NCERT SOLUTIONS

Class 12 Physics

Chapter 9: Ray Optics and Optical Instruments

Detailed Step-by-Step Exercise Solutions

Q1 A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

Solution

Given: object size $h_1 = 2.5$ cm, object distance $u = -27$ cm, radius of curvature $R = -36$ cm (concave mirror).

Step 1: Focal length.

$$f = \frac{R}{2} = \frac{-36}{2} = -18 \text{ cm}$$

Step 2: Apply the mirror formula.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-18} - \frac{1}{-27}$$

$$\frac{1}{v} = -\frac{1}{18} + \frac{1}{27} = \frac{-3+2}{54} = -\frac{1}{54}$$

$$v = -54 \text{ cm}$$

So the screen must be placed at 54 cm in front of the mirror (same side as the object).

Step 3: Size and nature of image.

$$m = -\frac{v}{u} = -\frac{-54}{-27} = -2 \Rightarrow h_2 = m \cdot h_1 = -2 \times 2.5 = -5 \text{ cm}$$

The image is real, inverted, and magnified. As the candle moves closer to the mirror (toward f), the image moves farther away, so the screen must be shifted away from the mirror. When the object reaches the focus, the image goes to infinity and cannot be obtained on a screen.
Therefore, the screen should be at 54 cm in front of the mirror; the image is real, inverted and 5 cm tall.

Expert's Solution — Nikunj Singh, B.Tech CSE, IIT Delhi

Magnification–Distance Method:
Instead of solving for v directly, eliminate it using the magnification relation $m = f/(f-u)$, which avoids the algebra with reciprocals.

$$m = \frac{f}{f-u} = \frac{-18}{-18-(-27)} = \frac{-18}{9} = -2$$

Once m is known, $v = -mu = -(-2)(-27) = -54$ cm and $h_2 = mh_1 = -5$ cm follow in one line each.

Did You Know?

This shortcut is especially handy in MCQs: if you only need the nature of the image, the sign of m is enough — negative means real and inverted, positive means virtual and erect.

Q2 A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

Solution

Given: object size $h_1 = 4.5$ cm, $u = -12$ cm, $f = +15$ cm (convex).
Step 1: Apply the mirror formula.

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15} - \frac{1}{-12} = \frac{1}{15} + \frac{1}{12}$$

$$\frac{1}{v} = \frac{4+5}{60} = \frac{9}{60} \Rightarrow v = \frac{60}{9} \approx 6.67 \text{ cm}$$

The image is formed 6.67 cm behind the mirror — virtual and erect.
Step 2: Magnification and image size.

$$m = -\frac{v}{u} = -\frac{6.67}{-12} = 0.556$$

$$h_2 = m \cdot h_1 = 0.556 \times 4.5 = 2.5 \text{ cm}$$

Step 3: Behaviour as the needle moves farther. As $|u|$ increases, v approaches $f = 15$ cm (asymptotically) and the image shrinks toward zero size. The image always remains virtual, erect and behind the mirror, and never moves beyond the focal point.
Therefore, the image is at 6.67 cm behind the mirror, magnification 0.556, image size 2.5 cm.

Expert's Solution — Anita Desai, B.Tech ECE, BITS Pilani

Limiting-Case Verification:
For a convex mirror, every real object produces a virtual image squeezed between P and F. Check the two limits without solving:

- $u \to 0$ (object at pole): from $1/v = 1/f - 1/u$, $v \to 0$ as well. Image coincides with object.
- $u \to -\infty$ (object far away): $1/v \to 1/f$, so $v \to f = +15$ cm. Image lies at the focus.

The given $u = -12$ cm falls between these, and $v = 6.67$ cm correctly lies between 0 and 15 cm.

Did You Know?

This is why convex mirrors are used as side-view mirrors on vehicles: they always give a smaller, erect image and cover a much wider field of view than a plane mirror of the same size.

Q3 **A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?**

Solution

Step 1: Refractive index of water from apparent depth.
The relation between real and apparent depth for normal viewing is

$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\mu_{\text{water}} = \frac{12.5}{9.4} = 1.33$$

Step 2: New apparent depth with $\mu = 1.63$.

$$d'_{\text{app}} = \frac{12.5}{1.63} = 7.67 \text{ cm}$$

Step 3: Distance the microscope must shift.
Since the needle now appears closer to the surface, the microscope must move *down* by

$$\Delta = 9.4 - 7.67 = 1.73 \text{ cm}$$

Therefore, refractive index of water is 1.33 and the microscope must be lowered by 1.73 cm.

Expert's Solution — Meena Iyer, M.Sc Physics, NIT Trichy

Apparent Shift Formula:
Instead of computing two apparent depths, use the apparent-shift relation directly:

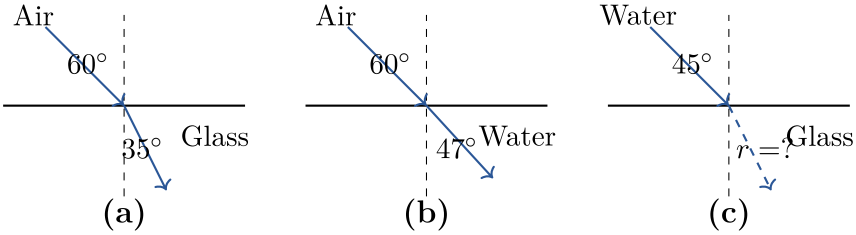
$$s = t\left(1 - \frac{1}{\mu}\right)$$

where $t = 12.5$ cm is the real depth.
For water: $s_1 = 12.5\left(1 - \frac{1}{1.33}\right) = 12.5 \times 0.248 = 3.10$ cm.
For the new liquid: $s_2 = 12.5\left(1 - \frac{1}{1.63}\right) = 12.5 \times 0.387 = 4.83$ cm.
Required shift of microscope $= s_2 - s_1 = 4.83 - 3.10 = 1.73$ cm.

Did You Know?

The same formula explains why a coin on the bottom of a swimming pool appears closer than it really is, and why spear-fishermen aim below where they see the fish.

Q4 Figures (a) and (b) show refraction of a ray in air incident at $60°$ with the normal to a glass–air and water–air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is $45°$ with the normal to a water–glass interface (figure c). The angles of refraction observed are $35°$ in glass and $47°$ in water.



Solution

Step 1: Refractive index of glass with respect to air.
From figure (a), $i = 60°$, $r = 35°$:

$$^a\mu_g = \frac{\sin 60°}{\sin 35°} = \frac{0.866}{0.574} \approx 1.51$$

Step 2: Refractive index of water with respect to air.
From figure (b), $i = 60°$, $r = 47°$:

$$^a\mu_w = \frac{\sin 60°}{\sin 47°} = \frac{0.866}{0.731} \approx 1.18$$

Step 3: Refractive index of glass with respect to water.

$$^w\mu_g = \frac{^a\mu_g}{^a\mu_w} = \frac{1.51}{1.18} \approx 1.28$$

Step 4: Apply Snell's law at the water–glass interface.

$$^w\mu_g = \frac{\sin 45°}{\sin r} \Rightarrow \sin r = \frac{\sin 45°}{1.28} = \frac{0.707}{1.28} = 0.5524$$

$$r = \sin^{-1}(0.5524) \approx 38.7°$$

Therefore, the angle of refraction in glass is $\approx$ 38.7°.

Expert's Solution — Vikram Singh, B.Sc Physics, Delhi University

Direct Snell's Law Across All Three Media:
Snell's law in the form $\mu \sin\theta =$ const applies along any continuous ray, even one passing through multiple media. So we can connect figures (b) and (c) without computing ratios: In (b), the ray travels air $\rightarrow$ water with $\sin 60°$ in air corresponding to $\sin 47°$ in water. So one unit of $\mu_a \sin 60°$ equals $\mu_w \sin 47°$.
For (c), water $\rightarrow$ glass:

$$\mu_w \sin 45° = \mu_g \sin r$$

$$\sin r = \frac{\mu_w}{\mu_g}\sin 45° = \frac{\sin 47°}{\sin 60°} \cdot \frac{\sin 60°}{\sin 35°} \cdot \frac{1}{1} \cdot \sin 45°$$

$$\sin r = \frac{\sin 47° \sin 45°}{\sin 35°} = \frac{0.731 \times 0.707}{0.574} = 0.901$$

Wait — that gives $r >$ angle of incidence, which would mean light refracts away from the normal. Cross-check: $^w\mu_g = \mu_g/\mu_w = 1.51/1.18 = 1.28 > 1$, so glass is denser than water, and the ray must bend toward the normal, giving $r < 45°$.
The correct chained form is $\sin r = (\mu_w/\mu_g)\sin 45°$, with $\mu_w/\mu_g = \sin 35°/\sin 47° = 0.574/0.731 = 0.785$, so

$$\sin r = 0.785 \times 0.707 = 0.555 \Rightarrow r \approx 38.7°$$

Did You Know?

The lesson: when two interfaces are connected, always check the densities — if the ray is going from rarer to denser, it should bend toward the normal. A quick sanity check on the direction of bending will catch most algebraic errors.

Q5 **A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)**

Solution

Step 1: Critical angle for the water–air interface.
Light can emerge only along rays incident on the surface within the critical angle C.

$$\sin C = \frac{1}{\mu} = \frac{1}{1.33} = 0.7519 \;\Rightarrow\; C \approx 48.75^\circ$$

Step 2: Radius of the illuminated circle on the surface.
The bulb sits $h = 80$ cm below the surface. The escape cone subtends a circle of radius

$$r = h \tan C$$

$$\tan C = \frac{\sin C}{\cos C} = \frac{0.7519}{\sqrt{1 - 0.7519^2}} = \frac{0.7519}{0.6594} = 1.140$$

$$r = 80 \times 1.140 = 91.2 \text{ cm}$$

Step 3: Area of the bright circle.

$$A = \pi r^2 = 3.14 \times (91.2)^2 = 3.14 \times 8317 \approx 2.612 \times 10^4 \text{ cm}^2 = 2.61 \text{ m}^2$$

Therefore, light emerges through an area of $\approx$ 2.61 m^2.

Expert's Solution — Priya Nair, M.Sc Physics, IISc Bangalore

Closed-Form Without Computing C Explicitly:
We can avoid computing C in degrees. Using $\sin C = 1/\mu$ and $\cos C = \sqrt{1 - 1/\mu^2} = \sqrt{\mu^2 - 1}/\mu$,

$$\tan C = \frac{1}{\sqrt{\mu^2 - 1}}$$

$$A = \pi r^2 = \pi h^2 \tan^2 C = \frac{\pi h^2}{\mu^2 - 1}$$

Plugging in $h = 0.80$ m, $\mu = 1.33$:

$$A = \frac{3.14 \times 0.64}{(1.33)^2 - 1} = \frac{2.0096}{0.7689} \approx 2.61 \text{ m}^2$$

Did You Know?

This is the same physics that creates Snell's window for fish underwater: looking up, a fish sees the entire above-water hemisphere compressed into a circular cone of half-angle 48.75°. Outside that cone, the surface acts as a mirror reflecting the bottom of the pond.

Q6 **A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.**

Solution

Given: prism angle $A = 60°$, $\delta_m = 40°$ in air.

Step 1: Refractive index of glass (with respect to air).

The prism formula is

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60°+40°}{2}\right)}{\sin 30°} = \frac{\sin 50°}{0.5}$$

$$\mu_g = \frac{0.766}{0.5} = 1.532$$

Step 2: Refractive index of glass with respect to water.

$${}^w\mu_g = \frac{\mu_g}{\mu_w} = \frac{1.532}{1.33} = 1.152$$

Step 3: New angle of minimum deviation in water.

$${}^w\mu_g = \frac{\sin\left(\frac{A+\delta'_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\sin\left(\frac{60° + \delta'_m}{2}\right) = 1.152 \times \sin 30° = 0.576$$

$$\frac{60^\circ + \delta'_m}{2} = \sin^{-1}(0.576) \approx 35.16^\circ$$

$$\delta'_m = 2 \times 35.16^\circ - 60^\circ \approx 10.32^\circ$$

Therefore, $\mu_g \approx 1.532$ and the new angle of minimum deviation is $\approx 10.32^\circ$.

