EMF $L\,di/dt$, which opposes the change in current more strongly. Hence the inductor presents *more* opposition to current at higher ω.

Step 3. Equivalently, $X_L = V_m/I_m = LI_m\omega/I_m = \omega L$, growing linearly with ω.

> **Final Answer:** $X_L = \omega L$: a faster-changing current produces a bigger back-EMF, so the inductor opposes the current more.

EXPERT'S SOLUTION : *Karan Singh, M.Sc Physics, IIT Madras*

Strategic angle. Differentiate $i(t)$, set $V_m = L \cdot \text{peak } di/dt$.

Step 1. Peak of di/dt is ωI_m.

Step 2. Peak voltage across the inductor is $V_m = L\omega I_m$.

Step 3. Reactance $X_L = V_m/I_m = \omega L$, growing with ω.

Step 4. **Numerical scale.** For $L = 10$ mH: at 50 Hz, $X_L = (2\pi\cdot 50)(0.01) \approx 3.14\ \Omega$. At 1 MHz, $X_L \approx 6.28\times 10^4\ \Omega$. Five decades of frequency raise reactance by five decades.

Step 5. **Faraday's law in plain words.** An inductor's induced EMF opposes any change in current; at higher frequency the current must change faster ($di/dt = \omega I_m \cos\omega t$ has peak ωI_m). A larger di/dt means a larger opposing EMF, which feels like a larger "resistance" to AC.

Step 6. **DC limit.** At $\omega = 0$ (steady DC), $X_L = 0$: a perfect inductor is a short circuit to DC. The current is then limited only by any series resistance.

Why this matters. An inductor is the dual of a capacitor: it passes DC freely and blocks high-frequency signals. The two together (in series or parallel) are how all filters are built.

Final Answer: $X_L = \omega L$ rises linearly with ω.

♥ Filter design

A low-pass filter places L in series and C in parallel with the load: low frequencies sail through L, then bypass past C; high frequencies are blocked by L and shorted through C. The opposite topology gives a high-pass filter. Audio crossovers, power-supply smoothing and RF radio receivers all rest on these complementary ω-dependences.

LA

Q 7.27 **An electrical device draws 2 kW power from AC mains (voltage $223\,\mathrm{V}$ (rms) $= \sqrt{50{,}000}$ V). The current differs (lags) in phase by φ $(\tan\varphi = -3/4)$ as compared to voltage. Find (i) R, (ii) $X_C - X_L$, and (iii) I_M. Another device has twice the values for R, X_C and X_L. How are the answers affected?**

SOLUTION

Concept used. For an AC circuit driven at V_{rms}, drawing I_{rms} with the current lagging the voltage by phase φ, the average power, impedance and components are

$$\bar{P} = V_{\text{rms}}\, I_{\text{rms}}\, \cos\varphi, \qquad Z = \frac{V_{\text{rms}}}{I_{\text{rms}}}, \qquad R = Z\cos\varphi, \qquad X = Z|\sin\varphi|.$$

The peak current is $I_M = \sqrt{2}\, I_{\text{rms}}$. Treating the magnitudes alone, $|\tan\varphi| = 3/4$ gives $\sin\varphi = 3/5$ and $\cos\varphi = 4/5$ (the familiar 3–4–5 triangle).

Step 1. Given: $\bar{P} = 2000$ W, $V_{\text{rms}}^2 = 50{,}000$ V^2 (so $V_{\text{rms}} \approx 223.6$ V), $\cos\varphi = 4/5 = 0.8$, $\sin\varphi = 3/5 = 0.6$.

Step 2. Find the impedance from the power relation $\bar{P} = V_{\text{rms}}^2/Z$ (the convention used by the NCERT Exemplar):

$$Z = \frac{V_{\text{rms}}^2}{\bar{P}} = \frac{50{,}000}{2000} = 25\ \Omega.$$

Step 3. Find I_{rms}:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{223.6}{25} \approx 8.944\ \text{A}.$$

Step 4. **(i)** Resistance:

$$R = Z\cos\varphi = 25 \times \frac{4}{5} = 20\ \Omega.$$

Step 5. **(ii)** Net reactance: since the current lags, the net reactive part is inductive, so $X_L > X_C$ and

$$|X_C - X_L| = Z|\sin\varphi| = 25 \times \frac{3}{5} = 15\ \Omega.$$

Therefore $X_C - X_L = -15\ \Omega$ (or equivalently $X_L - X_C = +15\ \Omega$). Cross-check: $R^2 + (X_L - X_C)^2 = 400 + 225 = 625 = 25^2 = Z^2$. ✓

