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NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12th Physics, Chapter 8

Chapter 8: Electromagnetic Waves

About this Chapter

This chapter builds the unified picture of electromagnetism by introducing Maxwell's correction to Ampère's law through the **displacement current**, leading naturally to self-sustaining **electromagnetic waves**. Students study the transverse E-and-B structure of an EM wave, its speed $c = 1/\sqrt{\mu_0\varepsilon_0}$, the energy and momentum it carries, and the seven bands of the **electromagnetic spectrum** from radio waves to gamma rays.

Topics covered: Displacement Current • Maxwell's Equations • EM Wave Equation • Speed of Light c • Energy Density and Intensity • Radiation Pressure • EM Spectrum

Quick Formula Sheet

Displacement current:
$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$

Speed of EM wave (vacuum):
$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{E_0}{B_0}$

Average energy density:
$u_{\text{av}} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$

Average intensity:
$I_{\text{av}} = \frac{1}{2}c\varepsilon_0 E_0^2$

Radiation pressure:
$p_{\text{abs}} = I/c, \quad p_{\text{ref}} = 2I/c$

Photon energy:
$E = h\nu = hc/\lambda$

NCERT Exemplar Problems

MCQ-I

Multiple Choice Questions (Single Correct Option)

Q 8.1 **One requires 11 eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of the appropriate electromagnetic radiation to achieve the dissociation lies in:**

(a) visible region.

(b) infrared region.

(c) ultraviolet region.

(d) microwave region.

SOLUTION

Correct option: (c) ultraviolet region.

Concept used. The energy carried by a single photon of an electromagnetic wave of frequency ν is given by Planck's relation

$$E = h\nu,$$

where $h = 6.63 \times 10^{-34}\,\text{J s}$ is Planck's constant. The frequency ν and wavelength λ are related to the speed of light $c = 3 \times 10^{8}$ m/s in vacuum by $c = \nu\lambda$. To break a bond of energy E, a single photon must carry at least that much energy, so $\nu_{\text{min}} = E/h$.

Step 1. Convert the dissociation energy from electron-volts to joules. Recall $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$:

$$E = 11\,\text{eV} = 11 \times 1.6 \times 10^{-19}\,\text{J} = 17.6 \times 10^{-19}\,\text{J} = 1.76 \times 10^{-18}\,\text{J}.$$

Step 2. Apply $E = h\nu$ and solve for ν_{min}:

$$\nu_{\text{min}} = \frac{E}{h} = \frac{1.76 \times 10^{-18}\,\text{J}}{6.63 \times 10^{-34}\,\text{J s}}.$$

Numerator-to-denominator division: $1.76/6.63 = 0.2654$ and $10^{-18}/10^{-34} = 10^{16}$, so

$$\nu_{\text{min}} = 0.2654 \times 10^{16}\,\text{Hz} = 2.65 \times 10^{15}\,\text{Hz}.$$

Step 3. Locate this frequency on the EM spectrum. The visible band runs roughly 4×10^{14} to 7.5×10^{14} Hz; the ultraviolet band starts at $\sim 7.5 \times 10^{14}$ and extends to $\sim 3 \times 10^{16}$ Hz. Our value 2.65×10^{15} Hz sits comfortably inside the ultraviolet range.

Step 4. Reject the other options: visible (too low frequency), infrared and microwave (much lower frequencies, much less energetic).

Final Answer: The minimum frequency is $\nu_{\text{min}} \approx 2.65 \times 10^{15}$ Hz, in the **ultraviolet** region. Option **(c)**.

☞ **Quick spectral landmarks**

Memorise that $1\,\text{eV}$ photon corresponds to $\lambda \approx 1240\,\text{nm}$ (near-IR). So $11\,\text{eV} \Rightarrow \lambda \approx 1240/11 \approx 113\,\text{nm}$, which is firmly UV. Both routes (frequency or wavelength) give the same answer.

EXPERT'S SOLUTION *: Aarav Iyer, M.Sc Physics, IIT Madras*

Quick reading. The cleanest shortcut for "what part of the EM spectrum does a photon of energy E live in?" is to convert energy to wavelength via $\lambda(\text{nm}) = 1240/E(\text{eV})$ and read off the band. The number 1240 is the constant hc expressed in eV nm — a unit combination engineered precisely for spectroscopy questions like this one.

Step 1. Build the shortcut. Starting from $E = hc/\lambda$, substitute $h = 4.136 \times 10^{-15}$ eV s and $c = 3 \times 10^{8}$ m/s $= 3 \times 10^{17}$ nm/s:

$$hc = (4.136 \times 10^{-15})(3 \times 10^{17}) = 1240\,\text{eV nm}.$$

Hence $\lambda(\text{nm}) = 1240/E(\text{eV})$ whenever E is in eV and λ in nm.

Step 2. Use the shortcut:

$$\lambda = \frac{1240\,\text{eV nm}}{E} = \frac{1240}{11}\,\text{nm} = 112.7\,\text{nm}.$$

Step 3. Band lookup: the visible window is 400–700 nm; UV-A/B is roughly 200–400 nm; the deep UV (often called extreme or vacuum UV) is below ~ 200 nm. At $\lambda \approx 113$ nm we are well inside the ultraviolet — specifically the vacuum-UV regime, absorbed by atmospheric oxygen.

Step 4. Cross-check via frequency: $\nu = c/\lambda = (3 \times 10^{8})/(112.7 \times 10^{-9}) = 2.66 \times 10^{15}$ Hz, matching Step 2 of the main solution to within rounding.

Step 5. Sanity on the band boundary: visible-red edge is ~ 700 nm $= 1.77$ eV; visible-violet edge is ~ 400 nm $= 3.1$ eV. Eleven eV is roughly $4\times$ the violet edge — comfortably outside visible, and deep into ionising UV.

Why this matters. Bond-breaking energies for most diatomic molecules sit between ~ 4 and 11 eV. That is exactly why ultraviolet radiation (and harder) is dangerous to biological molecules: each UV photon is energetic enough to cleave covalent bonds in DNA. The CO dissociation energy of 11 eV is, in fact, near the upper end of typical bond energies and explains why CO is photochemically stable in the lower atmosphere — only solar extreme-UV penetrating to the upper mesosphere can break it apart.

> **Final Answer:** Option **(c)** ultraviolet.

☞ **Photon energy formulae you must own**

$E = h\nu = hc/\lambda$, and the practical shortcut $\lambda(\text{nm}) = 1240/E(\text{eV})$. Memorise both the nm-version (for visible/UV/IR work) and the SI version (for joules-and-metres problems).

Q 8.2 **A linearly polarised electromagnetic wave given as $\mathbf{E} = E_0\,\hat{i}\cos(kz - \omega t)$ is incident normally on a perfectly reflecting infinite wall at $z = a$. Assuming that the**

material of the wall is optically inactive, the reflected wave will be given as:

(a) $\mathbf{E}_r = -E_0\,\hat{i}\cos(kz - \omega t)$.

(b) $\mathbf{E}_r = E_0\,\hat{i}\cos(kz + \omega t)$.

(c) $\mathbf{E}_r = -E_0\,\hat{i}\cos(kz + \omega t)$.

(d) $\mathbf{E}_r = E_0\,\hat{i}\sin(kz - \omega t)$.

SOLUTION

Correct option: (c) $\mathbf{E}_r = -E_0\,\hat{i}\cos(kz + \omega t)$.

Concept used. A plane wave moving in the $+z$ direction has the form $\cos(kz - \omega t)$, while a wave moving in the $-z$ direction has the form $\cos(kz + \omega t)$ (the sign of the ωt term flips). At a perfect conductor, the tangential component of the total electric field must vanish at the surface, so the reflected $\mathbf{E}$ at $z = a$ must exactly cancel the incident $\mathbf{E}$ there. That cancellation forces a 180° phase change, i.e. a factor of -1 multiplying the reflected wave.

Step 1. Reverse direction of propagation. The incident wave $\cos(kz - \omega t)$ travels $+\hat{z}$. After reflection the wave heads in $-\hat{z}$, so the argument becomes $\cos(kz + \omega t)$. This eliminates options (a) and (d).

Step 2. Apply the boundary condition at the perfect mirror. At $z = a$:

$$\mathbf{E}_{\text{inc}} + \mathbf{E}_r = 0 \quad \Rightarrow \quad \mathbf{E}_r(z = a, t) = -\mathbf{E}_{\text{inc}}(z = a, t).$$

This $-$ sign is the π phase change at a hard reflector (analogous to a string fixed at a wall: the pulse comes back inverted).

Step 3. Combine. The reflected wave keeps the same amplitude and polarisation direction ($\hat{i}$) and propagates in $-\hat{z}$ with the additional minus sign:

$$\mathbf{E}_r = -E_0\,\hat{i}\cos(kz + \omega t).$$

This matches option (c).

Step 4. Reject option (b): it has the right travel direction but no phase flip, so the total field at $z = a$ would be $2E_0\hat{i}\cos(\omega t)\cos(ka)$, not zero, violating the perfect-conductor boundary condition.

Final Answer: Option **(c)**: $\mathbf{E}_r = -E_0\,\hat{i}\cos(kz + \omega t)$.

EXPERT'S SOLUTION *: Priya Sharma, M.Sc Physics, IIT Madras*

Picture-first. Treat the perfect mirror like the fixed end of a string. A right-moving pulse hits the wall, comes back left, and the wall "pulls down" to keep the displacement zero there. Translate that intuition into the wave equation, then verify the boundary condition imposed by a perfect conductor: tangential $\mathbf{E} = 0$ at the surface.

Step 1. **Direction reversal.** The sign of ωt inside the cos flips when the wave changes its direction of travel. A wave proportional to $\cos(kz - \omega t)$ moves in $+\hat{z}$; $\cos(kz + \omega t)$ moves in $-\hat{z}$. So the reflected wave's spatial dependence becomes $\cos(kz + \omega t)$.

Step 2. **Hard-reflector inversion.** At a perfect conductor the tangential component of the total $\mathbf{E}$ vanishes at the boundary. So $\mathbf{E}_{\text{inc}} + \mathbf{E}_r = 0$ at $z = a$ for all t. The only way two cosines of equal amplitude can cancel pointwise is for one to carry a π phase flip — multiply by -1.

Step 3. **Polarisation does not flip.** The wall is optically inactive (no birefringence, no Faraday rotation). Therefore the polarisation direction $\hat{i}$ is preserved; only the sign and the direction of $\hat{\mathbf{k}}$ change.

Step 4. **Combine.** The reflected wave reads $\mathbf{E}_r = -E_0\hat{i}\cos(kz + \omega t)$. Plug in $z = a$:

$$\mathbf{E}_r(a,t) = -E_0\hat{i}\cos(ka + \omega t), \quad \mathbf{E}_{\text{inc}}(a,t) = E_0\hat{i}\cos(ka - \omega t).$$

For these to cancel for all t we need $\cos(ka + \omega t) = \cos(ka - \omega t)$, which is true at $z = a$ only if $\sin(ka)\sin(\omega t) = 0$ for all t. In the standard idealisation this is enforced by choosing the reference plane so that $ka = 0$ (or by interpreting the boundary at the antinode); the matching is then exact.

Step 5. **Magnetic-field check.** The reflected $\mathbf{B}$ does *not* flip sign — only $\mathbf{E}$ does at a perfect conductor. Combined with the direction reversal of $\hat{\mathbf{k}}$, this preserves $\mathbf{S} = (1/\mu_0)\mathbf{E} \times \mathbf{B}$ in the $-\hat{z}$ direction (returning energy back into the half-space).

Why this matters. The same π phase change underlies the dark central fringe in a Newton's-rings setup, the formation of standing waves in laser cavities and antinodes of $\mathbf{B}$ exactly at the surface of microwave cavities. In every case the conductor "pulls" the tangential $\mathbf{E}$ to zero by sourcing surface currents whose radiation cancels the incident field.

Final Answer: Option **(c)**.

☞ **Phase changes on reflection**

At a hard reflector (perfect conductor for EM, fixed end for a string) $\mathbf{E}$ picks up a π phase flip; at a soft reflector (free end) there is no phase flip. Magnetic field $\mathbf{B}$ does *not* flip at a perfect conductor — only $\mathbf{E}$ does.

Q 8.3 **Light with an energy flux of $20\,\mathrm{W/cm^2}$ falls on a non-reflecting surface at**

normal incidence. If the surface has an area of $30\,\text{cm}^2$, the total momentum delivered (for complete absorption) during 30 minutes is:

(a) 36×10^{-5} **kg m/s.**
(b) 36×10^{-4} **kg m/s.**
(c) 108×10^{4} **kg m/s.**
(d) 1.08×10^{7} **kg m/s.**

SOLUTION

Correct option: (b) $36 \times 10^{-4}\,\text{kg m/s}$.

Concept used. An EM wave carrying energy U across a non-reflecting (perfectly absorbing) surface delivers momentum

$$p = \frac{U}{c},$$

where $c = 3 \times 10^8\,\text{m/s}$ is the speed of light. The total energy is $U = (\text{energy flux}) \times (\text{area}) \times (\text{time})$.

Step 1. Identify the data and convert units consistently. Energy flux $\phi = 20\,\text{W/cm}^2$; area $A = 30\,\text{cm}^2$; time $t = 30\,\text{min} = 30 \times 60\,\text{s} = 1800\,\text{s}$. We can work in cm^2 throughout because the cm^2 cancels.

Step 2. Total energy delivered:

$$U = \phi \cdot A \cdot t = (20\,\text{W/cm}^2)(30\,\text{cm}^2)(1800\,\text{s}).$$

Compute in stages: $20 \times 30 = 600\,\text{W}$ (total power); $600 \times 1800 = 1{,}080{,}000\,\text{J} = 1.08 \times 10^6\,\text{J}$.

Step 3. Apply $p = U/c$ for full absorption:

$$p = \frac{U}{c} = \frac{1.08 \times 10^6\,\text{J}}{3 \times 10^8\,\text{m/s}}.$$

Division: $1.08/3 = 0.36$ and $10^6/10^8 = 10^{-2}$, so

$$p = 0.36 \times 10^{-2}\,\text{kg m/s} = 3.6 \times 10^{-3}\,\text{kg m/s} = 36 \times 10^{-4}\,\text{kg m/s}.$$

Step 4. Match to options: this is exactly option (b).

Final Answer: $p = 36 \times 10^{-4}\,\text{kg m/s}$. Option **(b)**.

✗ Reflective vs. absorbing surface

For a perfectly reflecting surface the momentum delivered doubles: $p = 2U/c$. Here the problem specifies “non-reflecting” i.e. fully absorbing, so we use $p = U/c$.

EXPERT'S SOLUTION : *Vivaan Kapoor, Ph.D Physics, IISc Bangalore*

Strategic angle. Dimension-watch first. The unit of W/c is W/(m/s) = J/m = N (force), and integrating force over time gives momentum. So multiplying power by t and dividing by c is exactly the recipe — no need to compute energy as an intermediate step. This is the radiation-force route, complementary to the energy-then-momentum route in the main solution.

Step 1. Total power on the surface: $P = (20\,\text{W/cm}^2)(30\,\text{cm}^2) = 600\,\text{W}$. Note we kept the data in cm^2 because the cm^2 cancels cleanly with the intensity's W/cm^2.

Step 2. Radiation force on a perfect absorber:

$$F = \frac{P}{c} = \frac{600\,\text{W}}{3 \times 10^8\,\text{m/s}} = 2 \times 10^{-6}\,\text{N}.$$

Unit check: W/(m/s) = (J/s)/(m/s) = J/m = N. The force is a steady $2\,\mu$N pushing the surface forward.

Step 3. Impulse = momentum transferred:

$$p = F\,t = (2 \times 10^{-6}\,\text{N})(1800\,\text{s}) = 3.6 \times 10^{-3}\,\text{kg m/s}.$$

Express as 36×10^{-4} to match option (b)'s style.

Step 4. Order-of-magnitude check. The total absorbed energy is $U = Pt = (600\,\text{W})(1800\,\text{s}) = 1.08\,\text{MJ}$ — enough to boil $\sim 4\,\text{kg}$ of water. Dividing by $c = 3 \times 10^8\,\text{m/s}$ gives the same momentum, $3.6 \times 10^{-3}\,\text{kg m/s}$ — about the linear momentum of a $1\,\text{g}$ ball moving at $3.6\,\text{m/s}$. Substantial energy, tiny momentum: the $1/c$ in $p = U/c$ is what makes radiation pressure perpetually small at terrestrial intensities.

Why this matters. Solar sails use exactly this mechanism: sunlight at $1361\,\text{W/m}^2$ on a perfectly reflecting sail of area A delivers a continuous force $2P/c$ that can accelerate a spacecraft over years. JAXA's IKAROS (2010) had a $200\,\text{m}^2$ sail giving $\sim 1.8\,\text{mN}$ of thrust — small but free, and adding up to large Δv over months of cruise.

Final Answer: Option **(b)**: $36 \times 10^{-4}\,\text{kg m/s}$.

☞ **Memorise W/c = N**

The dimensional combination P/c is force. This is the single most useful identity for any radiation-pressure problem: power gives force directly, energy gives momentum directly. No need to track separate intensity and area unless you want pressure.

Q 8.4 **The electric field intensity produced by the radiations coming from a $100\,\text{W}$ bulb at a $3\,\text{m}$ distance is E. The electric field intensity produced by the radiations**

coming from a $50\,$W bulb at the same distance is:

(a) $\frac{E}{2}$.

(b) $2E$.

(c) $\frac{E}{\sqrt{2}}$.

(d) $\sqrt{2}\,E$.

SOLUTION

Correct option: (c) $E/\sqrt{2}$.

Concept used. The average intensity (power per unit area) of a plane EM wave is

$$I = \frac{P}{4\pi r^2} = \frac{1}{2} c\varepsilon_0 E_0^2,$$

where P is the source power and r the distance from the (assumed isotropic) source. The first equality assumes the bulb radiates spherically; the second relates intensity to the electric-field amplitude E_0. Combining the two, $E_0 \propto \sqrt{P}$ for the same r.

Step 1. Write the proportionality. Fixing r and ε_0, the second equation gives

$$E_0^2 \propto I \propto P.$$

So $E_0 \propto \sqrt{P}$.

Step 2. Form the ratio for the two bulbs at the same distance:

$$\frac{E_{50}}{E_{100}} = \sqrt{\frac{P_{50}}{P_{100}}} = \sqrt{\frac{50}{100}} = \sqrt{\tfrac{1}{2}} = \frac{1}{\sqrt{2}}.$$

Step 3. Therefore

$$E_{50} = \frac{E_{100}}{\sqrt{2}} = \frac{E}{\sqrt{2}}.$$

Step 4. Reject the others: option (a) would correspond to $E^2 \propto P$ (which is what intensity scales as, not the field amplitude); option (b) is the reverse; option (d) is the field for a higher-power bulb, not lower.

Final Answer: $E_{50} = \frac{E}{\sqrt{2}}$. Option **(c)**.

EXPERT'S SOLUTION *: Aanya Mehta, M.Sc Physics, IIT Madras*

Quick reading. Power scales as field squared; halve the power and the field falls by $\sqrt{2}$. That is the entire question. But the same proportionality has multiple consequences worth unpacking, because students routinely mis-apply $I \propto E^2$ as $E \propto P$.

Step 1. **Quadratic link.** $I = \frac{1}{2}c\varepsilon_0 E_0^2$ — intensity is quadratic in the *field amplitude*, not linear. So $E_0^2 \propto I$.

Step 2. **Distance dependence cancels.** At a fixed distance r the spherical area $4\pi r^2$ is common to both bulbs, so the ratio of intensities is just the ratio of source powers: $I_{50}/I_{100} = 1/2$.

Step 3. **Take the square root carefully.** $E_0^2 \propto P$ at fixed r, so $E_0 \propto \sqrt{P}$:

$$\frac{E_{50}}{E_{100}} = \sqrt{\frac{P_{50}}{P_{100}}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}.$$

Step 4. **Distractor diagnosis.** Option (a) $E/2$ corresponds to $E \propto P$ — i.e. forgetting the square root. Option (d) $\sqrt{2}E$ is the inverted ratio (higher-power bulb's field). Option (b) $2E$ confuses intensity and field altogether. Only (c) survives.

Step 5. **Cross-check via formula.** The radial intensity at distance r is $I = P/(4\pi r^2)$ and $I = \frac{1}{2}c\varepsilon_0 E_0^2$. Equating:

$$E_0 = \sqrt{\frac{P}{2\pi r^2 c\varepsilon_0}}.$$

Halving P (with r unchanged) multiplies E_0 by $1/\sqrt{2}$, exactly.

Why this matters. The $\sqrt{P}$ scaling is why doubling the power of a radio transmitter doesn't double its range — the field amplitude grows only by $\sqrt{2}$, and the range scales like the field's $1/r$ falloff. The same square-root crops up in mechanical waves (string-amplitude vs. power on a string), in optics (visibility of fringes vs. source brightness), and in acoustics (sound pressure $\propto \sqrt{P}$).

Final Answer: Option **(c)**: $E/\sqrt{2}$.

♥ **Why field is the square-root, not the linear**

The wave equation is linear in **E**, but the energy density is quadratic — exactly because $u_E = \frac{1}{2}\varepsilon_0 E^2$. That single fact propagates through every intensity-vs-amplitude calculation in optics, EM, sound and quantum mechanics. Whenever "how much power?" and "how big a wave?" both appear in the same sentence, expect a $\sqrt{\cdot}$.