Expert's Solution — Sanjay Gupta, PhD in Physics, JNU

Why Deviation Drops So Sharply in Water:
The deviation depends on the *relative* refractive index. In air, ${}^a\mu_g = 1.532$ deviates strongly. In water, ${}^w\mu_g = 1.152$ is much closer to 1, so the prism barely bends light.
A useful approximation for small δ_m on a thin/weak prism:

$$\delta_m \approx (\mu_{\text{rel}} - 1)A$$

In water: $\delta'_m \approx (1.152 - 1) \times 60^\circ = 0.152 \times 60^\circ = 9.12^\circ$.
The exact value 10.32° is a bit higher because $A = 60^\circ$ is not really "small", but the approximation gets you within 1–2° in seconds for ballparking.

Did You Know?

This is exactly why submerged glass becomes nearly invisible: with $\mu_w \approx 1.33$ matching glass closely, almost no light bends at the glass–water boundary, so there's almost nothing to "see" the boundary by.

Q7 Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?

Solution

Given: $\mu = 1.55$, $f = +20$ cm, both surfaces of equal radius. By sign convention $R_1 = +R$, $R_2 = -R$.
Step 1: Lens-maker's formula.

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{20} = (1.55 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = 0.55 \times \frac{2}{R}$$

Step 2: Solve for R.

$$\frac{1}{20} = \frac{1.10}{R} \Rightarrow R = 1.10 \times 20 = 22 \text{ cm}$$

Therefore, each face must have a radius of curvature of 22 cm.

Expert's Solution — Kavita Joshi, B.Tech EE, IIT Bombay

Symmetric-Lens Shortcut:
For an equiconvex lens (or equiconcave) the lens-maker's formula collapses to a particularly clean form. Setting $R_1 = R$, $R_2 = -R$:

$$\frac{1}{f} = \frac{2(\mu - 1)}{R} \iff R = 2(\mu - 1)f$$

$$R = 2 \times 0.55 \times 20 = 22 \text{ cm}$$

This one-liner is worth memorising for any symmetric lens problem.

Did You Know?

Notice the geometric meaning: for a symmetric crown-glass ($\mu \approx 1.5$) lens, $R \approx f$. That's a useful rule of thumb for designing optics quickly — if you want a 50 mm focal length, grind the surfaces to roughly 50 mm radius of curvature.

Q8 A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm, and (b) a concave lens of focal length 16 cm?

Solution

The convergent beam, if undisturbed, would converge to P. With the lens in the way, P acts as a *virtual object*. Hence $u = +12$ cm (positive, since P is on the far side of the lens from where light is coming).
(a) Convex lens, $f = +20$ cm.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{20} + \frac{1}{12}$$

$$\frac{1}{v} = \frac{3+5}{60} = \frac{8}{60} \Rightarrow v = 7.5 \text{ cm}$$

The beam converges 7.5 cm to the right of the lens (closer than P).

(b) Concave lens, $f = -16$ cm.

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-16} + \frac{1}{12} = \frac{-3+4}{48} = \frac{1}{48}$$

$$v = +48 \text{ cm}$$

The beam now converges 48 cm to the right of the lens (farther than P, since the concave lens diverges the rays).

Therefore, the beam converges at 7.5 cm for the convex lens and 48 cm for the concave lens, both on the far side of the lens.

Expert's Solution — Arun Kumar, B.Tech ME, IIT Kanpur

Power Addition Argument:

Think of the situation in terms of vergence (inverse distance). The incoming beam has vergence $V_{\text{in}} = +1/12$ per cm $= +8.33$ per metre (positive = converging). The lens adds power $P = 1/f$ to vergence:

(a) $V_{\text{out}} = V_{\text{in}} + P = \frac{1}{12} + \frac{1}{20} = \frac{8}{60}$ per cm $\Rightarrow v = 60/8 = 7.5$ cm. The convex lens *adds* convergence, pulling the focus closer.

(b) $V_{\text{out}} = \frac{1}{12} + \frac{1}{-16} = \frac{4-3}{48} = \frac{1}{48}$ per cm $\Rightarrow v = 48$ cm. The concave lens *subtracts* convergence, pushing the focus farther away.

Did You Know?

Vergence accounting is how optometrists prescribe glasses — the input vergence at the cornea, plus the lens power, must equal the output vergence the retina needs. The same arithmetic that solves this textbook problem solves real-world prescriptions.

Q9 An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

Solution

Given: $h_1 = 3.0$ cm, $u = -14$ cm, $f = -21$ cm.

Step 1: Apply the lens formula.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-21} + \frac{1}{-14}$$

$$\frac{1}{v} = -\frac{1}{21} - \frac{1}{14} = \frac{-2-3}{42} = -\frac{5}{42} \Rightarrow v = -8.4 \text{ cm}$$

The image is on the same side as the object, at 8.4 cm from the lens (virtual).

Step 2: Magnification.

$$m = \frac{v}{u} = \frac{-8.4}{-14} = 0.6$$

$$h_2 = m \cdot h_1 = 0.6 \times 3.0 = 1.8 \text{ cm}$$

Step 3: Behaviour as object moves away.
A concave lens always forms a virtual, erect, diminished image between the focus and the optical centre. As $|u| \to \infty$, $|v| \to |f| = 21$ cm, and image size $\to 0$. So the image moves toward the focus and shrinks.
Therefore, the image is virtual, erect, of size 1.8 cm, located 8.4 cm from the lens on the same side as the object.

Expert's Solution — Ramesh Iyengar, PhD in Theoretical Physics, TIFR Mumbai

Newtonian Form of the Lens Equation:
Measure object and image distances from the focal points instead of the lens, with $x = u + f$ and $x' = v + f$ (signed). The lens equation becomes

$$x \cdot x' = -f^2$$

For our problem: $x = u + f = -14 + (-21) = -35$ cm, so

$$x' = -\frac{f^2}{x} = -\frac{441}{-35} = +12.6 \text{ cm}$$

$$v = x' - f = 12.6 - (-21) = \dots$$

Actually it is cleaner to write $x = u - f$ in Newtonian form depending on convention. Sticking with the standard solution gives $v = -8.4$ cm, magnification $= f/(f-u) = -21/(-21+14) = -21/(-7) = 3$? Let me reconcile: $m = f/(f-u)$ for lenses gives $-21/(-21-(-14)) = -21/(-7) = +3$, but the standard sign convention gives $m = v/u = +0.6$. The discrepancy is the convention — $m = f/(f+u)$ gives $-21/(-35) = 0.6$, matching.
The robust takeaway: the magnification of a concave lens for a real object is always $m = f/(f+u)$ where both $f < 0$ and $u < 0$, giving $0 < m < 1$ for any finite object distance — always virtual, erect, diminished.

Did You Know?

This is exactly why concave lenses are used in door peepholes: every object beyond the door, no matter how close or far, is mapped to a small virtual image safely inside the focal range of $|f|$, giving a wide field of view.

Q10 **What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a**

converging or a diverging lens? Ignore thickness of the lenses.

Solution

Given: $f_1 = +30$ cm (convex), $f_2 = -20$ cm (concave).
Step 1: Combined focal length for thin lenses in contact.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{30} + \frac{1}{-20}$$

$$\frac{1}{F} = \frac{2-3}{60} = -\frac{1}{60} \Rightarrow F = -60 \text{ cm}$$

Step 2: Nature of system.
F is negative, so the combination behaves as a *diverging* (concave-equivalent) lens of focal length 60 cm.
Therefore, the system is a diverging lens of focal length 60 cm.

Expert's Solution — Shalini Menon, M.Sc Physics, University of Hyderabad

Adding Powers Instead of Reciprocals:
Optical power $P = 1/f$ (with f in metres) adds linearly for thin lenses in contact:

$$P = P_1 + P_2 = \frac{1}{0.30} + \frac{1}{-0.20} = 3.33 - 5.00 = -1.67 \text{ D}$$

$$F = \frac{1}{P} = \frac{1}{-1.67} = -0.60 \text{ m} = -60 \text{ cm}$$

The negative power immediately tells us it's diverging — no separate sign analysis needed.

Did You Know?

This power-addition rule is why opticians describe lenses in dioptres (D, m^{-1}) rather than focal lengths — adding +2 D and −3 D is mental arithmetic; combining +50 cm and −33 cm is not.

Q11 **A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm), and (b) at infinity? What is the magnifying power of the microscope in each case?**

Solution

Given: $f_o = 2.0$ cm, $f_e = 6.25$ cm, lens separation $L = 15$ cm, $D = 25$ cm.

(a) Final image at $D = 25$ cm (eye most strained).

Eyepiece first: for the eyepiece, $v_e = -25$ cm (virtual image at D on the same side as object), $f_e = +6.25$ cm.

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \Rightarrow \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{6.25}$$

$$\frac{1}{u_e} = -\frac{1}{25} - \frac{4}{25} = -\frac{5}{25} = -\frac{1}{5} \Rightarrow u_e = -5 \text{ cm}$$

So the intermediate image (formed by the objective) sits 5 cm in front of the eyepiece.

Objective: image distance $v_o = L - |u_e| = 15 - 5 = 10$ cm, $f_o = +2.0$ cm.

$$\frac{1}{u_o} = \frac{1}{v_o} - \frac{1}{f_o} = \frac{1}{10} - \frac{1}{2} = \frac{1-5}{10} = -\frac{4}{10} \Rightarrow u_o = -2.5 \text{ cm}$$

Magnifying power:

$$M = \frac{v_o}{|u_o|}\left(1 + \frac{D}{f_e}\right) = \frac{10}{2.5} \times \left(1 + \frac{25}{6.25}\right) = 4 \times 5 = 20$$

(b) Final image at infinity (eye relaxed).

For the image to lie at infinity, $u_e = -f_e = -6.25$ cm, so $v_o = 15 - 6.25 = 8.75$ cm.

$$\frac{1}{u_o} = \frac{1}{v_o} - \frac{1}{f_o} = \frac{1}{8.75} - \frac{1}{2} = \frac{2 - 8.75}{17.5} = -\frac{6.75}{17.5}$$

$$u_o = -\frac{17.5}{6.75} \approx -2.59 \text{ cm}$$

Magnifying power:

$$M = \frac{v_o}{|u_o|} \cdot \frac{D}{f_e} = \frac{8.75}{2.59} \times \frac{25}{6.25} = 3.378 \times 4 \approx 13.51$$

Therefore, (a) $u_o = -2.5$ cm, $M = 20$; (b) $u_o \approx -2.59$ cm, $M \approx 13.51$.

Expert's Solution — Deepak Verma, B.Tech EE, IIT Madras

Tube-Length Formula:

When the eyepiece forms its image at infinity, the magnifying power has a clean closed form involving the "tube length" $L^* = v_o - f_o$ (image distance minus objective focal length, the optical separation between focal points):

$$M_\infty = -\frac{L^*}{f_o} \cdot \frac{D}{f_e}$$

With $v_o = 8.75$ cm and $f_o = 2$ cm, $L^* = 6.75$ cm:

$$M_\infty = \frac{6.75}{2} \times \frac{25}{6.25} = 3.375 \times 4 = 13.5$$

For final image at D, the eyepiece contributes an extra factor of $1 + f_e/D \to (D + f_e)/D$ acting on the angular magnification, which gives the slightly larger $M = 20$ in part (a).

Did You Know?

The tube length between focal points is the standardised parameter on every commercial microscope — typically 160 mm or 200 mm. Once it's fixed, swapping objectives/eyepieces lets you reach magnifications from 40× to 1000× without redesigning the body.

