NCERT SOLUTIONS

Class 12 Physics

Chapter 8: Electromagnetic Waves

Detailed Step-by-Step Exercise Solutions

Q1 **Figure 8.6 shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A.**
(a) Calculate the capacitance and the rate of change of potential difference between the plates.
(b) Obtain the displacement current across the plates.
(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

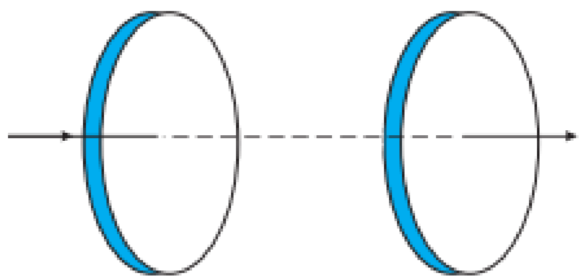


FIGURE 8.6

Solution

Given:

- Radius of each circular plate:
$$r = 12 \text{ cm} = 0.12 \text{ m}$$
- Separation between plates:
$$d = 5.0 \text{ cm} = 0.05 \text{ m}$$
- Charging current:
$$I = 0.15 \text{ A}$$
- Permittivity of free space:
$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

Step 1: Capacitance of Parallel Plate Capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

Area of circular plate:

$$A = \pi r^2 = \pi(0.12)^2$$

$$A = 0.0452 \text{ m}^2$$

Now:

$$C = \frac{8.85 \times 10^{-12} \times 0.0452}{0.05}$$

$$C = 8.0 \times 10^{-12} \text{ F}$$

$$C = 8.0 \text{ pF}$$

Step 2: Rate of Change of Potential Difference
Using:

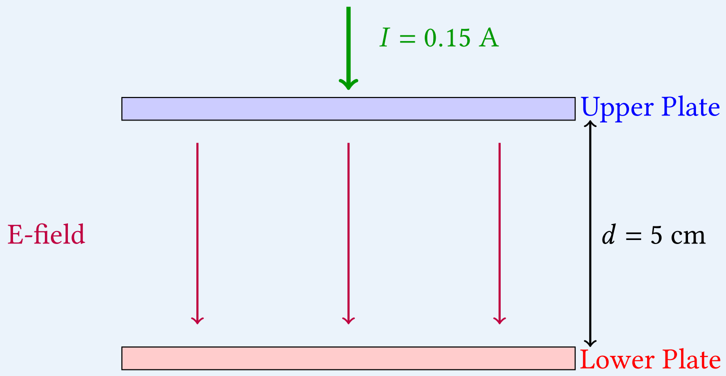
$$I = C\frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{I}{C}$$

$$\frac{dV}{dt} = \frac{0.15}{8.0 \times 10^{-12}}$$

$$\frac{dV}{dt} = 1.875 \times 10^{10} \text{ V/s}$$

Visual Representation



Step 3: Displacement Current
According to Maxwell:

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

For a charging capacitor:

$$I_d = I$$

$$I_d = 0.15 \text{ A}$$

Step 4: Kirchhoff's First Rule
At each plate:

- Conduction current enters the plate through wire.
- Between plates, displacement current continues.

Thus total current remains same.
So Kirchhoff's first rule is valid if displacement current is included.

$$(\boldsymbol{a})\ C = 8.0 \times 10^{-12} \text{ F} = 8.0 \text{ pF}$$

$$\left(\frac{\boldsymbol{dV}}{\boldsymbol{dt}}\right) = 1.875 \times 10^{10} \text{ V/s}$$

$$(\boldsymbol{b})\ \boldsymbol{I_d} = 0.15 \text{ A}$$

(c) Yes, Kirchhoff's rule is valid when displacement current is considered.

Expert's Solution — Vivek Jain, B.Tech Civil, IIT Kanpur

Shortcut Method:
Use:

$$C = \frac{\varepsilon_0 A}{d} \qquad I = C\frac{dV}{dt}$$

For charging capacitor:

$$I_d = I$$

Hence current continuity is maintained.

Did You Know?

Key Points to Remember:

- Displacement current exists in capacitor gap
- It is not due to actual charge flow
- Maxwell introduced it to preserve Kirchhoff's law
- In charging capacitor:
$$I = I_d$$

Q2 **A parallel plate capacitor (Fig. 8.7) made of circular plates each of radius $R = 6.0$ cm has a capacitance $C = 100$ pF. The capacitor is connected to a 230 V ac supply with an (angular) frequency of 300 rad s^{-1}.**
(a) What is the rms value of the conduction current?
(b) Is the conduction current equal to the displacement current?
(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.

