Chapter 8

Electromagnetic Waves

The Big Picture

Light is a wave of electric and magnetic fields, oscillating together, travelling through empty space. Predicted by Maxwell (1865), confirmed by Hertz (1887).

* What was missing?

Ampere's law fails for a charging capacitor. Maxwell added displacement current to fix it, and out fell the wave equation for light !

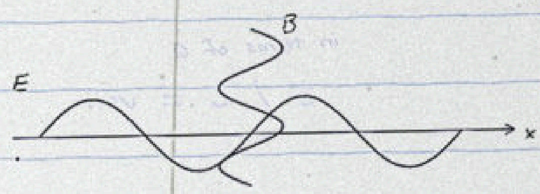


Fig. E and B oscillate in step, both perpendicular to direction of travel.

Roadmap

Displacement current ; Maxwell eqns ; Spectrum.

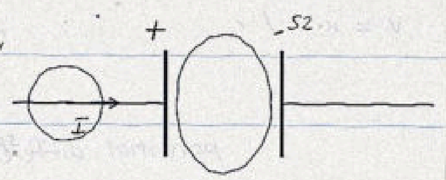
Need for Displacement Current

Consider a capacitor being charged by a current I in the wires. We apply Ampere's law on a loop around the wire :

Loop 1 (wire) : integral $B \cdot dl = \mu_0 I$

Now choose a different surface bounded by the SAME loop , bulging between the plates of the capacitor. No conduction current crosses this surface :

Loop 2 (plates) : integral $B \cdot dl = 0$?



Same loop , two surfaces , two answers ?

Ampere's law is incomplete.

Maxwell : there must be a ~~phantom~~ hidden current

Displacement Current - Definition

Between the plates, E grows as charge builds up. Maxwell associated a current with this changing flux of E.

$$\boxed{I_d = e_0 \; dF_E/dt}$$ <- displacement <- current

where Φ_E = integral $E \cdot dA$ is the flux of electric field through area.

Check : it equals I

Between plates : $E = Q / (\varepsilon_0 . A)$

So $\Phi_E = E . A = Q / \varepsilon_0$

$dF_E / dt = (1/\varepsilon_0) \, dQ/dt = I / \varepsilon_0$

$$I_d = \varepsilon_0 . (I / \varepsilon_0) = I.$$

So I_d matches the conduction current!

Continuity is restored : Ampere's law works with the new term added.

Physical meaning

A time - varying E field acts like a current for producing magnetic fields. No charges actually move across the gap.

Ampere - Maxwell Law

Ampere's law, corrected, becomes :

$$\boxed{\text{integral } B \cdot dl = u_0 \left(I_c + e_0 \, dF_E/dt \right)}$$

<- I_c = conduction current ; the $e_0 \, dF/dt = I_d$

Symbolically :

$$\text{integral } B \cdot dl = \mu_0 (I + I_d)$$

Both currents matter

In a steady wire : $I_d = 0$, only I.

Between plates : $I = 0$, only I_d.

In a transient mix : both contribute.

Symmetry hint

Faraday : changing B produces E.

Maxwell : changing E produces B.

These two together let E and B sustain each other - travelling outward as a wave.

No medium needed : EM waves move in ~~aether~~ vacuum.
This was the great triumph of Maxwell.

Maxwell's Four Equations

Integral form, in vacuum + charges :

1. Gauss for E

integral E . dA = Q / ε_0

Total electric flux through closed surface equals enclosed charge / e_0.

2. Gauss for B

integral B . dA = 0

No magnetic monopoles - B lines close on themselves.

3. Faraday

integral E . dl = - $d\phi_B/dt$

Changing magnetic flux drives an EMF.

4. Ampere - Maxwell

integral B . dl = $\mu_0 I + \mu_0 \varepsilon_0 \, d\phi_E/dt$

Current + changing E flux -> magnetic field.

In free space

Set Q = 0, I = 0. Only the time-derivative terms remain. Combine 3 & 4 -> wave equation for E and for B (next page).

Wave Equation from Maxwell

In vacuum (no charges or currents) :

curl E = - dB/dt
curl B = $\mu_0 \varepsilon_0 \, dE/dt$

Take curl of the first , substitute the second :

$d^2E \,/\, dx^2 = \mu_0 \varepsilon_0 \, d^2E \,/\, dt^2$

Standard wave form : d 2 y / dx 2 = (1/v 2) d 2 y / dt 2 . Comparing :

| v = 1 / sqrt(u_0 e_0) | <-wave speed <-in vacuum

Plug numbers :
$\mu_0 = 4\pi \times 10^{-7}$ T m / A
$\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2 / N m^2$

| v = c = 3 x 10 8 m / s | <-= speed <-of light !