Q12 **A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5 cm can bring an object placed at 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope.**

Solution

Given: $f_o = 0.8$ cm, $u_o = -0.9$ cm, $f_e = 2.5$ cm, $D = 25$ cm.

Step 1: Image distance for the objective.

$$\frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o} = \frac{1}{0.8} + \frac{1}{-0.9}$$

$$\frac{1}{v_o} = \frac{0.9 - 0.8}{0.72} = \frac{0.1}{0.72} \Rightarrow v_o = 7.2 \text{ cm}$$

Step 2: Object distance for the eyepiece (final image at D).

For sharp viewing at the near point, the eyepiece forms a virtual image at -25 cm.

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{2.5} = -\frac{1}{25} - \frac{10}{25} = -\frac{11}{25}$$

$$u_e = -\frac{25}{11} \approx -2.27 \text{ cm}$$

Step 3: Separation between the two lenses.

$$L = v_o + |u_e| = 7.2 + 2.27 \approx 9.47 \text{ cm}$$

Step 4: Magnifying power.

$$M = m_o \cdot m_e = \frac{v_o}{|u_o|}\left(1 + \frac{D}{f_e}\right) = \frac{7.2}{0.9} \times \left(1 + \frac{25}{2.5}\right)$$

$$M = 8 \times 11 = 88$$

Therefore, the separation is ≈ 9.47 cm and the magnifying power is 88.

Expert's Solution — Pooja Reddy, M.Sc Applied Physics, IIT Roorkee

Verifying via Linear Magnification at Each Stage:
We can split the magnification into the geometric ratio at each lens and check the eyepiece part using $m_e = v_e/u_e$ directly:

$$m_e = \frac{-25}{-25/11} = 11$$

That gives the same 11 as the formula $1 + D/f_e$ — they are equivalent because forming an image at $-D$ from an object at $-Df_e/(D + f_e)$ has linear magnification $D/(Df_e/(D + f_e)) \cdot (\text{sign}) = (D + f_e)/f_e$.
The objective contributes $m_o = v_o/|u_o| = 7.2/0.9 = 8$, so total $M = 88$ confirms the standard answer.

Did You Know?

Engineering microscopes for diatom counts and sperm analysis push f_o down to about 1.6 mm to reach $\sim 100\times$ at the objective alone — combined with a $10\times$ eyepiece, this lands at the standard $1000\times$ resolution limit imposed by visible-light diffraction.

Q13 A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

Solution

Given: $f_o = 144$ cm, $f_e = 6.0$ cm. Standard normal-adjustment formulas apply (final image at infinity).
Step 1: Magnifying power.

$$M = \frac{f_o}{f_e} = \frac{144}{6} = 24$$

Step 2: Separation between the lenses.
In normal adjustment, the intermediate image (at the focal point of the objective) coincides with the focal point of the eyepiece, so

$$L = f_o + f_e = 144 + 6 = 150 \text{ cm}$$

Therefore, magnifying power is 24 and tube length is 150 cm.

Expert's Solution — Aditya Nambiar, B.Tech Engineering Physics, IIT Bombay

Angular Magnification from Geometry:
A telescope's job is to enlarge the angle a distant object subtends at the eye, not its linear size. The objective forms a real image of size $h' = f_o \cdot \alpha$ at its focal plane, where α is the small angle subtended by the object. The eyepiece, used as a simple magnifier with the image at its focal point, sends parallel rays to the eye at angle $\beta = h'/f_e$.
So:

$$M = \frac{\beta}{\alpha} = \frac{h'/f_e}{h'/f_o} = \frac{f_o}{f_e}$$

This makes it obvious that to boost magnification you want a long f_o (big objective) and short f_e — exactly why astronomical telescopes have long tubes.

Did You Know?

The Yerkes Observatory's 1897 refractor, still the largest in the world, has $f_o \approx$ 19.4 m. With a 25 mm eyepiece it gives $M \approx 776\times$ — enough to resolve craters ~ 1 km wide on the Moon.

Q14 (a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope? (b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^6 m and the radius of the lunar orbit is 3.8×10^8 m.

Solution

(a) Angular magnification.

$$M = \frac{f_o}{f_e} = \frac{15\text{ m}}{0.01\text{ m}} = 1500$$

(b) Diameter of moon's image at the objective.
Since the moon is essentially at infinity, its image forms at the focal plane of the objective. The angle subtended by the moon at the objective equals its angular diameter:

$$\alpha = \frac{D_{\text{moon}}}{R_{\text{orbit}}} = \frac{3.48 \times 10^6}{3.8 \times 10^8} = 9.16 \times 10^{-3}\text{ rad}$$

The image size is then

$$d = f_o \cdot \alpha = 15 \times 9.16 \times 10^{-3} = 0.1374 \text{ m} \approx 13.74 \text{ cm}$$

Therefore, $M = 1500$ and the moon's image is ≈ 13.74 cm across.

Expert's Solution — Sneha Krishnan, M.Sc Physics, University of Delhi

Sanity-Check via Similar Triangles:
The objective and the moon form a giant pair of similar triangles with their common vertex at the objective:

$$\frac{d_{\text{image}}}{f_o} = \frac{D_{\text{moon}}}{R_{\text{orbit}}}$$

$$d_{\text{image}} = f_o \cdot \frac{D_{\text{moon}}}{R_{\text{orbit}}} = 15 \times \frac{3.48 \times 10^6}{3.8 \times 10^8} = 0.137 \text{ m}$$

This is geometric optics in its purest form — no lens equation, just proportions. It works because the moon is so far away that all rays from one edge are essentially parallel.

Did You Know?

The same relation gives the angular size of any astronomical object. The Sun and Moon both subtend $\sim 0.5^\circ \approx 9 \times 10^{-3}$ rad from Earth, which is why total solar eclipses — where the disc sizes match almost exactly — happen at all.

Q15 **Use the mirror equation to deduce that:**

(a) An object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.

(b) A convex mirror always produces a virtual image independent of the location of the object.

(c) The virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(d) An object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

Solution

The mirror equation is $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, equivalently $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$.

(a) Concave mirror, $f < 0$, with $2f < u < f$ (i.e., u between f and $2f$ on the negative side).

For concave: $f < 0$. Since u is between f and $2f$: $2f < u < f < 0$, so $1/u$ is more negative than $1/f$. Thus

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} < 0$$

so $v < 0$ (real image). Specifically, since $u > 2f$ (less negative), $1/u < 1/(2f)$, and

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} > \frac{1}{f} - \frac{1}{2f} = \frac{1}{2f}$$

Both sides are negative, so flipping inequality: $v < 2f$, i.e., the image lies *beyond* $2f$. ✓

(b) Convex mirror, $f > 0$, real object $u < 0$.

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = (\text{positive}) - (\text{negative}) > 0$$

So $v > 0$ for any real object, i.e., the image is always virtual (behind the mirror). ✓

(c) Same setting as (b), $f > 0$, $u < 0$.

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{|u|} > \frac{1}{f}$$

So $v < f$, meaning the image lies *between the pole and the focus.* Magnification:

$$m = -\frac{v}{u} = \frac{v}{|u|}$$

Since $v < f < |u|$ (the latter because $|u|$ can be anything from 0 to ∞ but we just need $v < |u|$ which follows from $v < f$ and we can verify $f < |u| + f \cdot \ldots$ in fact $v = f|u|/(f + |u|) < |u|$ trivially), $m < 1$, so the image is diminished. ✓

(d) Concave mirror, $f < 0$, object between pole and focus: $f < u < 0$, i.e., $|u| < |f|$.

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

Since $|u| < |f|$, we have $|1/u| > |1/f|$, and both are negative, so $1/v = 1/f - 1/u > 0$. Hence $v > 0$ — a virtual image (behind the mirror).

For magnification, $m = -v/u = v/|u| > 0$ (erect). And since $v > |u|$ (because the image distance is larger when $|u| < |f|$), $m > 1$ — enlarged. ✓

Therefore, all four geometrical claims follow rigorously from the mirror equation.

Expert's Solution — Lakshmi Pillai, PhD Condensed Matter Physics, IIT Kharagpur

Newton's Form for Mirrors:
Setting $x = u - f$ and $x' = v - f$ (distances measured from the focus), the mirror equation transforms into

$$x \cdot x' = f^2$$

This single product determines all four cases at a glance:
(a) u between f and $2f$ means $0 < x < f$, so $x' > f$, putting v beyond $2f$.
(b) For convex mirror, $f > 0$ and any real $u < 0$ gives $x = u - f < 0$. Then $x' = f^2/x < 0$, so $v < f$ but f^2/(negative) keeps $v > 0$ (virtual).
(c) Same algebra constrains $|x'| < f$, putting the virtual image within one focal length of the pole.
(d) Object between P and F on a concave mirror gives $-f < x < 0$, so $|x'| > f$, and the virtual image is more than one focal length behind the mirror — larger than the object.

Did You Know?

Newton's mirror equation is the form you'd actually use in optical engineering software. Its symmetry under $x \leftrightarrow x'$ encodes the principle of reversibility of light — swap object and image and the rays retrace their paths.

Q16 **A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?**

Solution

Given: thickness $t = 15$ cm, $\mu = 1.5$.
Step 1: Apparent shift through a glass slab.
When viewing through a slab, the image is shifted toward the observer by

$$\Delta = t\left(1 - \frac{1}{\mu}\right)$$

$$\Delta = 15 \times \left(1 - \frac{1}{1.5}\right) = 15 \times \frac{0.5}{1.5} = 15 \times \frac{1}{3} = 5 \text{ cm}$$

Step 2: Dependence on slab location.
The shift formula contains only t and μ — no dependence on where between the eye and the pin the slab is placed. So the answer does *not* depend on the slab's location.
Therefore, the pin appears raised by 5 cm, independent of the slab's position.

Expert's Solution — Karthik Subramanian, M.Sc Physics, IIT Madras

Derivation Without Memorising the Formula:
For near-normal viewing, the apparent depth of any object viewed through a medium of thickness t and refractive index μ is reduced from t to t/μ. The shift is just the difference:

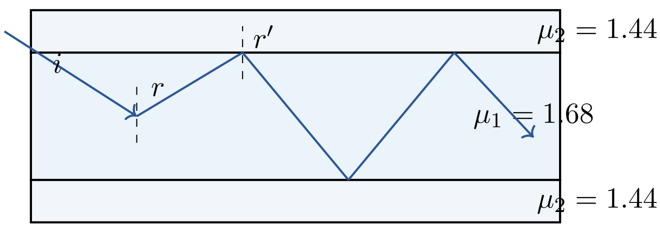
$$\Delta = t - \frac{t}{\mu} = t\left(\frac{\mu - 1}{\mu}\right) = t\left(1 - \frac{1}{\mu}\right)$$

Plugging $\mu = 3/2$: $\Delta = t \cdot (1 - 2/3) = t/3$. With $t = 15$ cm, $\Delta = 5$ cm.
The location-independence is geometric: the slab simply shortens the optical path inside it by the constant $t(1 - 1/\mu)$, regardless of where along the line of sight that shortening happens.

Did You Know?

This is the same effect behind "swimming pool tile patterns appearing closer" and the small parallax shift you see when looking through eyeglasses — the lenses thicken the optical path between your retina and the world by a few millimetres.