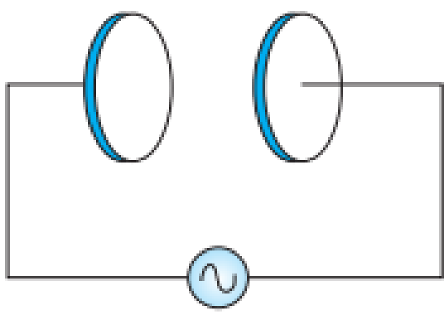


FIGURE 8.7

Solution

Given:

- Radius of plate:

$$R = 6.0 \text{ cm} = 0.06 \text{ m}$$

- Distance from axis:

$$r = 3.0 \text{ cm} = 0.03 \text{ m}$$

- Capacitance:

$$C = 100 \text{ pF} = 100 \times 10^{-12} \text{ F}$$

- RMS supply voltage:

$$V_{\text{rms}} = 230 \text{ V}$$

- Angular frequency:

$$\omega = 300 \text{ rad/s}$$

- Permeability of free space:

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

Step 1: RMS Conduction Current
For capacitor in AC circuit:

$$I_{\text{rms}} = V_{\text{rms}} \omega C$$

$$I_{\text{rms}} = 230 \times 300 \times 100 \times 10^{-12}$$

$$I_{\text{rms}} = 6.9 \times 10^{-6} \text{ A}$$

$$I_{\text{rms}} = 6.9 \ \mu\text{A}$$

Step 2: Conduction Current vs Displacement Current
In a capacitor connected to AC source:

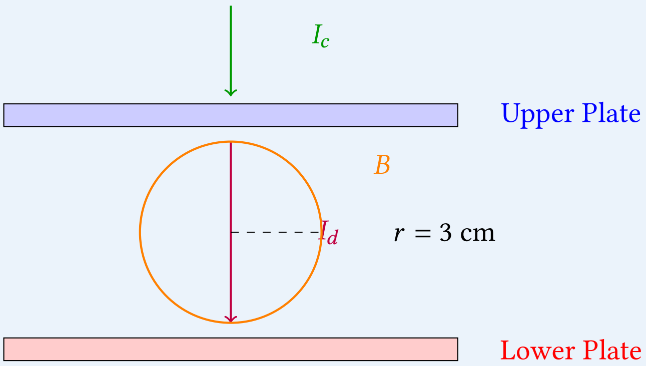
- Conduction current flows in connecting wires.
- Displacement current flows in the gap between plates.

According to Maxwell:

$$I_d = I_c$$

Hence displacement current is equal to conduction current at every instant.

Visual Representation



Step 3: Magnetic Field Amplitude at $r = 3$ cm
Use Ampere-Maxwell law:

$$B(2\pi r) = \mu_0 I_{\text{enc}}$$

Current enclosed inside radius r:

$$I_{\text{enc}} = I_0 \frac{r^2}{R^2}$$

Current amplitude:

$$I_0 = \sqrt{2}\, I_{\text{rms}}$$

$$I_0 = \sqrt{2}(6.9 \times 10^{-6}) = 9.76 \times 10^{-6} \text{ A}$$

Now:

$$I_{\text{enc}} = 9.76 \times 10^{-6} \times \frac{(0.03)^2}{(0.06)^2}$$

$$I_{\text{enc}} = 9.76 \times 10^{-6} \times \frac{1}{4} = 2.44 \times 10^{-6} \text{ A}$$

Now:

$$B = \frac{\mu_0 I_{\text{enc}}}{2\pi r}$$

$$B = \frac{4\pi \times 10^{-7} \times 2.44 \times 10^{-6}}{2\pi \times 0.03}$$

$$B = 1.63 \times 10^{-11} \text{ T}$$

$$\textbf{(a) } I_{\text{rms}} = 6.9\ \mu\text{A}$$
$$\textbf{(b) } I_c = I_d$$
$$\textbf{(c) } B_0 = 1.63 \times 10^{-11} \text{ T}$$

Expert's Solution — Vivek Jain, B.Tech Civil, IIT Kanpur

Shortcut Method:
Use capacitor current formula:

$$I_{\text{rms}} = \omega C V_{\text{rms}}$$

For capacitor gap:

$$I_d = I_c$$

Magnetic field inside plates:

$$B = \frac{\mu_0 I_0 r}{2\pi R^2}$$

(when $r < R$)

Did You Know?

Key Points to Remember:

- AC capacitor allows alternating current
- No charge crosses dielectric gap
- Displacement current maintains continuity
- Magnetic field forms concentric circles

Q3 **What physical quantity is the same for X-rays of wavelength 10^{-10} m, red light of wavelength 6800 Å and radiowaves of wavelength 500 m?**

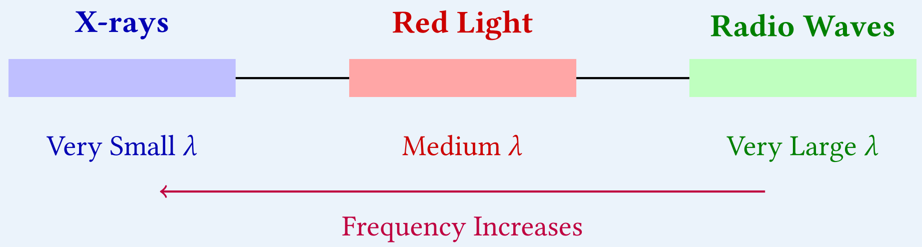
Solution

Given:

- X-rays wavelength = 10^{-10} m
- Red light wavelength = 6800 Å
- Radiowaves wavelength = 500 m

Step 1: Identify the nature of all three waves
X-rays, visible red light, and radiowaves all belong to the **electromagnetic spectrum**.