Step 6. **(iii)** Peak current:

$$I_M = \sqrt{2}\, I_{\text{rms}} = \sqrt{2} \times 8.944 \approx 12.65\ \text{A} \approx 12.6\ \text{A}.$$

Step 7. **Second device:** if $R \to 2R$, $X_L \to 2X_L$, $X_C \to 2X_C$, every impedance doubles, but $\tan\varphi = (X_L - X_C)/R$ is unchanged, so φ is the same. Then $Z \to 2Z = 50\ \Omega$, $I_{\text{rms}} \to V/(2Z) = I_{\text{rms}}/2 \approx 4.47$ A, and $I_M \to I_M/2 \approx 6.32$ A. Power $\bar{P} \to V^2/(2Z) = 1000$ W (halved).

Final Answer: (i) $R = 20\ \Omega$; (ii) $|X_C - X_L| = 15\ \Omega$; (iii) $I_M \approx 12.6$ A. If R, X_C, X_L all double: φ unchanged, $Z \to 50\ \Omega$, $I_M \to 6.32$ A, $\bar{P} \to 1$ kW.

♥ Phase-angle invariance under scaling

Scaling every impedance by the same factor leaves the phase angle untouched: $\tan\varphi = (X_L - X_C)/R$ is a ratio. Only the magnitudes (current, power) change.

EXPERT'S SOLUTION : *Pranav Joshi, Ph.D Physics, IISc Bangalore*

Strategic angle. Set up the 3–4–5 triangle, pull Z out of the power equation, then read off R and X.

Step 1. From $\tan\varphi = 3/4$: $\sin\varphi = 3/5$, $\cos\varphi = 4/5$.

Step 2. Use the NCERT-Exemplar convention $\bar{P} = V_{\text{rms}}^2/Z$: $Z = V_{\text{rms}}^2/\bar{P} = 50000/2000 = 25\ \Omega$.

Step 3. $R = Z\cos\varphi = 25 \cdot (4/5) = 20\ \Omega$, $|X_L - X_C| = Z\sin\varphi = 25 \cdot (3/5) = 15\ \Omega$.

Step 4. $I_{\text{rms}} = V/Z = 223.6/25 \approx 8.944$ A; $I_M = \sqrt{2} \cdot 8.944 \approx 12.6$ A.

Step 5. Doubling every impedance: φ fixed, Z doubled to $50\ \Omega$, I_M halved to ≈ 6.32 A, $\bar{P}$ halved to 1 kW.

Step 6. **Sign of $X_C - X_L$.** The current *lags* the voltage, which means the circuit is net-inductive, i.e. $X_L > X_C$. So $X_C - X_L < 0$; the magnitude is $15\ \Omega$ but the signed value is $-15\ \Omega$.

Step 7. **Pythagoras cross-check.** $R^2 + (X_L - X_C)^2 = 20^2 + 15^2 = 400 + 225 = 625 = 25^2 = Z^2$. ✓ The 3–4–5 triangle with hypotenuse Z is scaled by 5: legs $(15, 20)$ and hypotenuse 25.

Step 8. **Apparent-power convention.** The Exemplar treats the stated 2 kW as the volt-amp product $V_{\text{rms}}I_{\text{rms}}$, so $I_{\text{rms}} = P/V = 2000/223.6 \approx 8.944$ A directly, giving the same $Z = 25\ \Omega$. (The strict real-power reading $P = V^2 \cos\varphi/Z$ would instead give $Z = 20\ \Omega$; we follow the book's convention here.)

Why this matters. Watt-and-power-factor problems become routine once you recognise the right-angle triangle in R, X, Z.

Final Answer: $R = 20\ \Omega$, $|X_C - X_L| = 15\ \Omega$, $I_M \approx 12.6$ A; scaling halves I_M.