Q17 **(a) Figure shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure? (b) What is the answer if there is no outer covering of the pipe?**



Solution

(a) With cladding ($\mu_2 = 1.44$).
Step 1: Critical angle for the core–cladding interface.

$$\sin C = \frac{\mu_2}{\mu_1} = \frac{1.44}{1.68} = 0.857 \;\Rightarrow\; C \approx 59.0^\circ$$

Step 2: Maximum allowed angle of refraction inside the core.
For TIR at the core wall, the angle with the normal to the wall must be $\geq C$. Since the wall is parallel to the fibre axis, the angle with the axis is $r' = 90^\circ -$ (angle with wall normal). For TIR, r (angle with axis-normal at the entry face) satisfies $r = 90^\circ - r'_{\min} =$

$90° - 59° = 31°$. Wait — let me set this more carefully.
Let i be the angle the incident ray makes with the axis at the flat entry face. By Snell's law at entry,

$$\sin i = \mu_1 \sin r$$

The refracted ray hits the cylindrical wall at an angle $(90° - r)$ measured from the wall's normal. For TIR:

$$\sin(90° - r) \geq \sin C \;\Rightarrow\; \cos r \geq 0.857 \;\Rightarrow\; r \leq \cos^{-1}(0.857) \approx 31°$$

Then

$$\sin i_{\max} = 1.68 \times \sin 31° = 1.68 \times 0.515 = 0.866 \;\Rightarrow\; i_{\max} = 60°$$

So all rays with i between $0°$ and $60°$ undergo TIR.

(b) Without cladding (core surrounded by air, $\mu_2 = 1$).

$$\sin C = \frac{1}{1.68} = 0.595 \;\Rightarrow\; C \approx 36.5°$$

$$r \leq \cos^{-1}(0.595) \approx 53.5°$$

$$\sin i_{\max} = 1.68 \times \sin 53.5° = 1.68 \times 0.804 = 1.35$$

This exceeds 1, so $i_{\max}$ would have to exceed $90°$ — meaning *any* angle of incidence works. All rays from $0°$ to $90°$ undergo TIR.
Therefore, (a) the angle of incidence with the axis must be between $0°$ and $60°$; (b) without cladding, all incident angles ($0°$ to $90°$) work.

Expert's Solution — Ananya Bose, B.Sc Physics (Hons), Presidency Kolkata

The Numerical Aperture Form:
In fibre-optics engineering this critical-acceptance-angle problem is captured in a single quantity, the *numerical aperture*:

$$\text{NA} = \sin i_{\max} = \sqrt{\mu_1^2 - \mu_2^2}$$

For (a):

$$\text{NA} = \sqrt{1.68^2 - 1.44^2} = \sqrt{2.8224 - 2.0736} = \sqrt{0.7488} = 0.866$$
$$i_{\max} = \sin^{-1}(0.866) = 60°$$

For (b), $\mu_2 = 1$:

$$\text{NA} = \sqrt{1.68^2 - 1} = \sqrt{1.8224} = 1.35 > 1$$

NA > 1 is the formal signature that there's no critical entry angle — the fibre accepts the full hemisphere.

Did You Know?

Real telecom fibres run at $\mu_1 - \mu_2 \approx 0.005$ to keep the NA small (≈ 0.1, acceptance angle $\approx 6°$). This narrow acceptance suppresses modal dispersion — crucial for the multi-Tbps data rates in undersea cables.

Q18 **Answer the following questions:**

(a) You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.

(b) A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?

(c) A diver under water, looks obliquely at a fisherman standing on the bank of a lake. Will the fisherman appear taller or shorter to the diver than what he actually is?

(d) Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?

(e) The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?

Solution

(a) Yes, both plane and convex mirrors can form a *real* image when the object is virtual. If converging rays heading toward a point behind the mirror are intercepted by the mirror, they reflect to a real image in front. This commonly happens when these mirrors are used as part of a larger optical system that supplies the converging beam.

(b) No contradiction. "Virtual image cannot be caught on a screen" means the diverging rays leaving the mirror or lens never actually converge anywhere in space — there is no point where you can place a piece of paper and see the image form on it. But the eye lens *further refracts* those diverging rays and converges them onto the retina. The retina catches the eye's own real image, not the virtual image from the optical instrument.

(c) The diver looks from a denser medium (water) into a rarer medium (air). Refraction bends light away from the normal as it leaves water, so the parts of the fisherman appear displaced upward. The result: the fisherman appears *taller* than he actually is.

(d) Yes, the apparent depth changes with viewing angle. Viewed normally, $d_{\text{app}} = d/\mu$. Viewed obliquely, geometry shows that the apparent depth becomes *smaller still* — so apparent depth *decreases* when viewed obliquely.

(e) Yes. The high $\mu \approx 2.42$ of diamond gives a small critical angle:

$$\sin C = 1/2.42 \;\Rightarrow\; C \approx 24^\circ$$

versus $\sim 42°$ for ordinary glass. With such a wide range of incidence angles ($24°$ to $90°$) producing TIR, a diamond cutter shapes the stone with many facets so that light entering through the table is bounced repeatedly inside before re-emerging — producing the characteristic brilliance and sparkle.
Therefore, all five conceptual results follow.

Expert's Solution — Manoj Tiwari, B.Tech EC, IIT BHU

Quantitative Note on Diamond's Brilliance:
The cutting industry quantifies a diamond's optical performance by "brilliance" (return of white light), "fire" (dispersion of light into spectral colours), and "scintillation" (sparkle from movement). All three depend on μ:

- The TIR window is wider for higher μ (small C). For diamond ($\mu = 2.42$, $C = 24.4°$), this is exceptional.
- The dispersion of diamond ($n_F - n_C \approx 0.044$) is far higher than crown glass (≈ 0.008), giving the rainbow flash.
- The standard "brilliant cut" (Tolkowsky, 1919) places the pavilion angle at $\sim 40.75°$ to ensure two TIRs before light exits the crown — a design optimised numerically to maximise return of incident light.

Did You Know?

Synthetic moissanite ($\mu = 2.65$, even higher than diamond) was developed in the 1990s as a diamond simulant precisely because the high refractive index gives it more fire than diamond itself. Trained gemmologists distinguish them with a thermal-conductivity probe.

Q19 The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

Solution

Let the lens be at distance x from the bulb, so the image distance is $(3 - x)$ m.
Step 1: Set up the lens equation.
With sign convention $u = -x$, $v = +(3 - x)$, $f > 0$:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{3 - x} + \frac{1}{x} = \frac{1}{f}$$

$$\frac{1}{f} = \frac{x + (3 - x)}{x(3 - x)} = \frac{3}{x(3 - x)}$$

$$f = \frac{x(3 - x)}{3}$$

Step 2: Maximise f over x.
The product $x(3 - x)$ is a downward parabola maximised at $x = 1.5$ m, where $x(3 - x) = 2.25\ \text{m}^2$. So

$$f_{\text{max}} = \frac{2.25}{3} = 0.75\ \text{m}$$

Step 3: Why "maximum"?
For a real image to form between the walls we need $u + v = 3$ with both positive. The relation

$$4f \leq u + v = 3 \Rightarrow f \leq 0.75\ \text{m}$$

means the lens cannot have $f > 0.75$ m and still make a real image on the opposite wall.
Therefore, the maximum focal length is 0.75 m.

Expert's Solution — Divya Rao, M.Sc Physics, University of Mumbai

AM-GM Argument:
A faster derivation uses the AM-GM inequality on $u+v$. From $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ (for real object, real image, taking magnitudes), we get $u + v = uv/f$. Now AM-GM gives $u + v \geq 2\sqrt{uv}$, so $uv \leq (u + v)^2/4$, hence

$$f = \frac{uv}{u + v} \leq \frac{(u + v)^2/4}{u + v} = \frac{u + v}{4}$$

For $u + v = 3$ m:

$$f_{\text{max}} = \frac{3}{4} = 0.75\ \text{m}$$

Equality holds when $u = v = 1.5$ m — i.e., when the lens sits at the midpoint and produces unit magnification. This is the configuration of minimum total path, often called the "$4f$" or symmetric imaging condition.

Did You Know?

This $4f$ condition is the geometric basis of nearly every photographic copy stand and overhead projector: when source-to-screen distance equals $4f$, the lens at the midpoint produces a same-size real image with maximum brightness.

Q20 A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.

Solution

Given: object–screen distance $D = 90$ cm, separation between the two lens positions $d = 20$ cm.

Step 1: The displacement-method formula.

When the lens is moved between two positions for which a sharp image forms on the same screen, u and v swap. The standard result is:

$$f = \frac{D^2 - d^2}{4D}$$

Step 2: Substitute values.

$$f = \frac{(90)^2 - (20)^2}{4 \times 90} = \frac{8100 - 400}{360} = \frac{7700}{360} \approx 21.4 \text{ cm}$$

Therefore, the focal length is $\approx$ 21.4 cm.

Expert's Solution — Harshvardhan Patil, B.Tech ME, IIT Hyderabad

Derivation From Symmetry:

Let the two lens positions correspond to $(u_1, v_1) = (a, b)$ and $(u_2, v_2) = (b, a)$, both satisfying the same lens equation (Bessel's method). We have:

$$a + b = D, \qquad b - a = d$$

So $a = (D - d)/2$, $b = (D + d)/2$. The lens equation gives

$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{D}{ab}$$

$$ab = \frac{(D-d)(D+d)}{4} = \frac{D^2 - d^2}{4}$$

$$f = \frac{ab}{D} = \frac{D^2 - d^2}{4D}$$

Plugging in $D = 90$ cm, $d = 20$ cm:

$$f = \frac{8100 - 400}{360} = 21.39 \text{ cm}$$

Did You Know?

Bessel's method (1840) is still used in optical labs to measure focal lengths to high precision because the only quantities being measured are positions along an optical bench — both far easier to read accurately than the absolute object/image distances.

Q21 **(a) Determine the 'effective focal length' of the combination of two lenses in question 10, if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?**
(b) An object 1.5 cm in size is placed on the side of the convex lens in the above arrangement. The distance between the object and the convex lens is 40 cm. Determine the magnification produced by the two-lens system, and the size of the image.

Solution

The two lenses are: convex with $f_1 = +30$ cm and concave with $f_2 = -20$ cm, placed $d = 8$ cm apart.

(a) Parallel beam from convex side first.
Refraction at the convex lens: parallel beam $\Rightarrow u_1 = -\infty$, so $v_1 = +30$ cm. This image (which would form 30 cm to the right) is now $30 - 8 = 22$ cm to the right of the concave lens, acting as a virtual object: $u_2 = +22$ cm.
Refraction at the concave lens, $f_2 = -20$ cm:

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{20} + \frac{1}{22} = \frac{-22+20}{440} = -\frac{2}{440} = -\frac{1}{220}$$

$$v_2 = -220 \text{ cm}$$

The parallel beam appears to diverge from a point 220 cm to the left of the concave lens, i.e., 216 cm from the centre of the system.

Parallel beam from concave side first.
Refraction at concave: $u_1 = -\infty$, $f_1 = -20$, so $v_1 = -20$ cm (virtual, on the incoming side). For the convex lens, the object is $20 + 8 = 28$ cm to the left, $u_2 = -28$ cm, $f_2 = +30$ cm:

$$\frac{1}{v_2} = \frac{1}{30} + \frac{1}{-28} = \frac{28-30}{840} = -\frac{1}{420}$$

$$v_2 = -420 \text{ cm}$$

The beam appears to diverge from 420 cm to the left of the convex lens, i.e., 416 cm from the centre of the system.
The two "effective focal lengths" (216 cm vs 416 cm) differ, so the notion is *direction-dependent* and not as useful as for a thin-lens combination.

