Chapter 9

Ray Optics & Optical Instruments

Light = electromagnetic wave , but for sizes much larger than its wavelength (550 nm) , it travels in straight lines : a ' ray '.

Speed of light

In vacuum : $c = 3 \times 10^8$ m / s (exactly)

Slower in any medium -> refractive index n.

Sign convention used

New Cartesian : pole P at origin , incident light along +x , heights up = +ve.

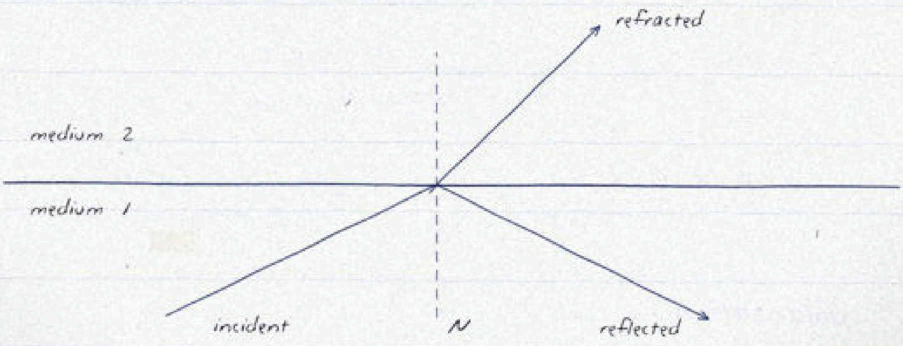

Fig. basic ray geometry at a boundary

Chapter map

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Laws of Reflection

The two laws

① Angle of incidence = angle of reflection :
$i = r$ (measured from normal)

② Incident ray, reflected ray and normal lie in the same plane.

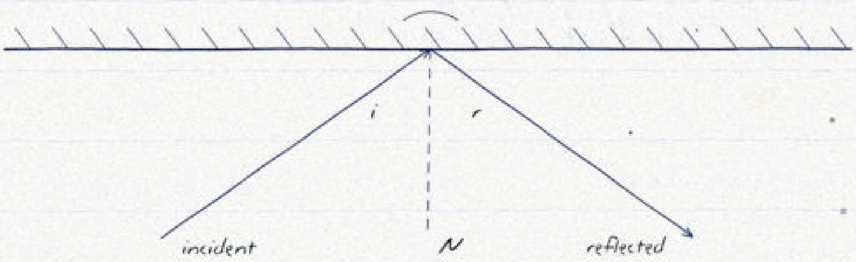

Image by a plane mirror

① Virtual (formed behind the mirror)
② Erect ; same size as object
③ Laterally inverted (left <-> right)
4. Image distance ~~longer~~ = object distance.

Used in periscope, kaleidoscope.

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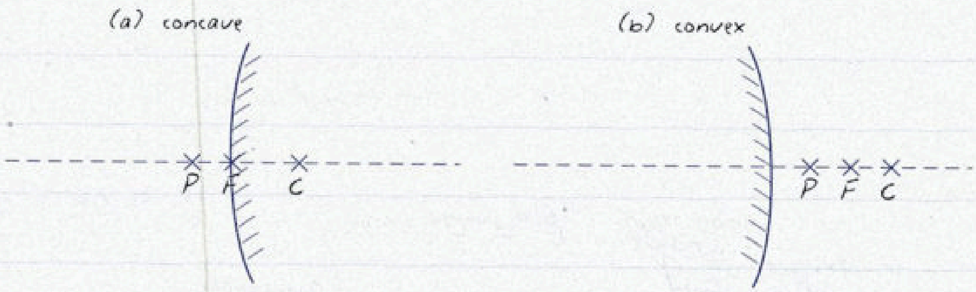
Spherical Mirrors

Types

1. Concave : reflecting side is the INSIDE of the sphere (converging).

2. Convex : reflecting side is the OUTSIDE of the sphere (diverging).



Key terms

1. Pole P : centre of mirror surface.
2. Centre of curvature C : centre of sphere.
3. Radius of curvature $R = PC$.
4. Principal axis : line through P and C.
5. Focus F : midpoint of PC ; $f = R / 2$.

$f > 0$ for concave , $f < 0$ for convex (Cartesian).

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New Cartesian Sign Convention

Rules

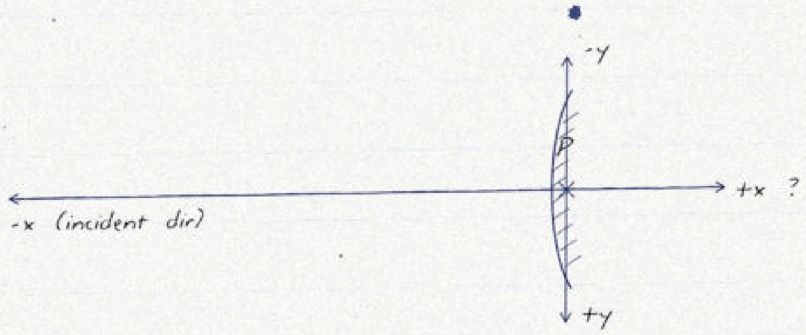
1. Pole P at origin ; principal axis along x.