Visual Representation: Electromagnetic Spectrum



Step 2: Common Property of Electromagnetic Waves
All electromagnetic waves travel in vacuum with the same speed:

$$c = 3 \times 10^8 \text{ m/s}$$

This speed is independent of wavelength.

Comparison Table

Wave Type	Wavelength	Speed in Vacuum
X-rays	10^{-10} m	3×10^8 m/s
Red Light	6800 Å	3×10^8 m/s
Radiowaves	500 m	3×10^8 m/s

Step 3: Conclusion
Though wavelength and frequency are different, their **speed in vacuum remains same**.

The same physical quantity is speed (velocity)

$$\boldsymbol{c = 3 \times 10^8} \text{ m/s}$$

Expert's Solution — Harshit Sharma, B.Tech CSE, NIT Bhubaneswar

Shortcut Method:
Whenever X-rays, visible light, infrared, UV, microwaves or radiowaves are given:

All are electromagnetic waves

Hence in vacuum:

$$c = 3 \times 10^8 \text{ m/s}$$

Did You Know?

Key Points to Remember:

- Wavelength changes from one wave to another
- Frequency also changes
- Speed in vacuum always same
- $c = \lambda\nu$
- Larger wavelength $\Rightarrow$ lower frequency

Q4 A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

Solution

Given:

- Direction of propagation = z-axis
- Frequency, $f = 30$ MHz $= 30 \times 10^6$ Hz
- Speed of electromagnetic wave in vacuum:

$$c = 3 \times 10^8 \text{ m/s}$$

Step 1: Direction of Electric and Magnetic Fields
For a plane electromagnetic wave:

$$\vec{E} \perp \vec{B} \perp \text{Direction of propagation}$$

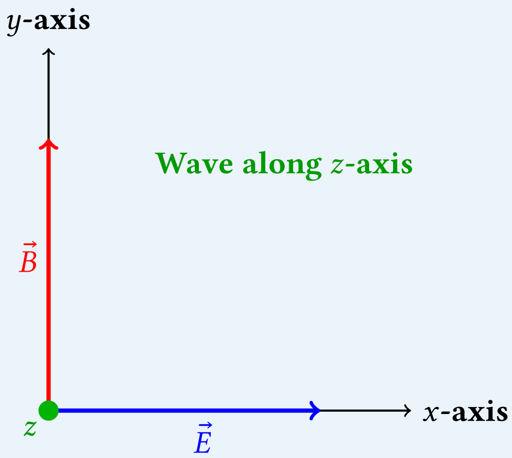
Since the wave travels along z-direction:

- Electric field lies in xy-plane
- Magnetic field lies in xy-plane
- Both are perpendicular to each other

Visual Representation

$$\text{Take } \vec{E} \text{ along } x\text{-axis}, \qquad \vec{B} \text{ along } y\text{-axis}$$

$$\vec{E} \times \vec{B} = \text{Direction of wave propagation}$$



Color Guide

- Blue = Electric field $\vec{E}$
- Red = Magnetic field $\vec{B}$
- Green = Direction of propagation

Step 2: Calculate Wavelength
Use relation:

$$c = f\lambda$$

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^8}{30 \times 10^6}$$

$$\lambda = 10 \text{ m}$$

Calculation Flow

$$30 \text{ MHz} = 30 \times 10^6 \text{ Hz}$$

$$\lambda = \frac{3 \times 10^8}{30 \times 10^6} = \frac{3}{30} \times 10^2 = 10 \text{ m}$$

$$\vec{E} \perp \vec{B} \perp z\text{-direction}$$

$$\vec{E} \parallel x\text{-axis}, \quad \vec{B} \parallel y\text{-axis}$$

$$\lambda = 10 \text{ m}$$

Expert's Solution — Vivek Jain, B.Tech Civil, IIT Kanpur

Shortcut Method:
Always remember:

$$\vec{E} \perp \vec{B} \perp \text{Wave Direction}$$

Use right-hand rule:

$$\vec{E} \times \vec{B} = \text{Propagation Direction}$$

For wavelength:

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^8}{30 \times 10^6} = 10 \text{ m}$$

Did You Know?

Key Points to Remember:

- EM waves are transverse waves
- $\vec{E}$ and $\vec{B}$ are mutually perpendicular
- Both are perpendicular to wave direction
- 1 MHz $= 10^6$ Hz

Q5 A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?