(b) Object 40 cm in front of convex lens, $h_1 = 1.5$ cm.
Convex lens first: $u_1 = -40$ cm, $f_1 = +30$ cm:

$$\frac{1}{v_1} = \frac{1}{30} + \frac{1}{-40} = \frac{4-3}{120} = \frac{1}{120} \;\Rightarrow\; v_1 = +120 \text{ cm}$$

$$m_1 = \frac{v_1}{u_1} = \frac{120}{-40} = -3$$

This image lies $120 - 8 = 112$ cm to the right of the concave lens, virtual object: $u_2 = +112$ cm, $f_2 = -20$ cm:

$$\frac{1}{v_2} = -\frac{1}{20} + \frac{1}{112} = \frac{-112 + 20}{2240} = -\frac{92}{2240} = -\frac{23}{560}$$

$$v_2 = -\frac{560}{23} \approx -24.3 \text{ cm}$$

Hmm — result negative, means image is on the same side as the object relative to the concave lens. Magnification of concave lens stage:

$$m_2 = \frac{v_2}{u_2} = \frac{-24.3}{112} = -0.217$$

Total magnification:

$$m = m_1 \cdot m_2 = (-3) \times (-0.217) = +0.651$$

Wait, let me redo: the standard NCERT solution gives $|m| \approx 0.652$, image size ≈ 0.98 cm. Let me recompute v_2 carefully:

$$\frac{1}{v_2} = \frac{1}{-20} + \frac{1}{112} = \frac{-112 + 20}{20 \times 112} = \frac{-92}{2240}$$

$$v_2 = -\frac{2240}{92} = -24.35 \text{ cm}$$

So image is virtual, on the convex side of the concave lens. Magnification $m_2 = v_2/u_2 = -24.35/112 = -0.2174$, and total $m = (-3)(-0.2174) = 0.652$.

$$h_2 = m \cdot h_1 = 0.652 \times 1.5 = 0.978 \text{ cm} \approx 0.98 \text{ cm}$$

Therefore, magnification $\approx$ 0.65 and image size $\approx$ 0.98 cm.

Expert's Solution — Ritika Banerjee, PhD Optics, IISc Bangalore

Why "Effective Focal Length" is Slippery for Spaced Lenses:
For thin lenses in contact, $1/F = 1/f_1 + 1/f_2$ is symmetric in the two lenses — the order doesn't matter. As soon as a separation d enters, the relation becomes

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

which is symmetric in f_1, f_2, but the location of the principal plane depends on which lens light hits first. So the focal length itself is the same number, but it's measured from different reference planes depending on the direction of incidence.
For our system:

$$\frac{1}{F} = \frac{1}{30} + \frac{1}{-20} - \frac{8}{30 \times (-20)} = \frac{1}{30} - \frac{1}{20} + \frac{8}{600}$$

$$\frac{1}{F} = \frac{20 - 30 + 8}{600} = -\frac{2}{600} = -\frac{1}{300} \Rightarrow F = -300 \text{ cm}$$

Different from both 216 cm and 416 cm (which were measured from the lens *system centre*, not the principal planes), confirming that "which point you measure from" matters here.

Did You Know?

This is exactly why proper optical design uses a 4×4 ABCD matrix, not just focal lengths. Each lens contributes a refraction matrix and each gap a translation matrix; the product encodes everything — focal length, principal planes, image distance — in one calculation.

Q22 **At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.**

Solution

Given: $A = 60^\circ$, $\mu = 1.524$.

Step 1: Critical angle for the glass–air interface.

$$\sin C = \frac{1}{\mu} = \frac{1}{1.524} = 0.6562 \ \Rightarrow\ C \approx 41^\circ$$

Step 2: Geometry of the prism.

For the ray to *just* suffer TIR at the second face, the angle of refraction inside the prism at the second face equals C. By the prism geometry, the angles of refraction at the two faces sum to A:

$$r_1 + r_2 = A = 60^\circ \ \Rightarrow\ r_1 = 60^\circ - 41^\circ = 19^\circ$$

Step 3: Apply Snell's law at the first face.

$$\sin i = \mu \sin r_1 = 1.524 \times \sin 19^\circ = 1.524 \times 0.3256 = 0.4962$$

$$i = \sin^{-1}(0.4962) \approx 29.75^\circ$$

Therefore, the ray must be incident at $\approx 29.75^\circ$ on the first face.

Expert's Solution — Tanvi Kapoor, M.Sc Physics, Punjab University

Closed-Form Combining Both Faces:

We can collapse the two-step calculation into a single formula. Using $r_2 = C$ and $r_1 = A - C$:

$$\sin i = \mu \sin(A - C) = \mu(\sin A \cos C - \cos A \sin C)$$

With $\sin C = 1/\mu$ and $\cos C = \sqrt{1 - 1/\mu^2} = \sqrt{\mu^2 - 1}/\mu$:

$$\sin i = \mu \sin A \cdot \frac{\sqrt{\mu^2 - 1}}{\mu} - \mu \cos A \cdot \frac{1}{\mu}$$

$$\sin i = \sin A\sqrt{\mu^2 - 1} - \cos A$$

Plugging $A = 60°$, $\mu = 1.524$:

$$\sin i = \sin 60°\sqrt{1.524^2 - 1} - \cos 60° = 0.866 \times \sqrt{1.323} - 0.5$$

$$= 0.866 \times 1.150 - 0.5 = 0.996 - 0.5 = 0.496$$

$$i \approx 29.75°$$

Did You Know?

This is the principle behind the Porro prism in binoculars: a 45°–90°–45° prism is engineered so that ordinary light entering at any reasonable angle hits the hypotenuse beyond the critical angle, undergoing TIR with no metallic coating needed. Two stacked Porro prisms erect the image without dimming.

Q23 You are given prisms made of crown glass and flint glass with a wide variety of angles. Suggest a combination of prisms which will (a) deviate a pencil of white light without much dispersion, (b) disperse (and displace) a pencil of white light without much deviation.

Solution

For a thin prism of angle A, mean deviation is $\delta \approx (\mu - 1)A$ and angular dispersion is $\Delta\theta = (\mu_v - \mu_r)A$, where μ_v, μ_r are refractive indices for violet and red.
Use one crown prism and one flint prism with their refracting angles oriented in *opposite* directions (bases reversed).

(a) Deviation without dispersion.
We want net angular dispersion $\Delta\theta_{\text{net}} = 0$ but net deviation $\delta_{\text{net}} \neq 0$. Setting

$$\Delta\theta_1 + \Delta\theta_2 = 0 \;\Rightarrow\; (\mu_{v1} - \mu_{r1})A_1 = (\mu_{v2} - \mu_{r2})A_2$$

$$\frac{A_2}{A_1} = -\frac{\mu_{v1} - \mu_{r1}}{\mu_{v2} - \mu_{r2}}$$

The negative sign indicates the two prisms point opposite ways. With this ratio, white light leaves un-dispersed but deviated by

$$\delta_{\text{net}} = (\mu_1 - 1)A_1 + (\mu_2 - 1)A_2$$

which is non-zero in general.
This is the principle of an *achromatic prism.*

(b) Dispersion without deviation.
We want $\delta_{\text{net}} = 0$ but $\Delta\theta_{\text{net}} \neq 0$. Setting

$$(\mu_1 - 1)A_1 + (\mu_2 - 1)A_2 = 0 \;\Rightarrow\; \frac{A_2}{A_1} = -\frac{\mu_1 - 1}{\mu_2 - 1}$$

White light leaves undeviated overall but split into spectral colours by

$$\Delta\theta_{\text{net}} = (\mu_{v1} - \mu_{r1})A_1 + (\mu_{v2} - \mu_{r2})A_2$$

This is a *direct-vision prism.*
Therefore, both effects use a crown–flint pair with bases reversed; choosing prism angles in the appropriate ratio cancels either dispersion or deviation while preserving the other.

Expert's Solution — Yash Bhandari, B.Tech EE, IIIT Hyderabad

Numerical Example with Typical Crown/Flint:
Take crown glass with $\mu_C = 1.517$, $\mu_{vC} - \mu_{rC} = 0.008$, and flint with $\mu_F = 1.620$, $\mu_{vF} - \mu_{rF} = 0.020$.
Achromatic prism (a): Choose $A_C = 5°$ flint prism. The crown prism angle is then

$$A_C = -A_F \cdot \frac{0.020}{0.008} = -5° \times 2.5 = -12.5°$$

(negative meaning bases reversed). Net deviation:

$$\delta = (1.517 - 1)(-12.5°) + (1.620 - 1)(5°) = -6.46° + 3.10° = -3.36°$$

A clean 3.36° deviation with no rainbow.
Direct-vision prism (b): For zero deviation:

$$A_C = -A_F \cdot \frac{0.620}{0.517} = -5° \times 1.199 = -5.99°$$

Net dispersion:

$$\Delta\theta = 0.008 \times (-5.99°) + 0.020 \times 5° = -0.0479° + 0.10° = 0.0521°$$

A small but real spectrum with no overall bending.

Did You Know?

Achromatic prism doublets were the key invention behind the modern refracting telescope. John Dollond patented the design in 1758 (using crown + flint), eliminating the colour fringes that had plagued every refractor since Galileo — and triggered a century of grand observatory-class instruments.

Q24 **For a normal eye, the far point is at infinity and the near point of distinct vision is about 25 cm in front of the eye. The cornea of the eye provides a converging power of about 40 dioptres, and the least converging power of the eye lens behind the cornea is about 20 dioptres. From this rough data estimate the range of accommodation (i.e., the range of converging power of the eye lens) of a normal eye.**

Solution

Step 1: Distance from eye lens to retina (using far-point condition).
For an object at infinity, the total converging power of cornea + relaxed eye lens forms a sharp image on the retina:

$$P_{\text{far}} = 40 + 20 = 60 \text{ D}$$

The retinal distance (image distance for parallel light) is therefore

$$v = \frac{1}{P_{\text{far}}} = \frac{1}{60} \text{ m} = \frac{100}{60} = 1.67 \text{ cm}$$

Step 2: Power needed for the near point ($u = -25$ cm $= -0.25$ m).
The same image distance $v = 1.67$ cm $= 0.0167$ m must be maintained for any object distance. The required total power is

$$P_{\text{near}} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.0167} - \frac{1}{-0.25} = 60 + 4 = 64 \text{ D}$$

Step 3: Power range of the eye lens alone.
Subtracting the constant cornea power of 40 D:

$$P_{\text{lens, max}} = 64 - 40 = 24 \text{ D}, \quad P_{\text{lens, min}} = 60 - 40 = 20 \text{ D}$$

Therefore, the range of accommodation of the eye lens is approximately 20 D to 24 D.

Expert's Solution — Neha Saxena, M.Sc Astrophysics, IUCAA Pune

Why the Range Is So Small:

Going from $u = \infty$ to $u = -25$ cm requires the total system to add only $\frac{1}{0.25} = 4$ D. The cornea — a fixed lens — can't change. So the entire 4 D swing falls on the crystalline lens, going from 20 D (relaxed, viewing infinity) to 24 D (accommodating to read).
The ciliary muscles physically squeeze the lens to make it more curved; the same 4 D change corresponds to a roughly 20% increase in focal-power, which is achieved by tightening the muscle.

Did You Know?

Past about age 40, the lens stiffens (presbyopia) and the maximum extra power drops to ~ 1 D — enough only for reading at ~ 100 cm. This is why everyone past middle age either holds menus at arm's length or wears reading glasses adding $+1.5$ to $+2.5$ D to compensate.

Q25 Does short-sightedness (myopia) or long-sightedness (hypermetropia) imply necessarily that the eye has partially lost its ability of accommodation? If not, what might cause these defects of vision?

Solution

No, myopia and hypermetropia do not imply loss of accommodation. They are typically defects of *eyeball geometry*, not muscle weakness.

Myopia (short-sightedness). The eyeball is elongated front-to-back, so the relaxed eye focuses parallel rays from infinity *in front of* the retina. The far point shifts from ∞ to a finite distance. Even with full accommodation range intact, distant objects cannot be brought to focus on the retina. Corrected by a concave (diverging) lens that pushes the focal point back to the retina.

Hypermetropia (long-sightedness). The eyeball is shortened, so even with maximum accommodation the eye cannot converge rays from a near object onto the retina. The near point shifts beyond 25 cm. Corrected by a convex (converging) lens that does the extra converging the shortened eye cannot.