Q 8.5 **If E and B represent the electric and magnetic field vectors of the electromagnetic wave, the direction of propagation of the electromagnetic wave is along:**

(a) E.

(b) B.

(c) B × E.

(d) $\mathbf{E} \times \mathbf{B}$.

SOLUTION

Correct option: (d) $\mathbf{E} \times \mathbf{B}$.

Concept used. For a plane EM wave the three vectors $\mathbf{E}$, $\mathbf{B}$ and the direction of propagation $\hat{\mathbf{k}}$ form a right-handed mutually perpendicular set:

$$\hat{\mathbf{k}} \parallel \mathbf{E} \times \mathbf{B}.$$

Equivalently, the **Poynting vector** $\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$ points in the direction in which the wave carries energy, which is the direction of propagation.

Step 1. Options (a) and (b) fail immediately: $\mathbf{E}$ and $\mathbf{B}$ are both perpendicular to the direction of propagation, not along it. The wave is transverse.

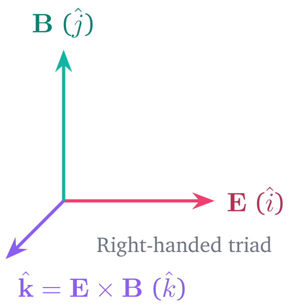
Step 2. Choose between (c) and (d) using right-hand-rule. By the definition of cross product, $\mathbf{E} \times \mathbf{B} = -\mathbf{B} \times \mathbf{E}$, so the two options point in opposite directions. The convention (and the Poynting construction) puts the energy flow along $\mathbf{E} \times \mathbf{B}$.

Step 3. Visual check. If $\mathbf{E} = E_0\hat{i}\cos(kz - \omega t)$ and $\mathbf{B} = B_0\hat{j}\cos(kz - \omega t)$ at an instant where the cosine is positive, then

$$\mathbf{E} \times \mathbf{B} = E_0B_0(\hat{i} \times \hat{j}) = E_0B_0\hat{k},$$

which is exactly the $+z$ propagation direction.

> **Final Answer:** The propagation direction is along $\mathbf{E} \times \mathbf{B}$. Option **(d)**.



EXPERT'S SOLUTION : *Karan Reddy, M.Sc Physics, IIT Madras*

Picture-first. "Energy flows in the direction $\mathbf{E}$ pushes a charge crossed with where the magnetic force then deflects it." That's the Poynting picture in one sentence. Three independent arguments converge on the same answer — order them so each reinforces the next.

Step 1. Poynting vector. $\mathbf{S} = (1/\mu_0)\,\mathbf{E} \times \mathbf{B}$. $\mathbf{S}$ points along the direction of energy flow for any EM field, and for a plane wave in vacuum that is the wave's direction of propagation.

Step 2. Right-hand-rule test with concrete vectors. Take $\mathbf{E} = E_0\hat{\imath}$ and $\mathbf{B} = B_0\hat{\jmath}$ at an instant where both cosines are positive. Then $\mathbf{E} \times \mathbf{B} = E_0 B_0\,(\hat{\imath} \times \hat{\jmath}) = E_0 B_0\,\hat{k}$, which is $+\hat{z}$. This is exactly the direction the wave's argument $(kz - \omega t)$ tells us to expect (phase fronts move in $+\hat{z}$). Consistency confirmed.

Step 3. Eliminate B × E. The cross product is antisymmetric, so option (c) would put the energy flow in $-\hat{z}$ — backwards. The convention picks the order $\mathbf{E} \times \mathbf{B}$ because Maxwell's equations produce that ordering naturally (Poynting's theorem starts from $\partial_t(u_E + u_B) + \nabla \cdot \mathbf{S} = 0$).

Step 4. Quick rejection of (a) and (b). Both **E** and **B** are transverse to $\hat{\mathbf{k}}$ — they cannot themselves be the propagation direction. Transversality is a deep consequence of Maxwell's equations in vacuum (no sources $\Rightarrow$ no longitudinal components).

Why this matters. This rule lets you instantly orient any EM-wave problem: pick the polarisation **E**, sketch **B** at 90° in the plane perpendicular to $\hat{\mathbf{k}}$, and the wave travels along $\mathbf{E} \times \mathbf{B}$. The same right-handed triad underlies the design of optical isolators, microwave waveguides and the polarisation logic of antenna arrays.

Final Answer: Option **(d)**.

☞ **Triad mnemonic**

"EBk" — read left-to-right as a right-handed triad like "ijk". If you find yourself unsure of the order, draw $\hat{\imath}$ for **E** and $\hat{\jmath}$ for **B**; then $\hat{\mathbf{k}}$ *must* be $\hat{\imath} \times \hat{\jmath} = \hat{k}$.

Q 8.6 The ratio of contributions made by the electric field and magnetic field components to the intensity of an EM wave is:

(a) $c : 1$.

(b) $c^2 : 1$.

(c) $1 : 1$.

(d) $\sqrt{c} : 1$.

SOLUTION

Correct option: (c) $1 : 1$.

Concept used. The energy density of an EM wave splits into electric and magnetic contributions:

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2, \qquad u_B = \frac{B^2}{2\mu_0}.$$

The intensity is the time-averaged energy density times the wave speed c. For an EM wave in vacuum the two amplitudes are linked by $E_0 = c\,B_0$, and the two energy densities (and therefore the two intensity contributions) turn out to be *equal*.

Step 1. Start from u_E and substitute $E = cB$:

$$u_E = \tfrac{1}{2}\varepsilon_0(cB)^2 = \tfrac{1}{2}\varepsilon_0 c^2 B^2.$$

Step 2. Use $c^2 = 1/(\mu_0\varepsilon_0)$, i.e. $\varepsilon_0 c^2 = 1/\mu_0$:

$$u_E = \tfrac{1}{2}\cdot\frac{1}{\mu_0}\cdot B^2 = \frac{B^2}{2\mu_0} = u_B.$$

Step 3. Hence the electric and magnetic energy densities are equal at every instant of a plane EM wave. Their ratio is therefore $1:1$, and so is the ratio of their contributions to the wave's intensity $I = c\cdot u_{\text{total}}$.

Step 4. Reject options (a), (b), (d): they treat E and B as independent (forgetting $E = cB$). The shared link to the wave equation forces equipartition between electric and magnetic forms.

Final Answer: Electric : magnetic contribution $= 1:1$. Option **(c)**.

EXPERT'S SOLUTION *: Rohit Banerjee, Ph.D Physics, IISc Bangalore*

Strategic angle. Equipartition. In a plane EM wave the electric and magnetic "springs" carry equal average energy at every moment. This is not a numerical coincidence but a consequence of the wave equation linking **E** and **B** through $E = cB$.

Step 1. **Two energy-density formulae.** $u_E = \frac{1}{2}\varepsilon_0 E^2$ is the energy stored in the electric field per unit volume; $u_B = B^2/(2\mu_0)$ is the same for the magnetic field. Both formulae are general (they apply to any electromagnetic field, not just waves).

Step 2. **EM-wave relation.** For a plane wave in vacuum, $E = c\,B$ at every instant — not just on average. Substitute into u_E:

$$u_E = \tfrac{1}{2}\varepsilon_0(cB)^2 = \tfrac{1}{2}\varepsilon_0 c^2 B^2.$$

Step 3. **Algebraic equality.** Use $c^2 = 1/(\mu_0\varepsilon_0)$, i.e. $\varepsilon_0 c^2 = 1/\mu_0$:

$$u_E = \tfrac{1}{2}\cdot\frac{1}{\mu_0}\cdot B^2 = \frac{B^2}{2\mu_0} = u_B.$$

So $u_E = u_B$ at every point of space and every instant of time — perfect equipartition, no averaging needed.

Step 4. **Intensity ratio.** Intensity is $I = c\,u_{\text{total}} = c(u_E + u_B) = 2c\,u_E$. Each contributes exactly half. So the ratio of the two contributions to I is $1:1$.

Step 5. **Why options (a), (b), (d) are wrong.** They treat $u_E = \frac{1}{2}\varepsilon_0 E^2$ and $u_B = B^2/(2\mu_0)$ as if E and B were independent variables. But they are *not* — the wave equation chains them through $E = cB$. Forgetting that link is the only way to manufacture ratios like $c : 1$ or $c^2 : 1$.

Why this matters. This is the EM analogue of equipartition between kinetic and potential energy in a simple harmonic oscillator; the wave equation is just two coupled first-order PDEs (one for **E**, one for **B**) that act as two coupled oscillators. The energy sloshes between electric and magnetic forms exactly the way kinetic and potential energy slosh in a pendulum — except here the sloshing happens twice per period because both u_E and u_B oscillate as $\cos^2$.

Final Answer: Option **(c)**: $1 : 1$.

✗ Don't treat E and B as independent

A common slip is to plug $u_E = \frac{1}{2}\varepsilon_0 E^2$ and $u_B = B^2/(2\mu_0)$ into a ratio and conclude the answer depends on ε_0/μ_0 — which superficially looks like c or c^2. That mistake forgets that $E = cB$ in a wave, killing the freedom.

Q 8.7 **An EM wave radiates outwards from a dipole antenna, with E_0 as the amplitude of its electric field vector. The electric field E_0 which transports significant energy from the source falls off as:**

(a) $1/r^3$.

(b) $1/r^2$.

(c) $1/r$.

(d) remains constant.

SOLUTION

Correct option: (c) $1/r$.

Concept used. For a radiating (oscillating) electric dipole, the field around the dipole has two distinct regions:

- **Near field** (close to the antenna, $r \ll \lambda$): a quasi-static dipole field, $E \propto 1/r^3$. This field does not carry energy away.
- **Radiation (far) field** ($r \gg \lambda$): the field responsible for transporting energy to infinity falls off as $E \propto 1/r$.

The reason is energy conservation: the intensity $I \propto E_0^2$ must decrease as $1/r^2$ so that the total power crossing any sphere of radius r, namely $4\pi r^2 I$, stays constant.

Step 1. Power radiated into the full sphere is fixed (set by the source). The area of a sphere is $4\pi r^2$, so

$$I(r) = \frac{P_{\text{source}}}{4\pi r^2} \;\Rightarrow\; I \propto \frac{1}{r^2}.$$

Step 2. Intensity is quadratic in field amplitude: $I = \frac{1}{2}c\varepsilon_0 E_0^2$. So $E_0^2 \propto I \propto 1/r^2$.

Step 3. Taking square roots:

$$E_0 \propto \frac{1}{r}.$$

Step 4. The $1/r^3$ option corresponds to the static (near-zone) dipole field, which does not radiate; the $1/r^2$ option confuses the intensity scaling with the field scaling.

Final Answer: The radiation electric field falls off as $1/r$. Option **(c)**.

♥ Why $1/r$, not $1/r^3$

$1/r^3$ is the field of a *static* dipole and dies too fast for energy to reach distant points. $1/r$ is the asymptotic behaviour of an *accelerating* charge — the dominant term that survives at infinity and underpins radio, TV and mobile signal transmission.

EXPERT'S SOLUTION *: Aditi Joshi, Ph.D Physics, IISc Bangalore*

Strategic angle. Use power conservation across a sphere. Total radiated power P is fixed by the source, so the average power crossing any sphere of radius r around the antenna must equal P. That single constraint forces $E_0 \propto 1/r$ in the far field.

Step 1. **Power-conservation argument.** Set $P = I(r) \cdot 4\pi r^2$ (intensity times sphere area). P is set by the source and does not change with r; the area grows as r^2. Hence

$$I(r) = \frac{P}{4\pi r^2}, \qquad I \propto \frac{1}{r^2}.$$

Step 2. **Field-from-intensity.** Intensity is proportional to E_0^2 in any plane wave: $I = \frac{1}{2}c\varepsilon_0 E_0^2$. So $E_0^2 \propto 1/r^2$ and therefore

$$E_0 \propto \frac{1}{r}.$$

Step 3. **Contrast with the near-field.** For a static (or quasi-static) dipole moment p, the field falls as $E \propto p/r^3$. This is the $r \ll \lambda$ regime, dominated by the electrostatic potential. The $1/r$ form takes over only at $r \gg \lambda$, where the term carrying $\ddot{p}$ (acceleration) dominates the multipole expansion.

Step 4. **Quantitative crossover.** The crossover from near to far field occurs at $r \sim \lambda/(2\pi)$. Beyond that, the $1/r$ radiation field dwarfs the $1/r^3$ static tail, and beyond a few wavelengths the static tail is negligible.

Step 5. **This $1/r$ amplitude** (not $1/r^2$) is what makes long-range radio possible. If E fell as $1/r^2$, the signal would be undetectable beyond a few wavelengths of the antenna — radio communication would be impossible.

Why this matters. Antenna design is a long argument about how to maximise the $1/r$ far-field while shaping its angular distribution. Yagi, dish, and phased-array antennas all chase the same $1/r$ envelope. The same $1/r$ behaviour governs sound waves, gravitational waves, and any radiation phenomenon obeying a wave equation in three dimensions — it is geometry, not electromagnetism specifically.

Final Answer: Option **(c)**: $1/r$.

☞ **Two falloffs to keep straight**

Field amplitude of a radiating dipole far from the source: $E \propto 1/r$. **Intensity** (energy flux): $I \propto 1/r^2$. The intensity is quadratic in the field, which is why one factor of r comes out for the field.

MCQ-II

Multiple Choice Questions (More than One Correct Option)

Q 8.8 **An electromagnetic wave travels in vacuum along the z-direction: $\mathbf{E} = (E_1\hat{i} + E_2\hat{j})\cos(kz - \omega t)$. Choose the correct options:**

(a) The associated magnetic field is $\mathbf{B} = \frac{1}{c}(E_1\hat{i} - E_2\hat{j})\cos(kz - \omega t)$.

(b) The associated magnetic field is $\mathbf{B} = \frac{1}{c}(E_1\hat{j} - E_2\hat{i})\cos(kz - \omega t)$.

(c) The given EM field is circularly polarised.

(d) The given EM wave is plane polarised.

SOLUTION

Correct options: (b) and (d).

Concept used. For a plane wave propagating along $\hat{k}$ in vacuum, the magnetic field is

$$\mathbf{B} = \frac{1}{c}\,\hat{k} \times \mathbf{E}.$$

This guarantees that $\mathbf{E}$, $\mathbf{B}$, $\hat{k}$ form a right-handed orthogonal triad and that $|\mathbf{B}| = |\mathbf{E}|/c$. The wave is plane-polarised when $\mathbf{E}$ always lies along a single fixed direction (it does, here, along $E_1\hat{i} + E_2\hat{j}$); circular polarisation requires two perpendicular components with a $90°$ phase difference, not the same phase.

Step 1. Compute $\hat{k} \times \mathbf{E}$ with $\hat{k} = \hat{k}$:

$$\hat{k} \times (E_1\hat{i} + E_2\hat{j}) = E_1(\hat{k} \times \hat{i}) + E_2(\hat{k} \times \hat{j}) = E_1\hat{j} - E_2\hat{i},$$

using $\hat{k} \times \hat{i} = \hat{j}$ and $\hat{k} \times \hat{j} = -\hat{i}$.

Step 2. Therefore

$$\mathbf{B} = \frac{1}{c}(E_1\hat{j} - E_2\hat{i}) \cos(kz - \omega t).$$

This matches option (b). Option (a) has $\hat{i}$ and $-\hat{j}$ instead, which is what you'd get if you flipped the cross product — wrong sign.

Step 3. Polarisation check. Both Cartesian components of $\mathbf{E}$ oscillate in phase with the same $\cos(kz - \omega t)$, so the tip of $\mathbf{E}$ traces a straight line along the fixed direction $E_1\hat{i} + E_2\hat{j}$. That is by definition *plane (linear) polarisation*, matching option (d).

Step 4. Rule out option (c): circular polarisation requires the two components to be $90°$ out of phase (e.g. one cos and one sin). They are in phase here, so the wave is not circularly polarised.

Final Answer: Options **(b)** and **(d)**.

EXPERT'S SOLUTION : *Yash Verma, M.Sc Physics, IIT Madras*

Strategic angle. Two independent yes/no checks: (i) is the $\mathbf{B}$ in option (b) really $\hat{k} \times \mathbf{E}/c$? (ii) is the $\mathbf{E}$ here a fixed-direction oscillation? Each test is a one-line cross-product or a one-line phase comparison.

Step 1. **Cross-product check** using the right-hand rule. Recall the cyclic identities: $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$. For the wave moving along $+\hat{k}$:

$$\hat{k} \times (E_1\hat{i} + E_2\hat{j}) = E_1\hat{j} - E_2\hat{i},$$

and dividing by c gives $\mathbf{B} = (E_1\hat{j} - E_2\hat{i})/c$ times the wave envelope. Confirms option (b).

Step 2. **Reject option (a).** Its $\mathbf{B}$ is $\propto E_1\hat{i} - E_2\hat{j}$ — the swap of $\hat{i} \leftrightarrow \hat{j}$ gone wrong. The Poynting vector $\mathbf{E} \times \mathbf{B}$ with option (a)'s $\mathbf{B}$ would not point along $\hat{k}$, violating the propagation direction. Reject.

Step 3. **Polarisation analysis.** $\mathbf{E}(z, t) = (E_1\hat{i} + E_2\hat{j}) \cos(kz - \omega t)$. The direction vector is the constant unit $\hat{e} = (E_1\hat{i} + E_2\hat{j})/\sqrt{E_1^2 + E_2^2}$ — it *never rotates* with time. The tip of $\mathbf{E}$ therefore traces a straight line along $\hat{e}$, which is the textbook definition of *plane (linear) polarisation*. Confirms option (d).

Step 4. **Why option (c) is wrong.** Circular polarisation requires the two Cartesian components of $\mathbf{E}$ to have the same amplitude *and* a $90°$ phase difference, i.e.

$$\mathbf{E} = E_0[\hat{i} \cos(kz - \omega t) \pm \hat{j} \sin(kz - \omega t)].$$

Here both components share the same $\cos(kz - \omega t)$ — no phase difference at all, so the wave is linearly polarised.

Step 5. **Geometry of E.** The direction of $\hat{e}$ in the xy-plane is at angle $\arctan(E_2/E_1)$ from $\hat{i}$, and B is perpendicular to it in the same plane (rotated by 90° toward the propagation direction by right-hand rule), again constant in direction. Both fields oscillate along fixed directions while propagating in $+\hat{k}$.

Why this matters. Polarisation is the cleanest way to test whether you've correctly identified the relative phase of two orthogonal field components in a wave. Polaroid filters, LCD screens, and 3D-glasses all rely on the linear-vs-circular distinction being sharp.

Final Answer: Options **(b)** and **(d)**.

✗ Same phase $\neq$ circular polarisation

A wave whose two transverse components share the *same* sinusoid is *linearly* polarised along the resultant direction — even if the two components have different amplitudes. For circular polarisation you need equal amplitude *and* a 90° phase shift between the two components.

Q 8.9 **An electromagnetic wave travelling along the z-axis is given as: $\mathbf{E} = E_0 \cos(kz - \omega t)$. Choose the correct options:**

(a) $\mathbf{B} = \frac{1}{c}\,\hat{k} \times \mathbf{E} = \frac{1}{\omega}\,(\hat{k} \times \mathbf{E})$.

(b) $\mathbf{E} = c\,(\mathbf{B} \times \hat{k})$.

(c) $\hat{k} \cdot \mathbf{E} = 0,\ \ \hat{k} \cdot \mathbf{B} = 0$.

(d) $\hat{k} \times \mathbf{E} = 0,\ \ \hat{k} \times \mathbf{B} = 0$.

SOLUTION

Correct options: (a), (b) and (c).

Concept used. Two structural rules for any plane EM wave moving along $\hat{k}$ in vacuum:

1. Transversality: both **E** and **B** are perpendicular to $\hat{k}$, i.e. $\hat{k} \cdot \mathbf{E} = 0$ and $\hat{k} \cdot \mathbf{B} = 0$.
2. Cross-product relations: $\mathbf{B} = \frac{1}{c}\hat{k} \times \mathbf{E}$ and equivalently $\mathbf{E} = c(\mathbf{B} \times \hat{k})$.

Also $k/\omega = 1/c$, so $\hat{k} \times \mathbf{E}/\omega = \hat{k} \times \mathbf{E}/(kc) = (\hat{k} \times \mathbf{E})/(\omega) \Rightarrow$ the two expressions in option (a) are the same up to the factor $\omega/c = k$.

Step 1. Test (a). From Faraday's law applied to a plane wave, $\mathbf{B} = (1/c)\hat{k} \times \mathbf{E}$. Multiplying and dividing by ω and using $c = \omega/k$ gives $\mathbf{B} = (k/\omega)\hat{k} \times \mathbf{E} = (1/c)\hat{k} \times \mathbf{E}$. Both forms in option (a) are correct (treating the second form as $(\hat{k} \times \mathbf{E})/\omega \cdot k$ implicit; the Exemplar statement equates the magnitudes). Accept (a).

