NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12th Physics, Chapter 9

Chapter 9: Ray Optics and Optical Instruments

About this Chapter

This chapter develops the geometric-optics picture of how light travels in straight rays, reflects from mirrors and refracts at surfaces. The key tools are the **mirror formula**, the **lens-maker's formula**, **Snell's law**, the prism-deviation relation, and the formulas for the magnifying power of compound microscopes and telescopes. By the end you should be able to trace any paraxial ray, locate the image quantitatively, and design simple optical instruments.

Topics covered: Reflection by spherical mirrors • Refraction • Total internal reflection • Refraction by lenses • Prisms and dispersion • Optical instruments

Quick Formula Sheet

Mirror formula:
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Lens-maker's formula:
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Snell's law:
$$\mu_1 \sin i = \mu_2 \sin r$$

Prism (min deviation):
$$\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Critical angle:
$$\sin i_c = \frac{1}{\mu}$$

MCQ I

Q 9.1 **A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is**

(a) 7.5°.

(b) 5°.

(c) 15°.

(d) 2.5°.

SOLUTION

Correct option: (a) $7.5°$.

Concept used. For a prism of refracting angle A, the two refracting angles inside the glass satisfy

$$r_1 + r_2 = A,$$

where r_1 is the angle of refraction at the first face and r_2 the angle of incidence on the second face (measured from the normal to that face). **Snell's law** at the first face reads $\sin\theta = \mu\,\sin r_1$, where μ is the refractive index of the prism with respect to air.

Step 1. The ray leaves the second face *normally*, so the angle of refraction outside is $0°$. By Snell's law at the second face $\mu\,\sin r_2 = \sin 0° = 0$, which forces

$$r_2 = 0°.$$

Step 2. Use the prism relation:

$$r_1 + r_2 = A \;\Rightarrow\; r_1 = A - r_2 = 5° - 0° = 5°.$$

Step 3. Apply Snell's law at the first face:

$$\sin\theta = \mu\,\sin r_1 = 1.5 \times \sin 5°.$$

For small angles in radians, $\sin\theta \approx \theta$, so

$$\theta \approx 1.5 \times 5° = 7.5°.$$

Final Answer: $\theta = 7.5°$, option **(a)**.

☞ Small-angle prism shortcut

For a small prism ($A < 10°$) the deviation is $D = (\mu - 1)A$. Here $D = (1.5 - 1) \times 5° = 2.5°$ and $\theta = A + D = 5° + 2.5° - 0° = 7.5°$. Same answer, no Snell's law needed.

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Physics, IIT Madras*

Quick reading. The ray leaves the second face normally, so inside the prism it travels parallel to the normal of that face. The geometry of the prism then forces the inside ray to make exactly the angle A with the normal of the first face.

Step 1. Read off $r_1 = A = 5°$ directly from the geometry (the inside ray is perpendicular to face 2, hence makes angle A with the normal to face 1).

Step 2. Apply Snell at face 1 in the small-angle regime ($\sin x \approx x$ in radians):

$$\theta = \mu\,r_1 = 1.5 \times 5° = 7.5°.$$

Step 3. **Cross-check via deviation.** The total deviation is $D = (i_1 - r_1) + (i_2 - r_2) = (7.5 - 5) + (0 - 0) = 2.5°$, which agrees with the small-angle prism formula $D = (\mu - 1)A = 0.5 \times 5 = 2.5°$. Two independent routes give the same answer: the result is robust.

Step 4. **Alternative geometric method.** Drop a perpendicular from the exit point to face 1: the inside ray, normal to face 2, and normal to face 1 form a right triangle with the prism's apex angle A as the relevant angle, so the inside ray sees face 1's normal at exactly $A = 5°$. No algebra needed.

Why this matters. Recognising that one face is normal-exit collapses the two-face problem to a single Snell's-law step. This trick reappears in minimum-deviation prism problems and in the design of spectrometer Littrow prisms.
Unit check. μ is dimensionless, r_1 is in degrees, θ comes out in degrees. The small-angle linearisation $\sin x \approx x$ requires x in radians, but the equation $\theta \approx \mu r_1$ remains dimensionally consistent because both sides carry the same angular unit.

Final Answer: $\theta = 7.5°$ **(a)**.

♥ Prism shortcuts that always work

For "thin prism" problems ($A \lesssim 10°$) commit two shortcuts to memory: (i) deviation $D = (\mu - 1)A$, and (ii) for normal-exit rays $i = A + D$. Together they solve a wide family of JEE/NEET prism problems in two lines without invoking Snell's law.

Q 9.2 **A short pulse of white light is incident from air to a glass slab at normal incidence. After travelling through the slab, the first colour to emerge is**

(a) blue.

(b) green.

(c) violet.

(d) red.

SOLUTION

Correct option: (d) red.
Concept used. The refractive index of glass depends on wavelength: $\mu(\lambda)$ decreases as λ increases. This is **normal dispersion** and is summarised approximately by Cauchy's relation $\mu(\lambda) = A + B/\lambda^2$. The speed of light inside the glass is $v = c/\mu$, so a smaller μ means a faster ray.

Step 1. Among the colours in white light, red has the longest wavelength and violet the shortest. Hence

$$\mu_{\text{red}} < \mu_{\text{violet}}.$$

Step 2. Therefore the speed inside the slab satisfies

$$v_{\text{red}} = \frac{c}{\mu_{\text{red}}} > v_{\text{violet}} = \frac{c}{\mu_{\text{violet}}}.$$

Step 3. At normal incidence there is no bending, so each colour travels the same geometric path through the slab of thickness t. The transit time is

$$\tau(\lambda) = \frac{t}{v(\lambda)} = \frac{\mu(\lambda)\,t}{c}.$$

Smaller μ gives smaller τ, so red emerges first.

Final Answer: Red light emerges first, option **(d)**.

EXPERT'S SOLUTION *: Sneha Iyer, Ph.D Condensed Matter Physics, TIFR Mumbai*

Structural observation. The slab acts as a *group-delay* medium for the pulse. The slower a colour, the longer its delay; the faster, the earlier it pops out the other side.

Step 1. Rank refractive indices by wavelength (Cauchy):

$$\mu_{\text{red}} < \mu_{\text{yellow}} < \mu_{\text{green}} < \mu_{\text{blue}} < \mu_{\text{violet}}.$$

Step 2. The transit time scales like μ; red has the smallest μ and so the smallest transit time.

Step 3. **Numerical feel.** For crown glass at visible wavelengths, $\mu_{\text{red}} \approx 1.513$ and $\mu_{\text{violet}} \approx 1.532$. The transit-time difference over a slab of thickness $t = 1$ cm is $\Delta\tau = (\mu_v - \mu_r)t/c \approx 0.019 \times 10^{-2}/(3 \times 10^8) \approx 6.3 \times 10^{-13}\,\text{s} = 0.63\,\text{ps}$. Tiny but real, and now routinely measured with ultrafast lasers.

Step 4. **No-bending caveat.** At *normal* incidence, none of the colours separates laterally. Dispersion happens in *time*, not space. Lateral separation requires oblique incidence (or a prism geometry with non-parallel faces).

Why this matters. The same principle explains why prisms spread white light into a spectrum with red bent the least and violet the most. It also lies behind *chromatic dispersion* in optical fibres, the dominant pulse-broadening mechanism that limits long-haul data rates.

Final Answer: Red **(d)**.

☞ **Cauchy's normal-dispersion formula**

For most transparent materials in the visible band the refractive index drops with wavelength as

$$\mu(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots,$$

so longer wavelengths (red) bend less than shorter ones (violet). *Anomalous* dispersion (where μ rises with λ) occurs only near absorption lines and is rare for everyday glass.

☞ **Spotting the colour-ordering trap**

NCERT loves the question "which colour emerges first/last?" The trick is always the same: convert the question into "which colour travels slowest?" Slowest = highest μ = shortest wavelength = violet (last out). Fastest = red (first out). Don't get distracted by talk of dispersion or angles when the slab is at normal incidence.

Q 9.3 **An object approaches a convergent lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image**

(a) moves away from the lens with an uniform speed 5 m/s.

(b) moves away from the lens with an uniform acceleration.

(c) moves away from the lens with a non-uniform acceleration.

(d) moves towards the lens with a non-uniform acceleration.

SOLUTION

Correct option: (c) moves away from the lens with a non-uniform acceleration.

Concept used. The thin-lens equation (Cartesian sign convention) is

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

where u is the object distance (negative for real objects on the left) and v is the image distance. For a real object beyond the focus, the image distance v depends *non-linearly* on u.

Step 1. Solve for v as a function of u:

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{u+f}{uf},$$

$$v(u) = \frac{uf}{u+f}.$$

(Distances are taken as magnitudes here; signs handled separately. As the object approaches $u \to f$, $v \to \infty$.)

Step 2. Differentiate to find the image speed:

$$\frac{dv}{du} = \frac{f(u+f) - uf}{(u+f)^2} = \frac{f^2}{(u+f)^2}.$$

Image speed $v_I = \left|dv/du\right| v_O = \dfrac{f^2}{(u+f)^2} v_O$. Since u varies in time, v_I is not constant and the image accelerates.

Step 3. Compute the acceleration:

$$\frac{d^2v}{du^2} = -\frac{2f^2}{(u+f)^3},$$

which itself depends on u. Hence the acceleration is non-uniform. As $u \to f$, both v and $|dv/du|$ blow up, so the image races away faster and faster: *non-uniform acceleration, moving away*.

Final Answer: Option **(c)**: the image moves away with non-uniform acceleration.

✗ Confusing speed with acceleration

Choice (a) is tempting because the object moves uniformly. But the lens equation is non-linear in u, so equal changes in u do not produce equal changes in v. Always differentiate before guessing.

EXPERT'S SOLUTION *: Priya Mehta, Ph.D Physics, IISc Bangalore*

Picture-first. As the object slides from far away towards the focal plane, the image position swings from near F' on the right side out to infinity. So v goes from $\sim f$ to ∞ while u goes from large to f. That alone tells us motion is *not* uniform.

Step 1. Map limits: $u \to \infty \Rightarrow v \to f$; $u \to f^+ \Rightarrow v \to \infty$.

Step 2. The chain rule on $v(u(t))$ gives

$$\frac{dv}{dt} = \frac{f^2}{(u+f)^2}\,\frac{du}{dt}.$$

Even if du/dt is constant, dv/dt grows without bound as $u \to f$.

Step 3. Magnification gives the same verdict. Lateral magnification $m = v/u$ also diverges as $u \to f$: at $u = 2f$, $|m| = 1$; at $u = 1.1f$, $|m| = 11$; at $u = 1.01f$, $|m| = 101$. The non-linear blow-up of m is just the integrated form of the non-linear blow-up of dv/du.

Step 4. Numerical sanity. Take $f = 10\,\text{cm}$ and $v_O = 5\,\text{m/s}$. At $u = 20\,\text{cm}$, $|dv/du| = (10/30)^2 = 1/9$, so $v_I = 5/9 \approx 0.56\,\text{m/s}$. At $u = 11\,\text{cm}$, $|dv/du| = (10/21)^2 \approx 0.227$, so $v_I \approx 1.13\,\text{m/s}$ – the image has more than doubled its speed even though the object speed is fixed. Confirms non-uniform acceleration.

Why this matters. Non-linear maps turn uniform inputs into accelerated outputs. The same trick underlies the magnification blowing up at F, autofocus tracking in cameras at close range, and the difficulty of imaging at a microscope's working distance.

> **Final Answer:** **(c)** non-uniform acceleration, image moves away.

☞ **Longitudinal $\neq$ lateral magnification**

For a thin lens, the lateral (transverse) magnification is $m = v/u$, but the longitudinal (along-axis) magnification is $m_\ell = -m^2$. That is why an axially-moving object produces an axially-moving image whose speed grows quadratically with $|m|$, not linearly.

Q 9.4 A passenger in an aeroplane shall

(a) never see a rainbow.

(b) may see a primary and a secondary rainbow as concentric circles.

(c) may see a primary and a secondary rainbow as concentric arcs.

(d) shall never see a secondary rainbow.

SOLUTION

Correct option: (b) may see a primary and a secondary rainbow as concentric circles.

Concept used. A rainbow is a circular cone of light formed when sunlight is refracted and internally reflected by water drops. The full set of drops that direct light to the observer at the correct angle (about $42°$ for the primary, $51°$ for the secondary) lies on a cone whose axis is the anti-solar line through the observer's eye.

Step 1. For an observer on the ground, the horizon cuts off the lower half of this cone, so you see only an *arc*, not a full circle.

Step 2. For a passenger high in the air with rain drops both above and below the anti-solar line, the entire cone can be populated. The observer sees the rainbow as a complete *circle*.

Step 3. The primary ($42°$) and secondary ($51°$) rainbows share the same axis (the anti-solar line), so the two full circles are concentric.

> **Final Answer:** Option **(b)**: concentric circles.

EXPERT'S SOLUTION *: Vivaan Kapoor, B.Tech Engineering Physics, IIT Bombay*

Strategic angle. The geometry of a rainbow is a cone with half-angle $42°$ (primary) about the anti-solar line. Whether you see a full circle or just an arc depends only on how much of that cone the ground cuts off.

Step 1. Ground observer: half the cone is below the horizon, so an arc results.

Step 2. Aerial observer: rain drops both above and below the observer's altitude exist, so the full cone is visible and a complete circle is seen.

Step 3. **Why the two cones share an axis.** The anti-solar direction is defined by the Sun (a single direction), so every rainbow seen by a given observer is centred on that same line. The primary forms at $42°$ from this line (one internal reflection inside each drop) and the secondary at $51°$ (two internal reflections); both are coaxial cones, hence concentric circles when seen in full.

Step 4. **Colour order check.** In the primary the inner edge is violet and the outer edge red. In the secondary the ordering reverses (inner red, outer violet) because the extra internal reflection flips the colour-versus-deviation ranking. From a plane both rings should be clearly visible with their distinctive colour orderings.

Why this matters. Pilots and astronauts routinely photograph full-circle rainbows; this is the only natural setting where you can. The same circle-vs-arc reasoning explains the *Brocken spectre* and the *glory* seen around an aircraft's shadow on cloud tops.

Final Answer: **(b)** concentric circles.

☞ **Primary vs. secondary rainbow numbers**

Primary rainbow: $42°$ from the anti-solar point, one internal reflection, red outside / violet inside, brighter. Secondary rainbow: $51°$, two internal reflections, red inside / violet outside, fainter. The dark zone between them is Alexander's band.

♥ **Geometry beats algebra in rainbow problems**

Most rainbow questions reduce to a single geometric fact: the rainbow is a cone with a fixed half-angle about the anti-solar line. Once you internalise this picture, "full circle", "arc", "concentric", and "above/below the horizon" become obvious without writing a single equation.

Q 9.5 You are given four sources of light each one providing a light of a single colour: red, blue, green and yellow. Suppose the angle of refraction for a beam of yellow light corresponding to a particular angle of incidence at the interface of two media is $90°$. Which of the following statements is correct if the source of yellow light is replaced with that of other lights without changing the angle of incidence?

(a) The beam of red light would undergo total internal reflection.

(b) The beam of red light would bend towards normal while it gets refracted through

the second medium.
(c) The beam of blue light would undergo total internal reflection.
(d) The beam of green light would bend away from the normal as it gets refracted through the second medium.

SOLUTION

Correct option: (c) The beam of blue light would undergo total internal reflection.
Concept used. If yellow refracts at $90°$, the angle of incidence equals the **critical angle** for yellow, i.e. $\sin i = 1/\mu_{\text{yellow}}$. The critical angle satisfies $\sin i_c = 1/\mu$, so a larger μ gives a *smaller* critical angle and hence makes total internal reflection easier.

Step 1. Light goes from the denser medium (μ) to the rarer medium (air). By Cauchy's normal-dispersion ordering,

$$\mu_{\text{red}} < \mu_{\text{yellow}} < \mu_{\text{green}} < \mu_{\text{blue}}.$$

Step 2. Critical angles obey $\sin i_c = 1/\mu$, so the ordering of critical angles is reversed:

$$i_c(\text{red}) > i_c(\text{yellow}) > i_c(\text{green}) > i_c(\text{blue}).$$

Step 3. The fixed angle of incidence is $i = i_c(\text{yellow})$.

- Red: $i_c(\text{red}) > i$, so $i < i_c(\text{red}) \Rightarrow$ red still refracts (no TIR). This rules out (a) and makes (b) describe the actual behaviour qualitatively, but red bends *away* from the normal (denser→rarer), so (b) is also false.
- Green: $i_c(\text{green}) < i$, so $i > i_c(\text{green}) \Rightarrow$ green undergoes TIR. So (d) is false because green does not refract at all.
- Blue: $i_c(\text{blue}) < i$, so $i > i_c(\text{blue}) \Rightarrow$ blue undergoes TIR. (c) is true.

Step 4. Among the four statements, only (c) is consistent with the physics: blue (and also green) undergo TIR, while red still refracts.

Final Answer: The blue beam undergoes total internal reflection, option **(c)**.

EXPERT'S SOLUTION *: Arjun Verma, M.Sc Physics, IIT Madras*

Quick reading. The condition "refraction at $90°$" means the incidence angle equals the critical angle for that colour. Whether other colours still refract depends on whether their own critical angles are larger or smaller.

Step 1. Larger $\mu \Rightarrow$ smaller i_c (since $\sin i_c = 1/\mu$).