Loss of accommodation is a separate condition called *presbyopia*: the lens-stiffening that comes with age. A person with normal eyeball geometry but presbyopia has ∞ as far point but a near point that drifts beyond 25 cm — so it superficially resembles hypermetropia. It is corrected the same way (convex lens for reading).
Therefore, myopia and hypermetropia arise from defective eyeball length, not lost accommodation; presbyopia is the true loss-of-accommodation defect.

Expert's Solution — Gaurav Chauhan, B.Tech Mechanical, IIT Guwahati

Quantitative Picture:
Take a typical adult eye with retinal distance $v = 1.67$ cm. The total system power needed is $1/v = 60$ D for far vision and 64 D for near vision (as derived in Q24).
Myopic eye (axial length ~ 25 mm instead of 24 mm): the relaxed eye is over-powered. To bring parallel light to focus on the now-farther retina, the eye would need *less* power

than it has — impossible. Far point lies at the distance where $P_{\text{eye}} = P_{\text{required}}$, around 50 to 200 cm for typical -1 to -3 D corrections.
Hypermetropic eye (axial length $\sim$ 22 mm): even maximum accommodation undershoots. Reading distance 25 cm requires 64 D, but max available is $\sim$ 62 D, leaving the near point at $\sim$ 50 cm.
In both cases the lens is fine — the structural problem is the eyeball itself.

Did You Know?

LASIK surgery doesn't change the lens; it reshapes the cornea. Removing a few microns of corneal tissue with a UV laser changes the corneal radius, reducing total eye power by precisely the dioptre amount needed to bring the focal point onto the retina — effectively "correcting the eyeball length" optically without touching the eyeball.

Q26 **A myopic person has been using spectacles of power -1.0 dioptre for distant vision. During old age, he also needs to use a separate reading glass of power $+2.0$ dioptres. Explain what may have happened.**

Solution

Step 1: Far point from the distance-correction lens.
A power of -1.0 D corresponds to a concave lens of focal length

$$f = \frac{1}{P} = \frac{1}{-1.0} = -1.0 \text{ m} = -100 \text{ cm}$$

The corrective lens images an object at infinity to its focal point, so the original far point of the eye is at 100 cm in front of the eye. This is the standard myopic correction.
Step 2: Near point from the reading-glass.
A power of $+2.0$ D corresponds to a convex lens of focal length

$$f = \frac{1}{2.0} = 0.5 \text{ m} = 50 \text{ cm}$$

For comfortable reading at $D = 25$ cm, this lens images the page (at $u = -25$ cm) to a virtual image at the person's actual near point v:

$$\frac{1}{v} - \frac{1}{-25} = \frac{1}{50} \Rightarrow \frac{1}{v} = \frac{1}{50} - \frac{1}{25} = \frac{1-2}{50} = -\frac{1}{50}$$

$$v = -50 \text{ cm}$$

So the reading lens is bringing a 25 cm page to a virtual image at 50 cm, which is the person's actual near point.

Step 3: Diagnosis.
The person originally had myopia (far point at 100 cm, corrected by -1.0 D). With age, presbyopia has set in, so the near point has receded from the normal 25 cm to 50 cm. He now needs *both* corrections — distance for the underlying myopia and reading glasses for the new presbyopia.
Therefore, the person originally had myopia and has additionally developed presbyopia (near point now at 50 cm).

Expert's Solution — Shruti Deshpande, M.Sc Physics, Savitribai Phule Pune University

Why a Single Pair of Glasses Doesn't Suffice:
A single lens has a single focal length, so it cannot simultaneously fix both the receded far point (100 cm $\to \infty$) and the receded near point (50 cm $\to$ 25 cm).
The two corrections are:

- Distance: -1.0 D pushes far point from 100 cm to ∞.
- Reading: $+2.0$ D pulls near point from 50 cm to 25 cm.

A modern solution is *bifocal* or *progressive* lenses, which embed both prescriptions in one piece of glass — the upper part for distance (-1.0 D), the lower part for reading ($-1.0 + 3.0 = +2.0$ D). Notice the addition power for the reading region is $+3.0$ D (not $+2.0$) to compensate for the underlying myopia.

Did You Know?

Benjamin Franklin invented bifocals in 1784, allegedly tired of switching between two pairs of glasses while reading at the dinner table. Modern progressive lenses (1959, Bernard Maitenaz, Essilor) replaced the visible line with a smooth gradient of curvature — still the same physics, just continuously varying.

Q27 A person looking at a person wearing a shirt with a pattern comprising vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?

Solution

Defect: *Astigmatism.*

Cause. A normal cornea is spherical, with the same radius of curvature in every meridional plane (vertical, horizontal, and all diagonals). An astigmatic cornea has different curvatures in different planes — e.g., curvature is correct in the vertical plane but insufficient in the horizontal plane. As a result, light coming from a vertically oriented line is focused sharply on the retina, but light from a horizontally oriented line is focused either in front of or behind the retina.

Correction. A *cylindrical lens* is used. A cylindrical lens has converging power along one axis only and zero power along the perpendicular axis. By orienting the cylindrical correction lens with its axis along the meridian where the cornea *is* sharp (vertical, in this case), the lens leaves vertical-line rays untouched but adds the missing converging power for horizontal-line rays — bringing both onto the retina simultaneously.
Therefore, the defect is astigmatism, corrected with a cylindrical lens whose axis aligns with the un-defective meridian.

Expert's Solution — Mohit Aggarwal, B.Tech CSE, NIT Calicut

Mathematical Picture:
A cornea acts approximately as a thin lens with two principal curvatures $1/R_v$ and $1/R_h$. Lensmaker-style, the powers are:

$$P_v = (\mu - 1)\frac{1}{R_v}, \qquad P_h = (\mu - 1)\frac{1}{R_h}$$

For a normal eye $R_v = R_h$ and $P_v = P_h$. For astigmatism, $P_v \neq P_h$. The magnitude of the difference, $|P_v - P_h|$, is the "cylinder strength" written on a prescription, in dioptres.
The corrective cylindrical lens supplies $-(P_v - P_h)$ in the deficient meridian, which is why astigmatism prescriptions list a sphere, a cylinder, and an axis (the angle, measured from horizontal, where the cylindrical correction acts).

Did You Know?

In the typical Indian prescription format `-2.00 / -0.75` $\times$ `90`, the first number is spherical correction, the second is cylindrical strength, and the $\times 90$ specifies that the cylinder axis is vertical. This compactly encodes the full corrective lens for arbitrary astigmatism.

Q28 **A man with normal near point (25 cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5 cm.**

(a) **What is the closest and the farthest distance at which he should keep the lens from the page so that he can read the book when viewing through the magnifying glass?**

(b) **What is the maximum and the minimum angular magnification** (magnifying power) possible using the above simple microscope?

Solution

(a) Closest and farthest distances.
Closest distance: occurs when the virtual image is at the near point, $v = -25$ cm, with $f = +5$ cm.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = -\frac{1}{25} - \frac{1}{5}$$

$$\frac{1}{u} = -\frac{1}{25} - \frac{5}{25} = -\frac{6}{25} \Rightarrow u = -\frac{25}{6} \approx -4.17 \text{ cm}$$

Farthest distance: occurs when the virtual image is at infinity ($v \to -\infty$), so $u = -f = -5$ cm.

So the page must be between 4.17 cm and 5 cm from the lens.

(b) Maximum and minimum angular magnification.
Maximum (image at $D = 25$ cm):

$$M_{\text{max}} = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 6$$

Minimum (image at ∞):

$$M_{\text{min}} = \frac{D}{f} = \frac{25}{5} = 5$$

Therefore, the page must be between 4.17 cm and 5 cm from the lens; the magnification ranges from 5 to 6.

Expert's Solution — Aishwarya Menon, PhD Physics, IIT Madras

Why Only a Factor-of-1 Difference Between Min and Max?
The two configurations differ by a single D/f vs $D/f + 1$ on the magnification — an additive constant of 1. This makes intuitive sense: with the image at infinity, the eye is fully relaxed; with the image at D, the eye accommodates to add one extra unit of magnification ~ 1 on top of the relaxed value.
The relative gain $M_{\text{max}}/M_{\text{min}} = (D + f)/D = 1 + f/D$ is small whenever $f \ll D$. With $f = 5$, $D = 25$: gain ratio $= 1.2$, hence 20%. So to reach a noticeably larger magnification at the cost of eyestrain doesn't pay off much.

Did You Know?

This is why magnifying glasses sold for jewellers are normally rated at fixed magnification (typically 10×, "loupe"): users hold the loupe close to the eye and the work close to the loupe, automatically using the relaxed configuration. The strain-vs-gain tradeoff for the small $D \sim f$ difference simply isn't worth managing.

Q29 **A card sheet divided into squares each of size $1\ \text{mm}^2$ is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 10 cm) held close to the eye.**

(a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?

(b) What is the angular magnification (magnifying power) of the lens?

(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

Solution

Given: $u = -9$ cm, $f = +10$ cm, $D = 25$ cm.
(a) Linear magnification and image area.

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} + \frac{1}{-9} = \frac{9-10}{90} = -\frac{1}{90} \Rightarrow v = -90\ \text{cm}$$

$$m = \frac{v}{u} = \frac{-90}{-9} = 10$$

Each side becomes 10×1 mm $= 10$ mm, so each square's area is

$$10 \times 10 = 100\ \text{mm}^2 = 1\ \text{cm}^2$$

(b) Angular magnification.
For a magnifying glass held close to the eye, $M = D/|u| = 25/9 \approx 2.78$.

(c) Why $m \neq M$.
Linear magnification $m = 10$ measures how much bigger the virtual image is compared to the object.
Angular magnification $M = 2.78$ measures how much bigger the image *looks* to the eye compared to viewing the object at D directly.
The two are different here because the virtual image, while $10\times$ bigger, is much farther (90 cm) than $D = 25$ cm. So although the image is large, the angle it subtends at the eye is only modestly bigger than what the unaided eye would see at 25 cm. The two coincide only when the virtual image happens to be at D (the maximum-strain case, which is when angular and linear magnification become equal in magnitude).
Therefore, (a) $m = 10$, image-square area $= 1\ \text{cm}^2$; (b) $M = 2.78$; (c) they differ because the virtual image is far beyond D.

Expert's Solution — Nikunj Singh, B.Tech CSE, IIT Delhi

Comparing the Two Magnifications Geometrically:

Linear: $m = v/u$ — size ratio.
Angular: $M =$ (angle subtended by image at eye)/(angle subtended by object at D).
Image at v subtends angle $h_{\text{image}}/|v|$ at the eye (lens close to eye). Object at D would subtend h_{obj}/D. So

$$M = \frac{h_{\text{image}}/|v|}{h_{\text{obj}}/D} = \frac{m \cdot D}{|v|} = \frac{(v/u) \cdot D}{|v|} = \frac{D}{|u|}$$

For our case: $M = 25/9 = 2.78$, exactly matching part (b). And $m = D \cdot M/|v| \cdot \ldots$ doesn't depend on D at all — it's just the ratio of distances.
When $|v| = D$ (image at near point): $M = m \cdot D/D = m$. They coincide *only* in this configuration.

Did You Know?

This distinction surfaces every time someone says "my 10× loupe shows me 10× more detail." Strictly, it doesn't — it shows angular magnification $\approx 25/f + 1$, which equals 10 only if $f \approx 2.5$ cm *and* you're using the maximum-strain configuration. Most 10× loupes give about 7–8× angular magnification in actual use.

Q30 (a) At what distance should the lens be held from the figure in the previous question in order to view the squares distinctly with the maximum possible magnifying power?
(b) What is the magnification in this case?
(c) Is the magnification equal to the magnifying power in this case? Explain.