2. Incident light always from LEFT -> RIGHT.

3. Distances measured FROM pole P.

4. Along incident light = +ve ;
   against incident light = -ve.

5. Heights above axis = +ve ; below = -ve.



Consequences

$u < 0$ for real object (in front of mirror).

$f < 0$ for concave , $f > 0$ for convex.

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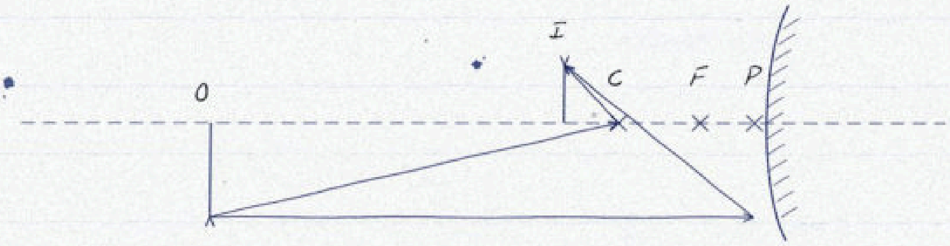
Mirror Formula - Derivation

Concave mirror, small aperture (paraxial).
Object O on axis -> image I on axis.



Using similar triangles

A'B' / AB = PB' / PB (~~FED~~ ABP A'B'P)

A'B' / AB = FB' / FP (FA'B' FED)

Equate the two ratios, substitute the Cartesian signs : u, v, f all measured from P.

$$1/v + 1/u = 1/f$$

<- mirror formula
<- f = R / 2

Valid for both concave and convex mirrors.

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Linear Magnification

Ratio of image height to object height.

$$m = h' / h = -v / u$$ <- for mirrors

Sign of m : what it tells us

1. $m < 0$: image is REAL & inverted.
2. $m > 0$: image is VIRTUAL & erect.
3. $|m| > 1$: magnified ; < 1 diminished.

Useful combined form

Differentiate $1/v + 1/u = 1/f$:

$$dv / du = -v^2 / u^2 = m^2$$

(useful for axial magnification of an object).

Worked check

$u = -30$ cm , $f = -20$ cm -> ?

$1/v = 1/f - 1/u = -1/20 + 1/30 = -1/60$

$v = -60$ cm ; $m = -v/u = -60/30 = -2$

Image : real , inverted , twice the size.

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Concave Mirror - Image Positions

Ray-diagram rules

(1) Ray parallel to axis -> through F.

(2) Ray through F -> parallel to axis.

(3) Ray through C -> retraces its path.

(4) Ray at P -> reflects symmetrically.

Image table

Object	Image	Nature	Size
Infinity	At F	Real , inv.	Highly
Beyond C	Between F,C	Real , inv.	Diminished
At C	At C	Real , inv.	Same
Between C,F	Beyond C	Real , inv.	Magnified
At F	Infinity	Real , inv.	Highly
Between F,P	Behind mirror	Virtual , erect	Magnified

Convex mirror

Always gives a virtual , erect , diminished image for a real object -> wide field of view.

Used as rear - view , road - safety mirrors.

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Refraction & Snell's Law

Observation

Light bends when it crosses a boundary between two media of different optical density.

Snell's Law

$$n_1 \sin i = n_2 \sin r$$

<- i , r from normal

Equivalently : $_1n_2 = \sin r / \sin i$

Refractive Index

$$n = c / v$$

<- v = speed in
<- the medium

Also : $n = \lambda_{vac} / \lambda_{med}$

(frequency does NOT change in refraction)

Optical density

Higher n => optically DENSER medium.

e.g. ~~water > glass~~ glass > water > air

Mass density and optical density are not same.

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Refraction Through a Glass Slab

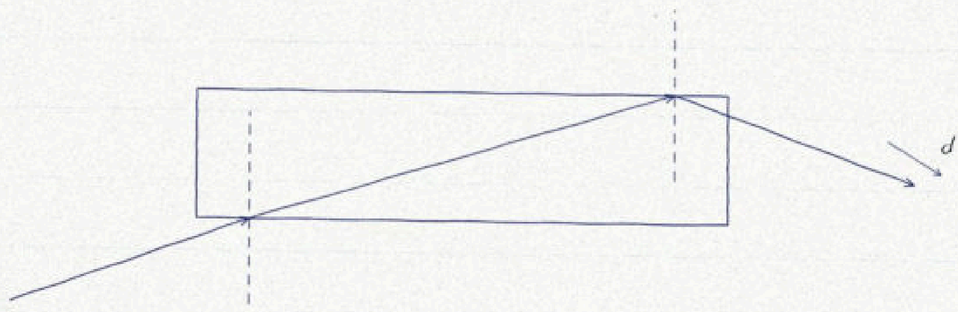
Light entering a parallel - faced slab :

1. Bends towards normal on entry.

2. Bends away from normal on exit.

3. Emergent ray is PARALLEL to incident ray.

4. Net effect : a lateral shift d.



Lateral shift formula

$d = t \sin(i - r) / \cos r$

t = slab thickness. d larger for greater i.