*

So light is an ~~aether~~ EM wave - Maxwell, 1865.

Confirmed experimentally by Hertz in 1887.

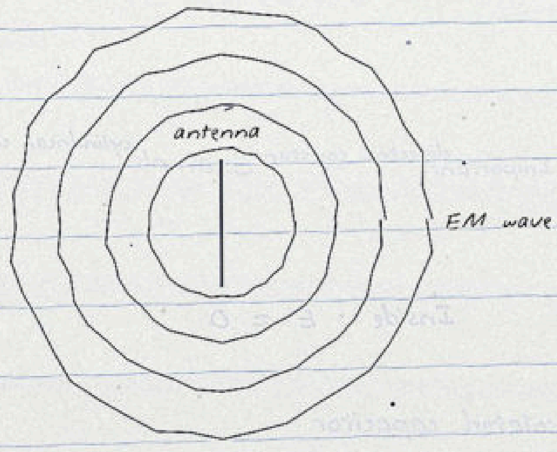
Source of EM Waves

A stationary charge produces only an E field.
A charge moving at constant velocity gives both E and B but no radiation.

Accelerated charge radiates

An accelerating charge produces a CHANGING B field around it, which produces a changing E field, which produces a changing B, ...

Wave-front spreads outward at speed c.



Frequency = source frequency

An LC oscillator at frequency f radiates EM waves of the SAME frequency f.

Wavelength : $\lambda = c / f$ in vacuum.

In medium : $v = f \times$ lambda , $v < c$.

Nature of EM Waves

Plane wave form

For a wave moving along + x with E along y and B along z :

$$\boxed{E\_y = E\_0 \sin(k x - w t)}$$ <- electric <- field

$$\boxed{B\_z = B\_0 \sin(k x - w t)}$$ <- magnetic <- field

k = 2 pi / lambda = angular wave number
w = 2 pi f = angular frequency
v = w / k = f λ = c (vacuum)

Three key properties

1. E and B in phase, peak together
2. E perp B (perpendicular vectors)
3. Both perp to direction of motion

EM waves are TRANSVERSE - no compressions or rarefactions, unlike sound waves.

EM Wave : Geometry

Draw the wave : E on y axis , B on z axis , propagation along + x. E x B points along v.

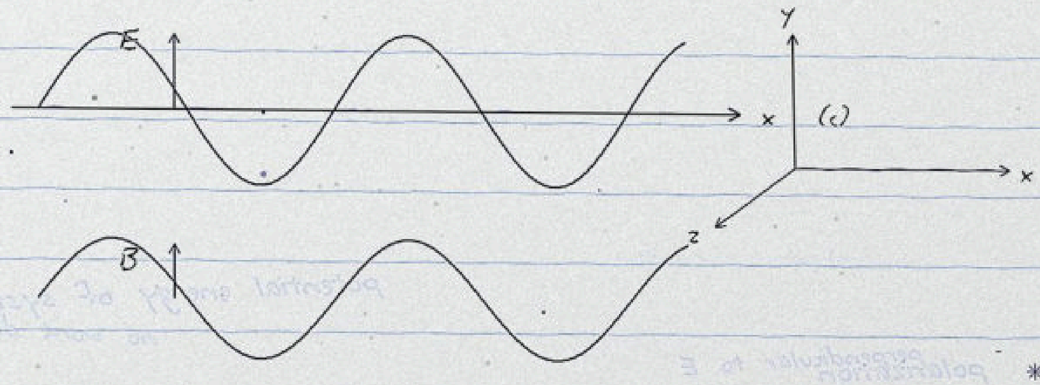


Fig. E (top wave) and B (bottom wave) , in step , perpendicular , both perp to x.

EO / BO = c

From Faraday's law applied to a strip :

$$\boxed{E_0 / B_0 = c}$$ <- amplitude ratio

$E0 / B0 = c$ - Derivation

Consider a rectangular loop in the x - y plane, length L along y, very thin dx along x.