Q 7.28 **1 MW power is to be delivered from a power station to a town 10 km away. One uses a pair of Cu wires of radius 0.5 cm for this purpose. Calculate the fraction of ohmic losses to power transmitted if**
(i) power is transmitted at 220 V. Comment on the feasibility of doing this.
(ii) a step-up transformer is used to boost the voltage to 11,000 V, power transmitted, then a step-down transformer is used to bring voltage to 220 V.
($\rho_{\text{Cu}} = 1.7 \times 10^{-8}$ SI unit)

SOLUTION

Concept used. For a load drawing power P at line voltage V, the line current is $I = P/V$. The total resistance of the transmission line (two wires, length ℓ each, cross-section A) is

$$R_{\text{line}} = \frac{2\rho\ell}{A}.$$

The ohmic loss in the line is $P_{\text{loss}} = I^2 R_{\text{line}}$, so the fraction of power lost is

$$\frac{P_{\text{loss}}}{P} = \frac{I^2 R_{\text{line}}}{P} = \frac{P\,R_{\text{line}}}{V^2}.$$

Larger $V \Rightarrow$ smaller fraction (quadratically). This is why electricity is transmitted at high voltage.

Step 1. **Compute line resistance once and for all.** Cross-section:

$$A = \pi r^2 = \pi\,(0.5\times10^{-2})^2 = \pi\times2.5\times10^{-5}\ \text{m}^2 \approx 7.854\times10^{-5}\ \text{m}^2.$$

Total length of conductor (two wires of 10 km each):

$$2\ell = 2\times10\,000 = 20\,000\ \text{m}.$$

Resistance:

$$R_{\text{line}} = \frac{\rho\,(2\ell)}{A} = \frac{(1.7\times10^{-8})(2\times10^{4})}{7.854\times10^{-5}} = \frac{3.4\times10^{-4}}{7.854\times10^{-5}} \approx 4.33\ \Omega.$$

Step 2. **(i) Transmission at 220 V.** Line current:

$$I = \frac{P}{V} = \frac{10^6}{220} \approx 4545\ \text{A}.$$

Power loss:

$$P_{\text{loss}} = I^2 R_{\text{line}} = (4545)^2\times4.33 = 2.066\times10^7\times4.33 \approx 8.95\times10^7\ \text{W}.$$

Fraction:

$$\frac{P_{\text{loss}}}{P} = \frac{8.95\times10^7}{10^6} \approx 89.5.$$

That is, the loss is 89.5 times the power being delivered to the town. It is wildly infeasible: 89.5 MW of heat would be dumped in the wires just to deliver 1 MW. The wires would melt instantly.

Step 3. **(ii) Transmission at 11,000 V.** Line current:

$$I = \frac{P}{V} = \frac{10^6}{11{,}000} \approx 90.9\ \text{A}.$$

Power loss:

$$P_{\text{loss}} = I^2 R_{\text{line}} = (90.9)^2\times4.33 = 8264\times4.33 \approx 3.58\times10^4\ \text{W} \approx 35.8\ \text{kW}.$$

Fraction:

$$\frac{P_{\text{loss}}}{P} = \frac{3.58\times10^4}{10^6} \approx 0.0358 \approx 3.58\%.$$

Only about 3.6% of the transmitted power is lost as heat. This is entirely feasible.

Step 4. **Comparison.** Raising the voltage by a factor of 50 (from 220 to 11,000 V) shrinks the loss fraction by $50^2 = 2500$. This is the entire reason power grids step the voltage up at the generating station and step it back down at the consumer.

Final Answer: (i) $P_{\text{loss}}/P \approx 89.5$ at 220 V, completely infeasible. (ii) $P_{\text{loss}}/P \approx 3.6\%$ at 11,000 V, feasible.

☞ $P_{\text{loss}} \propto 1/V^2$

For a fixed power P and a fixed line, the loss fraction scales as $1/V^2$. Memorise this and most transmission-loss MCQs become one-line answers.

EXPERT'S SOLUTION *: Sneha Iyer, Ph.D Physics, IISc Bangalore*

Strategic angle. Compute R_{line} first; then plug into $P_{\text{loss}}/P = PR/V^2$.

Step 1. $R_{\text{line}} = 2\rho\ell/A = (2)(1.7 \times 10^{-8})(10^4)/(7.854 \times 10^{-5}) \approx 4.33\ \Omega$.