Apparent depth = real depth / n.

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Apparent Depth & Lateral Shift

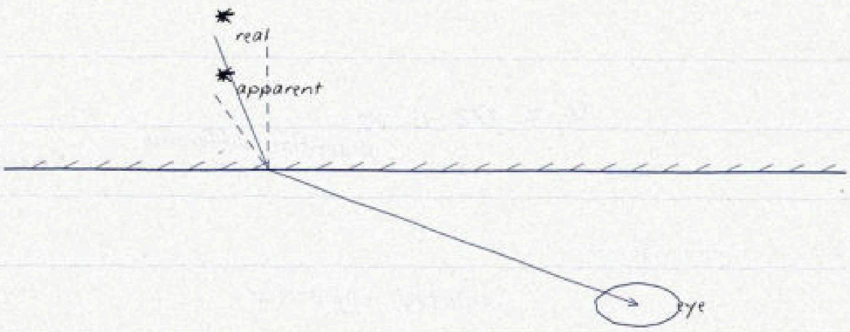
Looking into water

An object at depth h appears to be at h' :

$$h' = h / n$$

<- n = water / air

Apparent shift = $h - h' = h(1 - 1/n)$.



Reason for twinkling of stars

Atmospheric refractive index keeps varying with temperature -> apparent position fluctuates. Planets, having a disc, do NOT twinkle.

Advance sunrise / delayed sunset

Sun visible 2 min before / after horizon.

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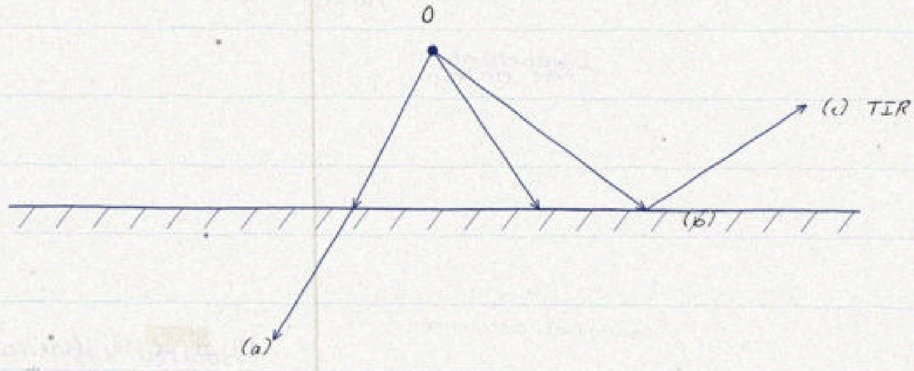
Total Internal Reflection

Setup

Light going from a denser to a rarer medium : ray bends AWAY from normal.

Increase i -> r grows faster -> at some $i = i_c$, $r = 90$ (ray grazes surface).

For $i > i_c$: no refraction -> total internal reflection (TIR).



Critical angle relation

$$\sin i_c = 1 / n$$

<- n = denser / rarer

For water -> air : i_c 48.6 ; glass -> air : 42.

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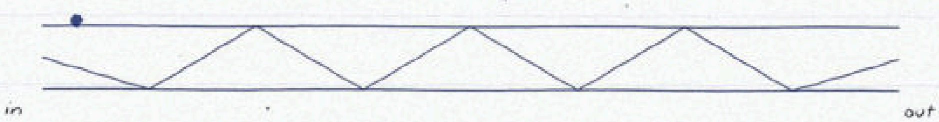
TIR - Applications

① Mirage (hot road / desert)

Hot air layers near ground are rarer ;
rays from sky bend up -> appear reflected.

② Optical fibre

Glass core (high n) clad with lower n.
Light entering core undergoes TIR repeatedly ,
guided along the fibre with minimal loss.



③ Prism for total internal reflection

Right - angled prism (45 - 45 - 90) acts as
100% reflector in binoculars , periscopes.

④ Brilliance of diamond

$n = 2.42$ (very high) -> i_c 24 degrees.
Light suffers many TIRs before emerging.

⑤ Endoscopy , communication via fibre bundles.

Bends in fibre have radius >> diameter.