Apply Faraday's law :

integral $E \cdot dl = - d\phi_B/dt$

LHS : $(E(x) - E(x+dx)) L = - (dE/dx) L \, dx$

RHS : $- d/dt (B L \, dx) = - (dB/dt) L \, dx$

So $dE/dx = - dB/dt$

For $E = E_0 \sin(kx - wt)$, $B = B_0 \sin(kx - wt)$

$k E_0 \cos(\ldots) = w B_0 \cos(\ldots)$

So $E_0 / B_0 = w / k =^* c$

$$\boxed{E_0 = c B_0}$$

<- key relation

Since c is huge , B_0 is tiny next to E_0 - in usual problems , E effects dominate.

EM Waves in a Medium

Inside a dielectric / magnetic medium, replace e_0 -> e and u_0 -> u.

$$\boxed{v = 1 / \mathrm{sqrt}(u\ e)}$$ <- speed in <- medium

For most transparent media, u u_0, so

v 1 / sqrt(μ_0 e)

Refractive index

$$\boxed{n = c / v = \mathrm{sqrt}(e\_r\ u\_r)}$$ <- Maxwell's <- relation

where e_r = e / e_0, u_r = u / u_0.

In glass : v 2 x 10 8 m/s, n 1.5.
In water : n 1.33 ; in air n ~~= 1.5~~ 1.0003

Frequency stays, wavelength shrinks

f does not change at a boundary.

$\lambda_{med} = \lambda_{vac} / n$

Color (= f) is the invariant, wavelength is not.

Energy in an EM Wave

An EM wave carries energy in both fields :

Energy density

$u_E = 1/2\ \varepsilon_0 E^2$ (electric)

$u_B = B^2 / (2 \mu_0)$ (magnetic)

Using $B = E / c$ and $c\ 2 = 1 / (u\_0\ e\_0)$:

$u_B = E^2 / (2 \mu_0 c^2) = 1/2\ \varepsilon_0 E^2 = u_E$

Equal share : Electric and magnetic parts carry the same energy density.

$\boxed{u\_total = e\_0\ E\ \ 2 = B\ \ 2\ /}$ u_0 <- instantaneous

Time-averaged density

$< u > = 1/2\ \varepsilon_0 E_0^2$ (average over cycle)

since $< \sin\ \ 2 > = 1/2$ over one full period.

Intensity: $I = u \cdot c$ (energy / area / sec)

$I = 1/2\ \varepsilon_0 c E_0^2$ (W / m^2)

Solar constant at Earth 1.36 kW / m 2.

Poynting Vector & Momentum

Energy flows in the direction of $E \times B$.
Define :

$$\boxed{S = (E \times B) / u_0}$$

<- Poynting
<- vector

Magnitude $S = E B / u_0$ = intensity (units W / m^2).

Direction of S = direction of propagation.

Linear momentum

EM waves carry momentum :

$$\boxed{p = U / c}$$

<- U = energy

Hence radiation pressure on a surface :

$P_{rad} = I / c$ (absorbed)
$P_{rad} = 2 I / c$ (fully reflected)

Small but real - confirmed by Nichols and Hull (1903). Powers ~~solar~~ sails in space.

Electromagnetic Spectrum

All EM waves obey c = f x lambda but differ in frequency / wavelength by huge factors.

Seven traditional bands, long lambda to short :

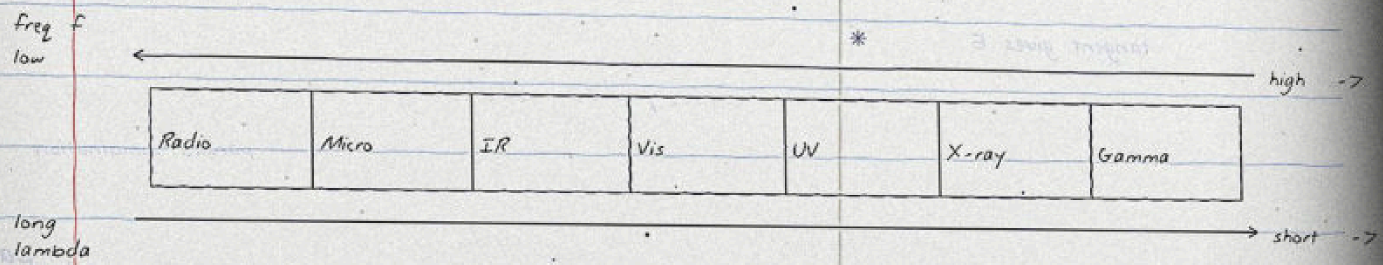


Fig. Bands overlap at the edges - no sharp boundaries between them.