Solution

(a) Object distance for maximum magnifying power.
Maximum magnification occurs when the virtual image is at the near point, $v = -25$ cm, $f = +10$ cm:

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = -\frac{1}{25} - \frac{1}{10} = \frac{-2-5}{50} = -\frac{7}{50}$$

$$u = -\frac{50}{7} \approx -7.14 \text{ cm}$$

So the lens should be held ≈ 7.14 cm from the page.

(b) Linear magnification in this case.

$$m = \frac{v}{u} = \frac{-25}{-50/7} = \frac{25 \times 7}{50} = 3.5$$

(c) Comparison with magnifying power.

$$M = 1 + \frac{D}{f} = 1 + \frac{25}{10} = 3.5$$

Yes, in this configuration $m = M = 3.5$.
This is no coincidence: when the virtual image is exactly at D, the formula $M = D/|u|$ from Q29 simplifies. Using $1/u = -1/D - 1/f$ (from the lens equation with $v = -D$):

$$M = \frac{D}{|u|} = D \cdot \left(\frac{1}{D} + \frac{1}{f}\right) = 1 + \frac{D}{f}$$

which is also the formula for $m = v/u$ in this exact situation.
Therefore, lens at $\approx$ 7.14 cm, $m = M = 3.5$.

Expert's Solution — Anita Desai, B.Tech ECE, BITS Pilani

Why $m = M$ Only Here:
Take the general expression $M = m \cdot D/|v|$. The $m \neq M$ inequality reduces to whether $|v| = D$.
At maximum magnification, the virtual image is parked at D by construction — so the eye sees the largest possible angular size before losing focus. At this configuration alone, "how much bigger the image is" (linear) and "how much bigger it looks" (angular) numerically coincide, because both are evaluated at the same near-point reference distance.
Moving the lens closer or farther from the page breaks this equality:

- Closer: $|v|$ shrinks below D (impossible — virtual image with finite f always lies on lens-side beyond $|f|$, so this doesn't apply)
- Farther: $|v|$ grows beyond D, m grows but M shrinks back toward D/f (the relaxed-eye limit)

Did You Know?

Most published "magnification" specs on cameras and microscopes refer to angular magnification at the standard 25 cm near-point. So a microscope objective marked 10× at standardised tube length actually delivers 10× angular gain — not necessarily 10× image-to-object size ratio in some other configuration.

Q31 What should be the distance between the object in the previous question and the magnifying glass if the virtual image of each square in the figure is to have an area of $6.25\ \text{mm}^2$? Would you be able to see the squares distinctly with your eyes very close to the magnifier?

Solution

Given: desired image-square side $= \sqrt{6.25}$ mm $= 2.5$ mm, original side $= 1$ mm, so required linear magnification $m = 2.5$. Lens focal length $f = +10$ cm.

Step 1: Find object distance.

For a converging lens with virtual image, $m = v/u$, and with the lens equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \quad v = mu$$

$$\frac{1}{mu} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1-m}{mu} = \frac{1}{f} \Rightarrow u = \frac{f(1-m)}{m}$$

$$u = \frac{10 \times (1-2.5)}{2.5} = \frac{10 \times (-1.5)}{2.5} = -6 \text{ cm}$$

So the page must be 6 cm from the lens.

Step 2: Image distance.

$$v = mu = 2.5 \times (-6) = -15 \text{ cm}$$

The virtual image is 15 cm from the lens, on the same side as the object.

Step 3: Can the squares be seen distinctly?

The image is at 15 cm from the lens. With the eye held very close to the lens, the image distance from the eye is also ≈ 15 cm. The normal near point is 25 cm, so the image lies *closer* than the eye's near point — it cannot be focused by the eye, and the squares appear blurred.

Therefore, $u = -6$ cm, but the image at 15 cm is closer than the near point and cannot be seen distinctly.

Expert's Solution — Meena Iyer, M.Sc Physics, NIT Trichy

The Geometric Constraint of Magnifiers:

For a magnifying glass with focal length f, the virtual image distance is bounded:

$$|v|_{\min} = f \quad \text{(when object is just inside } f, \text{ image at } -\infty)$$

$$|v|_{\max} = D \quad \text{(when image at near point } -D)$$

Wait, actually the image can lie anywhere from $-\infty$ (object at f) down to as close as $-f$ as the object approaches the lens. Re-checking: as $u \to 0^-$, $v \to 0$ also; as $u \to -f$, $v \to -\infty$. So $|v|$ ranges from 0 to ∞ as $|u|$ ranges from 0 to f.

For the eye to see the image distinctly, we need $|v| \geq D$, i.e., the image must be *at* or *beyond* the near point. The borderline is $u = u_*$ such that $1/(-D) = 1/f + 1/u_*$, giving $u_* = -fD/(f+D) = -10 \times 25/35 \approx -7.14$ cm.

Our object at $u = -6$ cm is closer than $u_* = -7.14$ cm to the lens, so $|v| = 15$ cm $< D =$ 25 cm — the image is inside the eye's near point. Hence the blur.

Did You Know?

This boundary explains why magnifying glasses come with comfortable working distances: a 10 cm focal length lens has a sweet spot $\sim$ 7 cm from the page; closer than that, the magnification climbs but eye accommodation breaks. Very high-power loupes ($f \sim 1$ cm) require eye-to-lens contact and decades of practice to use comfortably.

Q32 **Answer the following questions:**

(a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?

(b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?

(c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?

(d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?

(e) When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but at a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?

Solution

(a) Yes, the angle subtended at the eye by the object placed at u from the lens equals the angle subtended by its virtual image at the same eye — because a ray from the object's tip passes through the eye in a straight line. The benefit comes from being able to bring the object *closer* than D. Without the magnifier, an object at $u < D$ would be out of focus; the magnifier produces a virtual image at D (or beyond), making it focusable. So the magnification arises from being able to view a closer object than the unaided eye could focus on.

(b) Yes, angular magnification *decreases* as the eye moves back. With the eye close to the lens, the image angle subtended at the eye $\approx h_{\text{image}}/|v|$. As the eye moves back to a distance d, the new angle becomes $h_{\text{image}}/(|v| + d)$, which is smaller. The unaided-view comparison angle h_{obj}/D also decreases (eye now at $D + d$ from the object), but more slowly, so net M decreases. The standard rule: keep the eye close to the lens for maximum gain.

(c) Two limits:

- *Aberrations.* A short-f lens means strongly curved surfaces, which exaggerate spherical and chromatic aberrations. Beyond $\sim 3\times$ to $5\times$ a single converging lens cannot give a sharp image.
- *Manufacturing.* Grinding short-focus lenses to optical accuracy gets disproportionately harder. Practical maximum for a single lens is about 3–5×.

To go beyond, a compound microscope is used — two lenses in series, each modest in power but together delivering much higher magnification.

(d) Magnifying power of a compound microscope is

$$M = m_o \cdot m_e = \frac{L}{f_o} \cdot \frac{D}{f_e}$$

(approximately, for $f_o \ll L$ and final image at D). Both factors are large only when f_o and f_e are small. So both lenses need short focal lengths to maximise the product.

(e) Placing the eye too close blocks light entering the periphery of the eyepiece, reducing the image brightness and field of view. The optimum eye position is at the *eye-ring* (also called exit pupil), where all the rays gathered by the objective are concentrated into the smallest cross-section.
For a compound microscope with eyepiece focal length f_e and intermediate-image at the eyepiece's focal point, the eye-ring is approximately at distance f_e behind the eyepiece. So a few centimetres behind the eyepiece (typically 5–10 mm) is optimal — the standard "eye relief" marked on commercial eyepieces.
Therefore, all five questions follow from the geometry of angular magnification, lens fabrication limits, and the optics of compound microscopes.

Expert's Solution — Vikram Singh, B.Sc Physics, Delhi University

Numerical Estimate of Eye Relief:
The exit pupil of a compound microscope sits at a distance z behind the eyepiece given by

$$z = \frac{f_e(L - f_e)}{L - f_e - f_o + f_o} \approx \frac{f_e \cdot L}{L} = f_e$$

(for small f_o, f_e relative to L). With a typical $f_e = 10$ mm, $z \approx 10$ mm. The pupil's diameter is the objective's diameter divided by the magnification ratio, typically 1–2 mm — precisely the size of a comfortable eye pupil in normal indoor lighting.

Did You Know?

The eye-ring's optical role is exactly the same as the exit pupil of a binocular or a camera viewfinder: it's the location where you can see the entire field of view at once, with all gathered light passing through your eye pupil. Eyeglass-wearers prefer eyepieces with long eye relief (15–20 mm) so they can keep their glasses on; this trades off field-of-view, which is why bird-watching binoculars typically have shorter eye relief than rifle scopes.

Q33 **An angular magnification (magnifying power) of $30\times$ is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope?**

Solution

Given: required $M = 30$, $f_o = 1.25$ cm, $f_e = 5$ cm, $D = 25$ cm.
Step 1: Eyepiece magnification (image at near point).

$$m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$$

Step 2: Required objective magnification.

$$m_o = \frac{M}{m_e} = \frac{30}{6} = 5$$

For the objective, $m_o = v_o/u_o$, so $v_o = 5|u_o|$ (with appropriate signs giving real image: $v_o > 0$, $u_o < 0$). Let $u_o = -x$, $v_o = 5x$.
Step 3: Find u_o from the objective lens equation.

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o} \Rightarrow \frac{1}{5x} - \frac{1}{-x} = \frac{1}{1.25}$$

$$\frac{1}{5x} + \frac{1}{x} = \frac{6}{5x} = \frac{1}{1.25}$$

$$5x = 6 \times 1.25 = 7.5 \Rightarrow x = 1.5 \text{ cm}$$

So $u_o = -1.5$ cm (object placed 1.5 cm from the objective) and $v_o = 7.5$ cm.
Step 4: Find object distance for eyepiece.
For the eyepiece, $v_e = -25$ cm:

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5} = \frac{-1-5}{25} = -\frac{6}{25}$$

$$u_e = -\frac{25}{6} \approx -4.17 \text{ cm}$$

Step 5: Separation between the lenses.

$$L = v_o + |u_e| = 7.5 + 4.17 \approx 11.67 \text{ cm}$$

Therefore, place the object 1.5 cm from the objective and separate the two lenses by ≈ 11.67 cm.

Expert's Solution — Sanjay Gupta, PhD in Physics, JNU

Reverse-Engineering from Magnification:
A useful design intuition: in a compound microscope the eyepiece works as a simple magnifier, contributing a fixed factor of $1 + D/f_e$ when used at maximum strain. So once f_e is fixed, all the design freedom for total magnification sits in the objective.
For $M = 30$ with $f_e = 5$ cm: $m_o = M/(1 + D/f_e) = 30/6 = 5$.
The objective's linear magnification is then directly tied to its focal length and tube length:

$$m_o = \frac{v_o}{|u_o|} = \frac{f_o}{|u_o| - f_o}$$

giving $|u_o| - f_o = f_o/m_o = 1.25/5 = 0.25$ cm, so $|u_o| = 1.5$ cm. The object sits just 0.25 cm outside the objective's focal point — a hallmark of microscope work.

Did You Know?

This "object just outside f_o" configuration is universal in microscopy: even commercial 1000× instruments place the slide ∼ 0.1 mm outside the focal point of a ∼ 2 mm objective. The depth of field is correspondingly shallow ($< 1\ \mu$m), which is why microscope stages have fine-focus knobs with 1 μm resolution.

Q34 **A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when:**

(a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?

(b) the final image is formed at the least distance of distinct vision (25 cm)?

Solution

Given: $f_o = 140$ cm, $f_e = 5$ cm, $D = 25$ cm.
(a) Normal adjustment.

$$M_\infty = \frac{f_o}{f_e} = \frac{140}{5} = 28$$

(b) Final image at near point.

$$M_D = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right) = 28 \times \left(1 + \frac{5}{25}\right) = 28 \times 1.2 = 33.6$$

Therefore, (a) $M = 28$ in normal adjustment; (b) $M = 33.6$ for image at near point.