Solution

Given:

- Frequency range:

$$7.5 \text{ MHz to } 12 \text{ MHz}$$

- Speed of radio waves in vacuum:

$$c = 3 \times 10^8 \text{ m/s}$$

Step 1: Formula Used
For electromagnetic waves:

$$c = f\lambda$$

Hence,

$$\lambda = \frac{c}{f}$$

Step 2: Maximum Wavelength
Minimum frequency gives maximum wavelength.

$$f = 7.5 \text{ MHz} = 7.5 \times 10^6 \text{ Hz}$$

$$\lambda_{\text{max}} = \frac{3 \times 10^8}{7.5 \times 10^6}$$

$$\lambda_{\text{max}} = 40 \text{ m}$$

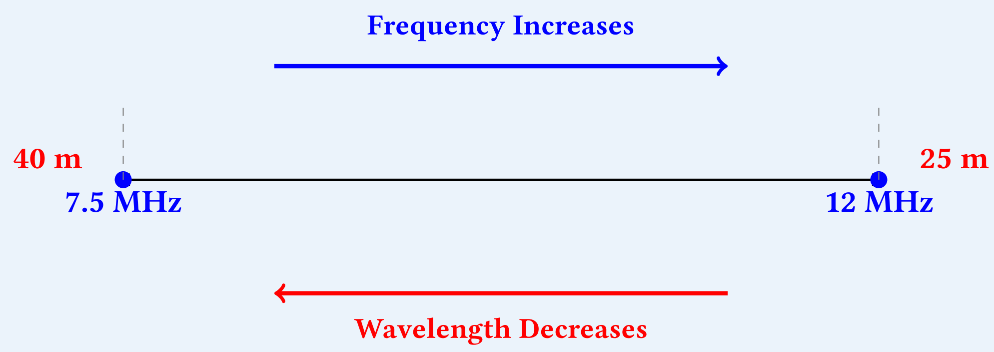
Step 3: Minimum Wavelength
Maximum frequency gives minimum wavelength.

$$f = 12 \text{ MHz} = 12 \times 10^6 \text{ Hz}$$

$$\lambda_{\text{min}} = \frac{3 \times 10^8}{12 \times 10^6}$$

$$\lambda_{\text{min}} = 25 \text{ m}$$

Visual Representation



Comparison Table

Frequency	Wavelength
7.5 MHz	40 m
12 MHz	25 m

$$\text{Corresponding wavelength band } = \mathbf{25} \text{ m to } \mathbf{40} \text{ m}$$

Expert's Solution – Harshit Sharma, B.Tech CSE, NIT Bhubaneswar

Shortcut Method:
For frequency in MHz:

$$\lambda(\text{m}) = \frac{300}{f(\text{MHz})}$$

So,

$$\lambda = \frac{300}{7.5} = 40 \text{ m}$$

$$\lambda = \frac{300}{12} = 25 \text{ m}$$

Hence,

$$25 \text{ m to } 40 \text{ m}$$

Did You Know?

Key Points to Remember:

- Frequency and wavelength are inversely proportional
- Lower frequency $\Rightarrow$ larger wavelength
- Higher frequency $\Rightarrow$ smaller wavelength
- Shortcut:

$$\lambda = \frac{300}{f(\text{MHz})}$$

Q6 A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. What is the frequency of the electromagnetic waves produced by the oscillator?

Solution

Given:

- Frequency of oscillating charged particle:

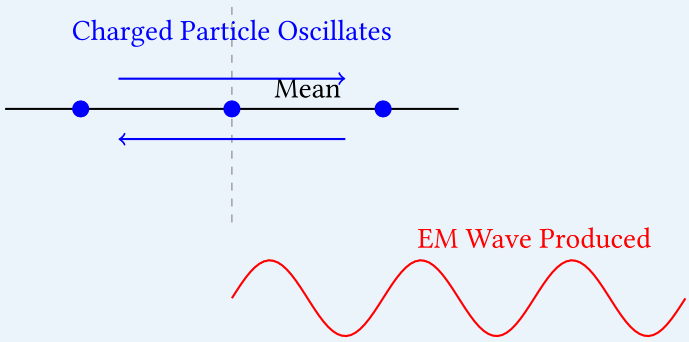
$$f = 10^9 \text{ Hz}$$

Step 1: Basic Concept

An accelerating or oscillating charged particle emits **electromagnetic waves**.
The frequency of emitted electromagnetic waves is equal to the frequency of oscillation of the charged particle.

$$f_{\text{wave}} = f_{\text{particle}}$$

Visual Representation



Step 2: Apply Formula

$$f_{\text{wave}} = 10^9 \text{ Hz}$$

Step 3: In MHz / GHz Form

$$10^9 \text{ Hz} = 1000 \text{ MHz} = 1 \text{ GHz}$$

$$\boldsymbol{f_{\text{EM wave}} = 10^9} \text{ Hz}$$

$$\boldsymbol{= 1} \text{ GHz}$$

Expert's Solution — Vivek Jain, B.Tech Civil, IIT Kanpur

Shortcut Method:
Whenever a charged particle oscillates:

$$\text{Frequency of emitted EM wave} = \text{Frequency of oscillation}$$

So directly:

$$f = 10^9 \text{ Hz}$$

Did You Know?