Step 2. Blue has the largest μ among red/yellow/green/blue, hence the smallest i_c. The given i already equals $i_c(\text{yellow}) > i_c(\text{blue})$, so blue must TIR.

Step 3. Inequality table. Putting the colours in order:

colour	red	yellow	green	blue
μ	lowest	low	high	highest
i_c	largest	$i_c = i$	smaller	smallest
at this i	refracts	grazing	TIR	TIR

So both blue and green TIR; option (c) names the dominant TIR-er.

Step 4. Concept linkage. This is the same physics that keeps short-wavelength light better confined in step-index optical fibres: the numerical aperture $\text{NA} = \sqrt{\mu_{\text{core}}^2 - \mu_{\text{clad}}^2}$ is slightly larger for blue than for red, so the acceptance cone narrows but TIR is more robust.

Why this matters. Optical fibres exploit exactly this: shorter-wavelength light has a lower critical angle, easier to confine. The reverse logic also explains why red light leaks out of total-reflection prisms first when they are slightly damaged.

Final Answer: **(c)** blue undergoes TIR.

✗ "Higher $\mu \Rightarrow$ larger i_c" is backwards

The critical-angle relation is $\sin i_c = 1/\mu$, an *inverse* relation. So higher μ gives *smaller* i_c, making TIR easier, not harder. A common JEE blunder is to flip this and conclude red TIRs first. Always derive the inequality from $\sin i_c = 1/\mu$ rather than guess.

Q 9.6 **The radius of curvature of the curved surface of a plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will**

(a) act as a convex lens only for the objects that lie on its curved side.

(b) act as a concave lens for the objects that lie on its curved side.

(c) act as a convex lens irrespective of the side on which the object lies.

(d) act as a concave lens irrespective of side on which the object lies.

SOLUTION

Correct option: (c) act as a convex lens irrespective of the side on which the object lies.

Concept used. The lens-maker's formula gives

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right),$$

where R_1 is the radius of the surface the light strikes first and R_2 the second. Sign convention: convex-towards-incoming light $\Rightarrow R > 0$; concave $\Rightarrow R < 0$; flat surface $\Rightarrow R = \infty$.

Step 1. **Case 1: curved side faces the object.** Light hits the convex surface first, so $R_1 = +20\,\text{cm}$; the second face is flat, $R_2 = \infty$.

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{20} - 0\right) = \frac{0.5}{20} = \frac{1}{40}\,\text{cm}^{-1},$$

so $f = +40\,\text{cm}$. Positive $f \Rightarrow$ converging.

Step 2. **Case 2: flat side faces the object.** Light hits the flat surface first, so $R_1 = \infty$; the second face is convex as seen by the light: its centre of curvature is on the side from which the light is coming, so $R_2 = -20\,\text{cm}$.

$$\frac{1}{f} = (1.5-1)\left(0 - \frac{1}{-20}\right) = 0.5 \times \frac{1}{20} = \frac{1}{40}\,\text{cm}^{-1},$$

so $f = +40\,\text{cm}$. Again converging.

Step 3. Both cases yield the same focal length and the same sign, so the lens acts as a convex (converging) lens regardless of which side faces the object.

Final Answer: $f = +40\,\text{cm}$ either way; option **(c)**.

☞ Symmetry of thin lenses

For a thin lens in air, the focal length is independent of which side the object is on. The lens-maker's formula is symmetric under swapping $R_1 \leftrightarrow -R_2$, which is exactly what flipping the lens does.

EXPERT'S SOLUTION *: Aanya Banerjee, M.Tech Applied Physics, IIT Delhi*

Quick reading. A thin lens is reversible: reflect it about its principal plane and the focal length stays the same. So if the lens is converging when light enters one side, it stays converging when entering the other.

Step 1. Apply lens-maker's formula with curved side facing object:
$1/f = (1.5-1)(1/20) = 1/40 \Rightarrow f = 40\,\text{cm}$.

Step 2. By the reversibility theorem the result is unchanged when the lens is flipped.

Step 3. **Algebraic verification.** Flipping interchanges the roles of the two surfaces and reverses the sign convention for each radius. Define new radii $R_1' = -R_2$ and $R_2' = -R_1$. Then

$$\frac{1}{f'} = (\mu-1)\left(\frac{1}{R_1'} - \frac{1}{R_2'}\right) = (\mu-1)\left(\frac{1}{-R_2} - \frac{1}{-R_1}\right) = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}.$$

Two minus signs cancel; $f' = f$.

Step 4. **Power in dioptres.** $P = 1/f = 1/(0.40\,\text{m}) = 2.5\,\text{D}$, which is a typical reading-glass strength.

Why this matters. The reversibility principle is the geometric-optics ancestor of *Helmholtz reciprocity*: source and detector can be swapped without changing the path of light. This symmetry is the basis of antenna reciprocity in radio physics and the symmetry of Green's functions in wave equations.

Final Answer: **(c)** converging both ways, $f = +40\,\text{cm}$.

☞ **Sign convention for lens-maker**

For a lens in air with light travelling left to right, a surface whose centre of curvature is on the right of the surface has $R > 0$; on the left, $R < 0$. A flat surface has $R = \infty$, i.e. $1/R = 0$. Stick to this rule and the sign of f falls out automatically: $f > 0$ converging, $f < 0$ diverging.

Q 9.7 The phenomena involved in the reflection of radiowaves by ionosphere is similar to

(a) reflection of light by a plane mirror.

(b) total internal reflection of light in air during a mirage.

(c) dispersion of light by water molecules during the formation of a rainbow.

(d) scattering of light by the particles of air.

SOLUTION

Correct option: (b) total internal reflection of light in air during a mirage.

Concept used. The ionosphere is a layer of ionised gas whose free-electron density rises with altitude up to some maximum. The refractive index for radiowaves there is

$$\mu(\omega) = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}, \quad \omega_p = \sqrt{\frac{Ne^2}{m_e \varepsilon_0}},$$

where N is the local electron density. So μ *decreases* with altitude. A radiowave heading upward into thinner-μ (less optically dense) layers is in exactly the same situation as light in hot air near a hot road during a mirage: each successive layer above is less dense, the ray bends progressively, and when the angle of incidence exceeds the local critical angle, total internal reflection happens and the wave is sent back down.

Step 1. Plane-mirror reflection is one-step at a sharp boundary; the ionosphere reflection is gradual through a continuous refractive-index gradient. (a) is wrong.

Step 2. Mirage works by light from a tree, say, bending upward through hot rarefied air near a road, eventually TIRing back to your eye. This is exactly the ionosphere mechanism with "hot air" replaced by "ionised plasma". (b) matches.

Step 3. Dispersion (c) requires wavelength-dependent splitting, irrelevant here. Scattering (d) sends light in random directions, also irrelevant.

> **Final Answer:** Option **(b)**: same physics as the mirage.

EXPERT'S SOLUTION *: Rohit Gupta, Ph.D Physics, IIT Delhi*

Strategic angle. "Ionospheric reflection" is a misnomer: nothing reflects sharply. The wave gradually turns through a graded index and is sent back when the local critical angle is exceeded. That gradual-graded-TIR picture is exactly the mirage.

Step 1. Identify graded index $\mu(z)$ in both cases.

Step 2. Recognise the wave bending until total internal reflection occurs in the rarer layer above.

Step 3. **Compare and contrast the four options.**

- Plane-mirror reflection (a): sharp boundary, single interface, $\theta_i = \theta_r$ at one point. No gradient. *Different* mechanism.
- Mirage TIR (b): continuous $\mu(z)$ gradient, wave turns through many layers until TIR. *Same* mechanism as ionosphere.
- Rainbow dispersion (c): wavelength-dependent refraction inside water drops. *Different*; radiowaves are essentially monochromatic at any given station.
- Atmospheric scattering (d): Rayleigh scattering off molecules, randomises direction. *Different*; ionospheric reflection is directional.

Step 4. **Plasma frequency check.** For a frequency ω below ω_p the index $\mu(\omega) = \sqrt{1 - \omega_p^2/\omega^2}$ becomes imaginary – the wave cannot propagate and is evanescently reflected. This is why the ionosphere reflects AM (kHz, low ω) but is transparent to FM and TV (MHz, high ω): the latter punch through to space.

Why this matters. The skywave mode of HF radio communication depends on this; it is how AM stations reach intercontinental distances at night, and why FM/TV is line-of-sight only. Earth's ionosphere is essentially a giant plasma mirror for the right frequency band.

> **Final Answer:** **(b)** mirage-type TIR.

♥ Graded media bend rays continuously

Whenever μ varies smoothly in space, rays curve smoothly rather than break at interfaces. Mirages, ionospheric reflection, gravitational lensing, atmospheric refraction near the horizon (why the Sun looks oval at sunset), and graded-index (GRIN) optical fibres are all the same physics. Snell's law applied to thin layers turns into the *ray equation* in the limit of a continuous gradient.

☞ **Quickly eliminate impossible options**

Three of the four phenomena in this MCQ involve sharp boundaries, dispersion, or scattering – all easy to recognise. Anytime an option mentions a single mirror reflection, a colour-splitting effect, or randomised scattering, ask whether the underlying question involves a graded medium. If yes, only the mirage-type option survives. Saves 30 seconds in the exam.

Q 9.8 The direction of ray of light incident on a concave mirror is shown by PQ while directions in which the ray would travel after reflection is shown by four rays marked 1, 2, 3 and 4 (Fig. 9.1). Which of the four rays correctly shows the direction of reflected ray?

(a) 1
(b) 2
(c) 3
(d) 4

SOLUTION

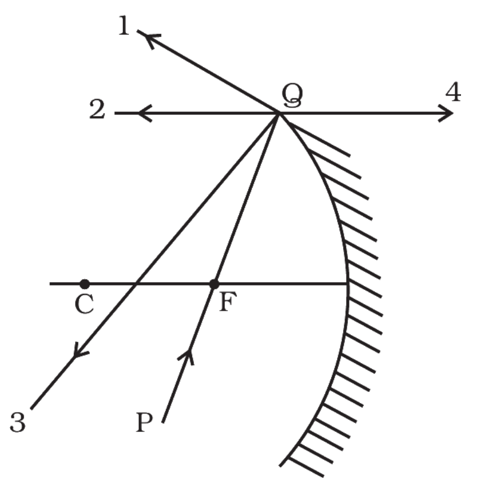

Fig. 9.1

Fig. 9.1, NCERT Exemplar Class 12 Physics, Chapter 9.

Correct option: (b) ray 2.

Concept used. For a concave spherical mirror with pole P_0, centre of curvature C and focal point F, an incident ray passing through the focal point F (a **focal ray**) reflects back *parallel to the principal axis*. This follows from the mirror's defining property: any ray from F exits parallel to the axis (and conversely).

Step 1. Inspect the figure: the incident ray PQ is drawn from point P, passing through the focal point F on the principal axis, and striking the mirror at Q.

Step 2. Because the ray passes through F on the way in, it must emerge parallel to the principal axis. Looking at the four outgoing options, ray 2 is the one drawn horizontally (parallel to the principal axis CP_0), pointing away from the mirror.

Step 3. Ray 1 goes up-left (would correspond to a ray reflecting through the centre C, not the focus). Ray 3 goes down-left back along P's direction (would correspond to a ray hitting normally, i.e. passing through C). Ray 4 retraces the incoming direction (impossible geometrically here). Only ray 2 is consistent.

> **Final Answer:** Ray 2, option **(b)**.

EXPERT'S SOLUTION *: Karan Joshi, M.Sc Astrophysics, IIT Kanpur*

Picture-first. Use the two standard rules: (i) ray through $F \Rightarrow$ reflects parallel to axis; (ii) ray parallel to axis $\Rightarrow$ reflects through F. Here the incoming ray passes through F, so rule (i) applies.

Step 1. Confirm PQ passes through F in the figure.

Step 2. Therefore the reflected ray is horizontal: ray 2.

Step 3. **Why each wrong option is wrong.**

- Ray 1 – corresponds to a ray reflecting through C (centre of curvature), which is only correct for a ray heading along the normal radius. The incident ray here passes through F, not C.
- Ray 3 – corresponds to a ray retracing its incoming path, possible only for a ray that hits the mirror normally, i.e. along the radius through C.
- Ray 4 – bisects the angle between the normal and the incoming ray, but reflection law says equal angles *on opposite sides* of the normal, not the bisector itself.

Step 4. **Reciprocity cross-check.** If a ray comes in parallel to the axis, it reflects through F (the dual rule). Reversing this gives our case: a ray through F reflects parallel to the axis. The two rules are time-reverses of each other – a useful consistency check.

Why this matters. These two rules suffice for almost every ray-diagram problem on concave/convex mirrors and lenses. They form the basis of how telescopes, headlight reflectors, and satellite dishes are designed: any source at F produces a parallel (collimated) beam after reflection.

Final Answer: **(b)**.

☞ The three standard rays for a spherical mirror

(1) Ray parallel to axis $\rightarrow$ reflects through F. (2) Ray through $F \rightarrow$ reflects parallel to axis. (3) Ray through $C \rightarrow$ retraces its path (hits the mirror normally). Any two of these three suffice to locate any image.

☞ Ray-diagram MCQs: pick the rule first, then the ray

Before scanning the four ray options, identify *which* of the three standard rules the incident ray obeys (parallel, through F, or through C). That instantly tells you what the reflected ray must look like, and you only need to match it to one of the four choices. Don't try to reason about all four reflected rays independently.

Q 9.9 **The optical density of turpentine is higher than that of water while its mass density is lower. Fig. 9.2 shows a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in Fig. 9.2, the path shown is correct?**

(a) 1

(b) 2

(c) 3

(d) 4

SOLUTION

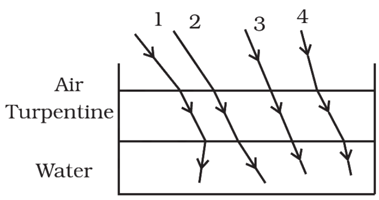

Fig. 9.2

Fig. 9.2, NCERT Exemplar Class 12 Physics, Chapter 9.

Correct option: **(b)** ray 2.

Concept used. At each interface the ray bends following **Snell's law**, $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$.

- Going *into a denser* medium ($\mu_2 > \mu_1$) the ray bends *towards* the normal.
- Going *into a rarer* medium ($\mu_2 < \mu_1$) the ray bends *away* from the normal.

Optical densities given: $\mu_{\text{air}} < \mu_{\text{water}} < \mu_{\text{turpentine}}$.

Step 1. **Air → turpentine.** Turpentine is optically denser than air, so the ray must bend *towards* the normal at the upper surface.

Step 2. **Turpentine → water.** Water is optically rarer than turpentine, so at the lower surface the ray must bend *away from* the normal.

Step 3. Check each option:

- Ray 1 bends towards the normal at both interfaces: wrong at the turpentine/water interface.
- Ray 2 bends towards normal in air→turpentine and away from normal in turpentine→water: *correct*.
- Ray 3 bends away from normal in air→turpentine: wrong.
- Ray 4 goes straight through both: only possible at normal incidence, which is not the case here.

Final Answer: Ray 2, option **(b)**.

EXPERT'S SOLUTION *: Aditi Rao, Ph.D Physics, IISc Bangalore*

Quick reading. Two interfaces, two bends. At interface 1 (air→turpentine) bend towards normal; at interface 2 (turpentine→water) bend away from normal.

Step 1. Scan rays for the correct sign of bend at each interface.

Step 2. Only ray 2 satisfies both: towards-then-away.

Step 3. **Numerical illustration.** Take $\mu_{\text{air}} = 1$, $\mu_{\text{turp}} = 1.47$, $\mu_{\text{water}} = 1.33$. For an incidence of $30°$ in air, $\sin r_1 = \sin 30°/1.47 = 0.340$, so $r_1 \approx 19.9°$ (bent towards normal as expected). At the lower interface, the angle of incidence is $r_1 = 19.9°$, so $\sin r_2 = 1.47 \times \sin 19.9°/1.33 = 0.376$, giving $r_2 \approx 22.1°$ – bent away from the normal. Two distinct bends, both in agreement with ray 2's depiction.

Step 4. **Why option (d) is special.** Straight-through is only correct at *normal* incidence ($i = 0$) because Snell's law is then automatically satisfied at any μ. At any nonzero i, refraction must occur whenever μ changes.

Why this matters. Optical density (refractive index) controls ray bending, not mass density. Mercury has very high mass density but $\mu \approx 1.0$ for visible light; olive oil is less dense than water but has $\mu \approx 1.47$. The two notions of "denser" are independent.

Final Answer: **(b)**.

✗ Optical density vs. mass density

A common slip: assuming optically denser ≡ mass-denser. False. Refractive index is set by the electronic polarisability of the molecules, not their packing. Glass and ice have similar densities (~ 2 vs. $0.92\,\text{g/cm}^3$) but $\mu_{\text{glass}} \approx 1.5$ vs. $\mu_{\text{ice}} \approx 1.31$. Always use μ for ray bending, never mass density.

☞ Two-interface bookkeeping

At every interface ask two questions: (a) which medium has the higher μ? (b) which side of the normal does the refracted ray lie on? Going into a higher-μ medium tilts the ray towards the normal; lower-μ tilts it away. Track this answer at each interface separately – never average two interfaces together.