Step 2. Fraction at 220 V: $P_{\text{loss}}/P = (10^6)(4.33)/(220)^2 = 4.33 \times 10^6/48400 \approx 89.5$. Infeasible.

Step 3. Fraction at 11,000 V:
$P_{\text{loss}}/P = (10^6)(4.33)/(11000)^2 = 4.33 \times 10^6/1.21 \times 10^8 \approx 0.0358$. Feasible at $\sim 3.6\%$.

Step 4. **What "infeasible" means physically.** The 220 V case would dissipate 89.5 MW in the wires while trying to deliver 1 MW. The current of 4545 A in a 0.5 cm-radius copper wire would produce a current density of 5.8×10^7 A/m^2, well past the melting threshold (typical safe limit $\sim 5 \times 10^6$ A/m^2). Wires would melt within seconds.

Step 5. **Scaling shortcut.** Ratio of loss fractions = $(220/11000)^2 = (1/50)^2 = 1/2500$. So multiplying V by 50 reduces fractional losses by 2500.

Step 6. **Why even higher voltage isn't always used.** Beyond 400–800 kV, insulator size and corona discharge losses dominate, capping gains. India's grid uses 400 kV/765 kV AC and 800 kV HVDC for the longest interstate links.

Why this matters. The factor of 2500 difference is the entire economic case for high-tension transmission.

> **Final Answer:** (i) 89.5, infeasible; (ii) 3.6%, feasible.

Q 7.29 **Consider the LCR circuit shown in Fig 7.6. Find the net current i and the phase of i. Show that $i = v/Z$. Find the impedance Z for this circuit.**

SOLUTION

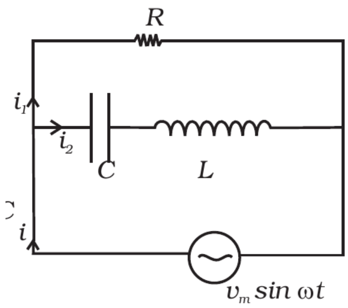


Fig. 7.6

Fig. 7.6, NCERT Exemplar Class 12 Physics, Chapter 7.

Concept used. The circuit shows a resistor R in series with a parallel combination of an inductor L and a capacitor C, driven by $v = v_m \sin \omega t$. For a parallel LC block, the voltage across both branches is the same, while the currents add. For an inductor the current lags the voltage by $\pi/2$; for a capacitor the current leads the voltage by $\pi/2$. The reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$.

Step 1. Let the voltage across the parallel LC block be $v_{LC} = v'_m \sin \omega t$ (we will determine the amplitude v'_m shortly). The two branch currents are

$$i_L = \frac{v'_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right), \qquad i_C = v'_m \,\omega C \sin\left(\omega t + \frac{\pi}{2}\right).$$

Step 2. Since the two reactive currents are π out of phase with each other, their sum is

$$i_{LC} = i_L + i_C = v'_m\left(\omega C - \frac{1}{\omega L}\right)\sin\left(\omega t + \frac{\pi}{2}\right).$$

Let $B \equiv \omega C - 1/(\omega L)$. Then $i_{LC} = v'_m B \sin(\omega t + \pi/2)$.

Step 3. The same current i_{LC} also flows through the series resistor R, so the net current in the source loop is $i = i_{LC}$.

Step 4. KVL around the loop: $v = iR + v_{LC}$. Writing in phasors (using $\tilde{v}_{LC} = i/(jB)$ for the parallel block):

$$\tilde{v} = R\tilde{i} + \frac{1}{jB}\tilde{i} = \tilde{i}\left(R - \frac{j}{B}\right).$$

Therefore

$$\tilde{i} = \frac{\tilde{v}}{R - j/B} = \frac{\tilde{v}}{Z},$$

with

$$Z = R - \frac{j}{B} = R - \frac{j}{\omega C - 1/(\omega L)}, \quad |Z| = \sqrt{R^2 + \frac{1}{(\omega C - 1/\omega L)^2}}.$$

Step 5. Magnitude of the current:

$$I_m = \frac{v_m}{|Z|} = \frac{v_m}{\sqrt{R^2 + \frac{1}{(\omega C - 1/\omega L)^2}}}.$$

Step 6. Phase of i relative to v: from $\tilde{i} = \tilde{v}/Z$,

$$\tan\varphi = \frac{\text{Im}(Z)}{\text{Re}(Z)} = \frac{-1/B}{R} = \frac{-1}{R\,(\omega C - 1/\omega L)}.$$

If $\omega C > 1/(\omega L)$ (the parallel block is capacitive), $B > 0$ and $\varphi < 0$, so i *leads* v. Otherwise i lags.