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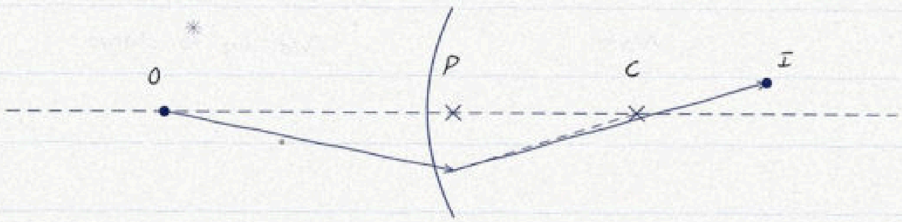
Refraction at Single Spherical Surface

Spherical surface separating media of n1 & n2.



Derivation outline

Small angles -> Snell's law $n1\, i = n2\, r$.

From triangle geometry :

$$i = \alpha + \theta$$

$$r = \theta - \phi$$

Use alpha = h / (-u) , theta = h / R , phi = h / v and Cartesian signs :

$$n_2/v - n/u = (n_2 - n) / R$$

Single - surface refraction formula.

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Lens Maker's Formula - Derivation

A thin lens has TWO refracting surfaces.
Apply the single - surface result twice.

Surface 1 (air -> glass)

$n / v_1 - 1 / u = (n - 1) / R_1$

Surface 2 (glass -> air)

Treat I_1 as object for second surface :

$1 / v - n / v_1 = (1 - n) / R_2$

Add the two equations

$1 / v - 1 / u = (n - 1) (1/R_1 - 1/R_2)$

When object at infinity , image at focus :

$$1 / f = (n - 1) (1/R_1 - 1/R_2)$$

<- lens <- maker

Sign conventions for R

R is +ve if centre of curvature is on the outgoing side ; -ve if on the incoming side.

Biconvex : $R1 > 0$, $R2 < 0$ -> $f > 0$.

Biconcave : $R1 < 0$, $R2 > 0$ -> $f < 0$.

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Thin Lens Formula

Derived directly from the previous result :

$$1/v - 1/u = 1/f$$ <- thin lens <- formula

Linear magnification

$$m = h'/h = v/u$$ <- for lenses

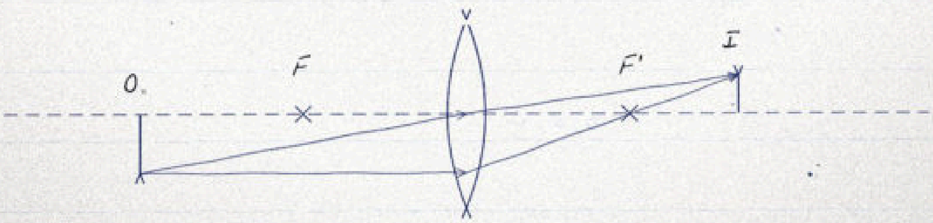
Note : for MIRRORS $m = -v/u$. (note sign).

Ray rules for lenses

(1) Ray parallel to axis -> through F.

(2) Ray through optical centre -> undeviated.

(3) Ray through F -> emerges parallel.



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Power of a Lens

Definition

A measure of converging / diverging ability.

$$P = 1 / f \quad (f \text{ in metres})$$

<- unit : diopter
<- 1 D = 1 / m

Sign

Convex (converging) lens : $P > 0$.

Concave (diverging) lens : $P < 0$.

Combination of thin lenses in contact

$$1 / F = 1 / f_1 + 1 / f_2 + \dots$$

$$P = P_1 + P_2 + \dots$$

<- powers add

Equivalent magnification

$m = m_1 \cdot m_2 \cdot m_3 \dots$

Example : +5 D and -2 D combination

$P = +5 - 2 = +3 \text{ D} \rightarrow F = 33.3 \text{ cm}$

Net effect : converging.

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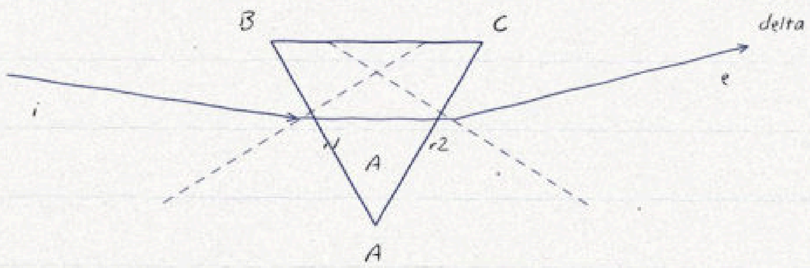
Refraction Through a Prism

Setup

Prism = glass block bounded by two refracting surfaces meeting at angle A (refracting angle).