Key Points to Remember:

- Accelerating charge emits electromagnetic radiation
- Oscillating charge produces periodic EM waves
- Source frequency = Wave frequency
- 10^9 Hz = 1 GHz

Q7 The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is $B_0 = 510$ nT. What is the amplitude of the electric field part of the wave?

Solution

Given:

- Magnetic field amplitude:

$$B_0 = 510 \text{ nT}$$

- Since $1 \text{ nT} = 10^{-9}$ T

$$B_0 = 510 \times 10^{-9} \text{ T}$$

- Speed of electromagnetic wave in vacuum:

$$c = 3 \times 10^8 \text{ m/s}$$

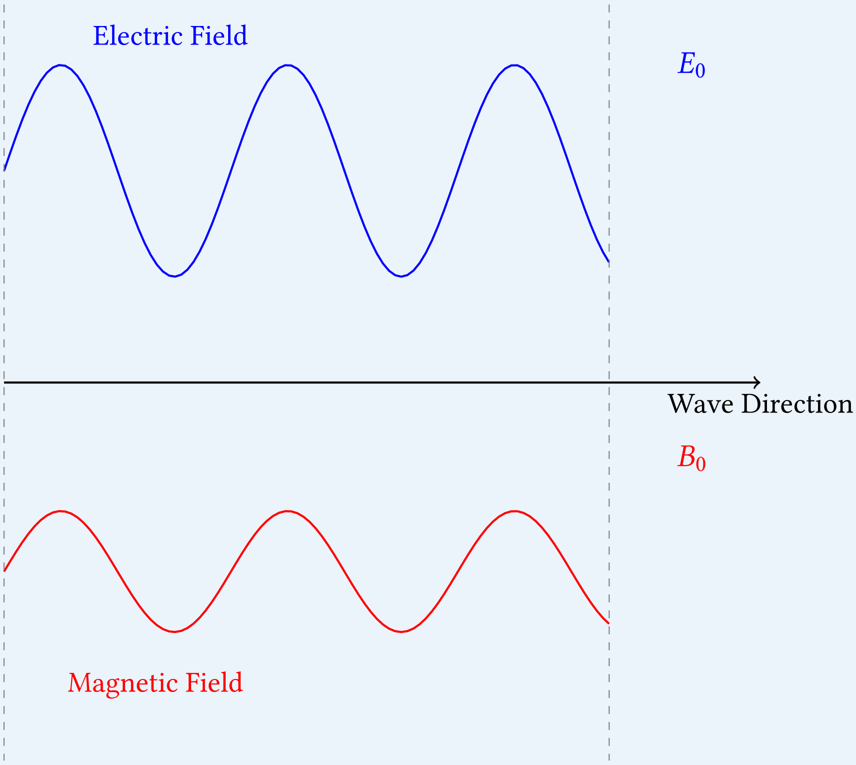
Step 1: Relation Between Electric and Magnetic Fields
For an electromagnetic wave in vacuum:

$$E_0 = cB_0$$

where:

- E_0 = Electric field amplitude
- B_0 = Magnetic field amplitude

Visual Representation



Step 2: Substitute Values

$$E_0 = (3 \times 10^8)(510 \times 10^{-9})$$

$$E_0 = 153 \times 10^{-1}$$

$$E_0 = 15.3 \text{ V/m}$$

Calculation Flow

$$3 \times 510 = 1530$$

$$10^8 \times 10^{-9} = 10^{-1}$$

$$E_0 = 1530 \times 10^{-1} = 15.3$$

$$\boldsymbol{E_0 = 15.3} \text{ V/m}$$

Expert's Solution — Kunal Pandey, B.Tech CSE, IIT Mumbai

Shortcut Method:
Use direct relation for EM waves:

$$E_0 = cB_0$$

$$E_0 = 3 \times 10^8 \times 510 \times 10^{-9}$$

$$E_0 = 15.3 \text{ V/m}$$

Did You Know?

Key Points to Remember:

- In vacuum:

$$E_0 = cB_0$$

- Electric and magnetic fields are perpendicular
- Both travel with same phase
- nT = nano tesla = 10^{-9} T

Q8 Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120$ N/C and that its frequency is $\nu = 50.0$ MHz.
(a) Determine $B_0,\ \omega,\ k$, and λ.
(b) Find expressions for $\vec{E}$ and $\vec{B}$.