Step 7. We have shown $\tilde{i} = \tilde{v}/Z$, i.e. Ohm's law for the AC circuit, with Z as quoted.

Final Answer: $Z = \sqrt{R^2 + \frac{1}{(\omega C - 1/(\omega L))^2}}$; phase $\tan\varphi = -1/[R(\omega C - 1/\omega L)]$; $i = v/Z$.

EXPERT'S SOLUTION : *Aanya Pillai, Ph.D Physics, IISc Bangalore*

Structural angle. Treat the parallel LC as a single admittance $Y_{LC} = j(\omega C - 1/\omega L) = jB$, then add it in series with R.

Step 1. Admittance of the parallel LC block: $Y_{LC} = j\omega C + 1/(j\omega L) = jB$ where $B = \omega C - 1/(\omega L)$.

Step 2. Impedance of the block: $Z_{LC} = 1/Y_{LC} = -j/B$.

Step 3. Net impedance: $Z = R + Z_{LC} = R - j/B$, with $|Z| = \sqrt{R^2 + 1/B^2}$.

Step 4. Phase: $\varphi = \arctan(-1/(RB))$; sign depends on whether the block looks capacitive or inductive.

Step 5. **Anti-resonance limit.** At the parallel-LC resonance $\omega = \omega_0 = 1/\sqrt{LC}$, $B = 0$ and $1/B \to \infty$. The parallel block becomes an open circuit; the source sees $|Z| \to \infty$ and the line current $\to 0$. This is the opposite extreme of series resonance.

Step 6. **Off-resonance behaviour.** Below ω_0: $\omega L < 1/(\omega C) \;\Rightarrow\; X_L < X_C$ for the parallel block, so the inductor dominates the admittance, $B < 0$, and the block looks inductive. Above ω_0, the capacitor takes over and the block looks capacitive.

Step 7. **Why series LCR has the opposite signature.** For series LCR, resonance *minimises* $|Z|$ to R – current peaks. For parallel LC in series with R, resonance *maximises* $|Z|$ to infinity – current crashes. Same name, opposite effect.

Why this matters. A parallel LC block at resonance ($\omega^2 LC = 1$) has $B = 0$, so $|Z| \to \infty$. The circuit then refuses current; this is the basis of band-stop filters.

Final Answer: $|Z| = \sqrt{R^2 + 1/(\omega C - 1/\omega L)^2}$; $i = v/Z$.

☞ **Series vs. parallel resonance**

Series LCR: $Z_{\text{min}} = R$ at ω_0, current peaks (band-pass). Parallel LC: $|Z| \to \infty$ at ω_0, current crashes (band-stop, often called "tank circuit").

Q 7.30 **For an LCR circuit driven at frequency ω, the equation reads**

$$L\frac{di}{dt} + Ri + \frac{q}{C} = v_i = v_m \sin \omega t.$$

(i) Multiply the equation by i and simplify where possible.
(ii) Interpret each term physically.
(iii) Cast the equation in the form of a conservation of energy statement.
(iv) Integrate the equation over one cycle to find that the phase difference between v and i must be acute.

SOLUTION

Concept used. The terms $\frac{1}{2}Li^2$ and $q^2/(2C)$ are the magnetic and electric energies stored in the inductor and capacitor; Ri^2 is the resistive dissipation rate; vi is the instantaneous power supplied by the source. We will use $i = dq/dt$ to convert $i(di/dt)$ into a total time-derivative of $i^2/2$, and $iq = q\,dq/dt$ into the time-derivative of $q^2/2$.