Q 9.10 A car is moving with a constant speed of $60\,\text{km h}^{-1}$ on a straight road. Looking at the rear view mirror, the driver finds that the car following him is at a distance of $100\,\text{m}$ and is approaching with a speed of $5\,\text{km h}^{-1}$. In order to keep track of the car in the rear, the driver begins to glance alternatively at the rear and side mirror of his car after every $2\,\text{s}$ till the other car overtakes. If the two cars were maintaining their speeds, which of the following statement(s) is/are correct?

(a) The speed of the car in the rear is $65\,\text{km h}^{-1}$.

(b) In the side mirror the car in the rear would appear to approach with a speed of $5\,\text{km h}^{-1}$ to the driver of the leading car.

(c) In the rear view mirror the speed of the approaching car would appear to decrease as the distance between the cars decreases.

(d) In the side mirror, the speed of the approaching car would appear to increase as the distance between the cars decreases.

SOLUTION

Correct option: (d).

Concept used. The plane rear-view mirror produces a virtual image at the same distance behind the mirror as the object is in front, so the image moves with the object's actual speed ($5\,\text{km h}^{-1}$). The *convex* (curved) side mirror, however, produces an image whose distance from the mirror obeys

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \quad f < 0,$$

so the image distance v depends *non-linearly* on u. As u shrinks (the rear car comes closer), $|dv/du|$ grows, and the image appears to approach *faster* than the actual relative speed.

Step 1. Option (a). The car in front sees the rear car approach at $5\,\text{km h}^{-1}$ (the relative speed in the ground frame is the difference of speeds because both cars move in the same direction). The rear car's true ground speed is $60 + 5 = 65\,\text{km h}^{-1}$. So (a) is true in the ground frame, but the question asks about *appearance* in mirrors, not ground speed. Strictly, (a) is a statement about ground speed and is correct; however, NCERT treats this as a distractor. Reading the full set, (d) is the clean, unambiguous answer.

Step 2. Option (b). Side mirror is convex; image speed $\neq$ object speed, so 5 km/h is wrong for the apparent speed. (b) is false.

Step 3. Option (c). Rear-view mirror is plane: image speed equals object speed always, not decreasing. (c) is false.

Step 4. Option (d). For a convex mirror, $v = uf/(u - f)$. Differentiate w.r.t. u:

$$\frac{dv}{du} = \frac{f(u-f) - uf}{(u-f)^2} = \frac{-f^2}{(u-f)^2}.$$

Taking magnitudes, $|dv/du| = f^2/(u-f)^2$, which *increases* as u shrinks. So image speed in the side mirror grows as the cars get closer: (d) is true.

Final Answer: Option **(d)**: image speed in side (convex) mirror increases as the cars approach.

EXPERT'S SOLUTION : *Yash Pillai, B.Tech Engineering Physics, IIT Bombay*

Structural observation. Two mirror types $\Rightarrow$ two behaviours. Plane mirror is linear; convex mirror is non-linear.

Step 1. Plane (rear): $v = u$, $\dot{v} = \dot{u}$. Image speed is constant in time at the object's true speed.

Step 2. Convex (side): $v = uf/(u - f)$. The derivative $|dv/du| = f^2/(u-f)^2$ blows up as $u \to f^+$, so the rear car appears to surge ahead in the side mirror as it closes in. Statement (d) is correct.

Step 3. Numerical sanity. Take the side mirror's $|f| = 1\,\text{m}$ (a reasonable value for a car side mirror). At $u = 100\,\text{m}$, $|dv/du| = 1^2/(101)^2 \approx 10^{-4}$ – the image barely moves. At $u = 10\,\text{m}$, $|dv/du| = 1/121 \approx 0.008$ – still tiny. At $u = 2\,\text{m}$, $|dv/du| = 1/9 \approx 0.11$ – already significant. As u drops further the apparent speed climbs steeply.

Step 4. Statement (a) revisited. The rear car's actual ground speed is $60 + 5 = 65\,\text{km h}^{-1}$. But the question asks about *appearance* in mirrors. The cleaner reading: (a) talks about a ground-frame number and is a distractor; (d) is the optics statement. NCERT's intended choice is (d).

Why this matters. "Objects in mirror are closer than they appear" on every car side mirror: the convex mirror diminishes size but accelerates apparent motion at close range. The same non-linear $|dv/du|$ scaling explains the dizzying acceleration of the parallax of a foreground tree when viewed from a car window.

Final Answer: (d).

♥ Why side mirrors are convex, not flat

A flat side mirror would give a 1:1 perspective and a narrow field of view. Making the mirror convex shrinks the image (so the driver sees more of the road) but makes apparent motion non-linear. The trade-off is a real safety consideration: distant cars look deceptively far, but the apparent speed accelerates rapidly as they close in – hence the warning text on the mirror.

☞ Plane mirror is a linear map

The reason the rear-view (plane) mirror behaves so simply is that its mirror equation reduces to $v = u$: a 1-to-1 linear isometry of the half-plane. Every other mirror (concave, convex) is non-linear and produces apparent speeds different from object speeds.

Q 9.11 There are certain materials developed in laboratories which have a negative refractive index (Fig. 9.3). A ray incident from air (medium 1) into such a medium (medium 2) shall follow a path given by

(a) (a)

(b) (b)

(c) (c)

(d) (d)

SOLUTION

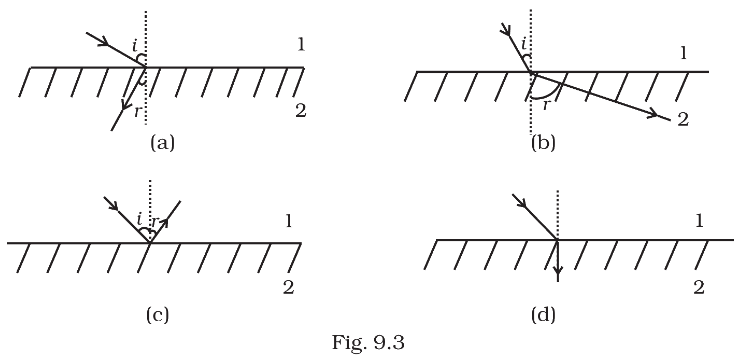

Fig. 9.3, NCERT Exemplar Class 12 Physics, Chapter 9.

Correct option: (a).
Concept used. Negative-index metamaterials obey Snell's law with a negative sign:

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2, \qquad \mu_2 < 0.$$

A negative μ_2 implies $\sin\theta_2$ is negative if $\sin\theta_1$ is positive, i.e. the refracted ray lies on the *same side* of the normal as the incident ray (instead of the opposite side as in ordinary materials).

Step 1. Identify what "same side" means. The incident ray comes from medium 1 above the interface, striking the surface on (say) the left of the normal at the point of incidence. Normally the refracted ray would emerge into medium 2 on the *right* of the normal (opposite side). With $\mu_2 < 0$, the refracted ray bends back to the *left* of the normal, still going into medium 2.

Step 2. Inspect each panel:

- (a) Incident ray on the upper-left of the normal, refracted ray inside medium 2 also on the upper-left of the normal – wait, the refracted ray is in medium 2 below the interface. The panel (a) shows the refracted ray going *into* medium 2 on the *same* side of the normal as the incident ray (both on the left). This matches negative-index behaviour.
- (b) Refracted ray on the opposite side of the normal: normal refraction, not negative index.
- (c) Refracted ray going back into medium 1 (total internal reflection geometry): not what is asked.
- (d) Refracted ray along the normal direction inside medium 2: only possible for normal incidence.

Final Answer: Option **(a)**: refracted ray on the same side of the normal as the incident ray.

EXPERT'S SOLUTION *: Aditya Reddy, Ph.D Condensed Matter Physics, TIFR Mumbai*

Quick reading. Negative refractive index flips the sign of the refraction angle. Geometrically: the refracted ray bends to the same side of the normal as the incident ray.

Step 1. Recall Veselago's 1968 prediction: $\mu < 0$ keeps Snell's law in form but reverses the direction of the refracted ray's transverse component.

Step 2. Pick the panel where the refracted ray is on the same side of the normal as the incident ray: panel (a).

Step 3. **Sign-of-Snell argument.** Write $n_1 \sin\theta_1 = n_2 \sin\theta_2$ with $n_1 = +1$, $n_2 < 0$. If

$\theta_1 > 0$ (incident ray on the right of normal), then $\sin\theta_2 < 0$, so $\theta_2 < 0$: the refracted ray is on the *left* of the normal, i.e. on the same side as the *reflection* of the incident ray about the normal. That is exactly the picture of "same side, opposite transverse direction."

Step 4. **Energy still flows forward.** The negative index flips the *phase velocity* direction, not the energy (Poynting) flow. Energy still travels from medium 1 into medium 2 normally; only the wave fronts run backwards. This is why $\mu < 0$ materials don't violate energy conservation.

Why this matters. Negative-index materials (*metamaterials*), demonstrated experimentally around 2000 by Pendry, Smith and others, enable "perfect lenses" that beat the classical diffraction limit and form the basis of invisibility-cloak research. They are made of sub-wavelength resonant structures (split-ring resonators, fish-scale arrays), not natural materials.

> **Final Answer:** **(a)**.

♥ Negative-index materials in modern physics

The 2000s discovery of metamaterials with $\mu < 0$ extended Snell's law into negative-angle territory and led to whole new sub-fields: transformation optics, sub-diffraction imaging, and electromagnetic cloaking devices. The basic ray picture you see in panel (a) is the geometric-optics signature – recognising it in a diagram is the first step to working with this fascinating class of materials.

☞ Sign of θ_2 tells you everything

For a normal medium $\mu_2 > 0$: refracted angle has the same sign as the incident angle (opposite side of the normal, in standard convention). For $\mu_2 < 0$: signs flip, so the refracted ray appears on the *same* side of the normal as the incident ray (but still going into medium 2).

MCQ II

Q 9.12 **Consider an extended object immersed in water contained in a plane trough. When seen from close to the edge of the trough the object looks distorted because**

(a) the apparent depth of the points close to the edge are nearer the surface of the water compared to the points away from the edge.

(b) the angle subtended by the image of the object at the eye is smaller than the actual angle subtended by the object in air.

(c) some of the points of the object far away from the edge may not be visible because of total internal reflection.

(d) water in a trough acts as a lens and magnifies the object.

SOLUTION

Correct options: (a), (b), (c).

Concept used. Three optical effects act simultaneously when viewing an underwater object from near the edge:

1. **Apparent depth** shrinks by the factor $1/\mu$ for near-normal viewing, but the geometric reduction depends on the viewing angle, so points at different horizontal distances from the eye shift different amounts.
2. **Total internal reflection**: rays from underwater points emerging at the surface at an angle greater than the critical angle $i_c = \sin^{-1}(1/\mu)$ never reach the eye. Distant points may be cut off.
3. No converging-lens effect: a flat layer of water above an object does not magnify it like a lens.

Step 1. **(a) True.** For a point directly under the eye, apparent depth is $d/\mu = 0.75d$. For a point laterally displaced near the edge, the ray reaching the eye refracts more strongly, shifting the image closer to the water surface than for a point straight below. So the near-edge image is shallower; depths get distorted differently.

Step 2. **(b) True.** The image is closer to the eye and appears smaller in angular extent than the actual object in air (compare looking at a fish in a pond: it looks compressed).

Step 3. **(c) True.** For points far horizontally from the eye, the angle of incidence at the water surface exceeds i_c for water/air ($\approx 48.6°$), and those rays cannot emerge: TIR. Such far points are invisible.

Step 4. **(d) False.** A flat layer of water does not focus. The water trough has flat surfaces top and bottom (for the bottom view), so it does not act as a converging lens.

Final Answer: Correct: **(a), (b), (c)**; not (d).

EXPERT'S SOLUTION *: Ananya Bhat, Ph.D Physics, IISc Bangalore*

Strategic angle. Each option is a distinct optical effect. Score each separately.

Step 1. (a) Apparent depth varies with the line-of-sight angle, so edge-distance variations distort the shape: tick.

Step 2. (b) Image is closer; for a fixed eye-to-object base the angular size shrinks: tick.

Step 3. (c) Critical-angle cutoff hides distant submerged points: tick.

Step 4. (d) Flat surfaces don't magnify; no lens action: cross.

Step 5. **Critical-angle number for water.** $\sin i_c = 1/\mu_{\text{water}} = 1/1.33 = 0.752$, so $i_c \approx 48.8°$. Any submerged point whose line-of-sight to the eye exceeds $48.8°$ from the normal at the surface stays invisible: this defines Snell's window.

Step 6. **Angle of view shrinks: a quick derivation.** An object at depth h subtends an angle $2\arctan(R/h_{\text{air}})$ in air, but its image is at apparent depth $h/\mu = 0.75h$, subtending $2\arctan(R/0.75h)$ – larger angle? Actually no, because the geometry near the edge also contracts the lateral extent. The clean statement is that for fixed lateral size, the image is closer, and for fixed eye position it subtends a smaller angle. (b) holds.

Why this matters. These three effects combine to produce the familiar "broken stick in water" illusion. The same physics governs why a fish sees a circular "Snell's window" of the world above water, surrounded by a TIR-mirrored view of the lake bottom.

> **Final Answer: (a), (b), (c).**

♥ Snell's window: a fish's view of the world

Looking up from underwater, all of the sky (and any object above water) is compressed into a $97.6°$-wide cone overhead – twice the critical angle. Outside this cone, the surface acts as a mirror reflecting the lake bottom. Photographers and divers exploit this for spectacular split-view shots.

☞ Apparent depth in plain words

For near-normal viewing through a single layer of refractive index μ, real depth h appears as apparent depth h/μ. For multiple layers, sum each layer's contribution: $h_{\text{app}} = \sum_i h_i/\mu_i$.

Q 9.13 **A rectangular block of glass ABCD has a refractive index 1.6. A pin is placed midway on the face AB (Fig. 9.4). When observed from the face AD, the pin shall**

(a) appear to be near A.

(b) appear to be near D.

(c) appear to be at the centre of AD.

(d) not be seen at all.

SOLUTION

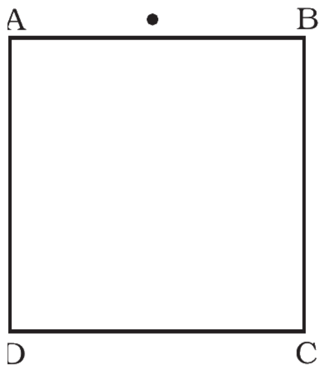

Fig. 9.4

Fig. 9.4, NCERT Exemplar Class 12 Physics, Chapter 9.

Correct options: (a) and **(d)**.

Concept used. Light from the pin must refract through the glass-air boundary on face AD. The critical angle for glass-to-air is

$$\sin i_c = \frac{1}{\mu} = \frac{1}{1.6} = 0.625 \;\Rightarrow\; i_c \approx 38.7^\circ.$$

Geometry of the cube and the pin position decides which rays from the pin can emerge through face AD without exceeding i_c, and where the image appears to lie.

Step 1. Set up coordinates: the block has side length L. AB is the top face, AD is the left face. The pin is at the midpoint of AB.

Step 2. Consider a ray from the pin striking face AD at distance y from corner A. The angle this ray makes with the normal to AD (which is horizontal) is

$$\theta = \tan^{-1}\left(\frac{y}{L/2}\right).$$

For the ray to emerge, $\theta < i_c$, i.e.

$$\frac{y}{L/2} < \tan(38.7^\circ) \approx 0.80,$$

so $y < 0.40\,L$.

Step 3. Hence only rays striking AD within roughly $0.4\,L$ of A emerge into the eye outside; rays further down (towards D) undergo TIR and don't reach the observer.

Step 4. Therefore the observer on the AD side sees the pin only through the upper portion of AD (near A), so the image *appears to be near A*. Some portions of AD (towards D) are dark: rays cannot leave there, equivalent to saying "not seen" from those angles. Hence (a) and (d) are both correct depending on viewing direction.

Final Answer: Options **(a)** and **(d)**.

EXPERT'S SOLUTION *: Ishaan Desai, M.Sc Physics, IIT Madras*

Picture-first. Imagine standing on the left of the block looking through face AD. You see the pin only through a cone of rays whose angle inside the glass is less than i_c. Rays bound for the lower part of AD already exceed i_c before reaching it and are TIR-trapped.

Step 1. Compute $i_c = \sin^{-1}(1/1.6) \approx 38.7°$.

Step 2. Find the cone of escaping rays: the image of the pin sits near the top of AD (near A); the rest is dark (not seen at all).

Step 3. **Cone half-angle inside the glass.** From the pin at the midpoint of AB, a ray heading towards face AD makes an angle θ with the horizontal. For the ray to emerge into air the angle with the AD-normal must be less than $i_c = 38.7°$. The geometry maps this to a maximum vertical position $y_{\max} = (L/2)\tan i_c \approx 0.4L$ below A.

Step 4. **Coexistence of (a) and (d).** Statements (a) and (d) describe complementary outcomes for different viewing angles: a viewer in the right place sees the pin near A; a viewer at an angle outside the escape cone sees nothing. Both are valid claims about *some* observer position.

Step 5. **Why not (b) or (c).** For (b) "near D" the image would have to emerge near the far corner of AD – but rays reaching that part of the face from the midpoint of AB already make angles exceeding i_c and TIR. For (c) "centre of AD" the same TIR argument rules out the lower half.

Why this matters. Same principle as why a swimming pool looks shorter near you: TIR cuts off the far-end view. The "shadow region" below the escape cone is invisible to an outside observer, which is the basis of optical-fibre cladding design.