Geometry results

$$r_1 + r_2 = A$$

$$\text{delta} = (i + e) - A$$

For a small - angle prism

$\sin i \quad i$, $\sin r \quad r$

$$\text{delta} = (n - 1) A$$

(n = refractive index of prism material)

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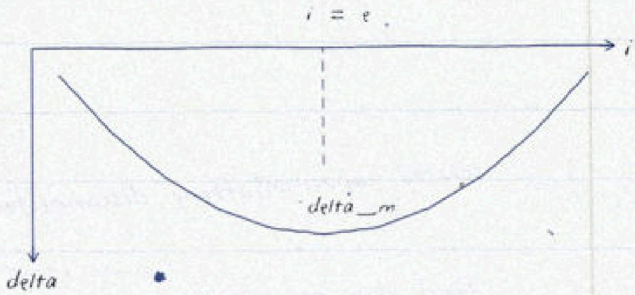
Minimum Deviation & Prism Formula

Observation

Plot delta vs i : has a unique minimum
at i = e -> r1 = r2 = A / 2.



Snell's law at minimum deviation

$i = (A + \delta_m) / 2$

$r = A / 2$

$$n = \sin[(A + \delta_m)/2] \; / \; \sin(A/2)$$

<- prism

Used in the lab to measure n of glass.

Symmetric path : ray inside is parallel to BC.

At delta_min , ray inside is ~~perpendicular~~ parallel to base.

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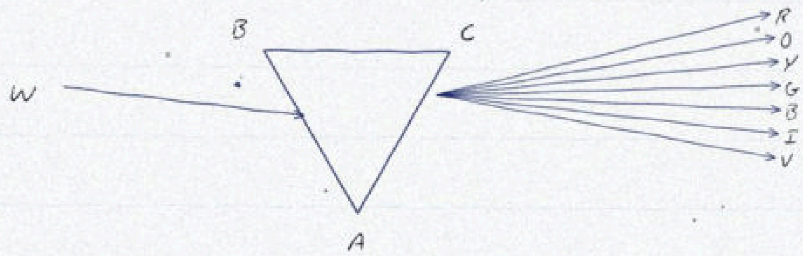
Dispersion of White Light

Refractive index n depends on wavelength :

$$n = A + B / \lambda^2 \quad \text{(Cauchy, empirical)}$$

So delta = (n - 1) A is wavelength dependent. White light passing through a prism splits into VIBGYOR (violet deviates most, red least).



Angular dispersion

$$\delta_v - \delta_r = (n_v - n_r) A$$

Dispersive power omega

$$\omega = (n_v - n_r) / (n - 1)$$

(n = mean refractive index)

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Rainbow & Scattering

Rainbow (qualitative)

① Primary rainbow

Sunlight enters drop -> refraction,
ONE internal reflection -> refraction out.
Red on top, violet at bottom (40 - 42 deg).

② Secondary rainbow

TWO internal reflections -> fainter, reverse colour order (red below, violet on top).

Scattering of light - Rayleigh

I (scattered) $1 / \lambda^4$

<- Rayleigh law

Short wavelengths scatter much more than long.

① Blue sky : blue light scatters most.

② Red sunrise / sunset : long path through atmosphere, blue scattered away, red passes.

③ White clouds : water drops are LARGE - Tyndall / Mie scattering, all colours equally.

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The Human Eye

Anatomy

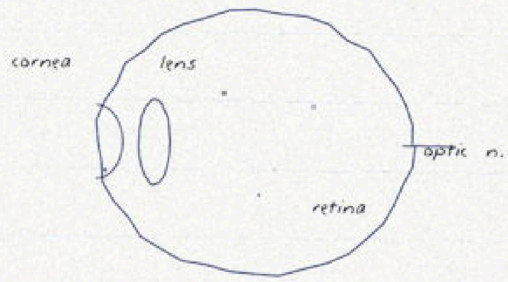
(1) Cornea : transparent front surface ; bulk of bending (two - thirds).

(2) Iris & pupil : control light intensity.

(3) Lens (crystalline) : fine - tunes focus by ciliary muscle action (accommodation).

(4) Retina : light - sensitive screen ; rods + cones.

(5) Optic nerve -> brain.



Key terms

Near point : D = 25 cm (least distance of distinct vision , normal adult).

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Defects of Vision

1. Myopia (short - sight)

Distant objects blurred ; far point < infinity.

Image of far object falls IN FRONT of retina.

Correction : concave lens of focal length

$$f \doteq -x \quad (x = \text{far point distance})$$

2. Hypermetropia (far - sight)

Near objects blurred ; near point > 25 cm.

Image of near object falls BEHIND retina.

Correction : convex lens with

$$1 / f = 1 / D - 1 / d \quad (d = \text{near point})$$

3. Presbyopia

Age - related loss of accommodation (lens hardens).

Often need BIFOCALS (concave + convex).

4. Astigmatism

Cornea curvature uneven -> different focus for vertical and horizontal lines.

Corrected with CYLINDRICAL lenses.

(Power required = corrective P in diopters.)