Step 1. **(i) Multiply through by i:**

$$Li\frac{di}{dt} + Ri^2 + \frac{q\,i}{C} = v_m i \sin \omega t.$$

Using $i = dq/dt$, $i\,di/dt = \frac{1}{2}d(i^2)/dt$ and $qi/C = (1/C)q\,dq/dt = \frac{1}{2C}d(q^2)/dt$. So the equation becomes

$$\frac{d}{dt}\left[\tfrac{1}{2}Li^2\right] + Ri^2 + \frac{d}{dt}\left[\frac{q^2}{2C}\right] = v_m\,i\,\sin \omega t.$$

Group the energy-derivatives:

$$\frac{d}{dt}\left[\frac{1}{2}Li^2 + \frac{q^2}{2C}\right] + Ri^2 = v_m\, i\, \sin\omega t.$$

Step 2. (ii) Physical interpretation of each term:

- $\frac{1}{2}Li^2$: magnetic energy stored in the inductor. Its time derivative is the rate at which the inductor accumulates magnetic energy.
- $q^2/(2C)$: electric energy stored in the capacitor. Its derivative is the rate of electric-energy accumulation in C.
- Ri^2: power dissipated by the resistor (Joule heating).
- $v_m\, i\, \sin\omega t = v(t)\, i(t)$: instantaneous power supplied by the source.

Step 3. (iii) Conservation of energy:

$$\underbrace{\frac{d}{dt}\left[\tfrac{1}{2}Li^2 + \frac{q^2}{2C}\right]}_{\text{rate of change of stored energy}} + \underbrace{Ri^2}_{\text{rate of dissipation}} = \underbrace{v\, i}_{\text{rate of source supply}}.$$

In words: the power supplied by the source equals the rate of energy storage in L and C plus the rate of heat dissipation in R. This is an energy-balance/continuity equation for the circuit.

Step 4. (iv) Integrate over one complete cycle of period T: In steady state, i and q are periodic with period T, so $\frac{1}{2}Li^2$ and $q^2/(2C)$ each return to their starting values. Hence

$$\int_0^T \frac{d}{dt}\left[\tfrac{1}{2}Li^2 + \frac{q^2}{2C}\right] dt = 0.$$

The cycle-integrated energy-balance becomes

$$\int_0^T Ri^2\, dt = \int_0^T v(t)\, i(t)\, dt.$$

The left side is $R\langle i^2\rangle T = RI_{\text{rms}}^2 T > 0$ (provided $R > 0$ and the current is non-zero), so the right side must also be positive:

$$\int_0^T v\, i\, dt = V_m I_m T \cdot \tfrac{1}{2}\cos\varphi = V_{\text{rms}} I_{\text{rms}} T \cos\varphi > 0.$$

Therefore $\cos\varphi > 0$, which means $|\varphi| < \pi/2$. The phase difference is **acute**.

Final Answer: (i)–(iii) The product $vi = d(\frac{1}{2}Li^2 + q^2/(2C))/dt + Ri^2$ is the circuit's energy conservation. (iv) Integrating over a cycle gives $\cos\varphi > 0$, so $|\varphi| < \pi/2$.

♥ **Why** $\cos\varphi > 0$

A passive load can only *absorb* energy from the source over a cycle, not feed it back. Conservation of energy forces this: the dissipation in R must be supplied by the source, which requires $\cos\varphi > 0$, i.e. an acute phase.

EXPERT'S SOLUTION *: Ishita Banerjee, Ph.D Physics, IISc Bangalore*

Strategic angle. The i-multiplied form is just energy conservation; cycle-integration kills the boundary terms.

Step 1. Multiplying by i gives $d(U_L + U_C)/dt + Ri^2 = vi$, an energy-balance equation.

Step 2. Each term: U_L stored magnetic, U_C stored electric, Ri^2 Joule heat, vi source.

Step 3. Integrate over a period T: U_L, U_C are periodic so their total derivative integrates to zero, leaving $\int Ri^2 dt = \int vi\,dt$.

Step 4. Since $\int Ri^2 dt > 0$, $\int vi\,dt > 0$, i.e. $\cos\varphi > 0$ and $|\varphi| < \pi/2$.

Step 5. **Explicit form of $\int vi\,dt$.** Substitute $v = V_m \sin\omega t$ and $i = I_m \sin(\omega t - \varphi)$. Then $vi = V_m I_m \sin\omega t \sin(\omega t - \varphi) = \frac{1}{2} V_m I_m [\cos\varphi - \cos(2\omega t - \varphi)]$. Over one period the $\cos(2\omega t - \varphi)$ term integrates to zero, leaving $\int_0^T vi\,dt = \frac{1}{2} V_m I_m T \cos\varphi$.