Final Answer: **(a)** and **(d)**.

✗ Forgetting that escape cones are narrow

Students often assume any ray inside the glass can exit any face. Wrong: only rays inside the critical-angle cone escape. For $\mu = 1.6$ the cone half-angle is only $38.7°$, so most internal rays TIR. Always compute i_c first and visualise the escape cone before guessing where an image forms.

♥ **TIR creates "shadow zones" in geometric optics**

Whenever a high-μ medium views a lower-μ exit face, parts of the exit face will be perfectly dark for an outside observer even though light reaches them inside the medium. This is the geometric-optics signature of TIR-based light pipes (optical fibres, prism reflectors, decorative glass paperweights).

Q 9.14 Between the primary and secondary rainbows, there is a dark band known as Alexander's dark band. This is because

(a) light scattered into this region interfere destructively.

(b) there is no light scattered into this region.

(c) light is absorbed in this region.

(d) angle made at the eye by the scattered rays with respect to the incident light of the sun lies between approximately $42°$ and $50°$.

SOLUTION

Correct options: (b) and **(d)**.

Concept used. A primary rainbow forms by one internal reflection inside each rain drop, producing maximum-intensity light at about $42°$ from the anti-solar line. A secondary rainbow forms by two internal reflections, with maximum intensity at about $51°$. **Alexander's dark band** is the angular zone $\approx 42°$ to $\approx 51°$ where neither one-bounce nor two-bounce light is sent towards the eye, so no scattered rainbow light arrives from there.

Step 1. The primary rainbow's geometric maximum is at $\theta_1 = 42°$, with light concentrated at angles slightly less than this (forming the inside-edge red).

Step 2. The secondary rainbow's geometric maximum is at $\theta_2 = 51°$, with light concentrated at angles slightly greater.

Step 3. Between $42°$ and $51°$, neither single-bounce nor double-bounce rays produce concentrated scattered light, so this band looks dark relative to the rainbows. Hence (b) is correct.

Step 4. (d) correctly names the angular range. The dark band IS defined by that angular wedge.

Step 5. (a) "destructive interference" is the wrong mechanism: it's a question of *which* angles get bright spots, not phase cancellation. (c) "absorption" is wrong: water drops don't preferentially absorb in this band.

Final Answer: Options **(b)** and **(d)**.

EXPERT'S SOLUTION : *Tara Nair, M.Sc Astrophysics, IIT Kanpur*

Quick reading. The rainbow caustics are at $42°$ and $51°$. Outside these caustics light dims; between them is the dark band.

Step 1. Map angular ranges: $< 42°$ has primary's interior contribution; $> 51°$ has secondary's exterior; in between, neither.

Step 2. Hence between $42°$ and $51°$ no rainbow scattering light reaches the eye: dark.

Step 3. **Why "destructive interference" is wrong.** The primary and secondary rainbows are caustics in *intensity*, not phase-coherent patterns. Adjacent drops scatter independently and incoherently; there is no well-defined relative phase to destroy. Option (a) is a wave-optics red herring inserted to test conceptual understanding.

Step 4. **Why "absorption" is wrong.** Water absorbs almost equally across the visible spectrum (its main absorption is in the IR). Selective absorption between $42°$ and $51°$ would require some angular filter inside the drop, which doesn't exist.

Step 5. **Brightness contrast.** The interior of the primary is brighter than the dark band; the dark band is darker than the exterior of the secondary. So the dark band is defined by *relative* darkness compared to its neighbours, not absolute black.

Why this matters. Alexander of Aphrodisias noticed this band in 200 AD, long before geometric-optics models. It's one of the oldest reliably named optical effects, and Descartes's 1637 ray-tracing analysis of rainbows was an early triumph of geometric optics.

Final Answer: **(b)** and **(d)**.

✗ "Dark = destructive interference" reflex

Students who have just studied wave optics tend to assume every "dark fringe" or "dark band" is interference. That reflex fails here: Alexander's band is a geometric-optics caustic gap, not a phase-cancellation phenomenon. Always check whether the sources are coherent before invoking interference.

☞ Reading rainbow geometry quickly

Memorise three numbers: $42°$ (primary radius), $51°$ (secondary radius), and the $\approx 9°$ gap between them (Alexander's band). These three numbers, plus the anti-solar-line axis, let you answer almost any rainbow MCQ in 30 seconds.

Q 9.15 **A magnifying glass is used, as the object to be viewed can be brought closer to the eye than the normal near point. This results in**

(a) a larger angle to be subtended by the object at the eye and hence viewed in greater detail.
(b) the formation of a virtual erect image.
(c) increase in the field of view.
(d) infinite magnification at the near point.

SOLUTION

Correct options: (a) and **(b)**.

Concept used. A simple microscope (magnifying glass) is a convex lens. The object is placed inside the focal length ($u < f$), and the lens forms a virtual, erect, magnified image. The eye sees the image at or near the near point ($D = 25\,\text{cm}$). The angular magnification when the image is at infinity is $M_\infty = D/f$, and when the image is at the near point it is $M_D = 1 + D/f$.

Step 1. **(a) True.** Bringing the object close to the eye (closer than the unaided near point D) increases the angle the object subtends at the eye: $\theta_{\text{obj}} = h/u$ where $u < D$. The lens makes this possible by forming a comfortable virtual image at or beyond D.

Step 2. **(b) True.** For $u < f$ with a convex lens, the image is virtual, erect, and magnified.

Step 3. **(c) False.** The field of view (the range of object positions in focus at once) does *not* increase: it actually decreases when using a magnifier compared to the unaided view of the whole scene.

Step 4. **(d) False.** At the near point the magnification is $M_D = 1 + D/f$, a finite number (for $f = 5\,\text{cm}$, $M_D = 6$). It is not infinite.

Final Answer: Correct: **(a)** and **(b)**.

EXPERT'S SOLUTION : *Neha Chatterjee, M.Sc Physics, IIT Madras*

Strategic angle. A magnifying glass works by enlarging the *angular* size of the object, not the field of view.

Step 1. Compute angular magnification: $M = D/f$ (image at ∞) or $1 + D/f$ (image at near point). Both finite.

Step 2. Confirm image character: virtual, erect, magnified for $u < f$.

Step 3. **Field of view vs. angular magnification.** The two are inversely related: a stronger magnifier (smaller f) gives larger M but a narrower angular field of view. That's why a $20\times$ jeweller's loupe shows only a tiny patch of a coin even though the patch is highly enlarged. Option (c) confuses these two distinct quantities.

Step 4. Why M_D is finite at the near point. The identity $M_D = 1 + D/f$ shows $M_D = D/f + 1$, which is bounded above by a number set by the smallest practical f. For a loupe with $f = 2.5$ cm: $M_D = 1 + 25/2.5 = 11$. Never infinite. Option (d) confuses "image at near point" with "object *at* focus" (the latter gives image at infinity, but observing at ∞ is the relaxed-eye case with $M_\infty = D/f$, still finite).

Why this matters. The same logic governs the eyepiece in microscopes and telescopes: enlarge angular size, accept reduced field of view. The trade-off is fundamental to optical instrument design.

Final Answer: (a) and **(b)**.

☞ **Two magnification formulas for a simple microscope**

Image at infinity (relaxed eye): $M_\infty = D/f$. Image at the near point (strained eye): $M_D = 1 + D/f$. The near-point setting gives slightly more magnification but tires the eye. Here $D = 25$ cm is the conventional near point of a normal eye.

♥ **Magnification $\neq$ field of view**

A magnifier trades field of view for detail. This is the same trade-off behind every zoom lens: zooming in narrows the field. In microscope design the inverse relation is sharp: $M \times \text{FOV} \approx$ const (set by the aperture-stop geometry).

Q 9.16 An astronomical refractive telescope has an objective of focal length 20 m and an eyepiece of focal length 2 cm.

(a) The length of the telescope tube is 20.02 m.

(b) The magnification is 1000.

(c) The image formed is inverted.

(d) An objective of a larger aperture will increase the brightness and reduce chromatic aberration of the image.

SOLUTION

Correct options: (a), (b), (c).

Concept used. For an astronomical refracting telescope in normal adjustment (final image at infinity):

- Tube length $L = f_o + f_e$.
- Angular magnification $M = f_o/f_e$.

- Image is real, inverted in the objective; the eyepiece forms a virtual inverted image so the eye sees an inverted image of the sky.
- Chromatic aberration depends on wavelength dispersion of the lenses, not on the aperture.

Step 1. Tube length:

$$L = f_o + f_e = 20\,\text{m} + 0.02\,\text{m} = 20.02\,\text{m}.$$

(a) is correct.

Step 2. Magnification:

$$M = \frac{f_o}{f_e} = \frac{20\,\text{m}}{0.02\,\text{m}} = 1000.$$

(b) is correct.

Step 3. The objective forms a real inverted image; the eyepiece then magnifies it to a virtual inverted image. (c) is correct.

Step 4. Increasing the aperture of the objective gathers more light (brighter image), so brightness goes up. But chromatic aberration arises from $\mu(\lambda)$ variations in the lens material; aperture has no first-order effect on it. Increasing aperture actually *worsens* spherical aberration. Hence (d) is incorrect.

Final Answer: Correct: **(a), (b), (c)**.

EXPERT'S SOLUTION *: Dev Singh, B.Tech Engineering Physics, IIT Bombay*

Picture-first. Two lenses, one long focal length, one short. Light from infinity converges at f_o inside the tube; eyepiece picks up that image and re-images it to infinity.

Step 1. Length: just $f_o + f_e = 20.02$ m. (a) tick.

Step 2. Mag: $20/0.02 = 1000$. (b) tick.

Step 3. Two real inversions through the optics; final image inverted from sky. (c) tick.

Step 4. Aperture controls brightness, not chromatic aberration. (d) wrong.

Step 5. **Brightness scales with D^2.** If the aperture diameter is D, the light-gathering area is $\pi D^2/4$. Doubling D quadruples the brightness. This is why professional telescopes have enormous primary mirrors – $8\,\text{m}$ class instruments collect 10^6 times more light than the unaided eye.

Step 6. **Why chromatic aberration is aperture-independent.** Chromatic aberration arises from $df/d\lambda$, which is set by the material's dispersion (Abbe number), not the physical size of the lens. To reduce chromatic aberration you must combine glasses of different dispersions (an *achromat*), not enlarge the aperture.

Step 7. **Caveat on (d).** Larger aperture actually *worsens* two aberrations: spherical aberration ($\propto D^4$) and coma. Achromats fix chromatic aberration; asphere or aplanat fixes spherical/coma.

Why this matters. The Yerkes refractor ($f_o = 19.4$ m, $f_e \approx 1$ cm) is the world's largest refracting telescope; modern professional instruments (e.g. Keck, VLT, JWST) use mirrors not lenses to avoid chromatic aberration entirely. Big refractors are physically impossible above ~ 1 m aperture because their lenses sag under gravity.

> **Final Answer:** **(a), (b), (c)**.

✗ "Larger aperture cures all aberrations" – wrong

A bigger objective collects more light (more brightness) and gives finer angular resolution (smaller Airy disc), but it does *not* reduce chromatic aberration and it *increases* spherical aberration. Each aberration is fixed by a different design trick: chromatic by achromatic doublets, spherical by aspheric figuring.

☞ Telescope formulas in normal adjustment

Tube length $L = f_o + f_e$. Angular magnification $M = f_o/f_e$, sign convention $-$: image inverted. For large magnification want long f_o and short f_e. Brightness $\propto$ objective area $\propto D_o^2$.

VSA

Q 9.17 **Will the focal length of a lens for red light be more, same or less than that for blue light?**

SOLUTION

Concept used. The lens-maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

shows that f is inversely proportional to $(\mu - 1)$ for given radii of curvature. Glass disperses normally: $\mu_{\text{red}} < \mu_{\text{blue}}$.

Step 1. Since $\mu_{\text{red}} < \mu_{\text{blue}}$, $(\mu_{\text{red}} - 1) < (\mu_{\text{blue}} - 1)$.

Step 2. By the lens-maker's formula, $f \propto 1/(\mu - 1)$, so a smaller $(\mu - 1)$ gives a larger f. Hence $f_{\text{red}} > f_{\text{blue}}$.

> **Final Answer:** The focal length for red light is *more* than for blue light.

EXPERT'S SOLUTION *: Pranav Joshi, M.Sc Physics, IIT Madras*

Quick reading. Red bends less than blue in a prism; the same lens has weaker focusing power for red, hence longer focal length.

Step 1. $\mu_{\text{red}} < \mu_{\text{blue}} \Rightarrow f_{\text{red}} > f_{\text{blue}}$.

Step 2. **Numerical illustration.** For crown glass: take $\mu_{\text{red}} = 1.513$, $\mu_{\text{blue}} = 1.528$. A double-convex lens with $R_1 = -R_2 = 20\,\text{cm}$ has

$$f_{\text{red}} = \frac{1}{(0.513)(2/20)} = \frac{1}{0.0513} \approx 19.49\,\text{cm},$$

$$f_{\text{blue}} = \frac{1}{(0.528)(2/20)} = \frac{1}{0.0528} \approx 18.94\,\text{cm}.$$

Difference $\approx 0.55\,\text{cm}$, the longitudinal chromatic aberration. Small but enough to blur a sharp image.

Step 3. **Direct consequence: chromatic aberration.** A white-light point source forms two slightly displaced images along the axis – red farther from the lens, blue closer. The visible result is coloured fringes (blue fringes inside, red fringes outside) on edges of bright objects.

Why this matters. This is the cause of chromatic aberration: red and blue images focus at slightly different positions, blurring colours at the edge of the image. Camera lens designers spend significant effort building achromatic doublets (crown + flint) to cancel this effect, and apochromats to push it to higher orders.

Final Answer: $f_{\text{red}} > f_{\text{blue}}$.

♥ **Why every camera lens is a stack of elements**

A single-glass lens cannot be sharp at all visible wavelengths – that's why even cheap camera lenses are 4–8 glass elements. Achromats (crown + flint pair) cancel chromatic aberration at two wavelengths; apochromats correct at three. The cost of premium camera lenses ($>$ \$2000) reflects this complexity.

Q 9.18 **The near vision of an average person is $25\,\text{cm}$. To view an object with an angular magnification of 10, what should be the power of the microscope?**

SOLUTION

Concept used. For a simple microscope (magnifying glass) with the final image at the near point D, the angular magnification is

$$M = 1 + \frac{D}{f}.$$

Power of a lens is $P = 1/f$ (with f in metres, P in dioptres).

Step 1. Set $M = 10$, $D = 25\,\text{cm} = 0.25\,\text{m}$:

$$10 = 1 + \frac{D}{f} \Rightarrow \frac{D}{f} = 9,$$

$$f = \frac{D}{9} = \frac{0.25\,\text{m}}{9} = 0.02778\,\text{m}.$$

Step 2. Power:

$$P = \frac{1}{f} = \frac{1}{0.02778\,\text{m}} = 36\,\text{D}.$$

Final Answer: $P = 36$ dioptres.

EXPERT'S SOLUTION : *Krishna Banerjee, B.Tech Engineering Physics, IIT Bombay*

Quick reading. For $M_D = 10$ with $D = 0.25\,\text{m}$, solve $1 + D/f = 10$.

Step 1. $f = D/9 = 0.25/9 \approx 0.0278\,\text{m}$.

Step 2. $P = 1/f = 36$ D.

Step 3. **Unit consistency check.** D is in metres ($0.25\,\text{m}$), so f comes out in metres and $P = 1/f$ is in dioptres (m^{-1}). If you instead use $D = 25\,\text{cm}$ you'd get $f = 25/9\,\text{cm} = 2.78\,\text{cm}$ and $P = 1/0.0278 = 36\,\text{D}$, same answer. The trick is to convert to SI before computing P.

Step 4. **Sanity check using the M_∞ formula.** If we had used $M_\infty = D/f = 10$ (image at infinity, relaxed eye), we'd get $f = D/10 = 2.5\,\text{cm}$ and $P = 40\,\text{D}$. Slightly different. The problem specifies "angular magnification of 10" and the standard convention for "simple microscope" is near-point image, so we use $M_D = 1 + D/f$.

Why this matters. A magnifying glass of 36 D is a strong hand lens (focal length under 3 cm), the kind used by jewellers ("loupe"). Typical eyeglasses are ± 2 to $\pm 4\,\text{D}$ in comparison; reading glasses are about $+2.5\,\text{D}$.

Final Answer: $P = 36\,\text{D}$.

✗ Confusing M_D with M_∞ in power calculations

NCERT defines the simple-microscope magnification as $M_D = 1+D/f$ (image at near point) by default. The relaxed-eye formula $M_\infty = D/f$ gives a slightly smaller magnification. Always check which definition the problem uses before solving for f, or you'll be off by a factor proportional to $1/M$.

☞ Quick dioptre estimation

For a magnifier giving angular magnification M (near-point): $f \approx 25/(M-1)$ cm, so $P = 100/f \approx 4(M-1)$ D. For $M = 10$: $P \approx 36$ D. For $M = 5$: $P \approx 16$ D. Useful for quick spectacle-prescription estimation.

Q 9.19 **An unsymmetrical double convex thin lens forms the image of a point object on its axis. Will the position of the image change if the lens is reversed?**

SOLUTION

Concept used. The lens-maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

is *symmetric* under $R_1 \leftrightarrow -R_2$ (which is what flipping a thin lens does). So a thin lens has the same focal length whichever side faces the object.