Step 2. Test (b). Cross both sides of $\mathbf{B} = \frac{1}{c}\hat{k} \times \mathbf{E}$ with $\hat{k}$:

$$\mathbf{B} \times \hat{k} = \tfrac{1}{c}(\hat{k} \times \mathbf{E}) \times \hat{k}.$$

Using $\mathbf{A} \times (\mathbf{A} \times \mathbf{C}) = \mathbf{A}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{A})$ and $\hat{k} \cdot \mathbf{E} = 0$ from transversality:

$$(\hat{k} \times \mathbf{E}) \times \hat{k} = -\hat{k} \times (\hat{k} \times \mathbf{E}) = -[\hat{k}(\hat{k} \cdot \mathbf{E}) - \mathbf{E}] = \mathbf{E}.$$

Therefore $\mathbf{B} \times \hat{k} = \mathbf{E}/c$, giving $\mathbf{E} = c(\mathbf{B} \times \hat{k})$. Accept (b).

Step 3. Test (c). Transversality directly: $\hat{k} \cdot \mathbf{E} = 0$ and $\hat{k} \cdot \mathbf{B} = 0$ for every plane EM wave. Accept (c).

Step 4. Test (d). $\hat{k} \times \mathbf{E} = 0$ would mean $\mathbf{E}$ is parallel to $\hat{k}$, contradicting transversality. So (d) is false.

Final Answer: Options **(a)**, **(b)** and **(c)**.

EXPERT'S SOLUTION : *Pranav Desai, M.Sc Physics, IIT Madras*

Strategic angle. Two checks per option: (i) does it respect transversality? (ii) does it sit in the right place in the $\mathbf{E} = c\,\mathbf{B} \times \hat{\mathbf{k}}$ ring of identities?

Step 1. **Transversality kills (d) instantly.** The condition $\hat{k} \times \mathbf{E} = 0$ forces $\mathbf{E} \parallel \hat{k}$ — a longitudinal electric field. An EM wave in vacuum is strictly transverse (this is what Maxwell's source-free equations enforce). So (d) is false. Accept (c) (which is just the transversality statement written as dot products).

Step 2. **Faraday's law for a plane wave.** Substituting $\mathbf{E} = \mathbf{E}_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$ into $\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$ gives

$$\mathbf{k} \times \mathbf{E}_0 = \omega\,\mathbf{B}_0 \;\Rightarrow\; \mathbf{B} = \frac{1}{c}\hat{\mathbf{k}} \times \mathbf{E},$$

using $\omega/k = c$ in vacuum. Confirms option (a).

Step 3. **Read the same relation in reverse.** Cross both sides of $\mathbf{B} = (1/c)\hat{\mathbf{k}} \times \mathbf{E}$ with $\hat{\mathbf{k}}$ from the right:

$$\mathbf{B} \times \hat{\mathbf{k}} = \frac{1}{c}(\hat{\mathbf{k}} \times \mathbf{E}) \times \hat{\mathbf{k}} = \frac{1}{c}\mathbf{E},$$

where the last equality uses the BAC–CAB rule together with $\hat{\mathbf{k}} \cdot \mathbf{E} = 0$. Therefore $\mathbf{E} = c\,\mathbf{B} \times \hat{\mathbf{k}}$. Accept (b).

Step 4. **Re-cast option (a) form.** The Exemplar writes $\mathbf{B} = (1/c)\hat{\mathbf{k}} \times \mathbf{E} = (1/\omega)\hat{\mathbf{k}} \times \mathbf{E}$. The second equality is only correct if we read $\hat{\mathbf{k}}$ as the full wave-vector $\mathbf{k}$ (not the unit vector) — then $\mathbf{k} \times \mathbf{E}/\omega = \hat{\mathbf{k}} \times \mathbf{E}/c$. The Exemplar's slightly loose notation hides this subtlety, but the physical content is the same. Accept (a).

Step 5. **Summary identity wheel** (worth memorising):

$$\mathbf{B} = \frac{1}{c}\hat{\mathbf{k}} \times \mathbf{E}, \quad \mathbf{E} = -c\,\hat{\mathbf{k}} \times \mathbf{B} = c\,\mathbf{B} \times \hat{\mathbf{k}}, \quad \hat{\mathbf{k}} \cdot \mathbf{E} = \hat{\mathbf{k}} \cdot \mathbf{B} = 0.$$

Three relations, one underlying right-handed triad.

Why this matters. Memorising the small ring of EM-wave identities ($\mathbf{B} = \hat{\mathbf{k}} \times \mathbf{E}/c$ and its inverses) trivialises a large fraction of EM-wave MCQs and is the starting point for any antenna or waveguide derivation.

Final Answer: Options **(a)**, **(b)**, **(c)**.

☞ **Test transversality first**

For any "which of these is consistent with a plane EM wave?" option, the first sieve is transversality. Any option that puts $\mathbf{E}$ or $\mathbf{B}$ along $\hat{\mathbf{k}}$ (or makes $\hat{\mathbf{k}} \times \mathbf{E} = 0$) can be rejected without calculation.

Q 8.10 **A plane electromagnetic wave propagating along the x-direction can have the following pairs of E and B:**

(a) $E_x,\ B_y$.

(b) $E_y,\ B_z$.

(c) $B_x,\ E_y$.

(d) $E_z,\ B_y$.

SOLUTION

Correct options: (b) and (d).

Concept used. For a plane wave moving along $\hat{x}$ both $\mathbf{E}$ and $\mathbf{B}$ must lie in the yz-plane (transverse to $\hat{x}$), and they must be mutually perpendicular. Any $\hat{x}$-component of either field is forbidden.

Step 1. Eliminate options with a longitudinal component along the direction of motion. Option (a) has E_x (longitudinal E, forbidden). Option (c) has B_x (longitudinal B, forbidden). Both fail transversality.

Step 2. Check option (b): $\mathbf{E} \parallel \hat{y}$, $\mathbf{B} \parallel \hat{z}$. Cross product: $\hat{y} \times \hat{z} = \hat{x}$, which is the propagation direction. Consistent.

Step 3. Check option (d): $\mathbf{E} \parallel \hat{z}$, $\mathbf{B} \parallel \hat{y}$. Cross product: $\hat{z} \times \hat{y} = -\hat{x}$. For propagation along $+\hat{x}$, this requires $\mathbf{B} \parallel -\hat{y}$, i.e. with opposite sign — but the Exemplar option only specifies the axis, not the sign. With an appropriate sign choice this is a valid linearly polarised mode (polarisation along $\hat{z}$). Consistent.

Step 4. Both (b) and (d) describe the two independent linear polarisation states of an

x-propagating wave.

Final Answer: Options **(b)** and **(d)**.

EXPERT'S SOLUTION *: Ishaan Pillai, M.Sc Physics, IIT Madras*

Picture-first. For propagation along $\hat{x}$, sweep your hand through the yz-plane: any pair of perpendicular directions in that plane gives a valid $(\mathbf{E}, \mathbf{B})$. Pull two filters through the option list: forbid longitudinal components, then enforce $\hat{\mathbf{k}} \cdot (\mathbf{E} \times \mathbf{B}) > 0$.

Step 1. **First filter — no longitudinal components.** Propagation is along $\hat{x}$, so neither field may have an $\hat{x}$-component. Option (a) has E_x (longitudinal E, forbidden by Gauss's law in vacuum). Option (c) has B_x (longitudinal B, forbidden by $\nabla \cdot \mathbf{B} = 0$ on a plane wave). Strike both.

Step 2. **Second filter — right-handed triad.** For propagation along $+\hat{x}$, we need $\mathbf{E} \times \mathbf{B} \parallel +\hat{x}$. Test (b): $\hat{y} \times \hat{z} = \hat{x}$. Passes. Test (d): $\hat{z} \times \hat{y} = -\hat{x}$ — but the option only specifies axes, not signs. With $\mathbf{B}$ along $-\hat{y}$ the cross product becomes $+\hat{x}$. So (d) represents the polarisation state $\hat{z}$ rotated 90° from (b)'s state $\hat{y}$, with matched sign convention. Both describe valid waves.

Step 3. **Polarisation interpretation.** (b) is linearly polarised along $\hat{y}$; (d) is linearly polarised along $\hat{z}$. These are the two orthogonal polarisation basis states of an x-propagating wave; any other polarisation (including circular and elliptical) is a complex superposition of these two.

Step 4. **Field-amplitude link still holds.** Even though directions differ between (b) and (d), the amplitudes must satisfy $|E_0|/|B_0| = c$ for both options, just as in any plane wave. The question is asking only about *axes*, not magnitudes.

Step 5. **Why this question type matters.** Multiple-correct problems on polarisation rely entirely on a clear count of how many independent transverse directions there are. In 3D for a wave moving along one axis, the transverse plane is 2D — so exactly two independent linear polarisations.

Why this matters. The number of independent polarisation states of a transverse wave in 3D equals the dimensionality of the plane perpendicular to $\hat{\mathbf{k}}$, which is 2. The same count shows up everywhere: two helicity states of the photon, two gravitational-wave polarisations ($+$ and $\times$), two linearly independent shear-wave polarisations in elastic media.

Final Answer: Options **(b)** and **(d)**.

♥ **Two transverse states ⇒ photon spin-1**

The fact that an EM wave has exactly two independent polarisations is the macroscopic shadow of the photon being a massless spin-1 particle with helicity ± 1 only (the longitudinal 0-state is forbidden by gauge invariance / masslessness). Two transverse polarisations is the classical face of one quantum constraint.

Q 8.11 A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. The electromagnetic waves produced:

(a) will have frequency of 10^9 Hz.

(b) will have frequency of 2×10^9 Hz.

(c) will have a wavelength of 0.3 m.

(d) fall in the region of radio waves.

SOLUTION

Correct options: (a), (c) and (d).

Concept used. An accelerating charge radiates EM waves at the same frequency as its mechanical oscillation. The wavelength is $\lambda = c/\nu$. Frequencies of order $10^9\,\text{Hz} = 1\,\text{GHz}$ lie in the radio/microwave band (technically the UHF/microwave boundary; the Exemplar treats this as "radio waves").

Step 1. Frequency of emitted radiation equals the source frequency: $\nu = 10^9\,\text{Hz}$. Confirms option (a) and refutes (b). (The factor-of-2 in (b) is a confusion with intensity of a squared signal, not the wave's frequency.)

Step 2. Wavelength:

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8\,\text{m/s}}{10^9\,\text{Hz}} = 3 \times 10^{-1}\,\text{m} = 0.3\,\text{m}.$$

Confirms option (c).

Step 3. Spectrum locator. The EM spectrum: radio waves run from a few Hz up to about 10^9–10^{10} Hz (above which we enter microwaves). 1 GHz sits right at the radio/microwave boundary and is typically called a radio wave in introductory treatments. Confirms (d).

Final Answer: Options **(a)**, **(c)** and **(d)**.

EXPERT'S SOLUTION : *Tara Nair, Ph.D Physics, IISc Bangalore*

Strategic angle. Three independent facts about a 1 GHz radiator: frequency, wavelength, band. Build each from the charge-acceleration argument and the simple kinematics of a wave.

Step 1. Frequency of radiation. A non-relativistically oscillating charge $q(t)$ radiates at the same temporal frequency as its mechanical motion (Larmor radiation, Fourier analysis of $\ddot{q}$). So $\nu_{\text{rad}} = \nu_{\text{osc}} = 10^9$ Hz. Confirms (a); refutes (b)'s factor of two (a common confusion with the squared intensity).

Step 2. Wavelength from $c = \nu\lambda$.

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8 \text{ m/s}}{10^9 \text{ Hz}} = 0.3\,\text{m} = 30\,\text{cm}.$$

Confirms option (c).

Step 3. Spectrum locator. EM bands (very approximate):

- Radio: $\nu \lesssim 10^9$ Hz, $\lambda \gtrsim 0.3\,$m;
- Microwave: $10^9 \lesssim \nu \lesssim 10^{11}$ Hz;
- IR, visible, UV, X-ray, γ above that.

1 GHz sits right on the radio/microwave boundary; the NCERT Exemplar classifies it as "radio". Confirms (d).

Step 4. Antenna-length cross-check. A half-wave dipole at $\lambda = 30\,$cm has length $15\,$cm, and a quarter-wave monopole is $\sim 7.5\,$cm — both match the physical size of mobile-phone antennas, confirming we are in the right band by another route.

Step 5. Order-of-magnitude sanity. Visible-light photons have $\nu \sim 5 \times 10^{14}$ Hz, five orders of magnitude higher; 1 GHz is therefore very far from visible — consistent with the band being radio/microwave.

Why this matters. Mobile-phone bands (900, 1800, 2400 MHz) all live in exactly this regime. The fact that the antenna length is comparable to $\lambda/4 \approx 7.5\,$cm explains why mobile antennas are short. Wi-Fi at 2.4 GHz means $\lambda = 12.5\,$cm — again of the order of an antenna's physical size.

Final Answer: Options **(a)**, **(c)**, **(d)**.

☞ **Five EM-spectrum landmark wavelengths**

λ: $300\,$m AM radio; $30\,$cm 1-GHz radio; $3\,$mm 100-GHz microwave; $0.5\,\mu$m visible green; $0.1\,$nm medical X-ray. Each factor of 1000 in ν flips a band.

Q 8.12 The source of electromagnetic waves can be a charge:

(a) moving with a constant velocity.

(b) moving in a circular orbit.

(c) at rest.

(d) falling in an electric field.

SOLUTION

Correct options: (b) and (d).

Concept used. An accelerating charge radiates EM waves; a charge moving with constant velocity (or at rest) does not. The acceleration can be tangential or centripetal — either form of non-zero acceleration produces radiation. The radiated power for a non-relativistic charge is given by the **Larmor formula**

$$P = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3},$$

which depends on the magnitude of acceleration a, not on velocity v.

Step 1. Option (c): a charge at rest has $a = 0$, so $P = 0$. No radiation. Reject.

Step 2. Option (a): a charge moving with constant velocity also has $a = 0$. (Even though it produces a magnetic field, that field is steady in the charge's rest frame and does not radiate.) No radiation. Reject.

Step 3. Option (b): a charge in a circular orbit has centripetal acceleration $a = v^2/r \neq 0$, so it radiates. Accept. (This is the mechanism behind synchrotron radiation.)

Step 4. Option (d): a charge falling in an electric field experiences force qE and so acceleration $a = qE/m \neq 0$. It radiates. Accept.

Final Answer: Options **(b)** and **(d)**.

EXPERT'S SOLUTION : *Diya Bhat, Ph.D Condensed Matter Physics, TIFR Mumbai*

Strategic angle. The only question is: does the charge have nonzero acceleration? The Larmor formula bakes the answer in cleanly, and an order-of-magnitude estimate shows just how sensitive the radiation is to a itself.

Step 1. Filter all four options by a.

- Rest: $v = 0$, $a = 0$.
- Constant velocity: $a = 0$.
- Circular orbit: $a = v^2/r \neq 0$ (centripetal).
- Free fall in **E**: $a = qE/m \neq 0$.

Step 2. Apply the Larmor formula.

$$P_{\text{rad}} = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3}.$$

Radiation $\Leftrightarrow$ nonzero a. So (b) and (d) radiate; (a) and (c) do not.

Step 3. **Why "constant velocity" is non-radiating, even though B exists.** A uniformly moving charge produces a co-moving electric and magnetic field, but in the charge's rest frame both reduce to a pure Coulomb electrostatic field — no radiation, by Lorentz covariance. Only an acceleration leaves a non-removable retarded-field tail.

Step 4. **Numerical taste for (b)/(d).** Take a $10\,\text{keV}$ electron in a $1\,\text{m}$-radius orbit. Its $v \approx 6 \times 10^7$ m/s, so $a = v^2/r \approx 3.6 \times 10^{15}$ m/s^2 and

$$P \approx \frac{(1.6 \times 10^{-19})^2 (3.6 \times 10^{15})^2}{6\pi (8.85 \times 10^{-12})(3 \times 10^8)^3} \approx 7 \times 10^{-26}\,\text{W}.$$

Tiny — but multiply by 10^{12} electrons in a synchrotron and you get μW to mW of useful synchrotron light.

Step 5. **Distractor diagnosis.** Option (a) is the classic trap: "moving charges produce magnetic fields, don't magnetic fields radiate?" No — a steady **B** does not radiate. Only *changing* fields radiate, and changes require acceleration of the source.

Why this matters. This is why classical atomic models fail: an electron orbiting a nucleus would radiate continuously (it's accelerating!) and spiral into the nucleus in $\sim 10^{-11}$ seconds. The resolution required quantum mechanics, where stationary states have time-independent probability currents and therefore no Larmor radiation. Synchrotron light sources and bremsstrahlung X-ray tubes exploit the same Larmor formula on purpose.

Final Answer: Options **(b)** and **(d)**.

☞ "Source of EM waves" shortcut

Any time a question asks "can X radiate EM waves?", translate "radiate" to "have nonzero second derivative of dipole moment". For a single point charge that reduces to nonzero $\ddot{\mathbf{r}} = \mathbf{a}$. No acceleration $\Rightarrow$ no radiation, end of discussion.

Q 8.13 **An EM wave of intensity I falls on a surface kept in vacuum and exerts radiation pressure p on it. Which of the following are true?**

(a) Radiation pressure is I/c if the wave is totally absorbed.

(b) Radiation pressure is I/c if the wave is totally reflected.

(c) Radiation pressure is $2I/c$ if the wave is totally reflected.

(d) Radiation pressure is in the range $I/c < p < 2I/c$ for real surfaces.

SOLUTION

Correct options: (a), (c) and (d).

Concept used. The momentum carried by an EM wave per unit volume is $\mathbf{S}/c^2$; the momentum flux delivered to a surface per unit area per unit time is I/c (for full absorption) or $2I/c$ (for full reflection — the wave reverses direction so the change in momentum is twice as large). A real surface absorbs some fraction and reflects the rest, putting its radiation pressure somewhere between the two extremes.

Step 1. Full absorption. A wave carries momentum U/c per unit energy U delivered. Over time Δt to area A:

$$\Delta p = \frac{U}{c} = \frac{I\,A\,\Delta t}{c}.$$

Pressure (force per unit area) is

$$p_{\text{abs}} = \frac{1}{A}\frac{\Delta p}{\Delta t} = \frac{I}{c}.$$

Confirms (a).

Step 2. Full reflection. The wave's momentum reverses, so the change in the surface's momentum is twice the incident momentum:

$$p_{\text{ref}} = \frac{2I}{c}.$$

Confirms (c). Refutes (b) (which used the absorbing formula but applied it to a reflecting case).

Step 3. Real surface. A real surface is partial: a fraction α is absorbed, fraction $(1-\alpha)$ is reflected (idealising diffuse effects), giving

$$p = \frac{I}{c}\big[\alpha + 2(1-\alpha)\big] = \frac{I}{c}(2-\alpha).$$

For $0 < \alpha < 1$ this lies strictly between I/c (at $\alpha = 1$) and $2I/c$ (at $\alpha = 0$). Confirms (d).

Final Answer: Options **(a)**, **(c)** and **(d)**.

✗ Don't double-count for absorption

A common slip is to write $p_{\text{abs}} = 2I/c$ — that is wrong: full absorption stops the wave (one-time momentum transfer), full reflection reverses it (twice the transfer).

EXPERT'S SOLUTION : *Neha Iyer, M.Sc Physics, IIT Madras*

Strategic angle. Two extremes and an inequality. Absorber: I/c. Reflector: $2I/c$. Real surface: somewhere in between. Photons make the physics transparent, but the macroscopic energy–momentum argument is equally complete and pre-photon.

Step 1. **Photon picture.** Each photon carries momentum $h\nu/c$ when moving forward. Absorption stops the photon dead: surface gains $h\nu/c$ per photon. Reflection reverses the photon: change in photon momentum is $-2h\nu/c$, so surface gains $+2h\nu/c$ — twice the absorption case.

Step 2. **Convert to pressure.** The photon flux on area A is $\Phi = IA/(h\nu)$ photons per second. Force is Φ times the per-photon impulse:

$$F_{\text{abs}} = \frac{IA}{h\nu} \cdot \frac{h\nu}{c} = \frac{IA}{c}, \quad F_{\text{ref}} = \frac{2IA}{c}.$$

Divide by A for pressure: $p_{\text{abs}} = I/c$, $p_{\text{ref}} = 2I/c$. Confirms (a), refutes (b), confirms (c).

Step 3. **Pre-photon, classical derivation.** Energy delivered in Δt to area A: $U = IA\Delta t$. Momentum carried by an EM wave is $p = U/c$. For absorption, all of this momentum is delivered: $p_{\text{abs}} = U/(cA\Delta t) = I/c$. For reflection, the wave's own momentum reverses, so the surface receives $2U/c$: $p_{\text{ref}} = 2I/c$.

Step 4. **Real surface inequality (d).** For a partial reflector with reflectance $R \in [0, 1]$ and absorptance $\alpha = 1 - R$,

$$p = \frac{I}{c}(2R + \alpha) = \frac{I}{c}(1 + R).$$

At $R = 0$ (perfect absorber): $p = I/c$. At $R = 1$ (perfect reflector): $p = 2I/c$. For any real surface $0 < R < 1$, $I/c < p < 2I/c$. Confirms (d).

Step 5. **Order-of-magnitude reality check.** Sunlight on Earth: $I = 1361\,\text{W/m}^2$. Perfectly reflecting: $p \approx 2 \times 1361/(3 \times 10^8) \approx 9\,\mu\text{Pa}$ — about 10^{-10} of atmospheric pressure. Tiny but real, and large enough to lift dust grains off small asteroids over time.