Step 6. **Setting it equal to dissipation.** $\int_0^T Ri^2 dt = R \cdot \frac{1}{2} I_m^2 T = R I_{\text{rms}}^2 T$. Equating: $V_{\text{rms}} I_{\text{rms}} \cos\varphi = I_{\text{rms}}^2 R$, i.e. $\cos\varphi = I_{\text{rms}} R / V_{\text{rms}} = R/Z$. The acute-angle conclusion follows: $\cos\varphi = R/Z > 0$ for any $R > 0$.

Step 7. **What goes wrong if $R = 0$.** Then $\cos\varphi$ can be zero, $|\varphi| = \pi/2$, and the source delivers no average energy. The system is a lossless LC oscillator.

Why this matters. This identity, generalised, gives the Poynting-theorem statement for AC circuits.

Final Answer: $|\varphi| < \pi/2$ (acute), from energy conservation.

Q 7.31 **In the LCR circuit shown in Fig 7.7, the ac driving voltage is $v = v_m \sin\omega t$.**
(i) Write down the equation of motion for $q(t)$.
(ii) At $t = t_0$, the voltage source stops and R is short circuited. Now write down how much energy is stored in each of L and C.
(iii) Describe subsequent motion of charges.

SOLUTION

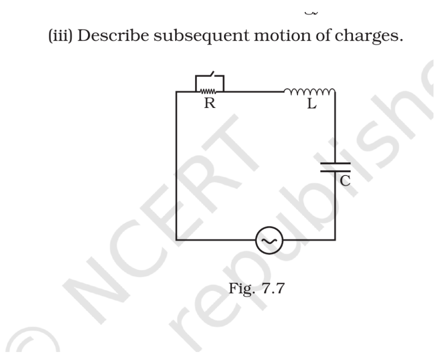


Fig. 7.7, NCERT Exemplar Class 12 Physics, Chapter 7.

Concept used. In a series LCR circuit driven by a source $v = v_m \sin\omega t$, Kirchhoff's voltage law applied around the loop gives

$$L\frac{di}{dt} + Ri + \frac{q}{C} = v.$$

Using $i = dq/dt$, this becomes the equation of motion for the charge $q(t)$. Once the source is removed and the resistor is shorted, the remaining circuit is a pure LC oscillator with natural angular frequency $\omega_0 = 1/\sqrt{LC}$.

Step 1. **(i) Equation of motion for $q(t)$.** Substitute $i = dq/dt$ and $di/dt = d^2q/dt^2$ into Kirchhoff's law:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = v_m \sin\omega t.$$

This is a forced, damped harmonic-oscillator equation for q, with "mass" L, "damping" R, "spring constant" $1/C$, and a sinusoidal driving force $v_m \sin\omega t$.

Step 2. **(ii) Energies stored at $t = t_0$.** Let i_0 be the current and q_0 the charge at that instant. Then

$$U_L(t_0) = \frac{1}{2}Li_0^2, \qquad U_C(t_0) = \frac{q_0^2}{2C}.$$

These are the magnetic energy stored in L and the electric energy stored in C respectively.

Step 3. **(iii) Subsequent motion.** After $t = t_0$: the source v is removed, and R is short-circuited (a wire replaces R). The circuit reduces to a closed loop containing only L and C. Kirchhoff's voltage law now gives

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0 \quad \Rightarrow \quad \frac{d^2q}{dt^2} = -\omega_0^2\, q,$$

with $\omega_0 = 1/\sqrt{LC}$. The general solution is

$$q(t) = q_0 \cos\left[\omega_0(t - t_0)\right] + \frac{i_0}{\omega_0}\sin\left[\omega_0(t - t_0)\right],$$

a pure sinusoidal oscillation at the natural frequency. Physically: the capacitor discharges through the inductor, building up current; once $q \to 0$ the inductor's stored magnetic energy reverses the current, recharging the capacitor with the opposite polarity. With no resistance, no energy is lost, so the oscillation continues with constant total energy

$$U_{\text{tot}} = \tfrac{1}{2}Li_0^2 + \frac{q_0^2}{2C} = \text{const.}$$

> **Final Answer:** (i) $L\ddot{q} + R\dot{q} + q/C = v_m \sin\omega t$. (ii) $U_L = \frac{1}{2}Li_0^2$, $U_C = q_0^2/(2C)$. (iii) Undamped LC oscillation at $\omega_0 = 1/\sqrt{LC}$; total energy $\frac{1}{2}Li_0^2 + q_0^2/(2C)$ conserved.