Lens prescriptions specify P not concave

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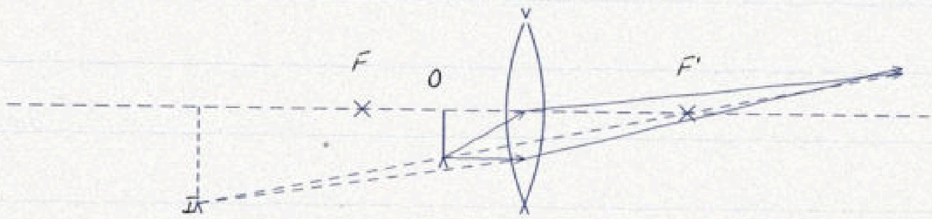
Simple Microscope (Magnifier)

A single convex lens of short focal length.
Object placed within F -> virtual, erect, magnified image on same side as object.



Magnifying power

M = angle (with lens) / angle (without lens)

$$M = 1 + D / f \quad \text{(image at D)}$$

<- max M (relaxed
<- but strained eye)

$$M = D / f \quad \text{(image at infinity)}$$

<- fully relaxed
<- (normal use)

D = 25 cm = least distance of distinct vision.

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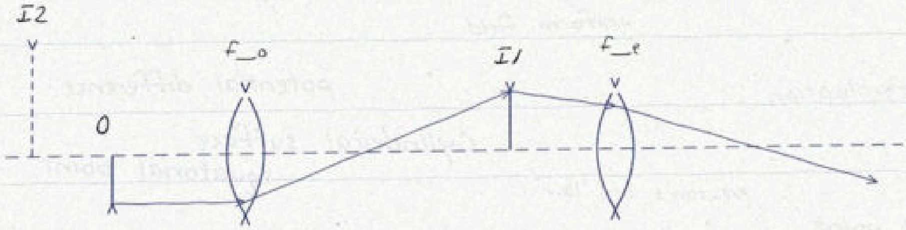
Compound Microscope

Construction

Two convex lenses arranged coaxially :

(1) Objective (small f_o , small aperture).

(2) Eyepiece (slightly larger f_e , acts as a simple microscope).



Magnifying power

$M = m_o \cdot m_e = (v_o/u_o)(1 + D/f_e)$

$$M \quad - (L / f_o)(1 + D / f_e)$$

<- image

L = tube length (distance between F_o and F_e)

Both f_o and f_e small -> large M (500 - 1000).

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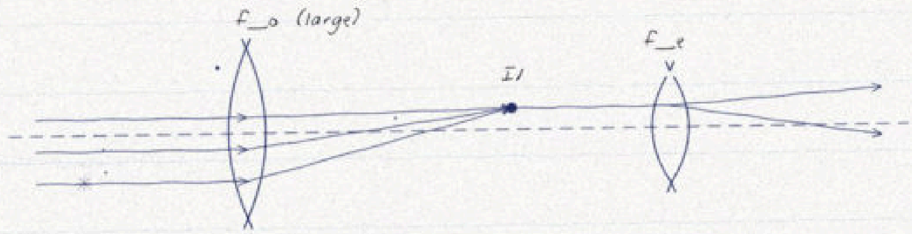
Astronomical Telescope

Construction

Two coaxial convex lenses :

① Objective : LARGE f_o , large aperture (more light + better resolution).

② Eyepiece : small f_e , acts as magnifier.



Normal adjustment

Final image at infinity -> least eye strain.

$$M = - f_o / f_e$$

<- normal <- adjustment

Tube length $L = f_o + f_e$.

For image at D : $M = - (f_o / f_e)(1 + f_e / D)$.

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Reflecting Telescope (Cassegrain)

Idea

Replace the heavy objective LENS with a concave PARABOLIC mirror.

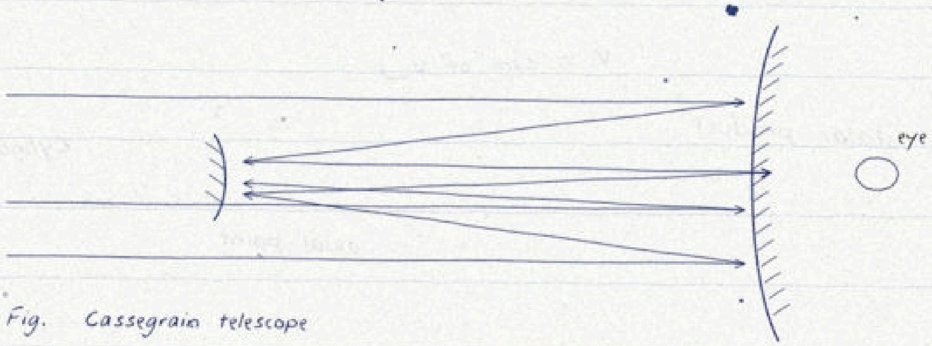

Fig. Cassegrain telescope

Advantages over refractor

1. No chromatic aberration (mirrors, not lenses).

2. Easier to make large mirrors than large lenses.

3. Lighter; can be supported from behind.
4. Higher resolving power for same aperture.

Hubble, JWST -> reflecting telescopes.