Solution

Given:

- Electric field amplitude:

$$E_0 = 120 \text{ N/C}$$

- Frequency:

$$\nu = 50.0 \text{ MHz} = 50 \times 10^6 \text{ Hz}$$

- Speed of light:

$$c = 3 \times 10^8 \text{ m/s}$$

Step 1: Magnetic Field Amplitude
For electromagnetic waves:

$$E_0 = cB_0$$

$$B_0 = \frac{E_0}{c}$$

$$B_0 = \frac{120}{3 \times 10^8} = 4.0 \times 10^{-7} \text{ T}$$

Step 2: Angular Frequency

$$\omega = 2\pi\nu$$

$$\omega = 2\pi(50 \times 10^6)$$

$$\omega = 3.14 \times 10^8 \text{ rad/s}$$

Step 3: Wavelength

$$\lambda = \frac{c}{\nu}$$

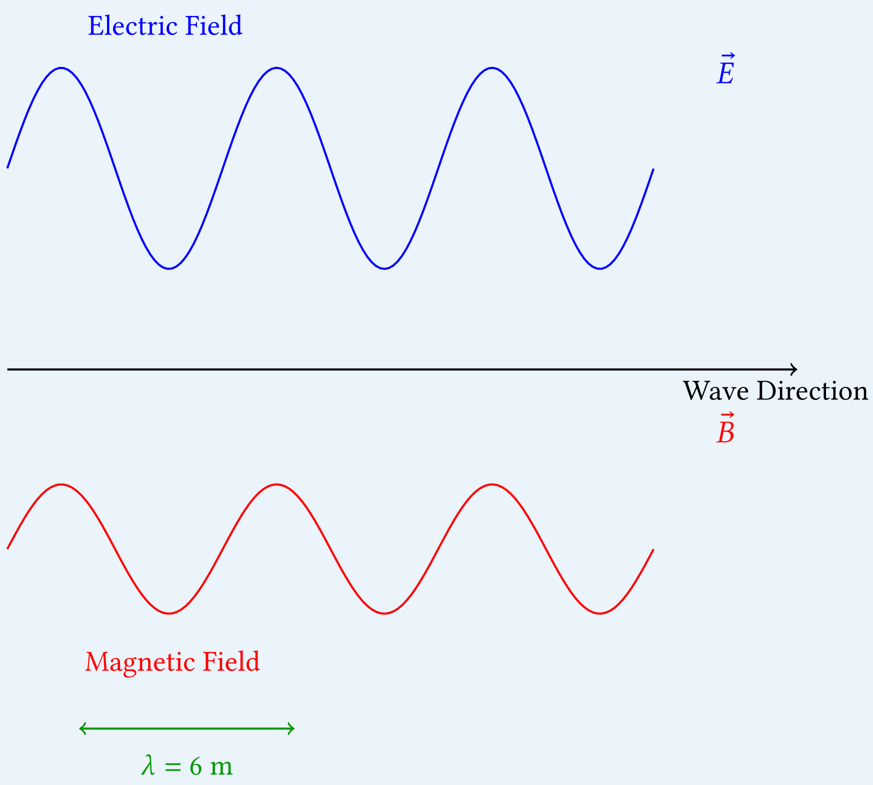
$$\lambda = \frac{3 \times 10^8}{50 \times 10^6} = 6 \text{ m}$$

Step 4: Wave Number

$$k = \frac{2\pi}{\lambda}$$

$$k = \frac{2\pi}{6} = 1.047 \text{ rad/m}$$

Visual Representation



Step 5: Expressions for Fields
Assume wave travels along $+x$-direction.

- $\vec{E}$ along y-axis
- $\vec{B}$ along z-axis

$$\vec{E} = E_0 \sin(kx - \omega t)\,\hat{j}$$

$$\vec{B} = B_0 \sin(kx - \omega t)\,\hat{k}$$

Substituting values:

$$\vec{E} = 120 \sin(1.047x - 3.14 \times 10^8 t)\,\hat{j}$$

$$\vec{B} = 4.0 \times 10^{-7} \sin(1.047x - 3.14 \times 10^8 t)\,\hat{k}$$

$$\begin{aligned}
\boldsymbol{B_0} &= 4.0 \times 10^{-7}\ \text{T} \\
\boldsymbol{\omega} &= 3.14 \times 10^8\ \text{rad/s} \\
\boldsymbol{k} &= 1.047\ \text{rad/m} \\
\boldsymbol{\lambda} &= 6\ \text{m} \\
\vec{\boldsymbol{E}} &= 120 \sin(1.047x - 3.14 \times 10^8 t)\,\hat{j} \\
\vec{\boldsymbol{B}} &= 4.0 \times 10^{-7} \sin(1.047x - 3.14 \times 10^8 t)\,\hat{k}
\end{aligned}$$

Expert's Solution — Vivek Jain, B.Tech Civil, IIT Kanpur

Shortcut Method:

$$B_0 = \frac{E_0}{c} \qquad \omega = 2\pi\nu \qquad \lambda = \frac{c}{\nu} \qquad k = \frac{2\pi}{\lambda}$$

Then:

$$\vec{E} = E_0 \sin(kx - \omega t)\hat{j}$$

$$\vec{B} = B_0 \sin(kx - \omega t)\hat{k}$$

Did You Know?