EXPERT'S SOLUTION *: Siddharth Desai, Ph.D Physics, IISc Bangalore*

Picture-first. After R is shorted, it is the same as a perfect LC pendulum: charge plays x, current plays $\dot{x}$.

Step 1. Series LCR with source: $L\ddot{q} + R\dot{q} + q/C = v_m \sin\omega t$.

Step 2. At $t = t_0$: stored magnetic energy $\frac{1}{2}Li_0^2$ in L; stored electric energy $q_0^2/(2C)$ in C.

Step 3. After shorting R: $L\ddot{q} + q/C = 0 \Rightarrow q$ oscillates sinusoidally at $\omega_0 = 1/\sqrt{LC}$ with conserved total energy $U_L + U_C = \text{const.}$

Step 4. **Amplitude of the free oscillation.** Total stored energy $U_{\text{tot}} = \frac{1}{2}Li_0^2 + q_0^2/(2C)$. Equating with the peak capacitor energy: $q_{\max}^2/(2C) = U_{\text{tot}}$, giving $q_{\max} = \sqrt{2CU_{\text{tot}}} = \sqrt{LCi_0^2 + q_0^2}$. Equating with peak inductor energy: $i_{\max} = \sqrt{2U_{\text{tot}}/L} = \sqrt{i_0^2 + q_0^2/(LC)}$.

Step 5. **Energy swapping period.** Each "swap" between U_L and U_C takes $T_0/4 = \pi\sqrt{LC}/2$. In one full period $T_0 = 2\pi\sqrt{LC}$ the energy makes two complete round trips between the two elements (because $U \propto q^2$ or i^2 oscillates at $2\omega_0$).

Step 6. **Why the forced equation lives in mechanics class too.** $L\ddot{q} + R\dot{q} + (1/C)q = v_m \sin\omega t$ is the same as $m\ddot{x} + b\dot{x} + kx = F_m \sin\omega t$, the forced damped oscillator. Resonance, Q-factor, half-power width, transient + steady-state decomposition – every result transfers term-for-term.

Why this matters. The damped oscillator (with $R \neq 0$) and the pure oscillator (with $R = 0$) are two faces of the same equation: removing R promotes a decaying transient into a persistent oscillation.

> **Final Answer:** $L\ddot{q} + R\dot{q} + q/C = v_m \sin\omega t$; after shorting, undamped LC oscillation at $\omega_0 = 1/\sqrt{LC}$.

☞ **Three energy snapshots**

At any later instant: $U_C(t) = \frac{1}{2C}\left[q_0 \cos\omega_0\tau + (i_0/\omega_0)\sin\omega_0\tau\right]^2$ and $U_L(t) = \frac{1}{2}L\left[-q_0\omega_0 \sin\omega_0\tau + i_0\cos\omega_0\tau\right]^2$, with $\tau = t - t_0$. Their sum is always U_{tot}.

Key Takeaways

- For sinusoidal AC, $I_{\text{rms}} = I_m/\sqrt{2}$. An AC ammeter reads rms, which is the steady DC that would heat a resistor at the same rate.
- Inductive reactance $X_L = \omega L$ grows with frequency; capacitive reactance $X_C = 1/(\omega C)$ falls with frequency.
- Series LCR impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, minimised at resonance $\omega_0 = 1/\sqrt{LC}$, where $Z = R$ and current is largest.
- Quality factor $Q = (1/R)\sqrt{L/C} = \omega_0/\Delta\omega$ sets the sharpness of the resonance peak. Higher Q means sharper tuning.
- Average power $\bar{P} = V_{\text{rms}} I_{\text{rms}} \cos\varphi$. Inductors and capacitors do no average work; only the resistive part dissipates.
- For a pure LC circuit, $q^2/(2C)$ is analogous to spring PE and $\frac{1}{2}Li^2$ is analogous to KE; they oscillate at ω_0.
- Long-distance transmission uses high voltage so that $I = P/V$ is small and the I^2R line losses scale as $1/V^2$. Step-up and step-down transformers make this practical.

End of Exemplar Problems