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Worked Example - Concave Mirror

Problem : An object is placed 18 cm in front of a concave mirror of focal length 12 cm. Find image position, nature and size if object height = 2 cm.

Sign convention

$u = -18$ cm , $f = -12$ cm , $h = +2$ cm.

Mirror formula

$1/v + 1/u = 1/f$

$1/v = 1/f - 1/u = -1/12 - (-1/18)$

$= -1/12 + 1/18 = (-3 + 2)/36$

$= -1/36$

$v = -36$ cm (in front of mirror)

Magnification & image height

$m = -v / u = -(-36) / (-18) = -2$

$h' = m h = -2 . 2 = -4$ cm

Conclusion

Image at 36 cm , real & inverted , height 4 cm (twice the size of object , on the same side).

Check : $v > u$ ~~AND~~ → magnified

(object between C and F gives such an image.)

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Worked Example - Lens

Problem : A biconvex lens of glass ($n = 1.5$) has $R1 = R2 = 20$ cm.
(a) Find f. (b) An object 30 cm in front - find image position & magnification.

(a) Lens maker's formula

$1/f = (n - 1)(1/R_1 - 1/R_2)$

$R1 = +20$ cm, $R2 = -20$ cm (biconvex)

$1/f = 0.5 \, (1/20 - (-1/20))$

$= 0.5 \cdot (2/20) = 0.05 \text{ cm}^{-1}$

$f = +20$ cm (converging)

(b) Image position

$u = -30$ cm, $f = +20$ cm.

$1/v - 1/u = 1/f$

$1/v = 1/f + 1/u = 1/20 - 1/30$

$= (3 - 2)/60 = 1/60$

$v = +60$ cm (real, opposite side)

Magnification

$m = v / u = 60 / (-30) = -2$

Image : real, inverted, twice the height.

Power : $P = 1 / f = 1 / 0.2 = 5$ D.

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Worked Example - Prism

Problem : Refracting angle $A = 60$.
Angle of minimum deviation delta_m = 30.
Find (a) refractive index of prism material ;
(b) angle of incidence at delta_m.

(a) Refractive index

$n = \sin[(A + \delta_m)/2] / \sin(A/2)$
$= \sin(45) / \sin(30)$
$= (1/\sqrt{2}) / (1/2)$
$= \sqrt{2}$ 1.414

(b) Angle of incidence

At minimum deviation : $i = e$
$2i = A + \delta_m = 60 + 30 = 90$
$i = 45$

Inside the prism : $r = A/2 = 30$.
Check : $\sin i / \sin r = \sin 45 / \sin 30$
$= (1/\sqrt{2})/(1/2) = \sqrt{2}$ OK

Dispersive remark

If white light : V and R deviate differently.
$\delta_v - \delta_r = (n_v - n_r) A$
n_v > n_r -> violet deviates more.

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Worked Example - Microscope

Problem : Compound microscope

$f_o = 2.0$ cm , $f_e = 6.25$ cm , tube $L = 15$ cm.

Object placed 2.5 cm from objective.

Image at $D = 25$ cm. Find M.

Step 1 : image by objective

$u_o = -2.5$ cm , $f_o = +2.0$ cm.

$1/v_o = 1/f_o + 1/u_o = 1/2 - 1/2.5$

$= (2.5 - 2)/5 = 0.1$

$v_o = +10$ cm

$m_o = v_o / u_o = 10 / -2.5 = -4$

Step 2 : image at D from eyepiece

Eye - piece object distance u_e :

$1/-D = 1/u_e + 1/f_e$

$u_e = -5$ cm (computed)

$m_e = 1 + D / f_e = 1 + 25/6.25 = 5$

Total magnification

$M = m_o \cdot m_e = (-4)(5) = -20$

Negative sign -> final image inverted.

$|M| = 20$ -> object appears 20 x larger.

(Larger f_o would reduce M ; smaller would help.)

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Aberrations of Lenses & Mirrors

1. Spherical aberration

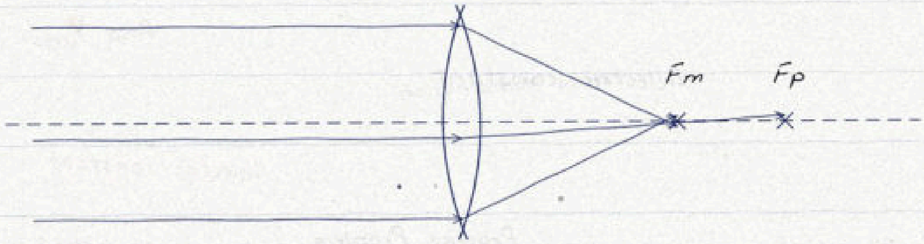
Marginal rays focus at a point DIFFERENT from paraxial rays -> blurred image.

Remedies : small aperture , parabolic mirror , stops , doublet lenses.