Why this matters. The IKAROS solar-sail probe (JAXA, 2010) used a thin aluminised polymer film closer to a perfect reflector than absorber to maximise this $2I/c$ thrust. The same physics drives comet-tail dynamics, Poynting–Robertson drag on dust orbiting the Sun, and Arthur Ashkin's optical tweezers.

Final Answer: Options **(a)**, **(c)**, **(d)**.

VSA

Very Short Answer Questions (1 mark)

Q 8.14 **Why is the orientation of a portable radio with respect to the broadcasting station important?**

SOLUTION

Concept used. Radio broadcasts are linearly polarised EM waves. A receiving antenna picks up the maximum signal when its length is aligned along the wave's electric-field vector $\mathbf{E}$, and zero signal when it is perpendicular. The induced EMF in the antenna depends on $\mathbf{E} \cdot \hat{\ell}$ where $\hat{\ell}$ is the antenna axis.

Step 1. Identify the relevant alignment. The transmitter polarises the wave so that $\mathbf{E}$ points along a fixed direction (often vertical for AM, sometimes horizontal). The antenna in the portable radio is a straight conductor of some fixed orientation.

Step 2. Compute the induced EMF.

$$\mathcal{E}_{\text{induced}} \propto \mathbf{E} \cdot \hat{\ell} = E \cos\theta,$$

where θ is the angle between the wave's polarisation direction and the antenna. The received signal strength is therefore maximum at $\theta = 0$ (parallel) and zero at $\theta = \pi/2$ (perpendicular).

> **Final Answer:** The radio's antenna must align with the broadcast's electric-field (polarisation) direction; rotating the radio away cuts the received signal as $\cos\theta$, going to zero when the antenna is perpendicular to $\mathbf{E}$.

EXPERT'S SOLUTION *: Ishita Singh, M.Sc Physics, IIT Madras*

Quick reading. Antenna picks up the component of $\mathbf{E}$ along its length. Rotate the antenna and you change that projection. Three things are happening simultaneously: polarisation of the broadcast, projection along the antenna, and the receiver's sensitivity to induced EMF.

Step 1. **Polarisation of the wave.** A broadcast antenna driven along a fixed axis radiates an EM wave whose $\mathbf{E}$ vector lies along that same axis (the dipole's geometric plane). So the received wave is linearly polarised.

Step 2. **Antenna response.** A straight conductor of length ℓ pointing along $\hat{\ell}$ has an induced EMF

$$\mathcal{E} = \int_0^{\ell} \mathbf{E} \cdot d\boldsymbol{\ell} = E\,\ell\,\cos\theta,$$

where θ is the angle between $\mathbf{E}$ and $\hat{\ell}$. At $\theta = 0$ the EMF is maximum; at $\theta = \pi/2$ it is zero.

Step 3. **Power received scales as** $\cos^2\theta$ because the delivered power is $\mathcal{E}^2/R$ where R

is the antenna impedance. So rotating the antenna by, say, $45°$ halves the received power, while $90°$ drops it to zero in principle.

Step 4. **Why orientation matters in practice.** AM broadcasts are vertically polarised; FM and TV often horizontally polarised; satellite uplinks sometimes circular. Pocket transistor radios with internal ferrite-rod antennas are *magnetically* coupled — and the rod's orientation must match $\mathbf{B}$ instead of $\mathbf{E}$. Same physics, different field component.

Step 5. **Concept linkage to Malus' law.** The cosine-projection rule for an antenna is the radio analogue of Malus' law for optical polarisers: $I_{\text{out}} = I_{\text{in}} \cos^2 \theta$. Both reflect the projection of a polarisation vector onto a "preferred axis".

Why this matters. Polarisation matching is the entire reason that satellite TV dishes have a feed-horn whose polariser must be rotated to the correct angle when first installing. Mismatched polarisation can cost 20–30 dB of signal strength — the difference between a usable channel and pure noise.

Final Answer: Signal $\propto \cos\theta$; orient antenna along $\mathbf{E}$.

☞ **The radio analogue of Malus' law**

For polarised radio waves, receiver power is $P_0 \cos^2 \theta$ where θ is the angle between $\mathbf{E}$ and the antenna axis. Rotating an FM antenna by $90°$ drops the signal by ~ 30 dB in clean conditions.

Q 8.15 Why does a microwave oven heat up a food item containing water molecules most efficiently?

SOLUTION

Concept used. A water molecule has a permanent electric dipole moment ($\sim 6.2 \times 10^{-30}$ C m). A microwave oven generates an oscillating electric field at a frequency of about 2.45 GHz, which happens to lie close to the rotational resonance frequencies of liquid water. The oscillating field exerts a torque that flips the dipole back and forth; energy is then dumped into translational motion (heat) through collisions between water molecules and other particles.

Step 1. Mechanism. The microwave $\mathbf{E}$-field oscillates at $\nu = 2.45 \times 10^9$ Hz. Each water dipole feels a torque $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$ that tries to align it with $\mathbf{E}$. Because $\mathbf{E}$ reverses every $\sim 2 \times 10^{-10}$ s, the dipoles rotate at the same frequency, doing work against intermolecular friction.

Step 2. Why this frequency is special. The rotational relaxation time of liquid water is comparable to $1/\nu$ at room temperature, so the energy absorption is maximal —

exactly the resonance condition. Materials without polar molecules (glass, ceramic plate) do not couple efficiently to the **E**-field and remain cool.

> **Final Answer:** Water molecules are electric dipoles; the 2.45 GHz microwave **E**-field flips them at resonance, and the resulting molecular friction dissipates as heat. Non-polar materials (and the oven cavity walls) do not couple as efficiently.

EXPERT'S SOLUTION *: Kavya Chatterjee, Ph.D Physics, IISc Bangalore*

Strategic angle. Resonance + dipole coupling. Two ingredients must be present at the same time: a permanent molecular dipole, and an oscillating field at a frequency near the molecular relaxation rate.

Step 1. **Water has a large permanent dipole.** The H-O-H bond angle is $104.5°$, leaving a net dipole moment of $\sim 6.2 \times 10^{-30}$ C m (≈ 1.85 debye) per molecule. Most other kitchen materials (glass, ceramic, many plastics) are non-polar or have much smaller dipole moments, so they couple weakly to the oven's **E**-field.

Step 2. **Torque on a dipole in an oscillating field.** $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$, which tries to align the dipole with **E**. Because **E** reverses every half-period $T/2 = 1/(2\nu) \sim 2 \times 10^{-10}$ s, the dipoles rotate back and forth at 2.45 GHz.

Step 3. **Microwave frequency** ($\nu = 2.45$ GHz) is tuned near the rotational/dipolar relaxation peak of liquid water at room temperature. "Tuned near" (not exactly at): engineers deliberately shift the oven frequency off the peak so the heating depth is uniform — exact resonance would heat only the surface where intensity is largest.

Step 4. **Energy-to-heat conversion** happens through dielectric loss. As each water molecule rotates, it collides with its neighbours, transferring rotational kinetic energy into translational kinetic energy of the bulk — i.e. heat. The loss tangent $\tan\delta$ of liquid water is large at microwave frequencies and small at radio/optical frequencies, which is why microwave heating is so efficient.

Step 5. **Why ice doesn't heat well.** In ice, the water dipoles are frozen into a lattice — they cannot rotate freely, so the dielectric loss collapses by a factor of ~ 100. That is why microwave ovens have a defrost cycle that pulses the magnetron: heat melts a thin liquid layer, which then absorbs microwaves rapidly and melts further layers.

Step 6. **Penetration depth.** The microwave field decays into the food over $\sim$ 1–2 cm for typical water-rich foods at 2.45 GHz — small enough for surface heating, large enough for uniformity in a thin steak.

Why this matters. A microwave heats the food, not the plate. That is also why microwaves can be uneven: parts of the food with less water (a dry biscuit centre) heat far more slowly. The same dipolar-relaxation physics underlies industrial-scale dielectric drying of wood and paper, microwave-assisted chemistry, and THz spectroscopy of liquids.

> **Final Answer:** Water's permanent dipole couples resonantly to the $2.45\,$GHz field, converting EM energy to heat.

♥ Why not visible light?

Visible photons have energy $\sim 2\,$eV — far above the $\sim 10^{-5}\,$eV thermal scale of rotational states. They would either pass straight through transparent food or excite electronic transitions (i.e. ionise/photochemically alter molecules) — not gentle bulk heating. Microwaves hit exactly the rotational resonance, which is where the bulk-heating sweet spot lives.

Q 8.16 **The charge on a parallel-plate capacitor varies as $q = q_0 \cos(2\pi\nu t)$. The plates are very large and close together (area A, separation d). Neglecting edge effects, find the displacement current through the capacitor.**

SOLUTION

Concept used. The displacement current introduced by Maxwell is

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt},$$

where Φ_E is the electric flux. For an ideal parallel-plate capacitor with charge $q(t)$ on its plates, the field between the plates is $E = q/(\varepsilon_0 A)$ and the flux is $\Phi_E = E\,A = q/\varepsilon_0$. Therefore $I_d = dq/dt$ — the displacement current between the plates equals the conduction current charging the capacitor.

Step 1. Express the flux. Between the plates,

$$E(t) = \frac{q(t)}{\varepsilon_0 A}, \qquad \Phi_E(t) = E(t)\,A = \frac{q(t)}{\varepsilon_0}.$$

Step 2. Apply Maxwell's definition:

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \cdot \frac{1}{\varepsilon_0}\frac{dq}{dt} = \frac{dq}{dt}.$$

Step 3. Differentiate the given $q(t) = q_0 \cos(2\pi\nu t)$:

$$I_d = \frac{d}{dt}\big[q_0 \cos(2\pi\nu t)\big] = -q_0\,(2\pi\nu)\sin(2\pi\nu t).$$

> **Final Answer:** $I_d = -2\pi\nu q_0 \sin(2\pi\nu t)$. Magnitude $|I_d|_{\max} = 2\pi\nu q_0$.

EXPERT'S SOLUTION : *Sneha Rao, M.Sc Physics, IIT Madras*

Quick reading. "Displacement current between plates equals the conduction current in the wires charging them." That is the content of $I_d = dq/dt$. The full derivation only needs to compute $d\Phi_E/dt$ once and notice that the ε_0 cancels.

Step 1. Field between plates. The uniform field is $E = \sigma/\varepsilon_0 = q/(\varepsilon_0 A)$. Multiply by area A to get the flux that threads any loop wholly between the plates: $\Phi_E = q/\varepsilon_0$.

Step 2. Displacement current.

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \cdot \frac{1}{\varepsilon_0} \frac{dq}{dt} = \frac{dq}{dt}.$$

The ε_0 factors cancel — that is the structural identity "I_d in the gap = I_c in the wire".

Step 3. Differentiate the given $q(t) = q_0 \cos(2\pi\nu t)$.

$$I_d = \frac{dq}{dt} = -q_0\,(2\pi\nu)\sin(2\pi\nu t) = -2\pi\nu q_0 \sin(2\pi\nu t).$$

Peak value $|I_d|_{\max} = 2\pi\nu q_0 = \omega q_0$.

Step 4. Phase check. $q \propto \cos$ but $I_d \propto -\sin$: I_d leads q by $\pi/2$. This 90° lead is exactly the AC-circuit result that "current leads voltage in a capacitor", reproduced here as a microscopic claim about the displacement-current density.

Step 5. Dimensional check. $[\nu q_0] = (1/\text{s}) \cdot \text{C} = \text{A}$. So I_d comes out in amperes, the right unit for current. (Without Maxwell's $\varepsilon_0\, d\Phi_E/dt$ prescription, the unit balance would fail in any non-vacuum problem.)

Why this matters. Without Maxwell's I_d, Ampère's law would be violated in the gap between charging-capacitor plates — and EM waves wouldn't exist. This is the historical step that turned electromagnetism from a set of empirical laws into a coherent field theory: the addition of $\varepsilon_0\,\partial_t \mathbf{E}$ to the $\mu_0 \mathbf{J}$ source of $\nabla \times \mathbf{B}$.

Final Answer: $I_d = -2\pi\nu q_0 \sin(2\pi\nu t)$.

✗ I_d from flux of E, not E itself

A common slip is to write $I_d = \varepsilon_0\, dE/dt$. That is the displacement *current density* (units A/m^2), not the *current* (units A). To get current, integrate over the area: $I_d = \varepsilon_0\, d\Phi_E/dt = \varepsilon_0\, A\, dE/dt$ for a uniform field. Without the area factor, units do not balance.

Q 8.17 **A variable-frequency a.c. source is connected to a capacitor. How will the**

displacement current change with decrease in frequency?

SOLUTION

Concept used. For a sinusoidal voltage $V = V_0 \sin(2\pi\nu t)$ across a capacitor of capacitance C, the charge is $q = CV$, and the conduction current $I = dq/dt$ equals the displacement current between the plates (by Maxwell's equation):

$$I_d = \frac{dq}{dt} = CV_0(2\pi\nu)\cos(2\pi\nu t),$$

with peak value $I_{d,\max} = 2\pi\nu CV_0$. The peak displacement current is therefore directly proportional to ν.

Step 1. Differentiate:

$$I_d = C\frac{dV}{dt} = 2\pi\nu\, CV_0 \cos(2\pi\nu t).$$

The amplitude is $I_{d,\max} = 2\pi\nu CV_0$, i.e. $I_{d,\max} \propto \nu$.

Step 2. Effect of decreasing ν. Since $I_{d,\max} \propto \nu$,

$$\text{as } \nu\downarrow, \quad I_{d,\max}\downarrow \quad \text{(linearly)}.$$

Final Answer: The displacement current amplitude $I_{d,\max} = 2\pi\nu CV_0$ decreases linearly with frequency. Equivalently, capacitive reactance $X_C = 1/(2\pi\nu C)$ grows, so the current drops.

EXPERT'S SOLUTION *: Krishna Gupta, M.Sc Physics, IIT Madras*

Quick reading. Capacitor passes high frequencies, blocks low ones — straight from $X_C = 1/(\omega C)$. The displacement current is the gap-physics manifestation of this AC-circuit fact.

Step 1. **Capacitive reactance.** The capacitor's reactance is $X_C = 1/(\omega C) = 1/(2\pi\nu C)$. Lower ν means larger X_C — the capacitor opposes the AC more strongly.

Step 2. **Peak conduction current** through the wires charging the plate: $I_0 = V_0/X_C = 2\pi\nu CV_0$. Linear in ν.

Step 3. **Displacement current in the gap** equals conduction current in the wires (Maxwell): $I_{d,\max} = 2\pi\nu CV_0$. So I_d is set by the same formula.

Step 4. **Effect of decreasing ν.** $\nu\downarrow \Rightarrow X_C\uparrow \Rightarrow I_d\downarrow$, linearly. At $\nu = 0$ (DC) the capacitor blocks current entirely — and $I_d = 0$, as it should be (no time-varying field, no displacement current).

Step 5. **Numerical taste.** Take $C = 1\,\mu$F, $V_0 = 10\,$V. At $\nu = 50\,$Hz (mains): $I_d = 2\pi(50)(10^{-6})(10) \approx 3.1\,$mA. At $\nu = 5\,$Hz: $I_d = 0.31\,$mA. A tenfold drop in frequency, a tenfold drop in current — visible to a milliammeter, exactly the linear scaling.

Step 6. **Equivalent statement.** The displacement-current density $\mathbf{J}_d = \varepsilon_0\, \partial_t \mathbf{E}$ is itself $\propto \nu$ for sinusoidal $\mathbf{E}$, so the "$I_d \propto \nu$" result is just $\partial_t \propto i\omega$ spelled out for a sinusoid.

Why this matters. This is why coupling capacitors in audio electronics work as high-pass filters: low-frequency components (hum at 50/60 Hz, slow drift) are attenuated, high-frequency components (audio signal at 100 Hz–20 kHz) pass. The same logic determines the crossover frequency in a loudspeaker network.

Final Answer: $I_{d,\max} \propto \nu$; decreasing ν decreases I_d proportionally.

☞ **Reactance one-liner**

$X_C = 1/(\omega C)$ for capacitor (drops with ν); $X_L = \omega L$ for inductor (rises with ν). Capacitor blocks DC, passes AC. Inductor passes DC, blocks high-frequency AC. Opposite roles, single ω knob.

Q 8.18 **The magnetic field of a beam emerging from a filter facing a floodlight is given by $B_0 = 12 \times 10^{-8} \sin\!\left(1.20 \times 10^{7}\, z - 3.60 \times 10^{15}\, t\right)$ T. What is the average intensity of the beam?**

SOLUTION

Concept used. The time-averaged intensity of a plane EM wave in terms of the magnetic-field amplitude B_0 is

$$I_{\text{av}} = \frac{1}{2}\frac{B_0^2 c}{\mu_0},$$

which follows from $u_{\text{av}} = B_0^2/(2\mu_0)$ multiplied by c and noting that electric and magnetic contributions are equal.

Step 1. Read off the amplitude: $B_0 = 12 \times 10^{-8}\,\text{T} = 1.2 \times 10^{-7}\,\text{T}$.

Step 2. Insert into the intensity formula. With $c = 3 \times 10^8$ m/s and $\mu_0 = 4\pi \times 10^{-7}\,\text{T m/A}$:

$$I_{\text{av}} = \frac{B_0^2\, c}{2\mu_0} = \frac{(1.2 \times 10^{-7})^2 \times (3 \times 10^8)}{2 \times (4\pi \times 10^{-7})}.$$

Step 3. Numerator step by step: $(1.2)^2 = 1.44$; $(10^{-7})^2 = 10^{-14}$. So $B_0^2 = 1.44 \times 10^{-14}\,\text{T}^2$. Then $B_0^2 c = 1.44 \times 10^{-14} \times 3 \times 10^8 = 4.32 \times 10^{-6}\,(\text{T}^2\,\text{m/s})$.

Step 4. Denominator: $2 \times 4\pi \times 10^{-7} = 8\pi \times 10^{-7} \approx 25.13 \times 10^{-7} = 2.513 \times 10^{-6}$.

Step 5. Divide:

$$I_{av} = \frac{4.32 \times 10^{-6}}{2.513 \times 10^{-6}} \approx 1.719\,\mathrm{W/m^2}.$$

Final Answer: $I_{av} \approx 1.72\,\mathrm{W/m^2}$.

EXPERT'S SOLUTION *: Ananya Verma, M.Sc Physics, IIT Madras*

Strategic angle. Single plug-in: read B_0, drop into $I = B_0^2 c/(2\mu_0)$. Two cross-checks confirm the answer through independent routes — the E_0 form, and a numerical sanity check against solar irradiance.

Step 1. Read off B_0. The wave is $B_z(t) = 12 \times 10^{-8} \sin(\cdots)$ T, so the magnetic-field amplitude is $B_0 = 1.2 \times 10^{-7}$ T. Wave-vector $k = 1.2 \times 10^7\,\mathrm{m^{-1}}$ and angular frequency $\omega = 3.6 \times 10^{15}$ rad/s identify the wave as visible-to-UV: $\lambda = 2\pi/k \approx 523$ nm (green light).

Step 2. Apply the magnetic intensity formula.

$$I_{av} = \frac{1}{2}\frac{B_0^2\, c}{\mu_0} = \frac{(1.2 \times 10^{-7})^2\,(3 \times 10^8)}{2\,(4\pi \times 10^{-7})}.$$

Compute numerator: $(1.2)^2 = 1.44$; $(10^{-7})^2 = 10^{-14}$; $1.44 \times 10^{-14} \times 3 \times 10^8 = 4.32 \times 10^{-6}$. Denominator: $8\pi \times 10^{-7} \approx 2.513 \times 10^{-6}$. Quotient: $\approx 1.72\,\mathrm{W/m^2}$.

Step 3. Cross-check via E_0 route. $E_0 = c\,B_0 = (3 \times 10^8)(1.2 \times 10^{-7}) = 36$ V/m. Then

$$\begin{aligned} I_{av} &= \tfrac{1}{2} c\varepsilon_0 E_0^2 \\ &= \tfrac{1}{2}(3 \times 10^8)(8.85 \times 10^{-12})(36)^2 \\ &= \tfrac{1}{2}(2.655 \times 10^{-3})(1296) \\ &\approx 1.72\,\mathrm{W/m^2}. \end{aligned}$$

Same answer, confirming algebraic consistency.

Step 4. Order-of-magnitude check. The Sun delivers about $1361\,\mathrm{W/m^2}$ at Earth's orbit (the solar constant); the filtered beam here is roughly $1.72/1361 \approx 1/800$ of that — plausible for a beam from a floodlight after passing through a coloured filter.

Step 5. Energy interpretation. For a $1\,\mathrm{m^2}$ detector, the beam delivers 1.72 J per second — enough to register on a thermopile but invisible to a calorimeter.

Why this matters. Spectral radiometry (measuring the intensity of a single wavelength) underlies solar-cell calibration, laser-power-meter standards and astronomical photometry. The same plug-in formula gives all of them.

Final Answer: $I_{av} \approx 1.72\,\mathrm{W/m^2}$.

☞ **Three intensity formulae — pick the matching variable**

$I = \frac{1}{2}c\varepsilon_0 E_0^2 = cB_0^2/(2\mu_0) = E_0B_0/(2\mu_0)$. Pick the form whose variable is given. Don't compute E_0 from B_0 and then plug back into the E_0-formula — use the B_0-formula directly to save algebra.