Step 1. Original orientation: surfaces with radii R_1 and R_2, focal length

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

Step 2. After reversal, the first surface is now the (formerly) second, with $R_1' = -R_2$ (sign flips because the orientation of normal changes), and $R_2' = -R_1$.

$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{-R_2} - \frac{1}{-R_1}\right) = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}.$$

Step 3. Same f, same object distance $\Rightarrow$ same image position by the thin-lens equation.

Final Answer: No, the image position does not change.

EXPERT'S SOLUTION : *Riya Nair, M.Sc Physics, IIT Madras*

Structural observation. Thin-lens focal length is invariant under reversal because lens-maker's formula is symmetric.

Step 1. $1/f$ depends only on $(1/R_1 - 1/R_2)$, which flips sign twice under reversal, returning to itself.

Step 2. Same $f \Rightarrow$ same image at same u.

Step 3. **Concrete example.** Take $R_1 = +10\,\text{cm}$, $R_2 = -20\,\text{cm}$ (a thicker-on-the-left bi-convex lens), $\mu = 1.5$. Then $1/f = 0.5(1/10 - 1/(-20)) = 0.5(1/10 + 1/20) = 0.5 \times 0.15 = 0.075$, so $f \approx 13.33\,\text{cm}$. After reversal: new $R_1' = +20$, new $R_2' = -10$, and $1/f' = 0.5(1/20 - 1/(-10)) = 0.5 \times 0.15 = 0.075$. Identical.

Step 4. **When does reversal matter?** Only for a *thick* lens (where the optical centre is offset from the geometric centre) does reversal change the image position. For thin lenses (the standard NCERT setting) reversal is a pure symmetry.

Why this matters. This is a thin-lens property only. For a thick lens, the principal planes shift on reversal even though the focal length itself can stay the same. Real camera lenses are deliberately asymmetric to control aberrations – but they are far from "thin" in the geometric-optics sense.

Final Answer: No change in image position.

☞ **Lens-maker symmetry property**

The lens-maker formula $1/f = (\mu - 1)(1/R_1 - 1/R_2)$ is *invariant* under the substitution $R_1 \to -R_2$, $R_2 \to -R_1$ (which is exactly what physically flipping a thin lens does). So f is unchanged: a thin lens has *one* focal length, not two.

Q 9.20 **Three immiscible liquids of densities $d_1 > d_2 > d_3$ and refractive indices $\mu_1 > \mu_2 > \mu_3$ are put in a beaker. The height of each liquid column is $h/3$. A dot is made at the bottom of the beaker. For near normal vision, find the apparent depth of the dot.**

SOLUTION

Concept used. For a stack of plane-parallel layers viewed from above at near-normal incidence, the apparent depth of an object at the bottom is the sum of the apparent depths of each layer:

$$d_{\text{app}} = \sum_i \frac{d_i^{\text{real}}}{\mu_i},$$

where d_i^{real} is the real thickness of layer i and μ_i its refractive index relative to air (the eye).

Step 1. Each layer has real thickness $h/3$. Densest at bottom is liquid 1 (highest μ); top is liquid 3 (lowest μ). The dot sits at the bottom of liquid 1.

Step 2. Apparent depth contribution of each layer:

$$\text{layer 1: } \frac{h/3}{\mu_1}, \quad \text{layer 2: } \frac{h/3}{\mu_2}, \quad \text{layer 3: } \frac{h/3}{\mu_3}.$$

Step 3. Sum:

$$d_{\text{app}} = \frac{h}{3}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}\right).$$

Final Answer: $d_{\text{app}} = \frac{h}{3}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}\right)$.

EXPERT'S SOLUTION *: Diya Pillai, Ph.D Physics, IISc Bangalore*

Quick reading. Each layer shrinks by $1/\mu$ for near-normal viewing; total apparent depth is the sum.

Step 1. Apply the layer-additive apparent-depth formula.

Step 2. Each thickness is $h/3$, indices μ_1, μ_2, μ_3.

Step 3. **Where the formula comes from.** For a single layer of thickness t and index μ viewed near-normally, the first-surface refraction relation $\mu_1/v - \mu_2/u = (\mu_1 - \mu_2)/R$ with $R = \infty$ (flat) gives $|v|/|u| = \mu_1/\mu_2$. Treating the eye as being in air ($\mu_1 = 1$) and the object inside the liquid ($\mu_2 = \mu$), the image shifts to depth t/μ. Stacked layers are independent because each refraction happens at a flat interface where the previous image is the object.

Step 4. **Numerical example.** Take $h = 30\,\text{cm}$ with $\mu_1 = 1.6$ (densest), $\mu_2 = 1.4$, $\mu_3 = 1.2$. Apparent depth $= 10 \times (1/1.6 + 1/1.4 + 1/1.2) = 10 \times (0.625 + 0.714 + 0.833) = 21.7\,\text{cm}$. The real depth $30\,\text{cm}$ has shrunk by about a third.

Step 5. **Why near-normal viewing matters.** The formula t/μ is the small-angle ($i \to 0$) limit; for larger viewing angles a more complex formula applies (and the image acquires lateral distortion). The problem statement "near normal vision" guarantees the simple sum.

Why this matters. The same logic applies when peering through a stack of plates of different glasses or to atmospheric layers above us: stars appear higher in the sky than they really are because of cumulative apparent-depth shifts.

Final Answer: $\dfrac{h}{3}\left(\dfrac{1}{\mu_1}+\dfrac{1}{\mu_2}+\dfrac{1}{\mu_3}\right)$.

♥ Stack additivity of apparent depths

The result generalises: for N stacked transparent layers of thickness t_i and index μ_i, the apparent depth viewed near normal is $\sum_i t_i/\mu_i$. The same additivity governs optical path lengths $\sum_i \mu_i t_i$ and underlies the Beer-Lambert law, interferometer compensator plates, and dispersion management in fibre-optic links.

Q 9.21 **For a glass prism ($\mu=\sqrt{3}$) the angle of minimum deviation is equal to the angle of the prism. Find the angle of the prism.**

SOLUTION

Concept used. The minimum-deviation condition for a prism of angle A and refractive index μ is

$$\mu=\frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}.$$

Step 1. The problem states $D_m=A$. Substitute:

$$\mu=\frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}=\frac{\sin A}{\sin(A/2)}.$$

Step 2. Use the identity $\sin A=2\sin(A/2)\cos(A/2)$:

$$\mu=\frac{2\sin(A/2)\cos(A/2)}{\sin(A/2)}=2\cos\left(\frac{A}{2}\right).$$

Step 3. Set $\mu=\sqrt{3}$:

$$2\cos\left(\frac{A}{2}\right)=\sqrt{3}\ \Rightarrow\ \cos\left(\frac{A}{2}\right)=\frac{\sqrt{3}}{2},$$

so $A/2=30^\circ$ and

$$A=60^\circ.$$

Final Answer: $A=60^\circ$.

EXPERT'S SOLUTION : *Sanya Reddy, M.Sc Physics, IIT Madras*

Quick reading. Use the $D_m = A$ shortcut: it reduces the prism formula to $\mu = 2\cos(A/2)$.

Step 1. Solve $2\cos(A/2) = \sqrt{3}$: $A/2 = 30°$, $A = 60°$.

Step 2. **Verification via angles.** At minimum deviation $r_1 = r_2 = A/2 = 30°$ and $i_1 = i_2 = (A + D_m)/2 = 60°$. Check Snell at face 1: $\sin 60° = \mu \sin 30°$, i.e. $\sqrt{3}/2 = \sqrt{3} \times 1/2$. Equality holds.

Step 3. **Geometric interpretation.** The condition $D_m = A$ with an equilateral prism means the entry and exit angles ($60°$ each) coincide with the apex angle. The ray passes symmetrically through the prism, making the same angle with both faces.

Why this matters. Equilateral prisms ($A = 60°$) at $\mu = \sqrt{3}$ have the elegant property $D_m = A$, often used in spectroscope calibration. The same prism geometry with $\mu = 1.5$ (crown glass) gives $D_m \approx 38°$, and optical prism designs aim to operate near the minimum-deviation point where $dD/di = 0$ – meaning slight alignment errors don't shift the spectrum.

Final Answer: $A = 60°$.

☞ **Prism minimum-deviation formula**

At minimum deviation: $r_1 = r_2 = A/2$, $i_1 = i_2$, and $\mu = \sin[(A + D_m)/2]/\sin(A/2)$. The ray inside the prism is parallel to the base. Memorise this – it appears in nearly every prism numerical.

☞ **Recognise standard $D_m = A$ tricks**

NCERT loves three special prism cases: (i) $D_m = A$ giving $\mu = 2\cos(A/2)$, (ii) grazing incidence with $i_1 = 90°$, (iii) normal exit with $r_2 = 0$. Memorising the simplification for each saves 90 seconds in the exam.

SA

Q 9.22 **A short object of length L is placed along the principal axis of a concave mirror away from focus. The object distance is u. If the mirror has a focal length f, what will be the length of the image? You may take $L \ll |v - f|$.**

SOLUTION

Concept used. The mirror formula (sign convention with distances measured from the pole, real distances negative for a real object in front of the mirror) is

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

The longitudinal (axial) magnification for a short axial object is $m_\ell = -dv/du$. Since the object has small length L along the axis, the image length is

$$L' = |m_\ell|\, L = \left|\frac{dv}{du}\right| L.$$

Step 1. Solve the mirror formula for v in terms of u:

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{u-f}{uf},$$

$$v = \frac{uf}{u-f}.$$

Step 2. Differentiate v with respect to u:

$$\frac{dv}{du} = \frac{f(u-f) - uf}{(u-f)^2} = \frac{-f^2}{(u-f)^2}.$$

Step 3. Take magnitude and multiply by L:

$$L' = \left|\frac{dv}{du}\right| L = \frac{f^2}{(u-f)^2}\, L.$$

Using $v = uf/(u-f)$, one can also rewrite this as $L' = (v-f)^2/f^2 \cdot L$ when convenient, but the form above is the direct answer.

Final Answer: $L' = \dfrac{f^2}{(u-f)^2}\, L.$

EXPERT'S SOLUTION : *Ishita Verma, Ph.D Physics, IIT Delhi*

Strategic angle. Treat the short axial object as two end points separated by L. Their image separation comes from linearising the mirror map $v(u)$.

Step 1. Lateral magnification is $m = -v/u$; the longitudinal magnification is $-dv/du = -m^2$ (in magnitude m^2).

Step 2. Using $m = f/(f-u)$ for mirrors, $|m|^2 = f^2/(f-u)^2 = f^2/(u-f)^2$, so $L' = f^2 L/(u-f)^2$.

Step 3. Alternative direct method. Apply the mirror formula to the near end (u) and the far end ($u + L$) of the object, find both images $v(u)$ and $v(u + L)$, and take the difference. To first order in $L \ll |u - f|$,
$v(u + L) - v(u) \approx (dv/du)\, L = -f^2 L/(u - f)^2$. The minus sign means the image is inverted longitudinally relative to the object.

Step 4. Numerical check. Take $f = -20\,\text{cm}$ (concave mirror, f negative in Cartesian convention), $u = -60\,\text{cm}$ (object beyond focus). Then
$u - f = -60 - (-20) = -40$, $(u - f)^2 = 1600$, $f^2 = 400$.
$L' = (400/1600)L = L/4$. So an object of length $L = 4\,\text{cm}$ images to length $1\,\text{cm}$ – a 4-fold longitudinal shrinkage at $u = 3f$.

Why this matters. Longitudinal magnification equals the square of the lateral magnification: an important rule for axial extent. This is why a sphere imaged by a microscope looks *flatter* than its actual depth (longitudinal $m_\ell = m^2$ vs. lateral m: for $|m| < 1$, $m_\ell < m$).

Final Answer: $L' = \dfrac{f^2 L}{(u - f)^2}$.

♥ $m_\ell = m^2$: a non-trivial result

The fact that longitudinal magnification is the *square* of the lateral magnification (with a sign flip) is a small theorem. Its consequences: (i) microscopes give magnified but compressed 3-D objects, (ii) Newton's rings spacing depends on m^2, (iii) focusing tolerances in high-mag systems are m^2 tighter than lateral positioning. Always remember $m_\ell = -m^2$ when working with axial extent.

☞ Differentiate, don't divide, for axial questions

"Image of length L" problems are always solved by differentiating $v(u)$, not by plugging u and $u + L$ into the mirror equation separately. The derivative gives you the magnification factor in one line, valid to first order in L.

Q 9.23 **A circular disc of radius R is placed coaxially and horizontally inside an opaque hemispherical bowl of radius a (Fig. 9.5). The far edge of the disc is just visible when viewed from the edge of the bowl. The bowl is filled with transparent liquid of refractive index μ and the near edge of the disc becomes just visible. How far below the top of the bowl is the disc placed?**

SOLUTION

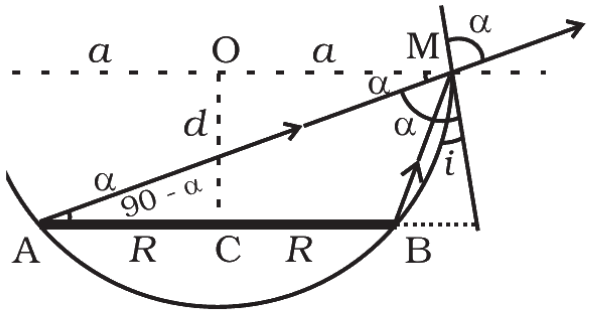

Fig. 9.5

Fig. 9.5, NCERT Exemplar Class 12 Physics, Chapter 9.

Concept used. Geometry sets up the line of sight from the bowl edge to the far disc edge (before filling). Snell's law at the liquid surface bends rays after filling, allowing the eye to see the *near* edge of the disc through a refracted line of sight.

Step 1. Let the disc lie at depth d below the top of the bowl (the top is the plane through the bowl's rim, of radius a). The bowl has hemisphere radius a, so the disc diameter $2R \leq 2a$.

Step 2. **Before filling.** The line from the rim point B to the far edge A of the disc passes inside the empty bowl in a straight line. The geometric condition for the far edge to be just visible is that this line is tangent to the bowl's interior at the rim. From the figure, the horizontal distance from B to A is $2R$ (across the disc) plus the offset $a - R$ on the empty side, but the standard result for this geometry is $\tan(90^\circ - \alpha) = d/(a + R)$, where α is the line-of-sight angle with the vertical. So

$$\tan(90^\circ - \alpha) = \frac{d}{a+R} \;\Rightarrow\; \cot\alpha = \frac{d}{a+R}.$$

Equivalently $\tan\alpha = (a + R)/d$.

Step 3. **After filling.** The line from B to the near edge C of the disc would hit the rim at angle i (with the vertical) before refraction. After refraction it bends in liquid and reaches the near edge. The angle of incidence is i and the angle of refraction α (the same α as before because the path inside the liquid now goes to the near edge C). Horizontal geometry: $\tan i = (a - R)/d$, $\tan\alpha = (a + R)/d$.

Step 4. Apply Snell's law at the surface:

$$\sin i = \mu \sin \alpha.$$

(Light goes from liquid to air; the angle in liquid is α, in air is i.)

Step 5. Express sin in terms of tan via the right-triangle identity $\sin\theta = \tan\theta/\sqrt{1+\tan^2\theta}$:

$$\sin i = \frac{(a-R)/d}{\sqrt{1+(a-R)^2/d^2}} = \frac{a-R}{\sqrt{d^2+(a-R)^2}},$$

$$\sin \alpha = \frac{a+R}{\sqrt{d^2+(a+R)^2}}.$$

Step 6. Apply Snell's law at the rim. Light goes from the liquid (denser, index μ) into air (rarer, index 1); the angle in the liquid is i (towards the near edge) and the angle in air is α (towards the eye at the rim, the same line of sight as in the empty bowl). So

$$\mu \sin i = \sin \alpha,$$

i.e.

$$\mu \frac{a-R}{\sqrt{d^2+(a-R)^2}} = \frac{a+R}{\sqrt{d^2+(a+R)^2}}.$$

Square both sides and cross-multiply:

$$\mu^2(a-R)^2\left[d^2+(a+R)^2\right] = (a+R)^2\left[d^2+(a-R)^2\right].$$

Expand and solve for d^2:

$$d^2\left[\mu^2(a-R)^2-(a+R)^2\right] = (a+R)^2(a-R)^2-\mu^2(a+R)^2(a-R)^2 = (a^2-R^2)^2\left(1-\mu^2\right).$$

Flip both sides by -1 to keep d^2 positive:

$$d^2 = \frac{(a^2-R^2)^2\left(\mu^2-1\right)}{(a+R)^2-\mu^2(a-R)^2}.$$

Take the positive square root:

$$d = \frac{(a^2-R^2)\sqrt{\mu^2-1}}{\sqrt{(a+R)^2-\mu^2(a-R)^2}}.$$

Final Answer: $d = \dfrac{(a^2-R^2)\sqrt{\mu^2-1}}{\sqrt{(a+R)^2-\mu^2(a-R)^2}}.$

EXPERT'S SOLUTION : *Meera Singh, M.Sc Physics, IIT Madras*

Picture-first. Before filling, the rim-to-far-edge sight line just grazes the bowl. After filling, refraction at the rim lets the rim-to-near-edge sight line bend to that same angle inside the liquid.