Ordering :

Radio < Microwave < IR < Visible < UV < X - ray < Gamma (by frequency)

Mnemonic : RM IR V UX G (Read ~~Mom~~ My Notes

Radio Waves & Microwaves

Radio Waves

$\lambda > 0.1$ m ; $f < 3$ GHz

Produced : oscillating LC circuits, transmitting antennas.

Detected : receiving antennas, tuned coils.

Used in : AM / FM radio, TV, cellular, RADAR (lower end of band).

Microwaves

λ 1 mm - 0.1 m ; f 1 - 300 GHz

Produced : klystrons, magnetrons, Gunn diodes.

Detected : crystal diode + point contact.

Used in : RADAR, satellite, microwave ovens, Wi-Fi, Bluetooth, GPS.

Why microwaves cook food

Water molecules have a permanent dipole. Microwave E oscillates the dipoles rapidly, friction heats the water.

Common oven uses 2.45 GHz (resonance tuned to water). Wavelength 12 cm.

Glass, plastic - transparent ; metal reflects.

Infrared Radiation

λ . 700 nm - 1 mm

f 3×10^{11} - 4×10^{14} Hz

Production

Emitted by hot bodies and molecules,
from vibrations of molecular bonds.
Sun, fires, LEDs, filament bulbs.

Detection

Thermopile, bolometer, IR photodiode,
specially blackened thermometer.

Uses

1. Heating : IR lamps, physiotherapy.
2. Remote controls - TV, AC.
3. Night vision, thermal imaging.
4. Earth's warmth - greenhouse effect.

Greenhouse effect

Sun's visible passes through atmosphere, Earth
re-emits IR, CO2 and H2O trap it -> warming.

Visible Light

λ 400 nm - 700 nm

f 4×10^{14} - 7.5×10^{14} Hz

Only band the human eye can detect.

Sub - bands (VIBGYOR)

Violet 400 - 440 nm

Indigo 440 - 460 nm

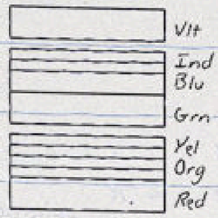
Blue 460 - 500 nm

Green 500 - 570 nm

Yellow 570 - 590 nm

Orange 590 - 620 nm

Red 620 - 700 nm



Production / Detection

Produced : atomic electronic transitions, incandescent objects, LEDs, lasers, Sun.

Detected : the eye, photographic film, CCDs, photomultiplier tubes.

Photon energy $E = hf$.

For $\lambda = 500$ nm : E 2.5 eV.

Above bond energies - drives photochemistry.

Ultraviolet Rays

λ 10 nm - 400 nm

f 7.5×10^{14} - 3×10^{16} Hz

Production

Sun (a lot), arc lamps, Hg vapour, very hot bodies ($T > 4000$ K).

Detection

Photographic film (fogs easily), fluorescent screens, photocells.

Effects

(1) Tan / sunburn - damages skin.

(2) Kills bacteria - used to sterilise.

(3) Causes fluorescence (UV - blue glow).

(4) Drives vitamin - D production in skin.

(5) Welder's arc gives strong UV - eye damage.

Ozone shield

O_3 in stratosphere absorbs most UV ; ~~CFCs~~ deplete it.

X - Rays

λ 10^{-3} - 10 nm

f 3×10^{16} - 3×10^{19} Hz

Production

High - speed electrons hitting a metal target (Coolidge tube , 1913). Two processes :

(i) Bremsstrahlung (braking radiation)

(ii) Characteristic K , L - line transitions

Detection

Photographic plate , Geiger - Muller tube , ionisation chamber , scintillation counter.

Properties / Uses

1. Highly penetrating - pass through flesh.

2. Ionising - damage DNA ; kill cells.

3. Medical imaging : bones , teeth , chest.

4. Crystal diffraction - atomic spacing X-ray wavelength. Bragg used this for structure.

5. Cancer therapy (radiotherapy , controlled).

Gamma Rays

$\lambda < 10^{-2}$ nm

$f > 3 \times 10^{19}$ Hz

Source

Nuclear transitions - emitted by radio-active nuclei when excited daughter nucleus falls to ground state.