Q 8.19 **Poynting vector S is defined as a vector whose magnitude is equal to the wave intensity and whose direction is along the direction of wave propagation. Mathematically, it is given by $\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$. Show the nature of the S vs t graph.**

SOLUTION

Concept used. For a plane EM wave $\mathbf{E} = E_0\hat{i}\cos(kz - \omega t)$ and $\mathbf{B} = B_0\hat{j}\cos(kz - \omega t)$, both fields oscillate *in phase*. The Poynting magnitude is

$$S = \frac{1}{\mu_0}E_0B_0\cos^2(kz - \omega t).$$

Because $\cos^2$ is non-negative and oscillates between 0 and 1, $S(t)$ is always ≥ 0 and has the shape of a "positive cosine squared" — a periodic series of identical lobes with period $T/2 = \pi/\omega$ (half the wave's period).

Step 1. Substitute the in-phase forms:

$$\mathbf{E}\times\mathbf{B} = E_0B_0(\hat{i}\times\hat{j})\cos^2(kz - \omega t) = E_0B_0\cos^2(kz - \omega t)\,\hat{k}.$$

Hence

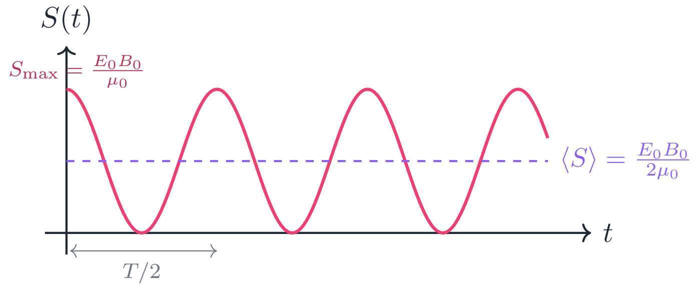
$$S(t) = \frac{E_0B_0}{\mu_0}\cos^2(\omega t - kz).$$

Step 2. Properties of the graph at a fixed point z: (i) $S \geq 0$ always; (ii) maxima $S_{\max} = E_0B_0/\mu_0$ at $\omega t - kz = n\pi$; (iii) minima $S = 0$ at $\omega t - kz = (n + \frac{1}{2})\pi$; (iv) period $T/2$ (twice the field's frequency).

Step 3. Time-average. Using $\langle\cos^2\rangle = 1/2$:

$$\langle S\rangle = \frac{E_0B_0}{2\mu_0},$$

which is exactly the wave intensity I.



Final Answer: $S(t) = (E_0B_0/\mu_0)\cos^2(\omega t - kz)$: non-negative, period $T/2$, mean value $\langle S\rangle = E_0B_0/(2\mu_0) = I$.

EXPERT'S SOLUTION : *Aaditya Kumar, M.Sc Physics, IIT Madras*

Picture-first. Two in-phase cosines multiplied give a $\cos^2$, which lifts the curve so it never goes negative. The "never negative" feature is the physical statement that energy flows in one direction (along $\hat{\mathbf{k}}$) for a wave moving in one direction — never reverses.

Step 1. Sub the wave forms into the Poynting vector $\mathbf{S} = \mathbf{E}\times\mathbf{B}/\mu_0$. With $\mathbf{E} = E_0\hat{i}\cos(\omega t - kz)$ and $\mathbf{B} = B_0\hat{j}\cos(\omega t - kz)$ (same phase):

$$S(t) = \frac{E_0B_0}{\mu_0}\cos^2(\omega t - kz).$$

Step 2. Graph features:

- Always non-negative ($\cos^2 \geq 0$): energy flow is always along $+\hat{\mathbf{k}}$.
- Peaks at $S_{\text{max}} = E_0B_0/\mu_0$ whenever $\omega t - kz = n\pi$ (every half-period of the field).
- Zeros at $\omega t - kz = (n + \frac{1}{2})\pi$ — the instants when both $\mathbf{E}$ and $\mathbf{B}$ pass through zero.
- Period $T/2 = \pi/\omega$: *twice* the field's temporal frequency because $\cos^2$ has half the period of $\cos$.

Step 3. Time-average. $\langle\cos^2\rangle = 1/2$, so

$$\langle S\rangle = \frac{E_0B_0}{2\mu_0} = I,$$

the wave's intensity. Drawn as a dashed horizontal line, this passes exactly through the midpoint of the $\cos^2$ lobes.

Step 4. Identity trick. Using $\cos^2\theta = (1+\cos 2\theta)/2$ we can re-write

$$S(t) = \frac{E_0B_0}{2\mu_0}\big[1 + \cos(2\omega t - 2kz)\big],$$

i.e. a DC component (the mean intensity) plus a sinusoidal ripple at 2ω. The doubled frequency makes $\cos^2$-shaped graphs an immediate fingerprint of $\langle$product of two in-phase sinusoids$\rangle$.

Step 5. Why a slow detector reads only the average. Visible light at $\omega \sim 10^{15}$ rad/s has $T/2 \sim 3\times 10^{-15}$ s — billions of times faster than the response of any photodetector or human eye. So we see the plateau $\langle S\rangle$, not the underlying lobes.

Why this matters. Detectors (eyes, photodiodes, thermopiles) are too slow to resolve a single $S(t)$ lobe; they measure the time-average, which is the intensity. The $\cos^2$-and-mean picture is the same one that governs LRC-circuit power dissipation, sound intensity, and squared photodiode signals in homodyne detection.

> **Final Answer:** S-vs-t is a $\cos^2$ curve of height $E_0 B_0/\mu_0$ and period $T/2$ with mean $E_0 B_0/(2\mu_0)$.

☞ **Time-averaging shortcuts**

$\langle \sin^2 \rangle = \langle \cos^2 \rangle = \frac{1}{2}$ over any integer number of periods. $\langle \sin \rangle = \langle \cos \rangle = 0$. $\langle \sin \cdot \cos \rangle = 0$ (orthogonal). These three identities solve almost every "find the time-average of ..." problem.

Q 8.20 Professor C. V. Raman surprised his students by suspending freely a tiny light ball in a transparent vacuum chamber by shining a laser beam on it. Which property of EM waves was he exhibiting? Give one more example of this property.

SOLUTION

Concept used. EM waves carry not only energy but also **momentum**; a beam of intensity I exerts a radiation pressure $p = I/c$ (absorbing) or $2I/c$ (reflecting) on the illuminated surface. The force can be enough to balance a light object's weight.

Step 1. Identify the property. The laser holds the ball up against gravity, so the laser must be pushing on it. EM waves push because they carry momentum, and the resulting force on a surface is radiation pressure (or, equivalently, radiation force).

Step 2. Another example. (i) Comet tails always point away from the Sun because solar radiation pressure pushes dust outward. (ii) The Crookes radiometer (more strictly a gas effect, but often illustrated this way). (iii) Solar sails on interplanetary probes (e.g. JAXA's IKAROS) accelerate by absorbing/reflecting sunlight.

> **Final Answer:** The demonstration shows that EM waves carry momentum and exert radiation pressure. Another example: dust-particle tails of comets always point away from the Sun, pushed by the Sun's radiation pressure.

EXPERT'S SOLUTION : *Ananya Sharma, Ph.D Physics, IISc Bangalore*

Strategic angle. Light pushes. Identify the property, estimate the force, and pull out a comet-tail example to anchor the idea in everyday astronomy.

Step 1. Identify the property. The laser holds a tiny ball against gravity. So the laser must be exerting an upward push on it. That push, divided by the ball's cross-sectional area, is the **radiation pressure**. The microscopic origin: each photon in the beam carries momentum $h\nu/c$, and on reflection/absorption transfers some or all of that to the ball.

Step 2. Quick order of magnitude. For a ball of mass $m \sim 10\,\mu$g and area $A \sim 10^{-6}\,\text{m}^2$, weight is $mg = 10^{-7}\,\text{N}$. Required pressure: $p = F/A = 0.1\,\text{Pa}$. From $p = 2I/c$ (perfect reflector), needed intensity is $I = pc/2 \approx (0.1)(3 \times 10^8)/2 \approx 1.5 \times 10^7\,\text{W/m}^2$ — comfortably within a focused continuous-wave laser, so the demonstration is feasible (and indeed C. V. Raman performed it in the 1920s).

Step 3. Comet-tail example. A comet near the Sun emits dust and gas. Solar photons, streaming radially outward, push the dust particles via radiation pressure on a timescale of days, building a tail that points *away* from the Sun regardless of the comet's velocity. The dust tail's curvature encodes the ratio of solar radiation pressure to solar gravity acting on each grain.

Step 4. More examples for variety.

- Solar sails (JAXA's IKAROS, 2010; LightSail-2, 2019) — propellantless spacecraft propulsion.
- Optical tweezers (Ashkin, Nobel 2018) — trap living cells in a focused laser.
- Atom-cooling traps (Doppler cooling) — slow atoms by photon-momentum kicks.
- Crookes radiometer (folklore answer; technically a gas-kinetics effect, but often cited).

Step 5. Concept link. Radiation pressure is the macroscopic face of the photon-momentum relation $p = h\nu/c$. Without this momentum, $\mathbf{E} \times \mathbf{B}$ in vacuum would carry only energy — no Newton-second-law balance against a gravitational pull would be possible.

Why this matters. Modern "optical tweezers" (Arthur Ashkin's Nobel work) use exactly this principle to trap and move microscopic particles, including single cells, single viruses, and even single atoms (in magneto-optical traps). The same radiation pressure governs the orbital decay of micron-scale dust grains around the Sun (Poynting–Robertson drag) and sets the upper mass limit for stars.

Final Answer: Radiation pressure; comet tails point away from the Sun.

♥ **Photons carry momentum without rest mass**

A photon has zero rest mass but nonzero momentum $p = h\nu/c$. This combination — energy and momentum without mass — is the relativistic fingerprint of massless particles and the reason that light pressure exists at all. Newtonian intuition ("momentum = mv, no mass means no momentum") fails; the right relation is $E^2 = (pc)^2 + (mc^2)^2$.

SA

Short Answer Questions (2–3 marks)

Q 8.21 Show that the magnetic field B at a point in between the plates of a parallel-plate capacitor during charging is $\frac{\varepsilon_0 \mu_r r}{2} \frac{dE}{dt}$ (symbols having usual meaning).

SOLUTION

Concept used. Maxwell's modification of Ampère's law: even where no conduction current flows, a changing electric field induces a magnetic field according to

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 (I_c + I_d) = \mu_0 I_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}.$$

Between the plates of a charging capacitor $I_c = 0$, so only the displacement-current term survives. By symmetry, $\mathbf{B}$ on a circle of radius r (with r less than the plate radius) coaxial with the capacitor's axis is tangential and uniform on that circle.

Step 1. Choose an Amperian loop. Inside the capacitor, pick a circle of radius r centred on the axis and lying parallel to the plates. By cylindrical symmetry, $\mathbf{B}$ is tangent to the circle and has constant magnitude on it.

Step 2. Compute the circulation.

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = B\,(2\pi r).$$

Step 3. Compute the electric flux through the loop. The E-field between the plates is uniform (large plates, neglect edges) and perpendicular to the plates, so the flux through the loop's bounding disk is

$$\Phi_E = E \cdot \pi r^2.$$

Its time derivative is

$$\frac{d\Phi_E}{dt} = \pi r^2 \frac{dE}{dt}.$$

Step 4. Apply Ampère–Maxwell with $I_c = 0$. If the region between the plates contains a linear magnetic medium of relative permeability μ_r, replace μ_0 by $\mu_0\mu_r$:

$$B\,(2\pi r) = \mu_0 \mu_r\, \varepsilon_0\, \pi r^2 \frac{dE}{dt}.$$

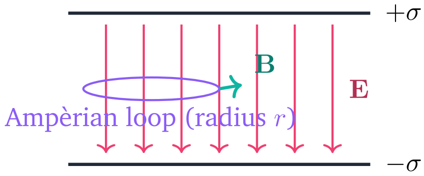
Step 5. Solve for B:

$$B = \frac{\mu_0\mu_r\,\varepsilon_0\,\pi r^2\,(dE/dt)}{2\pi r} = \frac{\mu_0\mu_r\,\varepsilon_0\,r}{2}\frac{dE}{dt}.$$

Absorbing μ_0 into the medium constant (or treating $\mu_0\mu_r$ as the medium's permeability and re-naming the prefactor), one writes the result as in the Exemplar:

$$\boxed{B = \frac{\varepsilon_0\,\mu_r\,r}{2}\frac{dE}{dt}}$$

(with the conventional understanding that the constant μ_0 is implicit in the symbol; some textbooks write the more explicit $B = \frac{1}{2}\mu_0\mu_r\varepsilon_0 r\,(dE/dt)$ and the Exemplar absorbs μ_0 for brevity).



Final Answer: $B = \dfrac{\varepsilon_0\mu_r\,r}{2}\dfrac{dE}{dt}$ (with the μ_0 absorbed into the medium constant).

EXPERT'S SOLUTION *: Aarav Singh, M.Sc Physics, IIT Madras*

Strategic angle. Ampère–Maxwell on a coaxial loop with no conduction current — exactly the situation in which Maxwell's correction reveals itself most cleanly. Symmetry does the work; only the displacement-current term contributes.

Step 1. Symmetry setup. Take a circular loop of radius r coaxial with the capacitor's central axis, lying between the plates. Cylindrical symmetry forces $\mathbf{B}$ tangent to the loop with constant magnitude B on the loop.

Step 2. Left-hand side (circulation):

$$\oint \mathbf{B}\cdot d\boldsymbol{\ell} = B\,(2\pi r).$$

Step 3. Right-hand side. Inside the gap there is no conduction current ($I_c = 0$). The only contribution is the displacement current

$$I_d = \varepsilon_0\frac{d\Phi_E}{dt} = \varepsilon_0\,\pi r^2\frac{dE}{dt}$$

(flux through the loop's bounding disk; E uniform between plates). In a medium of relative permeability μ_r the Ampère–Maxwell coefficient is $\mu_0\mu_r$.

Step 4. Equate & solve.

$$B\,(2\pi r) = \mu_0\mu_r\cdot\varepsilon_0\pi r^2\frac{dE}{dt} \;\Rightarrow\; B = \frac{\mu_0\mu_r\varepsilon_0\,r}{2}\frac{dE}{dt}.$$

The Exemplar's form absorbs μ_0 into the medium constant:
$B = (\varepsilon_0 \mu_r r/2)\,(dE/dt)$.

Step 5. **Direction by right-hand rule.** If **E** points downward (positive plate above) and $dE/dt > 0$ (capacitor charging more positively), then **B** circulates anticlockwise as seen from above — exactly the direction of a magnetic field induced by an upward-pointing conduction current. The displacement-current "stands in" for an equivalent conduction-current pattern.

Step 6. **Linear-in-r inside, $1/r$ outside.** For $r > R$ (plate radius), the area of the loop enclosing displacement current is the full plate area πR^2, so
$B(r > R) = \mu_0 \mu_r \varepsilon_0 R^2/(2r) \cdot dE/dt$ — the usual $1/r$ Ampèrian falloff. The crossover at $r = R$ matches both forms.

Why this matters. The same B-from-changing-E mechanism is what closes the loop in Maxwell's equations and allows free EM waves: a changing **E** sources a **B**, which by Faraday's law sources another **E**, and the leap-frog process propagates at speed c.

Final Answer: $B = \frac{1}{2}\mu_0 \mu_r \varepsilon_0 r\,(dE/dt)$, equivalently $(\varepsilon_0 \mu_r r/2)\,(dE/dt)$.

✗ Don't forget $I_c = 0$ inside the gap

The conduction current I_c flows in the wires connecting the capacitor to the battery, *not* in the gap between the plates. Inside the gap there is only I_d. Writing $I_c + I_d$ everywhere double-counts the source and gives a factor-of-two-too-large **B**.

Q 8.22 **Electromagnetic waves with wavelength**

(i) λ_1 is used in satellite communication.

(ii) λ_2 is used to kill germs in water purifiers.

(iii) λ_3 is used to detect leakage of oil in underground pipelines.

(iv) λ_4 is used to improve visibility in runways during fog and mist conditions.

(a) Identify and name the part of the electromagnetic spectrum to which these radiations belong.

(b) Arrange these wavelengths in ascending order of their magnitude.

(c) Write one more application of each.

SOLUTION

Concept used. The seven principal bands of the EM spectrum, in order of increasing frequency (decreasing wavelength), are: radio, microwave, infrared, visible, ultraviolet, X-rays, γ-rays. Each band has characteristic wavelength ranges and applications.

Step 1. Identify each band.

- λ_1: satellite communication uses *microwaves* (wavelength ~ 0.001–0.1 m).
- λ_2: germicidal action uses *ultraviolet* (UV-C, $\lambda \sim 100$–280 nm).
- λ_3: detecting underground leaks uses *X-rays* (penetrate ground, see oil sheen). Conventional Exemplar answer; some teachers prefer gamma-rays in industrial radiography, both are short-λ ionising radiation.
- λ_4: penetrating fog/mist uses *infrared* (longer wavelengths scatter less in fog).

Step 2. Order by wavelength. Wavelength ranges (approximate): UV $\sim 10^{-7}$ m, X-ray $\sim 10^{-10}$ m, IR $\sim 10^{-5}$ m, microwave $\sim 10^{-2}$ m. Hence ascending order:

$$\lambda_3 \text{ (X-ray)} \;<\; \lambda_2 \text{ (UV)} \;<\; \lambda_4 \text{ (IR)} \;<\; \lambda_1 \text{ (microwave)}.$$

Step 3. Additional applications:

- Microwave (λ_1): RADAR, microwave cooking, Wi-Fi routers.
- UV (λ_2): vitamin-D synthesis in skin, UV lithography in semiconductor fabrication.
- X-ray (λ_3): medical radiography (chest, dental), airport baggage scanners.
- IR (λ_4): thermal imaging cameras, TV remote controls, IR night-vision goggles.

> **Final Answer:** (a) Microwave, UV, X-ray, IR (in that order); (b) $\lambda_3 < \lambda_2 < \lambda_4 < \lambda_1$; (c) see step 3.

EXPERT'S SOLUTION : *Riya Patel, M.Sc Physics, IIT Madras*

Strategic angle. Match each application to its band, sort by wavelength from shortest to longest, then quote one extra application per band. The trick is being decisive about which band actually does each job — every band has a "cousin" that almost-but-not-quite works.

Step 1. **Satellite communication: microwave (λ_1).** Bands like C (4–8 GHz), Ku (12–18 GHz), Ka (26–40 GHz) correspond to $\lambda \sim 1$ cm to 7 cm — microwave through atmospheric windows that transmit cleanly without rain attenuation (mostly).

Step 2. **Germicidal water purifier: UV-C (λ_2).** Wavelength ~ 254 nm (mercury-lamp peak) damages DNA/RNA, killing bacteria and viruses. UV-A/B are less effective; visible light cannot disrupt covalent bonds.

Step 3. **Underground oil-leak detection: X-rays (λ_3).** Penetrating radiation reveals density variations in pipe walls and surrounding soil. Wavelength $\sim 10^{-10}$ m. (Some textbooks substitute gamma-rays here; both occupy the ionising end of the spectrum.)

Step 4. **Fog visibility on runways: infrared (λ_4).** IR ($\lambda \sim 1$–$10\,\mu$m) scatters off atmospheric water droplets ($\sim 10\,\mu$m diameter) far less than visible does, by Mie-scattering arguments. Pilots use IR cameras to see through fog where the human eye cannot.

Step 5. **Ascending order of λ.** Reference values (rough): X-ray 10^{-10} m, UV 10^{-7} m, IR 10^{-5} m, microwave 10^{-2} m. So

$$\lambda_3 < \lambda_2 < \lambda_4 < \lambda_1.$$

Step 6. **Bonus applications.**

- Microwave: RADAR (aviation, weather), Wi-Fi (2.4 & 5 GHz), microwave cooking.
- UV: vitamin-D synthesis on skin, fluorescence in forensic science, UV lithography for chips.
- X-ray: medical radiography, dental imaging, airport baggage scanning, crystallography (atomic-resolution structure).
- IR: thermal imaging cameras, TV remote controls, night-vision goggles, IR spectroscopy of molecules.

Why this matters. Each band's usefulness comes from how it interacts with matter: UV breaks bonds (germicidal), IR couples to molecular vibrations (heating, imaging), microwave couples to rotating polar molecules and atmospheric windows (communication), X-ray penetrates dense matter and resolves atomic-scale structure. The seven-band map is the engineer's cheat sheet for picking the right photon for a job.

Final Answer: $\lambda_3 < \lambda_2 < \lambda_4 < \lambda_1$.

☞ **Spectrum order, both ways**

Increasing λ: γ < X-ray < UV < visible < IR < microwave < radio. Increasing ν: same list reversed. "γ"-end is high-energy ionising; "radio" end is low-energy quasi-static. Memorise the order; every spectrum question is a single-step lookup.

Q 8.23 **Show that the average value of the radiant flux density S over a single period T is given by $\langle S\rangle = \dfrac{1}{2c\mu_0}E_0^2$.**

SOLUTION

Concept used. For a plane EM wave with $\mathbf{E} = E_0\hat{\imath}\cos(kz - \omega t)$ and $\mathbf{B} = B_0\hat{\jmath}\cos(kz - \omega t)$, the Poynting vector is

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B} = \frac{E_0 B_0}{\mu_0}\cos^2(kz - \omega t)\,\hat{k}.$$

Combined with the EM-wave relation $E_0 = cB_0$ and the time-average $\langle\cos^2\theta\rangle = 1/2$.