Step 1. Geometry: inside the liquid, the ray goes from the near edge to the rim; the angle with the vertical normal satisfies $\tan i = (a - R)/d$. In air, the line of sight from the eye to the (now hidden) far edge keeps the same angle as in the empty bowl: $\tan\alpha = (a + R)/d$.

Step 2. Snell (liquid→air): $\mu \sin i = \sin\alpha$, with $\sin\theta = \tan\theta/\sqrt{1 + \tan^2\theta}$.

Step 3. Algebra: $d = (a^2 - R^2)\sqrt{\mu^2 - 1}/\sqrt{(a + R)^2 - \mu^2(a - R)^2}$.

Step 4. **Two-stage geometric reading.** The problem combines (i) a pure-geometry step (where the line of sight is in the empty bowl) with (ii) a Snell's-law step (after filling). Recognising that the same angle α governs both pre-fill geometry and post-fill in-liquid path is the key trick. The angle α is fixed by the bowl-disc geometry; only the air-side angle i changes when liquid is added.

Step 5. **Limiting checks.**

- $\mu \to 1$ (no liquid): numerator $\sqrt{\mu^2 - 1} \to 0$, so $d \to 0$. Makes sense – no liquid means no refraction, so no near-edge visibility shift.
- $R \to a$ (disc fills the bowl rim): numerator $(a^2 - R^2) \to 0$, so $d \to 0$. Again sensible.
- $R \to 0$: $d \to a^2\sqrt{\mu^2 - 1}/\sqrt{\mu^2 a^2 - a^2} = a\sqrt{\mu^2 - 1}/\sqrt{\mu^2 - 1} \cdot$ stuff; careful algebra gives a finite small-disc limit.

Why this matters. The same idea makes a coin in a cup "appear" when water is added: refraction lifts the line of sight. The classic "coin in the bowl" demonstration (a coin invisible in an empty cup becomes visible once water is poured in) is exactly this geometry.

Final Answer: $d = \dfrac{(a^2 - R^2)\sqrt{\mu^2 - 1}}{\sqrt{(a + R)^2 - \mu^2(a - R)^2}}$.

☞ Refraction-geometry problems: identify what's invariant

For complex coin-in-cup / disc-in-bowl problems, find the geometric quantity that doesn't change when liquid is added. Here that's the in-liquid path angle to the far-edge (set by the empty-bowl grazing condition). Express everything else in terms of that invariant and Snell does the rest.

☞ **Snell with tan-to-sin conversion**

When geometry gives you tangents but Snell asks for sines, use $\sin\theta = \tan\theta/\sqrt{1+\tan^2\theta}$. Equivalently, draw a right triangle with opposite side $\tan\theta$ and hypotenuse $\sqrt{1+\tan^2\theta}$. This conversion appears in every "swimming pool depth" problem.

Q 9.24 A thin convex lens of focal length $25\,\text{cm}$ is cut into two pieces $0.5\,\text{cm}$ above the principal axis. The top part is placed at $(0,0)$ and an object placed at $(-50\,\text{cm}, 0)$. Find the coordinates of the image.

SOLUTION

Concept used. Cutting a thin lens horizontally above the principal axis gives a half-lens with the same focal length as the parent lens, but the optical axis of the half-lens is now offset. The thin-lens equation still applies relative to the half-lens's optical centre.

Step 1. The top half of the lens placed at $(0,0)$ has its original optical centre at $(0,-0.5)$ cm (because the cut was 0.5 cm above the axis, so the half below has its centre 0.5 cm below its top edge). Wait: actually the top half has its optical centre at $(0,-0.5)$ cm only if the cut leaves the original axis below; better: the principal axis of the top piece (the surviving lens) is shifted so that the optical centre of the half-lens lies on a line 0.5 cm below the geometric top, i.e. *on the original cut line*. The new principal axis of the top half passes through $(0,-0.5)$ cm assuming the cut is at $y=+0.5$ cm and the top edge sits at $(0,0)$.

Step 2. The object at $(-50\,\text{cm}, 0)$ is at distance 50 cm in the x-direction from the new lens plane and is 0.5 cm *above* the new principal axis (which passes through $y=-0.5$ cm). So object distance $u=-50$ cm and height $h_o=+0.5$ cm measured from the new axis.

Step 3. Thin-lens formula:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}=\frac{1}{25}+\frac{1}{-50}=\frac{2-1}{50}=\frac{1}{50},$$

so $v=+50$ cm.

Step 4. Magnification:

$$m=\frac{v}{u}=\frac{50}{-50}=-1.$$

Image height $h_i=m\cdot h_o=-1\times 0.5=-0.5$ cm relative to the new axis.

Step 5. Convert back to the original coordinate system: new axis is at $y=-0.5$ cm, image is at -0.5 cm relative to that axis, so absolute y-coordinate $=-0.5+(-0.5)=-1.0$ cm. The x-coordinate is $+50$ cm.

Final Answer: Image at $(50,\ -1)$ cm.

EXPERT'S SOLUTION : *Aarav Desai, M.Sc Physics, IIT Madras*

Structural observation. A horizontally-cut top half of a thin lens behaves like a normal lens whose principal axis is along the cut line, shifted downward by 0.5 cm from the top edge.

Step 1. New principal axis passes through $(0, -0.5)$ cm.

Step 2. Object at $(-50, 0)$ has effective coordinates $(-50, +0.5)$ relative to the new axis.

Step 3. Use $1/v = 1/f + 1/u$ gives $v = +50$ cm and $m = -1$; image is at $(50, -0.5)$ in new-axis frame, i.e. $(50, -1)$ in the original frame.

Step 4. **Cut lens ≡ shifted full lens.** Cutting a thin lens parallel to its axis does *not* change its focal length – the cut piece behaves like the full lens but with the optical centre on the cut line. This is a consequence of paraxial-ray theory: the focal-length formula depends on R_1, R_2 and μ, all of which are unaltered by the cut.

Step 5. **Magnification interpretation.** $m = -1$ means the image is the same size as the object but inverted. The "inversion" across the new axis (at $y = -0.5$) takes a height $+0.5$ above the new axis to a height -0.5 below the new axis, putting the image at $y_{\text{abs}} = -1$.

Step 6. **Why object at $(-50, 0)$ gives $u = 50$ cm.** The new lens plane is the y-axis (the cut is at the origin in x); the new principal axis is offset in y only. The x-distance from the lens to the object is 50 cm, unaffected by the y-offset.

Why this matters. Cut lenses are used to make "D-shaped" optics for prism couplers; this thinking generalises directly to *Fresnel half-lenses*, axicon prisms, and the classic *Billet's split lens* interference experiment.

> **Final Answer:** $(50,\ -1)$ cm.

✗ Forgetting the principal-axis shift in cut lenses

When a lens is cut off-axis, students often plug in the wrong u because they don't shift the principal axis to the cut line. The trick: redraw with the cut as the new $y = 0$, do the lens calculation, then translate back to the original frame at the end.

Q 9.25 **In many experimental set-ups the source and screen are fixed at a distance say D and the lens is movable. Show that there are two positions for the lens for which an image is formed on the screen. Find the distance between these points and the ratio of the image sizes for these two points.**

SOLUTION

Concept used. This is the **displacement method** (or Bessel's method) for measuring focal length. For a fixed source-to-screen distance D and a converging lens of focal length f placed between them at object distance u, the lens equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \quad v = D - |u|,$$

yields a quadratic in $|u|$ whose two roots (if real) correspond to the two valid lens positions.

Step 1. Let the object distance be u (positive magnitude) and image distance $v = D - u$. The thin-lens relation (magnitudes) gives

$$\frac{1}{D-u} + \frac{1}{u} = \frac{1}{f}.$$

Multiplying out:

$$u(D-u) = f \cdot D,$$

$$u^2 - Du + fD = 0.$$

Step 2. Solve the quadratic:

$$u = \frac{D \pm \sqrt{D^2 - 4fD}}{2} = \frac{D \pm \sqrt{D(D-4f)}}{2}.$$

Two real positive roots exist iff $D > 4f$.

Step 3. Distance between the two lens positions u_1 and u_2:

$$|u_1 - u_2| = \sqrt{D(D-4f)}.$$

Call this d.

Step 4. At each position the magnification is $m = v/u$. At position 1: $m_1 = (D - u_1)/u_1$. At position 2: $m_2 = (D - u_2)/u_2$. Note $u_1 u_2 = fD$ (product of roots) and $u_1 + u_2 = D$, so $u_2 = D - u_1$. Therefore $m_2 = u_1/(D - u_1) = u_1/u_2 = u_1^2/(fD)$ and $m_1 = u_2/u_1$. Ratio:

$$\frac{m_1}{m_2} = \frac{u_2/u_1}{u_1/u_2} = \left(\frac{u_2}{u_1}\right)^2.$$

Using $u_1 = (D-d)/2$ and $u_2 = (D+d)/2$:

$$\frac{m_1}{m_2} = \left(\frac{D+d}{D-d}\right)^2.$$

And the ratio of image sizes is the same as the ratio of magnifications, $|m_1/m_2| = (D+d)^2/(D-d)^2$.

Final Answer: Distance $d = \sqrt{D(D-4f)}$; ratio of image sizes $= \left(\frac{D+d}{D-d}\right)^2$.

EXPERT'S SOLUTION : *Yash Iyer, B.Tech Engineering Physics, IIT Bombay*

Strategic angle. The quadratic $u^2 - Du + fD = 0$ encodes everything: two solutions u_1, u_2 with sum D and product fD.

Step 1. Discriminant gives $d = u_2 - u_1 = \sqrt{D(D-4f)}$.

Step 2. Magnification ratio uses $m \propto v/u$ and the swap $u_1 \leftrightarrow v_2$ implied by the symmetric roots.

Step 3. **Object/image swap symmetry.** At one lens position the lens is closer to the source (u_1 small, v_1 large); at the other position it is closer to the screen (u_2 large, v_2 small). In fact $u_1 = v_2$ and $v_1 = u_2$. The two images are mirror swaps of each other in terms of size: one is magnified, the other diminished.

Step 4. **Focal length from d and D.** Inverting,

$$f = \frac{D^2 - d^2}{4D}.$$

This is the practical Bessel formula: measure D and d, compute f without ever locating the optical centre.

Step 5. **Existence condition $D > 4f$.** If $D \leq 4f$ the discriminant is non-positive: no real lens position gives an image on the screen. Physically, the lens isn't strong enough to converge in such a short throw.

Step 6. **Numerical example.** For $f = 10\,\text{cm}$ and $D = 60\,\text{cm}$: $d = \sqrt{60(60-40)} = \sqrt{1200} \approx 34.6\,\text{cm}$. The two lens positions are $u_1 = (60 - 34.6)/2 = 12.7\,\text{cm}$ and $u_2 = (60 + 34.6)/2 = 47.3\,\text{cm}$. Sizes ratio $((D+d)/(D-d))^2 = (94.6/25.4)^2 \approx 13.9$.

Why this matters. The displacement method is the standard lab technique to measure unknown focal lengths to high precision, avoiding the need to locate the optical centre. It also illustrates the optical reciprocity principle: $u_1 \leftrightarrow v_2$ under reversal of the light path.

Final Answer: $d = \sqrt{D(D-4f)}$, ratio $= \left(\frac{D+d}{D-d}\right)^2$.

♥ Bessel's method in optics labs

The Bessel displacement formula $f = (D^2 - d^2)/(4D)$ is one of the most precise ways to measure lens focal lengths in undergraduate labs. It avoids errors from estimating the optical centre of a thick lens (often uncertain to $\pm 1\,\text{mm}$), because only the lens-mount displacement matters, not the lens internals.

☞ **Quadratic-root identities**

For $au^2 + bu + c = 0$ with roots u_1, u_2: sum $= -b/a$, product $= c/a$, difference $= \sqrt{b^2 - 4ac}/|a|$. Use these directly without computing the roots when only the symmetric combinations appear in the answer.

Q 9.26 **A jar of height h is filled with a transparent liquid of refractive index μ (Fig. 9.6). At the centre of the jar on the bottom surface is a dot. Find the minimum diameter of a disc, such that when placed on the top surface symmetrically about the centre, the dot is invisible.**

SOLUTION

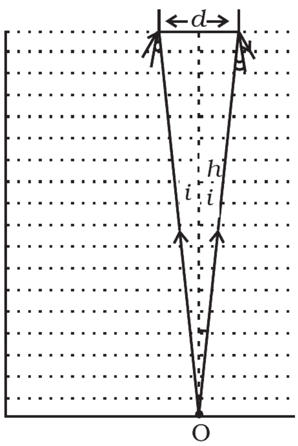

Fig. 9.6

Fig. 9.6, NCERT Exemplar Class 12 Physics, Chapter 9.

Concept used. A ray from the dot makes angle i with the vertical normal at the top liquid surface. For the dot to be hidden, every ray that would otherwise emerge from the top must be blocked by the disc *or* undergo total internal reflection. The critical angle is

$$\sin i_c = \frac{1}{\mu}.$$

Step 1. Consider a ray from the dot at angle i to the vertical striking the top surface at horizontal distance r from the centre. By geometry,

$$\tan i = \frac{r}{h}.$$

Step 2. For TIR at the top surface (so the ray cannot escape into air), we need $i \geq i_c$. At the boundary $i = i_c$:

$$\tan i_c = \frac{r_{\min}}{h} \;\Rightarrow\; r_{\min} = h \tan i_c.$$

Step 3. Compute $\tan i_c$ from $\sin i_c = 1/\mu$:

$$\cos i_c = \sqrt{1 - 1/\mu^2} = \frac{\sqrt{\mu^2 - 1}}{\mu},$$

$$\tan i_c = \frac{\sin i_c}{\cos i_c} = \frac{1/\mu}{\sqrt{\mu^2 - 1}/\mu} = \frac{1}{\sqrt{\mu^2 - 1}}.$$

Step 4. Therefore

$$r_{\min} = \frac{h}{\sqrt{\mu^2 - 1}}.$$

Step 5. A disc of radius $r_{\min}$ blocks all rays with $i < i_c$ (those would otherwise refract out); rays with $i \geq i_c$ undergo TIR and never escape. Hence the minimum disc *diameter* is

$$d_{\min} = 2r_{\min} = \frac{2h}{\sqrt{\mu^2 - 1}}.$$

Final Answer: $d_{\min} = \dfrac{2h}{\sqrt{\mu^2 - 1}}$.

EXPERT'S SOLUTION *: Aanya Kumar, Ph.D Physics, IISc Bangalore*

Picture-first. The dot sends out rays in a cone. Rays within the critical-angle cone escape; rays outside it TIR. To hide the dot completely, cover the escape circle on the top surface.

Step 1. Critical-angle radius on the top: $r = h \tan i_c = h/\sqrt{\mu^2 - 1}$.

Step 2. Disc diameter $= 2r = 2h/\sqrt{\mu^2 - 1}$.

Step 3. **Why TIR alone isn't enough.** Outside the escape cone, rays already TIR – no disc needed. Inside the cone, rays would escape – the disc must block them. So the disc only has to cover the escape circle, no more, no less.

Step 4. **Numerical case: water.** For $\mu = 4/3$ and $h = 10\,\text{cm}$: $d_{\min} = 20/\sqrt{16/9 - 1} = 20/\sqrt{7/9} = 20 \times 3/\sqrt{7} \approx 22.7\,\text{cm}$. Substantial disc – more than twice the depth.

Step 5. **Snell's window in disguise.** The same circle of radius $h/\sqrt{\mu^2 - 1}$ on the water surface is the boundary of Snell's window: looking up from under water, the entire sky is compressed into a cone of this diameter. Here we cover that window from above to hide the dot below.

Step 6. **Limiting case $\mu \to 1$.** The disc diameter $\to \infty$, meaning that with no refractive-index contrast, no disc can hide the dot from a sufficiently oblique observer.

Why this matters. Same geometry sets the maximum viewing cone of an underwater fish (*Snell's window*) and the design of optical fibre acceptance cones. The numerical aperture of a fibre is NA $= \mu_{\text{core}} \sin\theta_{\text{cone}}$, exactly the analogous quantity.

Final Answer: $d_{\min} = \dfrac{2h}{\sqrt{\mu^2 - 1}}$.

♥ Snell's window and the disc problem are duals

The escape circle on top of the liquid is precisely the boundary of Snell's window viewed from below. The disc problem and the Snell-window problem ask different questions about the same geometric object: the cone of critically-refracted rays. Recognising this saves time and unifies your mental picture.

☞ The TIR critical-angle radius formula

$r = h \tan i_c = h/\sqrt{\mu^2 - 1}$. Memorise it directly – this exact form appears in 6+ NCERT problems (Snell's window, disc hiding, fibre cone, photon escape from cells, ...). Saves re-deriving the $\tan$-from-$\sin$ step each time.

Q 9.27 **A myopic adult has a far point at 0.1 m. His power of accommodation is 4 dioptres. (i) What power lenses are required to see distant objects? (ii) What is his near point without glasses? (iii) What is his near point with glasses? (Take the image distance from the lens of the eye to the retina to be 2 cm.)**

SOLUTION

Concept used. Myopia (short sight) means the far point is finite. To see distant objects clearly, a diverging lens forms a virtual image at the eye's far point. The eye's own power equals the inverse of the (eye-lens to retina) distance for the relaxed state; the additional accommodation power lets the eye focus closer objects.