2. Chromatic aberration (lenses only)

Refractive index depends on wavelength -> different colours focus at different points.

Cured by an achromatic doublet (crown + flint).



Comparing the two

Spherical : geometric ; ~~colours~~ all colours.

Chromatic : wavelength dependent ; lens only.

Mirrors have NO chromatic aberration.

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Resolving Power

Telescope

Ability to see two close objects as separate.

$$d\theta = 1.22 \lambda / D$$

<- D = aperture

Larger D -> smaller d theta -> better resolution.

Microscope

$$d = \lambda / (2 n \sin \theta)$$

<- Abbe
<- criterion

n sin theta = numerical aperture (NA).

Oil immersion increases n -> better RP.

Magnifying power vs resolving power

High M but low RP -> bigger blob, still blurry.

Practical limit set by RP, not M.

Quick numbers

Naked eye 1 arc minute 3×10^{-4} rad.

100 m telescope 10^{-8} rad at lambda = 550 nm.

Electron microscope -> fraction of a nm.

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Useful Facts to Remember

Numbers worth memorising

1. Speed of light $c = 3 \times 10^{8}$ m / s.
2. Wavelength range : 400 - 700 nm (visible).
3. n : water 1.33 , glass 1.5 , diamond 2.42.
4. i_c : water 48.6 , glass 42 , diamond 24 .
5. Near point $D = 25$ cm (adult eye).
6. Power in $D = 1 / f$ (in m).

Common sign pitfalls

1. Mirror : $m = - v / u$ Lens : $m = v / u$.
2. Concave mirror has $f < 0$ (Cartesian).
3. Biconvex lens has $R1 > 0$, $R2 < 0$.
4. u is - ve for real object on left of mirror.

Concept ties

TIR + Rayleigh + dispersion -> rainbow.

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Formula Summary - I

Mirrors & sign

$1/v + 1/u = 1/f$; $m = - v/u$

$f = R / 2$

Refraction (single surface)

$n_2/v - n/u = (n_2 - n)/R$

Snell + TIR

$n_1 \sin i = n_2 \sin r$

$\sin i_c = 1 / n$

Apparent depth

$h' = h / n$; shift $= h (1 - 1/n)$

Lens maker & lens formula

$1/f = (n - 1)(1/R_1 - 1/R_2)$

$1/v - 1/u = 1/f$; $m = v / u$

Combination of lenses

$1/F = 1/f_1 + 1/f_2$; $P = P_1 + P_2$

Prism

$r_1 + r_2 = A$; delta $= i + e - A$

$n = \sin[(A + d_m)/2] / \sin (A/2)$

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Formula Summary - II

Dispersion & scattering

$\delta_v - \delta_r = (n_v - n_r) A$

$\omega = (n_v - n_r) / (n - 1)$

I (scatt.) $1 / \lambda^4$

Eye corrections

Myopia $f = -x$ (x = far pt)

Hyperm. $1/f = 1/D - 1/d$ (d = near pt)

Simple microscope

$M = 1 + D / f$ (image at D)

$M = D / f$ (image at infinity)

Compound microscope

$M = m_o \cdot m_e$

$M = -(L / f_o)(1 + D / f_e)$

Astronomical telescope

$M = -f_o / f_e$ (normal)

$M = -(f_o/f_e)(1 + f_e/D)$ (at D)

$L = f_o + f_e$

Resolving power

$d\theta = 1.22 \lambda / D$; $d = \lambda / (2 n \sin\theta)$

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Final Recap & Conceptual Tips

Big - picture flow

Reflection (plane, spherical)
-> refraction (Snell, slab, depth)
-> TIR (critical, fibres, diamond)
-> single surface -> thin lens
-> prism + dispersion -> scattering
-> eye + defects -> optical instruments.

Common student errors

1. Mixing up mirror vs lens magnification sign.

2. Forgetting that ~~v~~ - sign of u for real objects.

3. Using degrees / radians inconsistently.

4. Forgetting that f in $P = 1/f$ must be in metres.

5. Mixing R1, R2 sign in lens maker.

Strategy for problems

1. Always draw a small ray diagram first.
2. Apply sign convention, write u, v, f.
3. Substitute in the right formula.
4. Check sign / magnitude - cross-verify.

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End of Chapter 9

What we learned

1. How mirrors and lenses form images.
2. Sign convention and standard formulas.
3. Refraction, TIR - why diamonds shine.
4. Prism behaviour ; n and minimum deviation.
5. Dispersion, rainbow, Rayleigh scattering.
6. The eye and common vision defects.
7. Microscope (simple, compound).
8. Astronomical & Cassegrain telescopes.

Next chapter preview

Chapter 10 will tackle WAVE optics : Huygens' principle, interference, diffraction, polarisation - the wave nature of light.

Practice : NCERT Examples 9.1 - 9.12, Exercises 9.1 - 9.36 ; work all of them.

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