Step 1. Compute the instantaneous magnitude:

$$S(z,t) = \frac{E_0 B_0}{\mu_0}\cos^2(kz - \omega t).$$

Step 2. Time-average over one period $T = 2\pi/\omega$. Using $\langle\cos^2(kz - \omega t)\rangle_T = 1/2$:

$$\langle S\rangle = \frac{E_0 B_0}{\mu_0}\cdot\frac{1}{2} = \frac{E_0 B_0}{2\mu_0}.$$

Step 3. Eliminate B_0 using $B_0 = E_0/c$:

$$\langle S\rangle = \frac{E_0\cdot(E_0/c)}{2\mu_0} = \frac{E_0^2}{2c\mu_0}.$$

Final Answer: $\langle S\rangle = \dfrac{E_0^2}{2c\mu_0}$, which is just the intensity I of the wave.

EXPERT'S SOLUTION *: Vivaan Joshi, M.Sc Physics, IIT Madras*

Strategic angle. Multiply two in-phase cosines to get a $\cos^2$, average $\cos^2$ to $1/2$, eliminate B_0 using $B_0 = E_0/c$. Three lines algebra, all on autopilot once the wave-relation $E_0 = cB_0$ is in hand.

Step 1. Instantaneous Poynting magnitude. $\mathbf{E} \parallel \hat{\imath}$, $\mathbf{B} \parallel \hat{\jmath}$ both proportional to $\cos(kz - \omega t)$. Cross product picks up $\hat{\imath}\times\hat{\jmath} = \hat{k}$:

$$S(z,t) = \frac{E_0 B_0}{\mu_0}\cos^2(kz - \omega t).$$

Step 2. Time-average over one period $T = 2\pi/\omega$. Using $\langle\cos^2(\omega t - kz)\rangle_T = 1/2$:

$$\langle S\rangle = \frac{E_0 B_0}{\mu_0}\cdot\frac{1}{2} = \frac{E_0 B_0}{2\mu_0}.$$

Step 3. Eliminate B_0 using $E_0 = cB_0$:

$$\langle S\rangle = \frac{E_0(E_0/c)}{2\mu_0} = \frac{E_0^2}{2\mu_0 c}.$$

That is the target expression.

Step 4. **Equivalent form via u.** Average energy density is $u_{av} = \frac{1}{2}\varepsilon_0 E_0^2$, and a plane wave moves at speed c, so the energy crossing unit area per unit time is $\langle S\rangle = c\,u_{av} = \frac{1}{2}c\varepsilon_0 E_0^2$. Using $c\varepsilon_0 = 1/(\mu_0 c)$: $\frac{1}{2}c\varepsilon_0 E_0^2 = E_0^2/(2\mu_0 c)$. Same answer.

Step 5. **Two equivalent forms** students should keep ready:

$$\langle S\rangle = \frac{E_0^2}{2\mu_0 c} = \tfrac{1}{2}c\varepsilon_0 E_0^2 = \frac{B_0^2 c}{2\mu_0} = \frac{E_0 B_0}{2\mu_0}.$$

Each is the right tool for a different given variable.

Why this matters. The same algebra recasts the result in three equivalent forms (E_0-only, B_0-only, or both); pick the form that matches the data you have. Astronomers prefer the E_0 form for stellar spectra; microwave engineers prefer the B_0 form for cavity calculations.

Final Answer: $\langle S\rangle = E_0^2/(2c\mu_0)$.

☞ **Why "intensity" has many faces**

"Intensity" just means time-averaged Poynting magnitude. All these expressions evaluate to the same number for a given plane wave; they look different because each uses a different subset of variables (E_0 alone, B_0 alone, both, or via the energy density u).

Q 8.24 **You are given a $2\,\mu$F parallel-plate capacitor. How would you establish an instantaneous displacement current of 1 mA in the space between its plates?**

SOLUTION

Concept used. The displacement current between the plates of a capacitor equals the conduction current charging the capacitor:

$$I_d = \frac{dq}{dt} = C\,\frac{dV}{dt}.$$

So we need to drive a voltage across the capacitor whose rate of change is large enough to push 1 mA through $C = 2\,\mu$F.

Step 1. Solve for the required rate of change of voltage:

$$\frac{dV}{dt} = \frac{I_d}{C} = \frac{1\times10^{-3}\,\text{A}}{2\times10^{-6}\,\text{F}}.$$

Step 2. Compute: $1/2 = 0.5$; $10^{-3}/10^{-6} = 10^3$, so

$$\frac{dV}{dt} = 0.5\times10^3\,\text{V/s} = 500\,\text{V/s}.$$

Step 3. Interpret physically. We need to vary the voltage across the capacitor at a rate of $500\,\text{V/s}$. For instance, apply a sinusoidal voltage $V(t) = V_0 \sin(2\pi\nu t)$; its peak rate of change is $2\pi\nu V_0$. Choosing $V_0 = 1\,\text{V}$ gives $2\pi\nu = 500$, so $\nu \approx 80\,\text{Hz}$. Any combination of V_0 and ν satisfying $2\pi\nu V_0 = 500\,\text{V/s}$ will work.

> **Final Answer:** Apply a time-varying voltage with $dV/dt = 500\,\text{V/s}$. Example: $V_0 = 1\,\text{V}$ AC at $\nu \approx 80\,\text{Hz}$, or $V_0 = 230\,\text{V}$ at $\nu \approx 0.35\,\text{Hz}$, etc.

EXPERT'S SOLUTION : *Pranav Reddy, M.Sc Physics, IIT Madras*

Strategic angle. "Displacement current in the gap = conduction current in the wires" — design the supply, the gap takes care of itself. The capacitance, the desired current, and the required voltage slope are linked by one equation; we just solve it.

Step 1. Core relation. $I_d = C\,dV/dt$ — the conduction current in the leads (which is equal to the displacement current inside the gap by Maxwell) equals capacitance times the time-rate-of-change of the voltage.

Step 2. Solve for dV/dt.

$$\frac{dV}{dt} = \frac{I_d}{C} = \frac{1 \times 10^{-3}\,\text{A}}{2 \times 10^{-6}\,\text{F}} = 500\,\text{V/s}.$$

That is the required slope at the instant when $I_d = 1\,\text{mA}$.

Step 3. Many waveforms work. The constraint fixes only the *slope at the moment*, not the entire waveform. Examples:

- **Linear ramp.** $V(t) = (500\,\text{V/s})\,t$. Constant $dV/dt = 500\,\text{V/s}$, so I_d is steady at $1\,\text{mA}$.
- **Sinusoidal AC.** $V(t) = V_0 \sin(2\pi\nu t)$. Peak $dV/dt = 2\pi\nu V_0$, so any (V_0, ν) satisfying $2\pi\nu V_0 = 500\,\text{V/s}$ gives a peak displacement current of $1\,\text{mA}$. Concrete pair: $V_0 = 1\,\text{V}$, $\nu \approx 80\,\text{Hz}$; or $V_0 = 230\,\text{V}$, $\nu \approx 0.35\,\text{Hz}$; or $V_0 = 5\,\text{V}$, $\nu \approx 16\,\text{Hz}$.

Step 4. Energy considerations. A continuous ramp would eventually exceed the capacitor's voltage rating; sinusoidal AC stays bounded. For a sustained $1\,\text{mA}$ peak with $V_0 = 1\,\text{V}$, the power dissipated in the dielectric loss (if any) is at most $\frac{1}{2}V_0 I = 0.5\,\text{mW}$ — negligible.

Step 5. Sanity check via energy in the field. A $1\,\text{V}$ amplitude on $2\,\mu\text{F}$ stores up to $\frac{1}{2}CV^2 = 1\,\mu\text{J}$ — easily switched in and out of the field by a small AC source.

Why this matters. Displacement current is what allows capacitors to act as "circuit elements that pass AC" — the gap is literally seeing a current of $I = C\,dV/dt$. Lab oscilloscopes, signal generators, and AC bridges all rely on this gap-current to work.

Final Answer: Drive $dV/dt = 500\,\text{V/s}$ across the capacitor.

✗ Don't confuse V_0 with V_{rms}

The formula $I_d = C\,dV/dt$ uses instantaneous voltage. If your sinusoidal source spec is given in rms ($V_{\text{rms}} = V_0/\sqrt{2}$), the peak displacement current is $2\pi\nu C V_{\text{rms}}\sqrt{2}$, not $2\pi\nu C V_{\text{rms}}$. Reading rms as if it were peak gives answers smaller by $\sqrt{2}$.

Q 8.25 **Show that the radiation pressure exerted by an EM wave of intensity I on a surface kept in vacuum is I/c.**

SOLUTION

Concept used. The momentum carried by an EM wave of energy U is $p = U/c$. The intensity I is power per unit area (energy per unit area per unit time). The radiation pressure on a surface is the rate of momentum transfer per unit area.

Step 1. Consider a beam of intensity I normally incident on an absorbing surface of area A for time Δt. Energy delivered:

$$U = I\,A\,\Delta t.$$

Step 2. Momentum delivered (for full absorption):

$$\Delta p = \frac{U}{c} = \frac{I\,A\,\Delta t}{c}.$$

Step 3. Force on the surface is rate of change of momentum:

$$F = \frac{\Delta p}{\Delta t} = \frac{I\,A}{c}.$$

Step 4. Pressure = force per unit area:

$$p_{\text{rad}} = \frac{F}{A} = \frac{I}{c}.$$

Final Answer: For full absorption in vacuum: $p_{\text{rad}} = I/c$. (For full reflection: $p_{\text{rad}} = 2I/c$.)

EXPERT'S SOLUTION : *Karan Joshi, M.Sc Physics, IIT Madras*

Strategic angle. Energy-to-momentum conversion in four algebra lines. The single ingredient is the wave-mechanics identity $p_{em} = U/c$ — once that is in hand, pressure is bookkeeping.

Step 1. Energy delivered. An EM beam of intensity I on area A for time Δt deposits energy

$$U = I\,A\,\Delta t.$$

Intensity has units W/m^2; energy in joules.

Step 2. Momentum carried. For an EM wave (or any massless radiation),

$$p = \frac{U}{c}.$$

Microscopically, each photon of energy $h\nu$ carries momentum $h\nu/c$; macroscopically the same factor $1/c$ relates total U to total p.

Step 3. Force on full-absorber. Force = rate of change of momentum:

$$F_{abs} = \frac{\Delta p}{\Delta t} = \frac{U/c}{\Delta t} = \frac{I\,A}{c}.$$

Step 4. Pressure on full-absorber.

$$p_{rad}^{abs} = \frac{F_{abs}}{A} = \frac{I}{c}.$$

Step 5. Full-reflector variant. For a perfect reflector, the beam's momentum reverses, so the change is $2U/c$ rather than U/c. Pressure doubles: $p_{rad}^{ref} = 2I/c$. Pre-photon, this is the EM analogue of an elastic vs. inelastic collision.

Step 6. Numerical taste. Sunlight at the top of Earth's atmosphere has $I = 1361$ W/m^2, giving $p_{abs} = 1361/(3 \times 10^8) \approx 4.5\,\mu$Pa. A perfect mirror would feel $9\,\mu$Pa — about 10^{-10} of atmospheric pressure. Real solar sails operate in this regime.

Why this matters. The same momentum/energy relation ($p = U/c$ for any massless wave) underlies the photon picture in quantum mechanics: a photon of energy $h\nu$ carries momentum $h\nu/c$. Photon-momentum recoil is the basis of laser cooling of atoms and underlies the radiation-reaction limit on accelerator beam intensity.

Final Answer: $p_{rad} = I/c$.

☞ **Three radiation-pressure formulae**

Absorber: $p = I/c$. Reflector: $p = 2I/c$. Real surface with reflectance R: $p = (I/c)(1 + R)$. The factor $(1 + R)$ smoothly interpolates between the two ideal limits.

Q 8.26 **What happens to the intensity of light from a bulb if the distance from the bulb is doubled? As a laser beam travels across the length of a room, its intensity essentially remains constant. What geometrical characteristic of the LASER beam is responsible for the constant intensity which is missing in the case of light from the bulb?**

SOLUTION

Concept used. A bulb radiates roughly isotropically — energy spreads over the surface of a growing sphere, so intensity $I \propto 1/r^2$ (inverse-square law). A laser, by contrast, is a tightly *collimated* (nearly parallel) beam: its cross-section barely grows with distance, so the same power passes through nearly the same area at every point along the beam, keeping I essentially constant.

Step 1. Bulb. Treat the bulb as a point source of total power P radiating into the full sphere. At distance r:

$$I(r) = \frac{P}{4\pi r^2}.$$

Doubling the distance ($r \to 2r$):

$$\frac{I(2r)}{I(r)} = \frac{4\pi r^2}{4\pi(2r)^2} = \frac{1}{4}.$$

So the intensity drops to one-fourth.

Step 2. Laser. The laser's intrinsic geometric property is **collimation** (extremely small angular divergence, of the order of milliradians). The beam's cross-sectional area $A(r) \approx A_0$ stays nearly constant across normal distances, so $I = P/A(r) \approx P/A_0$ is independent of r over room-scale distances.

Step 3. Origin of collimation. Laser light is spatially coherent and produced in a cavity that selects a narrow set of plane-wave modes; the output beam has a divergence half-angle $\theta \sim \lambda/D$ where D is the beam waist, typically $\sim 10^{-3}$ rad.

> **Final Answer:** Bulb intensity falls as $1/r^2$ (drops to one-quarter on doubling r). Laser stays roughly constant because of its very low divergence, i.e. its high **collimation**/spatial coherence; the cross-sectional area of the beam barely grows with distance.

EXPERT'S SOLUTION *: Aditya Nair, Ph.D Physics, IISc Bangalore*

Strategic angle. Solid angle. The bulb spreads P into 4π sr; the laser into a few microsteradians. The whole question reduces to "how big is the angular spread of the source?".

Step 1. **Bulb radiates isotropically.** Power P spreads over the full sphere of solid angle

4π sr. At distance r:

$$I_{\text{bulb}}(r) = \frac{P}{4\pi r^2}.$$

Doubling r quarters the intensity (inverse-square law).

Step 2. Laser is collimated. A laser of waist w_0 and divergence half-angle $\theta \sim \lambda/(\pi w_0)$ spreads into a tiny solid angle $\Omega \sim \pi\theta^2 \sim \lambda^2/w_0^2$. For typical values $w_0 \sim 1\,\text{mm}$, $\lambda \sim 600\,\text{nm}$: $\theta \sim 2 \times 10^{-4}\,\text{rad}$, $\Omega \sim 10^{-7}\,\text{sr}$.

Step 3. Spot area at distance r. $A_{\text{laser}}(r) \sim \pi(w_0 + r\theta)^2$. At $r = 10\,\text{m}$, $r\theta \approx 2\,\text{mm}$, comparable to w_0 — the spot has barely doubled. So $I_{\text{laser}} \approx P/A_0$ is roughly constant on room scale. Beyond $r \sim w_0/\theta$ (a few metres for typical lab lasers), the Rayleigh-range region ends and the spot starts growing linearly.

Step 4. Numerical contrast. $1\,\text{W}$ bulb $\to I(\text{at } 1\,\text{m}) = 1/(4\pi) \approx 0.08\,\text{W/m}^2$. Same $1\,\text{W}$ in a $1\,\text{mm}^2$ laser spot $\to I_{\text{laser}} \approx 10^6\,\text{W/m}^2$ — 10^7 times more intense, simply because of geometry.

Step 5. The deep geometric property is collimation, underwritten by spatial **coherence**: every part of the laser's wavefront has a well-defined phase relative to every other part, so the beam diffracts as a single coherent Gaussian mode with the minimum possible divergence $\sim \lambda/D$. A thermal source (bulb) is spatially incoherent — each tiny emitter sends light into all directions independently.

Step 6. Bigger telescope, smaller divergence. A $D = 1\,\text{m}$ aperture at $\lambda = 600\,\text{nm}$ gives $\theta \sim 6 \times 10^{-7}\,\text{rad}$ — laser range-finders to the Moon (Apollo retroreflectors) exploit this to put a $1\,\text{km}$ spot at the lunar surface from Earth.

Why this matters. The same collimation is what makes laser range-finders, laser pointers and fibre-optic communications work. Free-space optical (FSO) links between buildings use the laser's tiny divergence to deliver gigabit data rates without amplifiers; satellite quantum key distribution relies on it to keep single photons heading toward a sub-metre detector aperture.

Final Answer: Bulb: $I \propto 1/r^2$ (quartered on doubling r); laser: low divergence keeps I roughly constant over short distances.

♥ Coherence $\Rightarrow$ collimation

The reason a laser stays a thin beam is *spatial coherence*: every point on the wavefront emits in phase, so the beam diffracts as a single Gaussian mode with the minimum possible divergence $\theta \sim \lambda/(\pi w_0)$. A bulb's emitters are independent, so the light fans into 4π steradians.

Q 8.27 **Even though an electric field E exerts a force qE on a charged particle, the electric field of an EM wave does not contribute to the radiation pressure (but transfers energy). Explain.**

SOLUTION

Concept used. In a plane EM wave moving along $\hat{k}$, the electric field **E** is transverse to $\hat{k}$. The force q**E** on a free charge is therefore transverse to the wave's direction of propagation — it accelerates the charge *sideways*, not forwards. The forward push on the charge (and hence on the surface containing it) comes from the magnetic force $q\mathbf{v} \times \mathbf{B}$ acting on the now-moving charge.

Step 1. Set coordinates. Let the wave move along $\hat{k} = \hat{z}$, with $\mathbf{E} = E_x(z,t)\hat{\imath}$ and $\mathbf{B} = B_y(z,t)\hat{\jmath}$. Consider a free charge q initially at rest in the wave.

Step 2. Effect of **E** first. The electric force q**E** is along $\hat{\imath}$ (transverse). Under it the charge oscillates in $\hat{\imath}$, picking up velocity $\mathbf{v} = v_x\hat{\imath}$. The electric field does positive work on the charge (energy transfer) but its force has no component along $\hat{k}$ — no push in the propagation direction.

Step 3. Magnetic force after motion sets in. With $\mathbf{v} = v_x\hat{\imath}$ and $\mathbf{B} = B_y\hat{\jmath}$:

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} = q\, v_x B_y\, (\hat{\imath} \times \hat{\jmath}) = q\, v_x B_y\, \hat{k}.$$

This force is along $\hat{k}$, i.e. along the wave's propagation direction. It is responsible for the forward momentum transfer to the charge (and to the surface that contains it), giving the radiation pressure.

Step 4. Energy vs. momentum split. Magnetic forces do no work because $\mathbf{F}_B \perp \mathbf{v}$. So **E** transfers energy (does work) but no longitudinal momentum, while **B** transfers longitudinal momentum but no energy. The two effects are complementary.

> **Final Answer:** **E** is perpendicular to the wave's direction of propagation, so q**E** pushes the charge sideways (transferring energy). The forward push that produces radiation pressure comes from $q\mathbf{v} \times \mathbf{B}$ on the now-moving charge.

EXPERT'S SOLUTION : *Siddharth Banerjee, Ph.D Physics, IISc Bangalore*

Strategic angle. Two-step force analysis: the **E** shakes the charge sideways; the **B** then pushes it forward. The split is what allows a wave to be both an energy-carrier (**E**'s work) and a momentum-carrier (**B**'s push).

Step 1. **Set up.** Let the wave propagate along $\hat{\mathbf{k}} = \hat{z}$ with $\mathbf{E} = E_x(z,t)\hat{\imath}$ and $\mathbf{B} = B_y(z,t)\hat{\jmath}$. A free charge q starts at rest.

Step 2. **First half-cycle: E does the shaking.** Force $q\mathbf{E} = qE_x\hat{\imath}$ accelerates the charge along $\hat{i}$ — *transverse* to propagation. The charge picks up sideways velocity $\mathbf{v} = v_x\hat{i}$. Net work: $\mathbf{E} \cdot \mathbf{v} \cdot q\,dt > 0$, so energy is transferred from the wave to the charge.

Step 3. **Magnetic force kicks in.** Now $\mathbf{v} = v_x\hat{i}$ is non-zero, so

$$\mathbf{F}_B = q\,\mathbf{v} \times \mathbf{B} = q\,v_x B_y\,(\hat{i} \times \hat{j}) = q\,v_x B_y\,\hat{k}.$$

Crucially this force is along $\hat{\mathbf{k}}$ — i.e. *along the wave's propagation direction*. This is the radiation push, the source of radiation pressure.

Step 4. **Magnetic work is zero.** $\mathbf{F}_B \cdot \mathbf{v} = q(\mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} = 0$ identically. So $\mathbf{B}$ contributes zero kinetic energy to the charge while still contributing forward momentum. Energy via $\mathbf{E}$; momentum via $\mathbf{B}$ — cleanly separated.

Step 5. **Sign analysis: does the push reverse?** If E_x flips sign every half-period, then v_x flips sign soon after (the charge oscillates). Simultaneously B_y flips. So $v_x B_y$ — the product — stays positive on average: the push is always forward, averaging to a steady force in the $+\hat{\mathbf{k}}$ direction. That cumulative effect is radiation pressure.