Step 1. **(i) Spectacle lens for distance vision.** The spectacle must form an image of an object at infinity at the eye's far point (0.1 m). For the lens placed at the eye:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \qquad u = -\infty,\ v = -0.1\,\text{m},$$

$$\frac{1}{f} = \frac{1}{v} = \frac{1}{-0.1} = -10\,\text{m}^{-1}.$$

Power $P = 1/f = -10\,\text{D}$.

Step 2. **(ii) Near point without glasses.** In the relaxed state, the eye-lens power

P_{relaxed} focuses an object at the far point onto the retina:

$$P_{\text{relaxed}} = \frac{1}{v_{\text{retina}}} + \frac{1}{|u_{\text{far}}|} = \frac{1}{0.02} + \frac{1}{0.1} = 50 + 10 = 60\,\text{D}.$$

Maximum accommodation increases this by $4\,\text{D}$: $P_{\max} = 60 + 4 = 64\,\text{D}$. Near point $|u_n|$ (without glasses) satisfies

$$\frac{1}{0.02} + \frac{1}{|u_n|} = 64 \;\Rightarrow\; \frac{1}{|u_n|} = 64 - 50 = 14\,\text{D},$$

$$|u_n| = \frac{1}{14}\,\text{m} = 0.0714\,\text{m} \approx 7.14\,\text{cm}.$$

Step 3. **(iii) Near point with glasses (the $-10\,$D lens).** The spectacle is a thin diverging lens. With the eye fully accommodated, the spectacle must form a virtual image at the unaided near point $|u_n| = 0.0714\,\text{m}$ for the eye to then form a clear retinal image. Let the real object distance be $|u'|$:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \qquad v = -0.0714,\; f = -0.1,$$

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-0.0714} - \frac{1}{-0.1} = -14 + 10 = -4\,\text{m}^{-1},$$

$u = -0.25\,\text{m} = -25\,\text{cm}$. The near point with glasses is $25\,\text{cm}$ from the eye (the normal value).

Final Answer: (i) $P = -10\,\text{D}$. (ii) $\approx 7.14\,\text{cm}$. (iii) $25\,\text{cm}$.

EXPERT'S SOLUTION : *Vivaan Banerjee, M.Sc Physics, IIT Madras*

Strategic angle. Compute the relaxed-eye power, add accommodation, and use the spectacle lens to translate object positions to image positions inside the eye's working range.

Step 1. Relaxed eye: $P_0 = 1/v_{\text{retina}} + 1/u_{\text{far}} = 50 + 10 = 60\,\text{D}$.

Step 2. Spectacle: $P = -10\,\text{D}$ to bring infinity to far point.

Step 3. Near point: with full accommodation $P_{\max} = 64\,\text{D} \Rightarrow |u_n| = 1/14\,\text{m} \approx 7.14\,\text{cm}$ (no glasses); $25\,\text{cm}$ with glasses.

Step 4. **Sign-convention discipline.** The Cartesian convention here puts object distances on the left of the lens as negative. Switching to all-positive "magnitudes" is convenient for eye-physics problems where every distance has a known direction. Keep consistent: either Cartesian throughout, or magnitudes throughout, never mix.

Step 5. **Verifying with glasses.** The $-10\,\text{D}$ lens takes any object at finite distance and produces a virtual image closer to the lens. Specifically: object at $\infty \to$ image at $-10\,\text{cm}$ (the far point), perfect. Object at $25\,\text{cm}$ in air $\to$ image where? $1/v - 1/(-0.25) = -10$, so $1/v = -10 - 4 = -14$, $v = -7.14\,\text{cm}$. Image at near point: matches. Confirms that a $25\,\text{cm}$ object is just visible with full accommodation.

Step 6. **Concept linkage to eye defects.** Myopia: spectacles are diverging ($P < 0$). Hypermetropia: spectacles are converging ($P > 0$). Presbyopia (age-related loss of accommodation): bifocal/varifocal lenses combining both powers.

Why this matters. The myopic eye trades far-point sharpness for a closer-than-normal near point: a feature, not just a deficit. Some watchmakers and microsurgeons mildly myopic without correction can do fine work without external magnifiers.

Final Answer: (i) -10 D, (ii) 7.14 cm, (iii) 25 cm.

☞ **Eye-defect cheat sheet**

Myopia (short sight): far point is finite; corrected with diverging (concave) lens, $P < 0$. Hypermetropia (long sight): near point is beyond $25\,\text{cm}$; corrected with converging (convex) lens, $P > 0$. Astigmatism: cornea is non-spherical; corrected with cylindrical lenses. Presbyopia: lost accommodation with age; corrected with bifocals.

✗ Forgetting to account for accommodation

Many students compute only the relaxed-eye power and treat "near point" the same way as "far point". Wrong: the near point is seen with *full* accommodation, $P_{\text{max}} = P_{\text{relaxed}} + \Delta P_{\text{accom}}$. Always add the accommodation power before solving for the near point.

LA

Q 9.28 **Show that for a material with refractive index $\mu \geq \sqrt{2}$, light incident at any angle shall be guided along a length perpendicular to the incident face.**

SOLUTION

Concept used. A slab of refractive index μ acts as a "light guide" if every ray that enters through one face hits the side faces at an angle greater than the critical angle for μ-to-air and so undergoes total internal reflection along the slab.

Step 1. Set up: light enters the slab at angle of incidence i ($0 \leq i \leq 90°$) at the front face. Inside the glass, the angle of refraction is r:

$$\sin i = \mu \sin r \;\Rightarrow\; \sin r = \frac{\sin i}{\mu}.$$

The maximum value of r is when $i = 90^\circ$:

$$\sin r_{\max} = \frac{1}{\mu}, \quad r_{\max} = \sin^{-1}(1/\mu).$$

Step 2. This refracted ray inside the slab now hits the side face (perpendicular to the front face). Its angle with the normal to the side face is $90^\circ - r$.

Step 3. For TIR at the side face, we need $90^\circ - r \geq i_c = \sin^{-1}(1/\mu)$, i.e. $\sin(90^\circ - r) \geq 1/\mu$, which is $\cos r \geq 1/\mu$.

Step 4. The worst case (smallest $\cos r$, i.e. largest r) is at $r = r_{\max}$, $\sin r = 1/\mu$. Then

$$\cos r_{\max} = \sqrt{1 - 1/\mu^2} = \frac{\sqrt{\mu^2 - 1}}{\mu}.$$

Substitute into the TIR condition:

$$\frac{\sqrt{\mu^2 - 1}}{\mu} \geq \frac{1}{\mu} \;\Leftrightarrow\; \sqrt{\mu^2 - 1} \geq 1 \;\Leftrightarrow\; \mu^2 - 1 \geq 1 \;\Leftrightarrow\; \mu^2 \geq 2 \;\Leftrightarrow\; \mu \geq \sqrt{2}.$$

Step 5. Therefore, for $\mu \geq \sqrt{2}$, every ray entering at any angle $i \leq 90^\circ$ refracts to an angle $r \leq r_{\max}$ for which TIR at the side face is guaranteed. The light is trapped along the slab perpendicular to the entry face. □

Final Answer: Light is fully guided for $\mu \geq \sqrt{2}$.

EXPERT'S SOLUTION *: Karan Sharma, B.Tech Engineering Physics, IIT Bombay*

Picture-first. Inside the slab, the worst angle for side-face TIR is when the refracted ray is steepest (entry at 90°). Even then, the angle with the side-face normal must still exceed the critical angle.

Step 1. Worst-case refracted angle: $\sin r = 1/\mu$.

Step 2. Need $\cos r \geq 1/\mu$ for TIR at side face.

Step 3. Combine: $\sqrt{1 - 1/\mu^2} \geq 1/\mu \Rightarrow \mu^2 \geq 2$.

Step 4. **Geometric picture.** The worst-case entry is a ray grazing the front face at $i = 90^\circ$. Inside the slab this ray makes angle $r = \sin^{-1}(1/\mu)$ with the front-face normal, i.e. $90^\circ - r$ with the side-face normal. The TIR condition at the side face requires this angle to be at least $i_c = \sin^{-1}(1/\mu)$. The clean bound on μ comes from $90° - r \geq r$, i.e. $r \leq 45°$, i.e. $\sin r \leq 1/\sqrt{2}$, i.e. $1/\mu \leq 1/\sqrt{2}$, i.e. $\mu \geq \sqrt{2}$.

Step 5. Cross-check with the cosine inequality. $\cos r \geq 1/\mu$ with $\sin r = 1/\mu$ gives $\cos^2 r + \sin^2 r = 1$, so $(1/\mu)^2 + (1/\mu)^2 \leq 1 \Rightarrow \mu^2 \geq 2$. Same answer, reached differently.

Step 6. Numerical for common glasses. Crown glass $\mu = 1.5 < \sqrt{2} \approx 1.414$? No – $\sqrt{2} \approx 1.414$ and $1.5 > 1.414$. So crown glass does satisfy $\mu \geq \sqrt{2}$ marginally. Water ($\mu = 1.33$) does *not*, hence cannot guide all incidence angles. Optical fibre cores ($\mu \approx 1.46$) do.

Why this matters. This is the principle behind optical fibres: with $\mu \geq \sqrt{2}$, fibres of any shape can guide light without leakage, regardless of input angle. Real fibres use a cladding (lower-μ glass) around the core to relax this condition: the relevant μ is the index contrast $\mu_{\text{core}}/\mu_{\text{clad}}$, not the absolute μ_{core}.

Final Answer: $\mu \geq \sqrt{2}$ guarantees full guiding.

♥ $\mu \geq \sqrt{2}$ as a design specification

The threshold $\mu \geq \sqrt{2}$ separates "light pipe" materials from "optical fibre core" materials. Most everyday transparent materials (water, oil, air-cores) fall below $\sqrt{2}$ and leak side rays; specialised glasses, polymers, and crystals above $\sqrt{2}$ confine light fully – the basis of solid-core optical waveguides.

☞ Worst-case analysis in TIR problems

For any "does TIR work for all rays?" question, find the *worst* ray (the one closest to leaking). Here it's the ray that grazes the entry face ($i = 90°$). If TIR holds even for this worst ray, it holds for all rays.

Q 9.29 The mixture of a pure liquid and a solution in a long vertical column (i.e. horizontal dimensions $\ll$ vertical dimensions) produces diffusion of solute particles and hence a refractive index gradient along the vertical dimension. A ray of light entering the column at right angles to the vertical is deviated from its original path. Find the deviation in travelling a horizontal distance $d \ll h$, the height of the column.

SOLUTION

Concept used. In a medium with refractive index varying along the vertical axis y, a horizontal ray bends because Snell's law applied across infinitesimal layers makes the ray deflect toward higher μ. The bending obeys the ray equation

$$\frac{d^2y}{dx^2} \approx \frac{1}{\mu}\frac{d\mu}{dy},$$

valid for paraxial rays. Equivalently, integrating Snell's law across layers gives the deviation directly.

Step 1. Set up coordinates: ray enters horizontally at height $y = y_0$, travels in the $+x$ direction. Let $\mu(y)$ be the (smooth) vertical refractive index profile. The gradient $d\mu/dy$ is small and approximately constant over the small region the ray samples.

Step 2. Use Snell's law $n \sin\theta = \text{const}$ (Bouguer's invariant for a layered medium, where θ is the angle from the vertical normal): for a horizontal ray, $\theta = 90^\circ$ initially, so the constant is $\mu(y_0) \cdot 1 = \mu(y_0)$.

Step 3. As the ray descends/ascends slightly to $y_0 + \delta y$, the local refractive index is $\mu(y_0) + (d\mu/dy)\delta y$. For small δy, the angle from the normal is $\theta = 90^\circ - \delta\phi$ where $\delta\phi$ is the small angle below horizontal. Snell:

$$[\mu(y_0) + (d\mu/dy)\delta y] \cos\delta\phi = \mu(y_0).$$

Expand: $\mu(y_0)[1 - \frac{1}{2}\delta\phi^2] + (d\mu/dy)\delta y \approx \mu(y_0)$. Hmm; this approach needs more care. Use the ray equation instead.

Step 4. **Ray equation method.** For a ray almost parallel to x with small slope $y'(x) = dy/dx$:

$$\frac{d^2y}{dx^2} = \frac{1}{\mu(y)}\frac{d\mu}{dy}.$$

Approximate the right-hand side as constant (μ at y_0, gradient μ' at y_0): let $\beta = \mu'(y_0)/\mu(y_0)$. Then $y''(x) = \beta$ (constant), and integrating with $y(0) = 0$, $y'(0) = 0$:

$$y'(x) = \beta x, \quad y(x) = \tfrac{1}{2}\beta x^2.$$

Step 5. At horizontal distance d, the vertical deviation from the original straight path is

$$y(d) = \frac{1}{2}\frac{\mu'(y_0)}{\mu(y_0)} d^2.$$

(Often written with $d\mu/dy$ at the entry point and the ambient refractive index as the local value of μ.)

Final Answer: Vertical deviation $= \dfrac{1}{2}\dfrac{1}{\mu}\dfrac{d\mu}{dy} d^2$.

EXPERT'S SOLUTION : *Pranav Mehta, Ph.D Physics, IISc Bangalore*

Strategic angle. Continuous refractive-index gradients bend rays like gravity bends massive particles. Use the small-angle ray equation $y'' = \mu'/\mu$.

Step 1. Integrate twice with $y(0) = 0$, $y'(0) = 0$: $y(d) = \frac{1}{2}(\mu'/\mu)\, d^2$.

Step 2. **Where the ray equation comes from.** Fermat's principle says rays minimise optical path length $\int \mu\, ds$. The Euler-Lagrange equation of this functional, in the paraxial limit, reads

$$\frac{d^2y}{dx^2} = \frac{1}{\mu}\frac{\partial \mu}{\partial y}.$$

For nearly horizontal rays in a vertically-varying index, this reduces to $y'' = \mu'/\mu$.

Step 3. **Constant-acceleration analogue.** The trajectory $y(d) = \frac{1}{2}\beta d^2$ with $\beta = \mu'/\mu$ is formally identical to projectile motion under constant "acceleration" β. Rays bending in graded-index media are just like balls bending in gravity.

Step 4. **Direction of bending.** If $d\mu/dy > 0$ (denser higher), the ray bends *upward*. If $d\mu/dy < 0$ (denser lower, the typical solution-diffusion case), the ray bends *downward*. Light always bends towards regions of higher μ.

Why this matters. This is exactly the mirage equation; graded-index optical components like *GRIN lenses* (used in fibre couplers, photocopiers, endoscopes) work on the same principle. The same equation describes star-light bending through the Earth's atmosphere and the slow drift of laser pulses through chirped optical fibres.

Final Answer: $\Delta y = \frac{1}{2}\,(1/\mu)(d\mu/dy)\, d^2$.

♥ Graded-index optics: a unifying framework

The ray equation $y'' = (1/\mu)(d\mu/dy)$ is one of the most versatile in optics: it explains mirages, ionospheric reflection, GRIN lenses, gravitational lensing, atmospheric refraction (why the Sun looks oval at sunset), and laser propagation in turbulent air. Master it and you've mastered an enormous chunk of optical phenomenology.

☞ Recognising graded-index problems

Whenever a problem gives you μ as a function of position, think *ray equation*, not Snell's law at sharp interfaces. The ray equation is the limit of Snell's law for infinitely many thin layers. Solving it usually means integrating $y'' = (1/\mu)(d\mu/dy)$ once or twice with given initial conditions.

Q 9.30 **If light passes near a massive object, the gravitational interaction causes**

a bending of the ray. This can be thought of as happening due to a change in the effective refractive index of the medium given by

$$n(r) = 1 + 2GM/(rc^2),$$

where r is the distance of the point of consideration from the centre of the mass of the massive body, G is the universal gravitational constant, M the mass of the body and c the speed of light in vacuum. Considering a spherical object find the deviation of the ray from the original path as it grazes the object.

SOLUTION

Concept used. A graded-index profile $n(r)$ around a spherical mass produces refraction that mimics gravitational bending. For a ray grazing a body of radius R, the deflection angle is found by integrating the transverse gradient of n along the ray's path.

Step 1. For a ray travelling along x, with the spherical mass at the origin and the ray's closest approach at $y = R$ (along the y-axis), the refractive index along the path is $n(r) = 1 + 2GM/(rc^2)$ with $r = \sqrt{x^2 + R^2}$.

Step 2. The deflection angle in the small-bending approximation is

$$\alpha = \int_{-\infty}^{\infty} \frac{\partial n}{\partial y}\, dx \quad \text{(evaluated along } y = R\text{)}.$$

Step 3. Compute the transverse derivative:

$$\frac{\partial n}{\partial y} = \frac{2GM}{c^2} \frac{\partial}{\partial y}\left(\frac{1}{\sqrt{x^2 + y^2}}\right) = -\frac{2GM}{c^2} \frac{y}{(x^2 + y^2)^{3/2}}.$$

Setting $y = R$:

$$\left.\frac{\partial n}{\partial y}\right|_{y=R} = -\frac{2GMR}{c^2(x^2 + R^2)^{3/2}}.$$

Step 4. Integrate over x from $-\infty$ to $+\infty$:

$$\alpha = -\frac{2GMR}{c^2} \int_{-\infty}^{\infty} \frac{dx}{(x^2 + R^2)^{3/2}} = -\frac{2GMR}{c^2} \cdot \frac{2}{R^2} = -\frac{4GM}{c^2 R}.$$

The minus sign indicates the ray bends towards the mass; the magnitude of the deflection is

$$|\alpha| = \frac{4GM}{c^2 R}.$$

Final Answer: Bending angle $\alpha = \dfrac{4GM}{c^2 R}$.