Also from cosmic events : supernovae, neutron stars, pulsars, GRBs.

Detection

Geiger - Muller counter, scintillator, Cerenkov detector, semiconductor diodes.

Uses

1. Cancer treatment - Co - 60 therapy.
2. Sterilise surgical instruments.
3. Food irradiation - long shelf life.
4. Probe nuclear structure, astrophysics.

Most penetrating - need lead shielding.

Spectrum - Quick Table

Compare wavelengths and photon energies :

Band	lambda	f (Hz)	E (eV)
Radio	> 0.1 m	< 3e9	< 1e-5
Micro	1mm - 0.1m	3e9 - 3e11	1e-5 - 1e-3
IR	700nm - 1mm	3e11 - 4e14	1e-3 - 1.7
Visible	400 - 700nm	4 - 7.5e14	1.7 - 3.1
UV	10 - 400nm	7.5e14 - 3e16	3.1 - 124
X-ray	0.01 - 10nm	3e16 - 3e19	124 - 1e5
Gamma	< 0.01 nm	> 3e19	> 1e5

Notes

Photon energy $E = h f = h c / \lambda$.

$h = 6.63 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV s.

$h c = 1240$ eV . nm (useful shortcut).

*

Bands overlap and have fuzzy edges.

Same physics ; only the energy scale changes.

All travel at c in vacuum - ~~but light wins~~ same speed.

Higher f -> more energetic -> more harmful.

Worked Example - 1

Q. An EM wave in vacuum has lambda = 500 nm.

(a) Find its frequency.

(b) If E_0 = 30 V/m, find B_0.

(c) Find intensity.

Solution

(a) $c = f \lambda \rightarrow f = c / \lambda$

$f = 3 \times 10^{8} / 500 \times 10^{-9}$

$f = 6 \times 10^{14}$ Hz

(b) $B_0 = E_0 / c$

$B_0 = 30 / (3 \times 10^{8})$

$B_0 = 10^{-7}$ T $=$ 100 nT

(c) $I = 1/2\, e_0\, c\, E_0\ 2$

$I = 0.5 \times 8.85e{-12} \times 3e8 \times (30)^2$

$I = 0.5 \times 8.85e{-12} \times 3e8 \times 900$

$I = 1.19$ W / m^2

Note on units

Always convert lambda to metres first ;

nm $= 10^{-9}$ m , μm $= 10^{-6}$ m , mm $= 10^{-3}$ m.

Common mistake : forgetting ~~a~~ factor of 10.

Cross - check by dimensions ?

Worked Example - 2

Q. A parallel - plate capacitor of plate area A is being charged so that the electric field between plates grows at $dE/dt = 10 \quad 10$ V / m / s . Find the displacement current density between the plates.

Solution

$I\_d = \varepsilon_0 \, d\phi_E / dt$

Current density $J\_d = I\_d / A$

$J\_d = \varepsilon_0 \, dE / dt$

$= 8.85 \times 10^{-12} \times 10^{10}$

$= 8.85 \times 10^{-2}$ A / m^2

J_d 0.0885 A / m^2

Remark

Displacement current is a real current for purposes of Ampere's law , though no charge actually flows across the gap.

It is the dE / dt term doing the work.

Summary - Key Formulae

Maxwell + speed

$I\_d = \varepsilon_0 \, d\Phi_E / dt$

integral $B \cdot dl = \mu_0 (I + I\_d)$

$c = 1 / \text{sqrt}(\mu_0 \varepsilon_0) = 3 \times 10^8$ m/s

Wave relations

$E = E_0 \sin(kx - wt)$

$B = B_0 \sin(kx - wt)$

$E_0 / B_0 = c$; $v = f \lambda$; $k = 2\pi / \lambda$

In a medium

$v = 1 / \text{sqrt}(\mu \varepsilon)$; $n = c / v$

$n = \text{sqrt}(\varepsilon_r \mu_r)$

Energy & momentum

$u = \varepsilon_0 E^2 = B^2/\mu_0$

$I = 1/2 \, \varepsilon_0 c E_0^2$

$S = (E \times B) / \mu_0$; $p = U / c$

$P_{rad} = I / c$ (abs) ; $2 I / c$ (refl)

Spectrum order

Radio < Micro < IR < Visible <

UV < X - ray < Gamma (by frequency).

All travel at c in vacuum. END - Ch 8.