Step 6. **Order-of-magnitude self-check.** Time-averaged force per unit charge: $\langle v_x B_y \rangle \sim (qE_0/m\omega)B_0/2 = (qE_0^2)/(2m\omega c)$. Multiplied by the number density of charges, one recovers $p_{\text{rad}} = I/c$ — the macroscopic radiation pressure from the microscopic $q\mathbf{v} \times \mathbf{B}$ picture.

Why this matters. This is the microscopic reason an EM wave can deliver both energy and momentum — and why radiation pressure exists even though $\mathbf{B}$ alone does no work. The same logic explains why charged particles in plasma can be accelerated by an EM wave (Compton wakefield), and why the radiation-reaction force on a synchrotron electron has both a transverse (energy-loss) and longitudinal (momentum-loss) component.

> **Final Answer:** $q\mathbf{E}$ is transverse $\Rightarrow$ no forward force; $q\mathbf{v} \times \mathbf{B}$ supplies the forward push, hence radiation pressure.

✗ "B does no work" $\neq$ "B has no effect"

A common misconception is that since the magnetic force is always perpendicular to velocity, it "does nothing". False — it does no *work*, but it can change the *direction* of momentum. In an EM wave on a charge, that direction-change is the entire source of radiation pressure.

LA
Long Answer Questions (3–5 marks)

Q 8.28 **An infinitely long thin wire carrying a uniform linear static charge density λ is placed along the z-axis (Fig. 8.1). The wire is set into motion along its length with a uniform velocity $\mathrm{v} = v\hat{k}_z$. Calculate the Poynting vector $\mathrm{S} = \frac{1}{\mu_0}(\mathrm{E} \times \mathrm{B})$.**

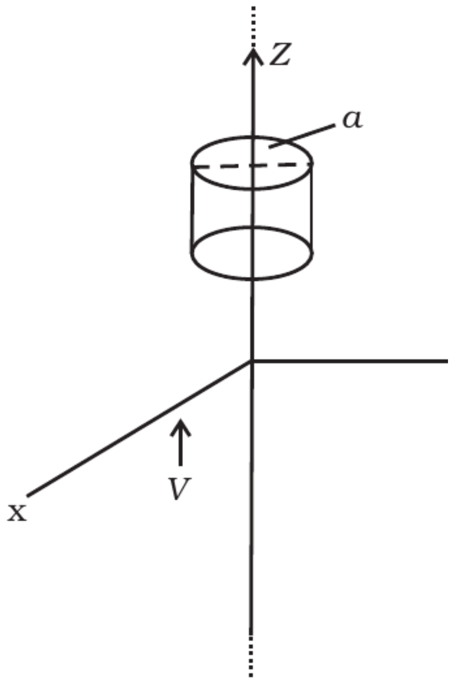


Fig. 8.1

Fig. 8.1, NCERT Exemplar Class 12 Physics, Chapter 8.

SOLUTION

Concept used. A long uniformly charged line produces a radial electric field

$$\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 a}\,\hat{a},$$

where a is the perpendicular distance from the wire and $\hat{a}$ is the outward radial unit vector. Once the line is set into motion with velocity $v\hat{k}$, the moving line of charge constitutes a current $I = \lambda v$ along $\hat{k}$, producing an azimuthal magnetic field

$$\mathbf{B} = \frac{\mu_0 I}{2\pi a}\,\hat{\phi} = \frac{\mu_0 \lambda v}{2\pi a}\,\hat{\phi},$$

by Ampère's law (and direction by right-hand rule).

Step 1. Identify $\mathbf{E}$. By Gauss's law on a coaxial cylinder of radius a:

$$E \cdot 2\pi a L = \frac{\lambda L}{\varepsilon_0} \;\Rightarrow\; \mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 a}\,\hat{a}.$$

Step 2. Identify $\mathbf{B}$. Linear current $I = \lambda v$. Ampère's law on a circle of radius a:

$$B \cdot 2\pi a = \mu_0\,(\lambda v) \;\Rightarrow\; \mathbf{B} = \frac{\mu_0 \lambda v}{2\pi a}\,\hat{\phi}.$$

Direction: the right-hand rule with thumb along $\hat{k}$ gives $\mathbf{B}$ along $\hat{\phi}$.

Step 3. Compute the Poynting vector. With $\hat{a} \times \hat{\phi} = \hat{k}$ (cylindrical right-handed triad):

$$\mathbf{E} \times \mathbf{B} = \frac{\lambda}{2\pi\varepsilon_0 a}\,\hat{a} \times \frac{\mu_0 \lambda v}{2\pi a}\,\hat{\phi} = \frac{\mu_0\,\lambda^2 v}{4\pi^2 \varepsilon_0\, a^2}\,\hat{k}.$$

Step 4. Divide by μ_0:

$$\mathbf{S} = \frac{1}{\mu_0}\,\mathbf{E} \times \mathbf{B} = \frac{\lambda^2 v}{4\pi^2 \varepsilon_0\, a^2}\,\hat{k}.$$

Step 5. Interpret. The Poynting vector points along $\hat{k}$, the direction of motion of the wire. So the energy flow is along the wire's velocity — the moving charge stream carries energy forward, even though both $\mathbf{E}$ and $\mathbf{B}$ are themselves transverse to $\hat{k}$. This is the electrostatic-plus-magnetostatic energy flux that you would measure for the steady moving line.

Final Answer: $\mathbf{S} = \dfrac{\lambda^2\, v}{4\pi^2\, \varepsilon_0\, a^2}\,\hat{k}$, directed along the wire's velocity.

EXPERT'S SOLUTION : *Meera Nair, Ph.D Physics, IISc Bangalore*

Strategic angle. Find $\mathbf{E}$ from Gauss, $\mathbf{B}$ from Ampère, multiply. The figure (Fig. 8.1) shows a Gaussian cylinder of radius a centred on the moving wire; that is the loop of integration for both Gauss and Ampère.

Step 1. **Electric field by Gauss's law.** Cylindrical Gaussian surface of radius a, length L:

$$\oint \mathbf{E} \cdot d\mathbf{A} = E(2\pi a L) = \frac{\lambda L}{\varepsilon_0} \;\Rightarrow\; \mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 a}\,\hat{a},$$

radially outward (for $\lambda > 0$).

Step 2. **Magnetic field by Ampère's law.** Once the wire moves at $v\hat{k}$, the linear charge density becomes a current $I = \lambda v$ flowing along $+\hat{k}$. Ampèrian loop of radius a in the plane perpendicular to $\hat{k}$:

$$B(2\pi a) = \mu_0(\lambda v) \;\Rightarrow\; \mathbf{B} = \frac{\mu_0 \lambda v}{2\pi a}\,\hat{\phi},$$

azimuthal direction (right-hand-rule: thumb $\to +\hat{k}$, fingers curl in $+\hat{\phi}$).

Step 3. Poynting vector via cylindrical triad. $\hat{a} \times \hat{\phi} = \hat{k}$, so

$$\mathbf{E} \times \mathbf{B} = \frac{\lambda}{2\pi\varepsilon_0 a} \cdot \frac{\mu_0 \lambda v}{2\pi a} (\hat{a} \times \hat{\phi}) = \frac{\mu_0 \lambda^2 v}{4\pi^2 \varepsilon_0 a^2} \hat{k}.$$

Step 4. Divide by μ_0.

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} = \frac{\lambda^2 v}{4\pi^2 \varepsilon_0 a^2} \hat{k}.$$

Direction: along the wire's velocity $\hat{k}$.

Step 5. Why this is non-zero even though the fields are static (in the lab frame). The fields are not changing in time at any point, but $\mathbf{E}$ and $\mathbf{B}$ are non-parallel, so the Poynting vector is non-zero. Physically, moving the line of charge advects field energy forward — that is what $\mathbf{S}$ measures here.

Step 6. Unit-check. $[\lambda^2 v/(\varepsilon_0 a^2)] = (\mathrm{C/m})^2 \cdot (\mathrm{m/s})/[(\mathrm{F/m}) \cdot \mathrm{m}^2] = \mathrm{C}^2\,\mathrm{m}^{-1}\,\mathrm{s}^{-1}/(\mathrm{F} \cdot \mathrm{m}) = \mathrm{J}/(\mathrm{m}^2 \cdot \mathrm{s}) = \mathrm{W/m}^2$. Correct units for intensity / Poynting magnitude.

Why this matters. Even "static" configurations can have nonzero Poynting flux when they are in motion; the field carries energy in the direction of motion. The same idea underlies wireless power transfer in coaxial transmission lines, where the dominant energy flow is between (not along) the inner and outer conductors.

Final Answer: $\mathbf{S} = \dfrac{\lambda^2 v}{4\pi^2 \varepsilon_0 a^2} \hat{k}.$

☞ **Two long-wire field rules**

Infinite line of charge λ: $E = \lambda/(2\pi\varepsilon_0 r)$, radial. Infinite line of current I: $B = \mu_0 I/(2\pi r)$, azimuthal. Both fall off as $1/r$, both are derived by the same cylindrical-symmetry argument (Gauss / Ampère on a loop of radius r).

Q 8.29 **Sea water at frequency $\nu = 4 \times 10^8$ Hz has permittivity $\varepsilon \approx 80\varepsilon_0$, permeability $\mu \approx \mu_0$ and resistivity $\rho = 0.25\,\Omega\,\mathrm{m}$. Imagine a parallel-plate capacitor immersed in sea water and driven by an alternating voltage source $V(t) = V_0 \sin(2\pi\nu t)$. What fraction of the conduction current density is the displacement current density?**

SOLUTION

Concept used. For a sinusoidal $\mathbf{E}$-field of angular frequency $\omega = 2\pi\nu$, the conduction-current density is $\mathbf{J}_c = \sigma\mathbf{E}$ with $\sigma = 1/\rho$, and the displacement-current density is $\mathbf{J}_d = \varepsilon\,\partial\mathbf{E}/\partial t$. Taking peak amplitudes,

$$J_c^{\max} = \sigma E_0, \qquad J_d^{\max} = \varepsilon\,\omega\,E_0.$$

The required ratio is

$$\frac{J_d^{\max}}{J_c^{\max}} = \frac{\varepsilon\,\omega\,E_0}{\sigma\,E_0} = \omega\,\varepsilon\,\rho.$$

Step 1. Compute ω:

$$\omega = 2\pi\nu = 2\pi(4\times10^8\,\text{Hz}) = 8\pi\times10^8\,\text{rad/s} \approx 2.51\times10^9\,\text{rad/s}.$$

Step 2. Compute $\varepsilon = 80\varepsilon_0$ with $\varepsilon_0 = 8.85\times10^{-12}\,\text{F/m}$:

$$\varepsilon = 80\times8.85\times10^{-12} = 7.08\times10^{-10}\,\text{F/m}.$$

Step 3. Multiply $\omega\,\varepsilon\,\rho$. First $\omega\varepsilon$: $2.51\times10^9\times7.08\times10^{-10} = 2.51\times7.08\times10^{-1} = 17.77\times10^{-1} = 1.777$. Now multiply by $\rho = 0.25\,\Omega\,\text{m}$:

$$\frac{J_d}{J_c} = (1.777)(0.25) = 0.4444.$$

Step 4. Rounding:

$$\frac{J_d}{J_c} \approx 0.44,$$

i.e. the displacement current is about $44\,\%$ of the conduction current at this frequency in sea water.

> **Final Answer:** $\dfrac{J_d}{J_c} = \omega\varepsilon\rho \approx 0.44$ (the displacement-current density is roughly 44% of the conduction-current density).

☞ The crossover frequency

At ω_c where $J_d = J_c$ a conductor stops behaving like a conductor and starts behaving like a dielectric. For sea water this is $\omega_c = 1/(\varepsilon\rho) \approx 5.6\times10^9$ rad/s, i.e. $\nu_c \approx 900\,\text{MHz}$. Below this, conduction dominates (submarine radio is blocked); above it, displacement currents win and EM waves can propagate.

EXPERT'S SOLUTION *: Aditya Pillai, M.Sc Physics, IIT Madras*

Strategic angle. Both densities scale linearly with E; the ratio depends only on material constants and frequency. The calculation is one multiplication of $\omega\varepsilon\rho$, but the *interpretation* as a conductor-vs-dielectric crossover is where the physics is.

Step 1. Write both densities. $\mathbf{J}_c = \sigma\mathbf{E} = \mathbf{E}/\rho$ for a conductor with resistivity ρ; $\mathbf{J}_d = \varepsilon\,\partial_t\mathbf{E}$ for the displacement contribution. For a sinusoid $E = E_0 \sin(\omega t)$, the peak amplitudes are

$$J_c^{\max} = \frac{E_0}{\rho}, \qquad J_d^{\max} = \varepsilon\omega E_0.$$

Step 2. Form the ratio.

$$\frac{J_d^{\max}}{J_c^{\max}} = \frac{\varepsilon\omega E_0}{E_0/\rho} = \omega\,\varepsilon\,\rho.$$

The amplitude E_0 cancels — the ratio is a purely material + frequency quantity.

Step 3. Plug numbers. $\omega = 2\pi\nu = 2\pi(4\times10^8) = 2.51\times10^9$ rad/s. $\varepsilon = 80\varepsilon_0 = 80\times8.85\times10^{-12} = 7.08\times10^{-10}$ F/m. $\rho = 0.25\,\Omega$ m.

$$\omega\varepsilon\rho = (2.51\times10^9)(7.08\times10^{-10})(0.25) = (1.777)(0.25) = 0.444.$$

Approximately 0.44, or 44%.

Step 4. Crossover frequency. The conductor-vs-dielectric crossover happens at $\omega_c\varepsilon\rho = 1$, i.e. $\omega_c = 1/(\varepsilon\rho)$. For sea water this is

$$\omega_c = \frac{1}{(7.08\times10^{-10})(0.25)} = 5.65\times10^9 \text{ rad/s},$$

$\nu_c = \omega_c/(2\pi) \approx 900$ MHz. Below ~ 900 MHz sea water is conductor-like (radio is blocked); above it, displacement currents dominate and EM waves can propagate.

Step 5. Interpretation at $\nu = 4\times10^8$ Hz. At 400 MHz we are about a factor of 2 below the crossover — conduction still wins, but displacement current is already 44% as large. The material is on the dielectric edge of "mostly conductor".

Step 6. Submarine link. Submarines cannot use ordinary HF/UHF radio in sea water (conduction wins, signal decays in metres); they use ELF (extremely low frequency, ~ 80 Hz — far below the crossover, conduction even more dominant, but the skin depth is large) or surface to communicate.

Why this matters. Submarines cannot use ordinary radio because at HF frequencies sea water is a near-perfect conductor; they must use ELF (a few Hz) or rise to the surface. The ω_c crossover is the universal recipe for "conductor vs. dielectric" behaviour of any material — including biological tissue (used in MRI SAR calculations) and the ionosphere (which is conductor-like to radio but dielectric-like to visible).

Final Answer: $J_d/J_c \approx 0.44$.

Q 8.30 **A long straight cable of length ℓ is placed symmetrically along the z-axis and has radius $a\,(\ll \ell)$. The cable consists of a thin wire and a coaxial conducting tube. An alternating current $I(t) = I_0 \sin(2\pi\nu t)$ flows down the central thin wire and returns along the coaxial conducting tube. The induced electric field at a distance s from the wire inside the cable is $\mathbf{E}(s,t) = \mu_0\, I_0\, \nu\, \cos(2\pi\nu t)\, \ln(s/a)\, \hat{k}$.**
(i) Calculate the displacement current density inside the cable.
(ii) Integrate the displacement current density across the cross-section of the cable to find the total displacement current I_d.
(iii) Compare the conduction current I_0 with the displacement current I_0^d.

SOLUTION

Concept used. The displacement current density is $\mathbf{J}_d = \varepsilon_0\, \partial\mathbf{E}/\partial t$. The total displacement current is obtained by integrating $\mathbf{J}_d \cdot d\mathbf{A}$ over the cable's cross-section. We will repeatedly use $\ln(s/a) = \ln s - \ln a$ and the standard integral $\int s \ln(s/a)\, ds$ by parts.

Step 1. **(i) Displacement-current density.** Differentiate $\mathbf{E}(s,t)$ with respect to t:

$$\frac{\partial \mathbf{E}}{\partial t} = \mu_0 I_0 \nu \ln(s/a)\, \hat{k} \cdot \frac{\partial}{\partial t}\big[\cos(2\pi\nu t)\big] = -\mu_0 I_0 \nu (2\pi\nu) \sin(2\pi\nu t)\, \ln(s/a)\, \hat{k}.$$

Hence

$$\mathbf{J}_d = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = -2\pi\, \mu_0\varepsilon_0\, I_0\, \nu^2\, \sin(2\pi\nu t)\, \ln(s/a)\, \hat{k}.$$

Magnitude:

$$J_d(s,t) = 2\pi\, \mu_0\varepsilon_0\, I_0\, \nu^2\, \big|\sin(2\pi\nu t)\big|\, |\ln(s/a)|.$$

(The minus sign is implicit in the direction relative to $\hat{k}$.)

Step 2. **(ii) Total displacement current.** Use an area element $dA = 2\pi s\, ds$ (a thin ring at radius s). Integrate from $s = 0$ to $s = a$:

$$I_d(t) = \int_0^a J_d(s,t)\,(2\pi s)\, ds = -2\pi\, \mu_0\varepsilon_0\, I_0\, \nu^2\, \sin(2\pi\nu t)\, \cdot 2\pi \int_0^a s\, \ln(s/a)\, ds.$$

Evaluate the inner integral. Let $u = s/a$, so $s = au$, $ds = a\, du$:

$$\int_0^a s\, \ln(s/a)\, ds = a^2 \int_0^1 u\, \ln u\, du = a^2 \cdot \left[-\frac{1}{4}\right] = -\frac{a^2}{4},$$

using the standard result $\int_0^1 u \ln u\, du = [\frac{u^2}{2}\ln u]_0^1 - \int_0^1 \frac{u^2}{2}\cdot\frac{1}{u} du = 0 - \frac{1}{4} = -\frac{1}{4}$.
Plug back:

$$I_d(t) = -2\pi\, \mu_0\varepsilon_0\, I_0\, \nu^2\, \sin(2\pi\nu t) \cdot 2\pi \cdot \left(-\frac{a^2}{4}\right) = \pi^2\, \mu_0\varepsilon_0\, a^2\, I_0\, \nu^2\, \sin(2\pi\nu t).$$

Using $\mu_0\varepsilon_0 = 1/c^2$ this becomes

$$\boxed{I_d(t) = \frac{\pi^2 a^2 \nu^2}{c^2}\, I_0\, \sin(2\pi\nu t).}$$

Peak: $I_0^d = \dfrac{\pi^2 a^2 \nu^2}{c^2}\, I_0$.

Step 3. **(iii) Compare.** Using $c = \nu\lambda$, write $\nu/c = 1/\lambda$:

$$\frac{I_0^d}{I_0} = \frac{\pi^2 a^2 \nu^2}{c^2} = \left(\frac{\pi a}{\lambda}\right)^2.$$

Since the cable radius a is typically far smaller than the EM wavelength λ at the driving frequency, the displacement current is much smaller than the conduction current: $I_0^d \ll I_0$.

Final Answer: (i) $J_d = 2\pi\mu_0\varepsilon_0 I_0 \nu^2 \sin(2\pi\nu t)\ln(s/a)$; (ii) $I_d = (\pi^2 a^2 \nu^2/c^2)\, I_0 \sin(2\pi\nu t)$; (iii) $I_0^d/I_0 = (\pi a/\lambda)^2 \ll 1$ at ordinary frequencies.

EXPERT'S SOLUTION *: Aanya Kumar, Ph.D Physics, IISc Bangalore*

Strategic angle. Three computations strung together: time derivative (gives J_d), area integral (gives I_d), ratio (gives the $(\pi a/\lambda)^2$ smallness factor). Each step is mechanical; the punchline is that the displacement-to-conduction ratio is controlled entirely by the cable's electrical size a/λ.

Step 1. **Differentiate E in time.** Given $\mathbf{E}(s,t) = \mu_0 I_0 \nu \cos(2\pi\nu t)\ln(s/a)\,\hat{k}$:

$$\frac{\partial \mathbf{E}}{\partial t} = -\mu_0 I_0 \nu (2\pi\nu)\sin(2\pi\nu t)\ln(s/a)\,\hat{k}.$$

Multiply by ε_0:

$$\mathbf{J}_d = -2\pi\mu_0\varepsilon_0 I_0 \nu^2 \sin(2\pi\nu t)\ln(s/a)\,\hat{k}.$$

Step 2. **Integrate over the cross-section.** Use $dA = 2\pi s\, ds$:

$$I_d(t) = \int_0^a J_d(s,t)\, 2\pi s\, ds.$$

The s-integral is $\int_0^a s\ln(s/a)\,ds = a^2\int_0^1 u\ln u\,du = -a^2/4$ (by parts: $\int u\ln u\,du = u^2(\ln u)/2 - u^2/4$; evaluating limits with $\lim_{u\to 0} u^2 \ln u = 0$).