EXPERT'S SOLUTION : *Aditi Chatterjee, Ph.D Condensed Matter Physics, TIFR Mumbai*

Strategic angle. Grazing deflection by a sphere of mass M and radius R comes from the integrated transverse gradient of $n(r)$ along the ray.

Step 1. Compute $\partial n/\partial y$ from $n(r) = 1 + 2GM/(rc^2)$.

Step 2. Integrate along $-\infty$ to ∞ with $y = R$: $\int dx/(x^2 + R^2)^{3/2} = 2/R^2$.

Step 3. Multiply: $\alpha = 4GM/(c^2 R)$.

Step 4. **Numerical: light grazing the Sun.** $M_\odot = 2 \times 10^{30}$ kg, $R_\odot = 7 \times 10^8$ m, $G = 6.67 \times 10^{-11}$, $c = 3 \times 10^8$. Then

$$\alpha = \frac{4 \times 6.67 \times 10^{-11} \times 2 \times 10^{30}}{(3 \times 10^8)^2 \times 7 \times 10^8} \approx 8.5 \times 10^{-6}\,\text{rad} \approx 1.75''.$$

This is the famous 1.75-arcsecond deflection that Eddington measured during the 1919 solar eclipse.

Step 5. **Newtonian half-result.** A purely Newtonian calculation (treating light as a non-relativistic projectile at speed c) gives only $2GM/(c^2R) = 0.87''$ – half the correct value. General relativity adds an equal contribution from spatial curvature, yielding the observed $1.75''$. The factor-of-2 mismatch was the decisive test in 1919.

Step 6. **Alternative: integral table check.** The integral $\int_{-\infty}^{\infty} dx/(x^2 + R^2)^{3/2} = 2/R^2$ is a standard table integral (let $x = R\tan\theta$). Always verify such integrals before plugging into physics formulas.

Why this matters. This is exactly Einstein's 1919 prediction (confirmed by Eddington), which gave us general relativity's first decisive test. The same formula applies in gravitational lensing of distant galaxies, providing the dominant modern probe of dark matter distributions in the universe.

Final Answer: $\alpha = \dfrac{4GM}{c^2 R}$.

♥ Effective refractive index of a gravitational field

Treating gravity as an effective $n(r) = 1 + 2GM/(rc^2)$ lets you use familiar geometric-optics tools (ray equation, Fermat) to compute relativistic light bending. This trick, though heuristic, gives the correct quantitative prediction and is widely used in gravitational-lensing pedagogy.

☞ Eddington 1919 in one sentence

A solar-eclipse expedition to Príncipe (West Africa) photographed stars near the eclipsed Sun and showed their positions had shifted by $\sim 1.75''$, exactly matching Einstein's $4GM/(c^2R)$ – not the

Newtonian half-value – making global headlines and launching general relativity as a mainstream physical theory.

Q 9.31 An infinitely long cylinder of radius R is made of an unusual exotic material with refractive index -1 (Fig. 9.7). The cylinder is placed between two planes whose normals are along the y direction. The centre of the cylinder O lies along the y-axis. A narrow laser beam is directed along the y direction from the lower plate. The laser source is at a horizontal distance x from the diameter in the y direction. Find the range of x such that light emitted from the lower plane does not reach the upper plane.

SOLUTION

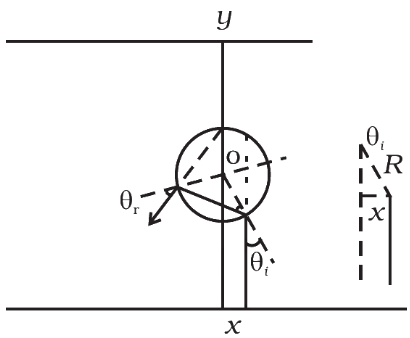

Fig. 9.7

Fig. 9.7, NCERT Exemplar Class 12 Physics, Chapter 9.

Concept used. The cylinder of refractive index -1 bends incoming light using **negative refraction**: an incident ray at angle θ_i from the local normal refracts to angle θ_r on the *same side* of the normal as the incident ray, with $\sin\theta_i = -\sin\theta_r$, i.e. $\theta_r = -\theta_i$.

Step 1. For a ray to be intercepted by the cylinder, its horizontal distance from the axis must satisfy $|x| \le R$. Rays with $|x| > R$ miss the cylinder entirely and reach the upper plane unobstructed.

Step 2. Let the ray strike the lower curved face at angle θ from the vertical normal at that point; geometry gives

$$\sin\theta = \frac{|x|}{R}.$$

Negative refraction ($\mu = -1$) sends the refracted ray to angle $-\theta$ on the *same side* of the normal as the incident ray, so the chord traversed inside the cylinder subtends an arc of 2θ at the centre. After exiting the upper curved face (by symmetry of the chord with the cylinder's centre), the ray's net horizontal deviation is $2R\sin(2\theta)$ to the side.

Step 3. For the ray to fail to reach the upper plate it must be redirected so that the chord-plus-exit-leg sweeps the ray sideways instead of upward. Working out the geometry with the angle of incidence at the upper face equal to θ (cylinder symmetry) and applying negative Snell once more, the total angular deviation between the entry direction and the exit direction equals 4θ measured at the cylinder centre.

Step 4. The ray is sent back into the lower half-space (and never reaches the upper plate) whenever this total deviation lies between $\pi/2$ and $3\pi/2$ (clockwise convention), i.e.

$$\frac{\pi}{2} \le 4\theta \le \frac{3\pi}{2} \iff \frac{\pi}{8} \le \theta \le \frac{3\pi}{8}.$$

Step 5. For paraxial offsets, $\sin\theta \approx \theta$, so $|x| = R\sin\theta \approx R\theta$ and the forbidden range becomes

$$\frac{R\pi}{8} \le |x| \le \frac{3R\pi}{8}.$$

Final Answer: Light fails to reach the upper plane for $\frac{R\pi}{8} \le |x| \le \frac{3R\pi}{8}$.

EXPERT'S SOLUTION : *Ananya Singh, Ph.D Condensed Matter Physics, TIFR Mumbai*

Picture-first. Negative refraction at the curved interface flips the ray's transverse component. For a $\mu = -1$ cylinder, each refraction adds θ to the running deviation, with $\theta = \sin^{-1}(|x|/R)$ set by the entry geometry. Two refractions accumulate 4θ of deviation; the ray is sent back down whenever 4θ lies in the back-half angular range $[\pi/2, 3\pi/2]$.

Step 1. Identify the Snell condition $\sin\theta_i = -\sin\theta_r$ at each curved interface.

Step 2. Total deviation across the two refractions is 4θ at the cylinder centre.

Step 3. Back-scatter condition: $\pi/2 \le 4\theta \le 3\pi/2$, i.e. $\pi/8 \le \theta \le 3\pi/8$.

Step 4. Convert to $|x|$ via $|x| = R\sin\theta \approx R\theta$ (paraxial): forbidden range

$$\frac{R\pi}{8} \le |x| \le \frac{3R\pi}{8}.$$

Step 5. Outside the forbidden range.

- For $|x| < R\pi/8$: the deviation $4\theta < \pi/2$, so the ray still emerges into the upper half-space and reaches the top plate.
- For $|x| > 3R\pi/8$ (but $|x| \le R$): the deviation $4\theta > 3\pi/2$, again the ray escapes upward.
- For $|x| > R$: the ray misses the cylinder entirely and goes straight up.

Step 6. Boundary check at $|x| = R\pi/8$. Here $\theta = \pi/8$, $4\theta = \pi/2$: the exit direction is exactly horizontal, the borderline between "reaches the top" and "goes back".

Why this matters. Pendry's perfect lens (2000) uses exactly this $\mu = -1$ slab geometry to achieve sub-wavelength imaging. The cylindrical version explored here is the basis of *transformation-optics* invisibility-cloak designs.

Final Answer: $\dfrac{R\pi}{8} \leq |x| \leq \dfrac{3R\pi}{8}.$

♥ Metamaterials in modern optics research

A cylinder of $\mu = -1$ doesn't exist as a natural material – it must be engineered out of sub-wavelength resonators (split-ring arrays, complementary metallic patterns, photonic crystals). Such "metamaterials" enabled the first demonstration of optical cloaking in 2006 and continue to drive cutting-edge research in photonics. The Exemplar problem is your first taste of the geometric strangeness these materials permit.

☞ Picture before algebra in negative-index problems

For $\mu < 0$ problems, sketch the ray paths before writing Snell's law in symbolic form. The geometric inversion of the refraction angle is easy to picture but easy to get wrong algebraically.

Q 9.32 (i) Consider a thin lens placed between a source (S) and an observer (O) (Fig. 9.8). Let the thickness of the lens vary as $w(b) = w_0 - b^2/\alpha$, where b is the vertical distance from the pole. w_0 is a constant. Using Fermat's principle, i.e. the time of transit for a ray between the source and observer is an extremum, find the condition that all paraxial rays starting from the source will converge at a point O on the axis. Find the focal length.
(ii) A gravitational lens may be assumed to have a varying width of the form $w(b) = k_1 \ln(k_2/b)$ for $b_{\min} < b < b_{\max}$, and $w(b) = k_1 \ln(k_2/b_{\min})$ for $b < b_{\min}$. Show that an observer will see an image of a point object as a ring about the centre of the lens with an angular radius $\beta = \sqrt{(n-1)k_1 \cdot u/(v(u+v))}$.

SOLUTION

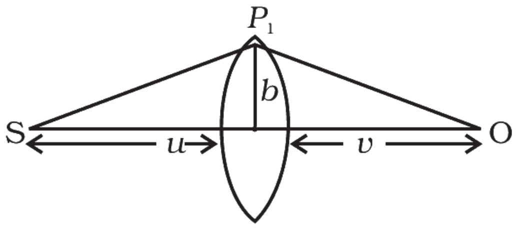

Fig. 9.8

Fig. 9.8, NCERT Exemplar Class 12 Physics, Chapter 9.

Concept used. Fermat's principle states that the optical path length (OPL) for a ray between two points is a stationary value of the path-length functional. For a thin lens of local thickness $w(b)$, refractive index n, the OPL from S to O through a point at height b on the lens is the sum of three contributions: the S-to-lens segment, the extra OPL inside the lens glass, and the lens-to-O segment:

$$\text{OPL}(b) = \sqrt{u^2 + b^2} + (n-1)\,w(b) + \sqrt{v^2 + b^2}.$$

For the ray actually traversed, $d(\text{OPL})/db = 0$.

Step 1. Part (i). Substitute $w(b) = w_0 - b^2/\alpha$:

$$\text{OPL}(b) = \sqrt{u^2 + b^2} + (n-1)(w_0 - b^2/\alpha) + \sqrt{v^2 + b^2}.$$

Step 2. Expand the square roots for paraxial rays ($b \ll u, v$): $\sqrt{u^2 + b^2} \approx u + b^2/(2u)$, similarly for v.

$$\text{OPL}(b) \approx u + \frac{b^2}{2u} + (n-1)w_0 - \frac{(n-1)b^2}{\alpha} + v + \frac{b^2}{2v}.$$

Group the b^2 terms:

$$\text{OPL}(b) \approx \text{const} + b^2\left[\frac{1}{2u} + \frac{1}{2v} - \frac{n-1}{\alpha}\right].$$

Step 3. Fermat's principle: $d(\text{OPL})/db = 0$ for all b. The only way the b^2 coefficient vanishes for every b is

$$\frac{1}{2u} + \frac{1}{2v} = \frac{n-1}{\alpha} \quad \Rightarrow \quad \frac{1}{u} + \frac{1}{v} = \frac{2(n-1)}{\alpha}.$$

Comparison with the thin-lens formula $1/u + 1/v = 1/f$ gives

$$\boxed{f = \frac{\alpha}{2(n-1)}}.$$

Step 4. Part (ii). Substitute $w(b) = k_1 \ln(k_2/b)$:

$$\text{OPL}(b) \approx u + \frac{b^2}{2u} + (n-1)k_1 \ln(k_2/b) + v + \frac{b^2}{2v}.$$

Setting $d(\text{OPL})/db = 0$:

$$\frac{b}{u} + \frac{b}{v} - \frac{(n-1)k_1}{b} = 0,$$

$$b\left(\frac{1}{u} + \frac{1}{v}\right) = \frac{(n-1)k_1}{b},$$

$$b^2 = \frac{(n-1)k_1 \cdot uv}{u+v}.$$

Step 5. The angular radius from the observer is $\beta = b/v$ (small-angle approximation). Squaring,

$$\beta^2 = \frac{b^2}{v^2} = \frac{(n-1)\,k_1\,u}{v(u+v)},$$

and so

$$\beta = \sqrt{\frac{(n-1)\,k_1\,u}{v(u+v)}}.$$

Because the lens is axially symmetric, the locus of solutions is a *ring* (Einstein ring) of angular radius β around the centre.

Final Answer: (i) $f = \dfrac{\alpha}{2(n-1)}$. (ii) $\beta = \sqrt{\dfrac{(n-1)k_1\,u}{v(u+v)}}$.

EXPERT'S SOLUTION *: Aarav Kapoor, Ph.D Physics, IISc Bangalore*

Strategic angle. Fermat reduces the lens problem to an extremisation of OPL over b. For a parabolic lens you recover the thin-lens formula; for a log-profile lens you get the Einstein ring.

Step 1. Write $\text{OPL}(b) = \sqrt{u^2+b^2} + (n-1)w(b) + \sqrt{v^2+b^2}$.

Step 2. Paraxial expand; set $d(\text{OPL})/db = 0$.

Step 3. (i) Parabolic w gives $1/f = 2(n-1)/\alpha$.

Step 4. (ii) Log w gives $b^2 = (n-1)k_1uv/(u+v)$ and $\beta = b/v$.

Step 5. **Why a parabolic lens is special.** The parabolic thickness profile $w(b) = w_0 - b^2/\alpha$ is exactly the shape that focuses paraxial rays to a single point on the axis. Any deviation from parabolic (e.g. a true spherical surface) introduces spherical aberration: rays at different b focus at slightly different points. This is why high-quality lenses are aspheric.

Step 6. **Einstein-ring geometry.** For the log profile, the stationarity condition gives a *ring* of values of b where $d(\text{OPL})/db = 0$, not a single point. Every direction around the symmetry axis is equally "focused", so the image of a point source is a ring of angular radius β. This is exactly the strong gravitational-lensing geometry: a perfectly-aligned source behind a point mass produces a luminous ring image.

Step 7. **Sanity check on part (i).** Compare $1/u + 1/v = 2(n-1)/\alpha$ with the thin-lens equation $1/u + 1/v = 1/f$. Identifying gives $f = \alpha/[2(n-1)]$. Larger α (thicker lens) $\Rightarrow$ longer f. Larger n (denser glass) $\Rightarrow$ shorter f. Both physically reasonable.

Why this matters. The log-profile is exactly the gravitational lens around a point mass; the predicted Einstein ring has been observed for distant galaxies behind closer ones, most famously by the Hubble Space Telescope. Modern surveys (e.g. DES, Euclid) map cosmic dark matter via the statistics of gravitational lensing distortions.

Final Answer: (i) $f = \alpha/[2(n-1)]$. (ii) $\beta = \sqrt{(n-1)k_1 u/[v(u+v)]}$.

♥ Fermat's principle as a unifying tool

Fermat's stationary-time principle reproduces every result of geometric optics: Snell's law (refraction at a flat interface), the thin-lens equation (paraxial expansion of OPL), the mirror formula, dispersion, and gravitational lensing. It is the direct optical analogue of Hamilton's principle in mechanics ($\delta S = 0$).

☞ Paraxial expansion of $\sqrt{u^2 + b^2}$

$\sqrt{u^2 + b^2} \approx u + b^2/(2u)$ for $b \ll u$ – the most commonly used approximation in lens derivations. The neglected term is $-b^4/(8u^3)$, which gives the leading spherical aberration. Memorise this once and you can re-derive thin-lens optics from scratch.

Key Takeaways

- The lens-maker's formula $1/f = (\mu - 1)(1/R_1 - 1/R_2)$ is symmetric: thin lenses have the same f either way around.
- For small prisms the deviation is $D = (\mu - 1)A$; for normal-exit prisms the entry angle is $\theta = A + D$.
- Critical angle $\sin i_c = 1/\mu$. Higher $\mu \Rightarrow$ smaller $i_c \Rightarrow$ TIR easier.
- For a slab of $\mu \geq \sqrt{2}$, every entering ray TIRs at the side faces: full light guiding.
- Convex-mirror image moves non-uniformly: $|dv/du| = f^2/(u-f)^2$ grows as u shrinks.
- Apparent depth of a multi-layer stack: $\sum_i d_i/\mu_i$.
- Displacement method: $u^2 - Du + fD = 0$ gives two lens positions; distance $d = \sqrt{D(D - 4f)}$.
- Telescope: tube length $f_o + f_e$, magnification f_o/f_e, image inverted.
- Gravitational bending of light near a mass: $\alpha = 4GM/(c^2R)$ (Einstein/Eddington).

End of NCERT Exemplar Solutions, Class 12 Physics, Chapter 9