Key Points to Remember:

- $\vec{E} \perp \vec{B} \perp$ direction of propagation
- $E_0 = cB_0$
- Both fields oscillate in same phase
- $\omega = 2\pi\nu$
- $k = 2\pi/\lambda$

Q9 The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula $E = h\nu$ (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

Solution

Given:

- Photon energy:

$$E = h\nu$$

- Planck's constant:

$$h = 6.63 \times 10^{-34} \text{ J s}$$

- Conversion:

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

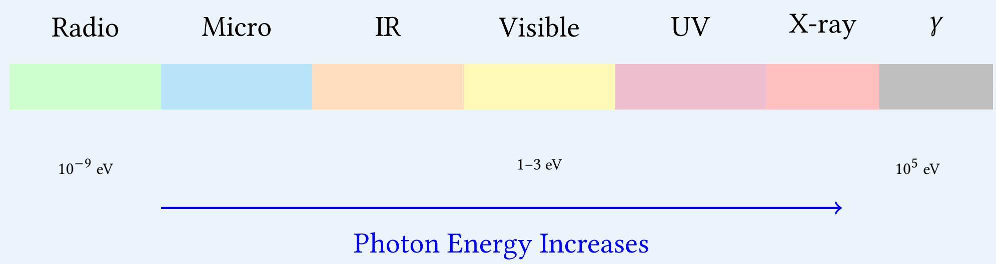
- Therefore:

$$E(\text{eV}) = \frac{h\nu}{1.6 \times 10^{-19}} \approx 4.14 \times 10^{-15}\nu$$

Step 1: Photon Energies for Different Regions

Region	Typical Frequency (Hz)	Energy (eV)
Radio Waves	10^6 to 10^9	10^{-9} to 10^{-6}
Microwaves	10^9 to 10^{11}	10^{-6} to 10^{-4}
Infrared	10^{12} to 10^{14}	10^{-3} to 10^{-1}
Visible Light	4×10^{14} to 7.5×10^{14}	1.6 to 3.1
Ultraviolet	10^{15} to 10^{17}	4 to 400
X-rays	10^{17} to 10^{19}	400 to 4×10^4
Gamma Rays	$> 10^{19}$	$> 4 \times 10^4$

Visual Representation



Step 2: Relation With Sources

- **Low energy photons** (radio, microwave) are produced by oscillating charges in antennas and electronic circuits.
- **Medium energy photons** (infrared, visible) are produced by hot bodies, atoms, molecules, lamps and stars.
- **High energy photons** (UV, X-rays, gamma rays) are produced in atomic transitions, inner-shell electron changes, nuclear reactions and cosmic events.

$$\begin{aligned} \text{Radio}: &\ 10^{-9} \text{ to } 10^{-6} \text{ eV} \\ \text{Microwave}: &\ 10^{-6} \text{ to } 10^{-4} \text{ eV} \\ \text{Infrared}: &\ 10^{-3} \text{ to } 10^{-1} \text{ eV} \\ \text{Visible}: &\ 1.6 \text{ to } 3.1 \text{ eV} \\ \text{UV}: &\ 4 \text{ to } 400 \text{ eV} \\ \text{X-rays}: &\ 10^{2} \text{ to } 10^{4} \text{ eV} \\ \gamma\text{-rays}: &> 10^{4} \text{ eV} \end{aligned}$$

Higher photon energy requires more energetic sources.

Expert's Solution — Rabi Jain, B.Tech Civil, IIT Delhi

Shortcut Method:
Use direct formula:

$$E(\text{eV}) = 4.14 \times 10^{-15}\nu$$

So higher frequency means higher photon energy.

$$\text{Radio} < \text{Visible} < \text{X-rays} < \gamma$$

Did You Know?

Key Points to Remember:

- Photon energy is directly proportional to frequency
- Lower wavelength ⇒ higher energy
- Visible photons have energy of few eV
- Nuclear sources produce gamma rays
- Electronic devices produce radio waves

Q10 **In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 V m^{-1}.**
(a) What is the wavelength of the wave?
(b) What is the amplitude of the oscillating magnetic field?
(c) Show that the average energy density of the E field equals the average energy density of the B field.
[$c = 3 \times 10^8$ m s^{-1}]

Solution

Given:

- Frequency:
$$\nu = 2.0 \times 10^{10}\ \text{Hz}$$
- Electric field amplitude:
$$E_0 = 48\ \text{V/m}$$
- Speed of light:
$$c = 3 \times 10^8\ \text{m/s}$$

- Permittivity of free space:

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

- Permeability of free space:

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

Step 1: Find Wavelength
Use:

$$\lambda = \frac{c}{\nu}$$

$$\lambda = \frac{3 \times 10^8}{2.0 \times 10^{10}}$$

$$\lambda = 1.5 \times 10^{-2} \text{ m}$$

$$\lambda = 0.015 \text{ m} = 1.5 \text{ cm}$$

Step 2: Find Magnetic Field Amplitude
For electromagnetic waves:

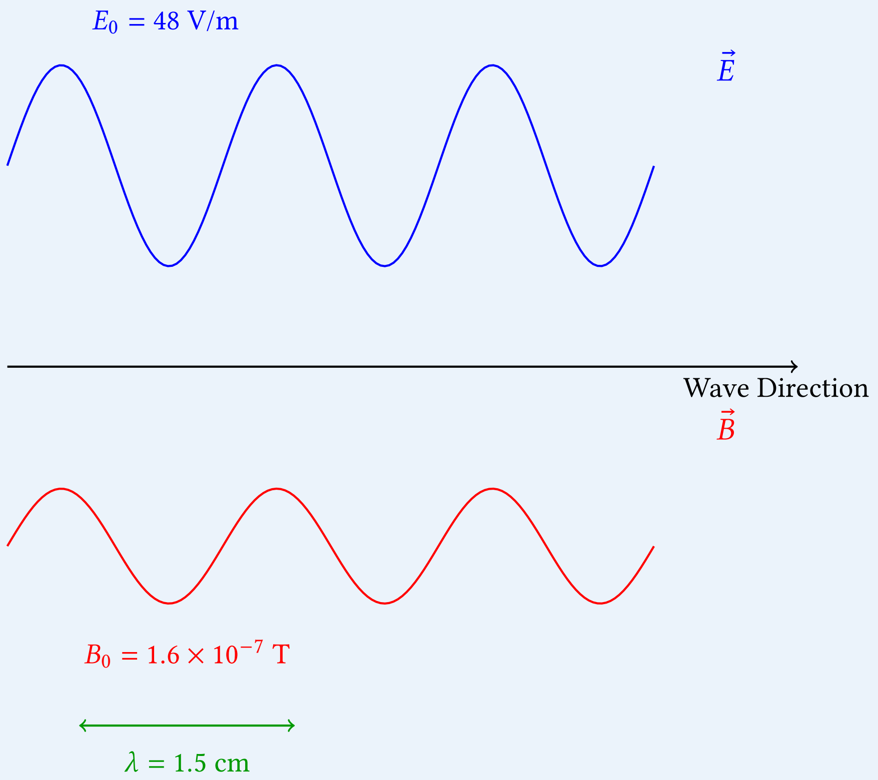
$$E_0 = cB_0$$

$$B_0 = \frac{E_0}{c}$$

$$B_0 = \frac{48}{3 \times 10^8}$$

$$B_0 = 1.6 \times 10^{-7} \text{ T}$$

Visual Representation



Step 3: Compare Average Energy Densities
Average electric energy density:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

For sinusoidal wave:

$$\langle u_E \rangle = \frac{1}{2}\varepsilon_0 \left\langle E^2 \right\rangle$$

Since:

$$\left\langle \sin^2 \theta \right\rangle = \frac{1}{2}$$

$$\langle u_E \rangle = \frac{1}{4}\varepsilon_0 E_0^2$$

Average magnetic energy density:

$$u_B = \frac{B^2}{2\mu_0}$$

$$\langle u_B \rangle = \frac{1}{4\mu_0}B_0^2$$

Using:

$$E_0 = cB_0$$

and

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

Then:

$$\langle u_B \rangle = \frac{1}{4\mu_0} \frac{E_0^2}{c^2} = \frac{1}{4\mu_0} E_0^2 (\mu_0 \varepsilon_0)$$

$$\langle u_B \rangle = \frac{1}{4} \varepsilon_0 E_0^2$$

Hence:

$$\boxed{\langle u_E \rangle = \langle u_B \rangle}$$

$$\boldsymbol{(a)}\ \boldsymbol{\lambda} = 1.5 \times 10^{-2}\ \text{m} = 1.5\ \text{cm}$$

$$\boldsymbol{(b)}\ \boldsymbol{B_0} = 1.6 \times 10^{-7}\ \text{T}$$

$$\boldsymbol{(c)}\ \boldsymbol{\langle u_E \rangle} = \boldsymbol{\langle u_B \rangle}$$

Average electric and magnetic energy densities are equal

Expert's Solution — Arab Sharma, B.Tech CSE, IIT Kanpur

Shortcut Method:
Use standard formulas:

$$\lambda = \frac{c}{\nu} \qquad B_0 = \frac{E_0}{c}$$

Energy densities:

$$\langle u_E \rangle = \frac{1}{4} \varepsilon_0 E_0^2 \qquad \langle u_B \rangle = \frac{1}{4\mu_0} B_0^2$$

For EM waves:

$$E_0 = cB_0 \Rightarrow \langle u_E \rangle = \langle u_B \rangle$$

Did You Know?

Key Points to Remember:

- Electric and magnetic fields share equal average energy
- $\vec{E} \perp \vec{B} \perp$ propagation direction
- $E_0 = cB_0$
- Higher frequency means shorter wavelength