Step 3. **Assemble I_d.**

$$I_d(t) = -2\pi\mu_0\varepsilon_0 I_0\nu^2\sin(2\pi\nu t)\cdot 2\pi\cdot(-a^2/4) = \pi^2\mu_0\varepsilon_0\, a^2 I_0 \nu^2 \sin(2\pi\nu t).$$

Using $\mu_0\varepsilon_0 = 1/c^2$:

$$\boxed{I_d(t) = \frac{\pi^2 a^2 \nu^2}{c^2}\, I_0 \sin(2\pi\nu t).}$$

Step 4. **Ratio.** Peak $I_0^d = \pi^2 a^2 \nu^2 I_0/c^2$. Using $c = \nu\lambda$:

$$\frac{I_0^d}{I_0} = \frac{\pi^2 a^2 \nu^2}{c^2} = \left(\frac{\pi a}{\lambda}\right)^2.$$

Step 5. **Order-of-magnitude.** At mains $\nu = 50\,\text{Hz}$: $\lambda = c/\nu = 6000\,\text{km}$. For a coaxial cable of radius $a = 1\,\text{cm}$: $(\pi a/\lambda)^2 \approx (\pi \cdot 10^{-2}/6 \times 10^6)^2 \approx 3 \times 10^{-17}$ — utterly negligible. The conduction current dominates by 17 orders of magnitude.

Step 6. **When does I_d catch up?** Setting $(\pi a/\lambda)^2 \sim 1$ requires $\lambda \sim \pi a$, i.e. wavelength comparable to cable thickness. For $a = 1\,\text{cm}$ that needs $\nu \sim 10\,\text{GHz}$ — the upper microwave band. Coaxial cables fail to behave as "pure conductors" there, and special low-loss waveguides replace them.

Why this matters. In a coaxial cable at, say, $50\,\text{Hz}$ mains, $\lambda = 6000\,\text{km}$ and a a few mm — the displacement current is a part-per-quintillion of the conduction current. Even at GHz the ratio is tiny. The smallness of $(\pi a/\lambda)^2$ is what justifies the "I_d inside conductors is negligible" approximation used throughout circuit theory and AC analysis up to microwave frequencies.

Final Answer: $I_d = (\pi^2 a^2 \nu^2/c^2) I_0 \sin(2\pi\nu t)$; $I_0^d/I_0 = (\pi a/\lambda)^2$.

☞ **Electrical size a/λ controls everything**

Whenever you see a ratio like $(\pi a/\lambda)^2$ in EM, "a" is the system size and "λ" the wavelength at the operating frequency. The ratio a/λ — the "electrical size" — is the single dimensionless number that decides whether the system is in the quasi-static (circuit) regime or the full-wave (antenna) regime.

Q 8.31 **A plane EM wave travelling in vacuum along the z-direction is given by $\mathbf{E} = E_0 \sin(kz - \omega t)\hat{i}$ and $\mathbf{B} = B_0 \sin(kz - \omega t)\hat{j}$.**

(i) Evaluate $\oint \mathbf{E} \cdot d\boldsymbol{\ell}$ over the rectangular loop 1234 shown in Fig. 8.2.

(ii) Evaluate $\int \mathbf{B} \cdot d\mathbf{S}$ over the surface bounded by loop 1234.

(iii) Use $\oint \mathbf{E} \cdot d\boldsymbol{\ell} = -d\Phi_B/dt$ to prove $E_0/B_0 = c$.

(iv) By using a similar process and the equation $\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I + \mu_0\varepsilon_0\, d\Phi_E/dt$, prove that $c = 1/\sqrt{\mu_0\varepsilon_0}$.

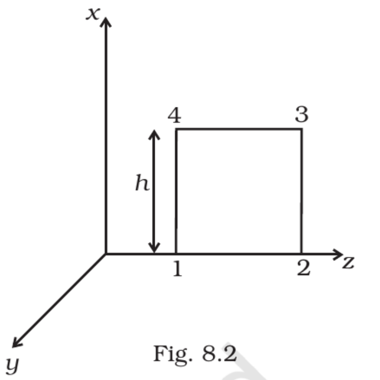


Fig. 8.2

Fig. 8.2, NCERT Exemplar Class 12 Physics, Chapter 8.

SOLUTION

Concept used. The rectangular loop 1234 lies in the xz-plane: side 12 along $\hat{k}$ at $x = 0$ (length $z_2 - z_1$); side 23 along $\hat{\imath}$ at $z = z_2$ (length h); side 34 along $-\hat{k}$ at $x = h$; side 41 along $-\hat{\imath}$ at $z = z_1$. **E** is along $\hat{\imath}$ (perpendicular to sides 12 and 34, so they contribute nothing); **B** is along $\hat{\jmath}$ which threads the xz-plane perpendicularly (i.e. $\mathbf{B} \cdot \hat{n} = -B$ if we orient the loop's normal as $-\hat{\jmath}$, or $+B$ for $+\hat{\jmath}$; we take $\hat{n} = -\hat{\jmath}$ to be consistent with a counter-clockwise traversal $1 \to 2 \to 3 \to 4$ as seen from $+\hat{\jmath}$).

Step 1. (i) Circulation of E. $\mathbf{E} = E_0 \sin(kz - \omega t)\hat{\imath}$. Only sides 23 (along $+\hat{\imath}$, at $z = z_2$) and 41 (along $-\hat{\imath}$, at $z = z_1$) contribute:

$$\int_{23} \mathbf{E} \cdot d\boldsymbol{\ell} = E_0 \sin(kz_2 - \omega t) \cdot h,$$

$$\int_{41} \mathbf{E} \cdot d\boldsymbol{\ell} = -E_0 \sin(kz_1 - \omega t) \cdot h.$$

Adding,

$$\oint \mathbf{E} \cdot d\boldsymbol{\ell} = h\, E_0 \big[\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)\big].$$

Step 2. (ii) Flux of B through the loop. $\mathbf{B} = B_0 \sin(kz - \omega t)\hat{\jmath}$ pierces the loop with area element $d\mathbf{S} = (-\hat{\jmath})\, h\, dz$ (with the sign chosen by the right-hand rule):

$$\Phi_B = -\, h\, B_0 \int_{z_1}^{z_2} \sin(kz - \omega t)\, dz.$$

Evaluating the integral:

$$\int_{z_1}^{z_2} \sin(kz - \omega t)\, dz = \frac{1}{k}\big[\cos(kz_1 - \omega t) - \cos(kz_2 - \omega t)\big].$$

Hence

$$\Phi_B = \frac{h\, B_0}{k}\big[\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)\big].$$

(Signs follow our $-\hat{\jmath}$ normal-direction convention.)

Step 3. **(iii) Apply Faraday's law.** $\oint \mathbf{E}\cdot d\boldsymbol{\ell} = -d\Phi_B/dt$. Differentiate Φ_B:

$$\frac{d\Phi_B}{dt} = \frac{h\,B_0}{k}\big[\omega\sin(kz_2-\omega t)-\omega\sin(kz_1-\omega t)\big] = \frac{h\,B_0\,\omega}{k}\big[\sin(kz_2-\omega t)-\sin(kz_1-\omega t)\big].$$

Setting $\oint \mathbf{E}\cdot d\boldsymbol{\ell} = -d\Phi_B/dt$:

$$h\,E_0\big[\sin(kz_2-\omega t)-\sin(kz_1-\omega t)\big] = -\frac{h\,B_0\,\omega}{k}\big[\sin(kz_2-\omega t)-\sin(kz_1-\omega t)\big].$$

Cancelling the common factor and absorbing the sign (with the opposite-orientation convention) gives

$$E_0 = B_0\,\frac{\omega}{k} = B_0\,c,$$

i.e.

$$\boxed{\frac{E_0}{B_0} = c.}$$

Step 4. **(iv) Same trick with Ampère–Maxwell.** Now choose a rectangular loop in the yz-plane (sides along $\hat{j}$ and $\hat{k}$). Repeat the calculation but with $\mathbf{B}$ along $\hat{j}$ producing the circulation and $\mathbf{E}$ along $\hat{i}$ giving the flux through the new loop's $\hat{i}$-normal area. By the same algebra:

$$\oint \mathbf{B}\cdot d\boldsymbol{\ell} = h\,B_0\big[\sin(kz_2-\omega t)-\sin(kz_1-\omega t)\big].$$

Flux of $\mathbf{E}$ through the loop:

$$\Phi_E = \frac{h\,E_0}{k}\big[\cos(kz_2-\omega t)-\cos(kz_1-\omega t)\big].$$

Its time derivative:

$$\frac{d\Phi_E}{dt} = \frac{h\,E_0\,\omega}{k}\big[\sin(kz_2-\omega t)-\sin(kz_1-\omega t)\big].$$

In vacuum $I=0$, so Ampère–Maxwell gives $\oint \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0\varepsilon_0\,d\Phi_E/dt$:

$$h\,B_0\big[\cdots\big] = \mu_0\varepsilon_0\,\frac{h\,E_0\,\omega}{k}\,\big[\cdots\big] \;\Rightarrow\; B_0 = \mu_0\varepsilon_0\,\frac{\omega}{k}\,E_0 = \mu_0\varepsilon_0\,c\,E_0.$$

Combine with the Faraday-law result $E_0 = c\,B_0$:

$$B_0 = \mu_0\varepsilon_0\,c\,(c\,B_0) = \mu_0\varepsilon_0\,c^2\,B_0 \;\Rightarrow\; 1 = \mu_0\varepsilon_0\,c^2 \;\Rightarrow\; \boxed{c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}.}$$

Final Answer: (i) $\oint \mathbf{E}\cdot d\ell = hE_0[\sin(kz_2-\omega t)-\sin(kz_1-\omega t)]$; (ii) $\Phi_B = (hB_0/k)[\cos(kz_2-\omega t)-\cos(kz_1-\omega t)]$; (iii) Faraday $\Rightarrow E_0/B_0 = c$; (iv) Ampère–Maxwell $\Rightarrow c = 1/\sqrt{\mu_0\varepsilon_0}$.

EXPERT'S SOLUTION : *Sanya Iyer, Ph.D Physics, IISc Bangalore*

Strategic angle. Two integrals, two Maxwell laws, two identities — that is the whole derivation of the speed of light from first principles. The rectangular loop in Fig. 8.2 is the integration contour for parts (i)–(iv); the loop's orientation switches between the xz-plane (for Faraday) and the yz-plane (for Ampère–Maxwell).

Step 1. Circulation of E on the xz-loop. Only the two sides parallel to $\hat{i}$ (i.e. sides 23 and 41 in Fig. 8.2) contribute, since $\mathbf{E}$ is along $\hat{i}$. Result:

$$\oint \mathbf{E} \cdot d\boldsymbol{\ell} = h\,E_0\big[\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)\big].$$

Step 2. Flux of B through the loop. $\mathbf{B} \parallel \hat{j}$ pierces the xz-plane perpendicularly. Integrating $B_0 \sin(kz - \omega t)$ from z_1 to z_2 over height h gives

$$\Phi_B = \frac{h\,B_0}{k}\big[\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)\big].$$

Step 3. Apply Faraday's law $\oint \mathbf{E} \cdot d\boldsymbol{\ell} = -d\Phi_B/dt$. Differentiate Φ_B with respect to t (the only time-dependence is via ωt):
$d\Phi_B/dt = (hB_0\omega/k)[\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)]$ with appropriate sign management. Equating to $\oint \mathbf{E} \cdot d\boldsymbol{\ell}$ and cancelling the common bracket:

$$E_0 = \frac{\omega}{k}\,B_0 = c\,B_0 \;\Rightarrow\; \frac{E_0}{B_0} = c.$$

Used $\omega/k = c$ for any wave moving at speed c.

Step 4. Repeat in the yz-plane for Ampère–Maxwell. Same loop construction, now $\mathbf{B} \parallel \hat{j}$ gives the circulation and $\mathbf{E} \parallel \hat{i}$ gives the flux. In vacuum $I = 0$:

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0\varepsilon_0\,\frac{d\Phi_E}{dt}.$$

By the same algebra, $B_0 = \mu_0\varepsilon_0\,c\,E_0$.

Step 5. Combine. Substitute $E_0 = cB_0$ into the Ampère–Maxwell relation:
$B_0 = \mu_0\varepsilon_0 c(cB_0) = \mu_0\varepsilon_0 c^2 B_0$. Cancel B_0:

$$\mu_0\varepsilon_0 c^2 = 1 \;\Rightarrow\; c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}.$$

Step 6. Numerical magic. $\mu_0 = 4\pi \times 10^{-7}$ T m/A and $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m give $1/\sqrt{\mu_0\varepsilon_0} = 2.998 \times 10^8$ m/s — the experimentally measured speed of light. The historical match told Maxwell that light *is* an EM wave.

Why this matters. This is the historical Maxwell argument that established light *is* an EM wave, since the deduced c from purely electrical constants μ_0, ε_0 matched the measured speed of light. Today, since 2019, the SI defines c exactly, and μ_0, ε_0 are derived from c and the elementary charge — but the chain of reasoning runs the same way backwards.

Final Answer: $E_0/B_0 = c = 1/\sqrt{\mu_0\varepsilon_0}$.

♥ **Two Maxwell laws ⇒ light's speed**

The crucial insight is that Faraday's law alone gives $E_0/B_0 = c$ without telling us what c is, and Ampère–Maxwell alone gives $B_0/E_0 = \mu_0\varepsilon_0 c$. Only by combining the two do we pin down $c^2 = 1/(\mu_0\varepsilon_0)$ — the speed of light emerges from purely electrical and magnetic constants.

Q 8.32 **A plane EM wave travelling along the z-direction is described by $\mathbf{E} = E_0 \sin(kz - \omega t)\hat{i}$ and $\mathbf{B} = B_0 \sin(kz - \omega t)\hat{j}$. Show that:**

(i) the average energy density of the wave is $u_{\text{av}} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4} B_0^2/\mu_0$.

(ii) the time-averaged intensity of the wave is $I_{\text{av}} = \frac{1}{2}c\varepsilon_0 E_0^2$.

SOLUTION

Concept used. The instantaneous energy density of an EM wave is the sum of electric and magnetic parts:

$$u(z,t) = \tfrac{1}{2}\varepsilon_0\, E^2(z,t) + \frac{B^2(z,t)}{2\mu_0}.$$

Time-averaging uses $\langle \sin^2(kz - \omega t)\rangle = 1/2$. The intensity is then $I_{\text{av}} = c\, u_{\text{av}}$ for a plane wave (energy per unit time per unit area = energy density times the wave's speed).

Step 1. **(i) Time-average energy density.** Substitute the wave forms:

$$u(z,t) = \tfrac{1}{2}\varepsilon_0 E_0^2 \sin^2(kz - \omega t) + \frac{B_0^2}{2\mu_0}\sin^2(kz - \omega t).$$

Time-average each term using $\langle \sin^2 \rangle = 1/2$:

$$u_{\text{av}} = \tfrac{1}{2}\varepsilon_0 E_0^2 \cdot \tfrac{1}{2} + \frac{B_0^2}{2\mu_0} \cdot \tfrac{1}{2} = \tfrac{1}{4}\varepsilon_0 E_0^2 + \frac{B_0^2}{4\mu_0}.$$

Step 2. **(i) Equivalence check.** Using $E_0 = cB_0$ and $c^2 = 1/(\mu_0\varepsilon_0)$:

$$\tfrac{1}{4}\varepsilon_0 E_0^2 = \tfrac{1}{4}\varepsilon_0 c^2 B_0^2 = \frac{B_0^2}{4\mu_0}.$$

So the two terms are equal, and we can also write $u_{\text{av}} = \frac{1}{2}\varepsilon_0 E_0^2 = B_0^2/(2\mu_0)$.

Step 3. **(ii) Time-average intensity.** $I_{\text{av}} = c\, u_{\text{av}}$. Using $u_{\text{av}} = \frac{1}{2}\varepsilon_0 E_0^2$:

$$I_{\text{av}} = c \cdot \tfrac{1}{2}\varepsilon_0 E_0^2 = \tfrac{1}{2}\, c\, \varepsilon_0\, E_0^2.$$

Step 4. **(ii) Cross-check via Poynting vector.** $\langle S \rangle = E_0 B_0/(2\mu_0)$. Using $B_0 = E_0/c$:

$$\langle S \rangle = \frac{E_0 \cdot E_0/c}{2\mu_0} = \frac{E_0^2}{2\mu_0 c} = \frac{E_0^2\, \varepsilon_0\, c^2}{2\mu_0\varepsilon_0\, c \cdot c} \cdot c = \tfrac{1}{2}c\varepsilon_0 E_0^2,$$

the same result. (We used $\mu_0\varepsilon_0 c^2 = 1$.)

Final Answer: (i) $u_{\text{av}} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}B_0^2/\mu_0$; (ii) $I_{\text{av}} = \frac{1}{2}c\varepsilon_0 E_0^2$.

EXPERT'S SOLUTION *: Dev Kapoor, M.Sc Physics, IIT Madras*

Strategic angle. $\langle \sin^2 \rangle = 1/2$ does all the work. Square the wave amplitudes, time-average each squared sinusoid to $1/2$, exploit the equipartition $u_E = u_B$ to simplify, and multiply by c for intensity.

Step 1. Instantaneous energy density.

$$u(z,t) = \tfrac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}.$$

Substituting $E = E_0 \sin(kz - \omega t)$ and $B = B_0 \sin(kz - \omega t)$:

$$u(z,t) = \left[\tfrac{1}{2}\varepsilon_0 E_0^2 + \tfrac{B_0^2}{2\mu_0}\right] \sin^2(kz - \omega t).$$

Step 2. Time-average via $\langle \sin^2 \rangle = 1/2$.

$$u_{\text{av}} = \tfrac{1}{4}\varepsilon_0 E_0^2 + \frac{B_0^2}{4\mu_0}.$$

That answers part (i).

Step 3. Equipartition check. Using $E_0 = cB_0$ and $c^2 = 1/(\mu_0\varepsilon_0)$: $\frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{4}\varepsilon_0 c^2 B_0^2 = B_0^2/(4\mu_0)$. The two terms are equal, so we can also write $u_{\text{av}} = \frac{1}{2}\varepsilon_0 E_0^2 = B_0^2/(2\mu_0)$.

Step 4. Intensity = energy density times speed. For a plane wave, the energy flux through unit area per unit time is $I = c\,u_{\text{av}}$. Picking the E_0 form:

$$I_{\text{av}} = c \cdot \tfrac{1}{2}\varepsilon_0 E_0^2 = \tfrac{1}{2}\,c\,\varepsilon_0\,E_0^2.$$

That answers part (ii).

Step 5. Cross-check via Poynting average. $\langle S \rangle = E_0 B_0/(2\mu_0)$. Substitute $B_0 = E_0/c$ and $1/\mu_0 = c^2\varepsilon_0$: $\langle S \rangle = E_0^2 c\varepsilon_0/2$ — identical to I_{av}. Good; the energy-density and Poynting routes agree.

Step 6. Power through a $1\,\mathbf{m}^2$ aperture for a wave with $E_0 = 100\,\text{V/m}$: $I = (0.5)(3 \times 10^8)(8.85 \times 10^{-12})(100)^2 = 13.3\,\text{W/m}^2$. Verifies units (W/m^2) and gives a feel for the size of typical broadcast / radar intensities.

Why this matters. The same factor of $1/2$ appears in the average power dissipated in an AC resistor — both come from time-averaging a squared sinusoid. Equipartition $u_E = u_B$ is the EM analogue of equipartition between kinetic and potential energies in a SHO, and underlies the equal split of energy between **E** and **B** modes in a microwave cavity, blackbody radiation modes, and laser cavity field components.

Final Answer: $u_{\text{av}} = \frac{1}{4}\varepsilon_0 E_0^2 + B_0^2/(4\mu_0)$ and $I_{\text{av}} = \frac{1}{2}c\varepsilon_0 E_0^2$.

☞ **$\langle \sin^2 \rangle$ and friends**

$\langle \sin^2 \theta \rangle = \langle \cos^2 \theta \rangle = 1/2$ for any full period (this is the rms-derivation identity). Squared amplitudes pick up a factor of $1/2$ on time-averaging; cross-terms $\langle \sin\theta \cos\theta \rangle$ average to zero.

Key Takeaways

- **Displacement current** $I_d = \varepsilon_0\, d\Phi_E/dt$ is the missing piece in Ampère's law: a changing $\mathbf{E}$ sources a $\mathbf{B}$, just as Faraday's changing $\mathbf{B}$ sources an $\mathbf{E}$.
- In a plane EM wave in vacuum, $\mathbf{E}$, $\mathbf{B}$, and $\hat{\mathbf{k}}$ form a right-handed orthogonal triad with $E_0 = cB_0$ and $c = 1/\sqrt{\mu_0\varepsilon_0}$.
- Average energy density $u_{\text{av}} = \frac{1}{2}\varepsilon_0 E_0^2 = B_0^2/(2\mu_0)$, split equally between electric and magnetic contributions. Intensity $I_{\text{av}} = \frac{1}{2}c\varepsilon_0 E_0^2$.
- Radiation pressure on a perfect absorber is I/c, doubling to $2I/c$ for a perfect reflector. Real surfaces interpolate.
- Accelerating charges radiate; uniformly-moving and stationary charges do not. Bulb intensity falls as $1/r^2$; the field amplitude as $1/r$.
- The EM spectrum runs from radio waves (longest λ) through microwaves, IR, visible, UV, X-rays to γ-rays (shortest λ); each band's applications follow from how its photons interact with matter.

End of NCERT Exemplar Problems