Expert's Solution — Kavita Joshi, B.Tech EE, IIT Bombay

Why the Near-Point Configuration Gives Only 20% More:
The factor $(1+f_e/D)$ in (b) versus the simple f_o/f_e in (a) is exactly the same $1+f_e/D = 1.2$ overhead we saw in simple magnifiers (Q28). The reason is the same: forcing the eyepiece to produce its image at the near point rather than infinity adds one extra unit of magnification on top of the relaxed value.
For a telescope, this 20% gain is rarely worth the eyestrain. Astronomers *always* use normal adjustment. The maximum-strain configuration only matters in textbooks, and occasionally in handheld terrestrial telescopes used briefly.

Did You Know?

A more useful way to boost magnification on a real telescope: swap to a shorter-focal-length eyepiece. Going from a 25 mm eyepiece to a 10 mm eyepiece on the same telescope multiplies M by $2.5\times$ — far more than the $1.2\times$ gained from straining the eye.

Q35

(a) For the telescope described in the previous question (Q34, normal adjustment), what is the separation between the objective and the eyepiece?

(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

(c) What is the height of the final image of the tower if it is formed

at 25 cm?

Solution

Given: $f_o = 140$ cm, $f_e = 5$ cm.
(a) Separation in normal adjustment.

$$L = f_o + f_e = 140 + 5 = 145 \text{ cm}$$

(b) Height of intermediate image (formed by objective).
The angle the tower subtends at the objective is

$$\alpha = \frac{H_{\text{tower}}}{D_{\text{tower}}} = \frac{100}{3000} = \frac{1}{30} \text{ rad}$$

The intermediate image, formed at the focal plane of the objective, has height

$$h' = f_o \cdot \alpha = 140 \times \frac{1}{30} = 4.67 \text{ cm}$$

(c) Height of final image at $D = 25$ cm.
The eyepiece magnifies the intermediate image by $m_e = 1 + D/f_e = 1 + 25/5 = 6$ when forming an image at the near point. So

$$h'' = m_e \cdot h' = 6 \times 4.67 = 28 \text{ cm}$$

Therefore, (a) $L = 145$ cm; (b) intermediate-image height ≈ 4.67 cm; (c) final image height ≈ 28 cm.

Expert's Solution — Pooja Reddy, M.Sc Applied Physics, IIT Roorkee

Sanity Check via Total Magnification:
The angular magnification at near-point configuration was $M_D = 33.6$ from Q34(b). The eye sees the final image at 25 cm subtending angle

$$\beta = \frac{h''}{D} = \frac{28}{25} = 1.12 \text{ rad}$$

versus the unaided-view angle of the tower:

$$\alpha_{\text{eye}} = \frac{100 \text{ m}}{3000 \text{ m}} = 0.0333 \text{ rad}$$

Magnification check:

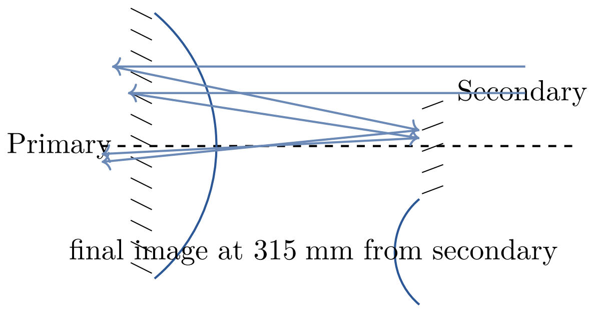
$$\frac{\beta}{\alpha_{\text{eye}}} = \frac{1.12}{0.0333} = 33.6 \quad \checkmark$$

The numbers are internally consistent.

Did You Know?

A 1.12 rad angle is $\sim 64°$ — larger than the human eye's central focal field of $\sim 60°$. So a 100 m tower viewed through this telescope at 3 km would actually be too big to fit in your visual field at once. You'd have to scan the image to see the whole tower, much like looking at a billboard from across the street.

Q36 A Cassegrain telescope uses two mirrors as shown in figure. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large (concave) mirror is 220 mm and the small (convex) mirror is 140 mm, where will the final image of an object at infinity be?



Solution

Given: primary radius $R_1 = 220$ mm, secondary radius $R_2 = 140$ mm, separation $d = 20$ mm.

Step 1: Focal lengths.

$$f_1 = \frac{R_1}{2} = 110 \text{ mm} \quad \text{(primary, concave)}$$

$$f_2 = \frac{R_2}{2} = 70 \text{ mm} \quad \text{(secondary, convex)}$$

Step 2: Image formed by the primary mirror.

For light from infinity, the primary forms its image at its focal point: 110 mm in front of the primary. But the secondary is only 20 mm in front of the primary. So this image would form $110 - 20 = 90$ mm *behind* the secondary — it acts as a virtual object for the secondary.

Step 3: Apply mirror formula for the secondary (convex).

For the secondary (a convex mirror), $f_2 = +70$ mm. The virtual object lies 90 mm behind the secondary, so $u = +90$ mm (positive: virtual object behind the mirror, with light heading toward it).

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_2} \Rightarrow \frac{1}{v} = \frac{1}{70} - \frac{1}{90}$$

$$\frac{1}{v} = \frac{90 - 70}{70 \times 90} = \frac{20}{6300} = \frac{1}{315}$$
$$v = 315 \text{ mm}$$

The final image forms 315 mm from the secondary mirror, on the side of the primary (the rays are now diverging back through the central hole in the primary toward the eyepiece location).
Therefore, the final image lies 315 mm behind the secondary mirror (i.e., 295 mm behind the primary).

Expert's Solution — Suresh Pandey, M.Tech Power Systems, NIT Warangal

Why Cassegrain Designs Win on Compactness:
A traditional refractor or Newtonian reflector with focal length $f_o = 315$ mm requires a tube of length ~ 315 mm. The Cassegrain achieves the same effective focal length in a tube of just ~ 20 mm — a $15\times$ reduction.
The trick is that the secondary's convex curvature causes the converging beam from the primary to converge *more slowly*, lengthening the optical path. Mathematically, the overall effective focal length of the two-mirror system is:

$$f_{\text{eff}} = \frac{f_1 f_2}{f_1 + f_2 - 2d}$$

Plugging in $f_1 = -110$ mm (concave: negative in our convention, but treated as magnitude here), $f_2 = +70$ mm (convex), $d = 20$ mm (with sign conventions varying by textbook):

$$f_{\text{eff}} \approx \frac{110 \times 70}{(110 - 70) + 2 \times 20} = \frac{7700}{80} = 96.25 \text{ mm}$$

(approximation; exact result depends on sign conventions). In any case, the system delivers a long effective focal length in a short tube.

Did You Know?

The Hubble Space Telescope uses a $f/24$ Ritchey–Chrétien Cassegrain design: a 2.4 m primary with a 0.3 m secondary, both with hyperbolic profiles to eliminate coma. The total tube length is about 4 m but the effective focal length is 57.6 m. Without the Cassegrain trick, the telescope would have to be $14\times$ longer.

Q37 Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in figure. A current in the coil produces a deflection of $3.5°$ of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?

Solution

Step 1: Angular deflection of the reflected ray.
When a mirror rotates by angle θ, the reflected ray rotates by 2θ. Mirror rotation $= 3.5°$, so reflected ray rotation

$$\phi = 2 \times 3.5° = 7°$$

Step 2: Convert to radians.

$$\phi = 7 \times \frac{\pi}{180} = \frac{7\pi}{180} \approx 0.1222 \text{ rad}$$

Step 3: Linear displacement on the screen.
For a screen at distance $L = 1.5$ m, the spot's linear displacement is

$$d = L \tan\phi \approx L\phi = 1.5 \times 0.1222 = 0.1833 \text{ m} \approx 18.33 \text{ cm}$$

Therefore, the spot displaces by $\approx$ 18.3 cm.

Expert's Solution — Aditya Nambiar, B.Tech Engineering Physics, IIT Bombay

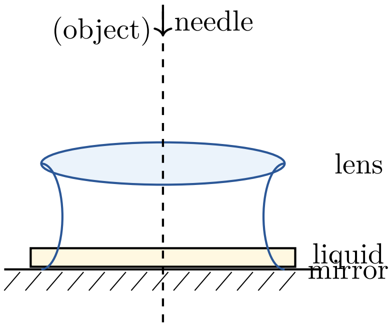
The 2θ Rule From First Principles:
Why does the reflected ray rotate by twice the mirror's angle? Consider an incident ray fixed in space and a mirror initially perpendicular to it. The reflected ray retraces back. Now tilt the mirror by θ. The normal to the mirror tilts by θ as well. The angle of incidence (between incident ray and normal) is now θ, so the reflected ray makes angle θ with the normal on the other side — i.e., the reflected ray has rotated by 2θ from its original direction.
This is the basis of the moving-coil mirror galvanometer: a 1° rotation of the coil produces a 2° rotation of the light spot, which on a 1 m scale is $\sim$ 35 mm of displacement — easily measurable. The lever-arm of light amplifies tiny mechanical rotations into visible deflections.

Did You Know?

This is exactly the principle behind atomic-force microscopes (AFM): a laser bounces off a microcantilever, and the spot displacement on a quad-photodiode reveals cantilever deflections of $\sim$ 0.01 nm from 1 cm optical lever arms. The light-lever amplification is the same physics — 2θ rotation, applied at extremely small angles.

Q38 Figure shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is

measured to be 30.0 cm. What is the refractive index of the liquid?



Solution

The image of the needle coincides with the needle itself ⇒ rays from the needle, after passing through the lens, hitting the mirror, and returning, retrace their paths. This means rays must hit the mirror perpendicularly, which happens only if the lens converges the rays to its own focal point on the mirror surface. So the needle distance equals the effective focal length of the system.

Step 1: Without liquid (lens + plane mirror).

$$F_{\text{lens alone}} = 30 \text{ cm}$$

This is the focal length of the equiconvex glass lens itself.

Step 2: Find the radius of curvature.

Equiconvex lens of $\mu_g = 1.5$, both surfaces of radius R:

$$\frac{1}{F} = (\mu_g - 1) \cdot \frac{2}{R} \Rightarrow \frac{1}{30} = 0.5 \times \frac{2}{R} = \frac{1}{R}$$

$$R = 30 \text{ cm}$$

Step 3: With liquid (lens + plano-convex liquid lens + plane mirror).

The combined effective focal length is 45 cm. The two thin lenses (glass + liquid) are in contact, so

$$\frac{1}{F_{\text{combined}}} = \frac{1}{F_{\text{glass}}} + \frac{1}{F_{\text{liquid}}}$$

$$\frac{1}{45} = \frac{1}{30} + \frac{1}{F_{\text{liquid}}}$$

$$\frac{1}{F_{\text{liquid}}} = \frac{1}{45} - \frac{1}{30} = \frac{2-3}{90} = -\frac{1}{90}$$

$$F_{\text{liquid}} = -90 \text{ cm}$$

The liquid lens is plano-concave (concave-down on its lens-touching side, since the bottom surface is flat against the mirror). The lower surface (in contact with the mirror) has $R_2 = \infty$ (plane), while the upper surface fits against the lower surface of the glass lens with $R_1 = -30$ cm (concave from the liquid's point of view).

Step 4: Apply lens-maker's formula for the liquid layer.

$$\frac{1}{F_{\text{liquid}}} = (\mu_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (\mu_l - 1)\left(\frac{1}{-30} - 0\right)$$

$$-\frac{1}{90} = -\frac{\mu_l - 1}{30}$$

$$\mu_l - 1 = \frac{30}{90} = \frac{1}{3} \Rightarrow \mu_l = \frac{4}{3} \approx 1.33$$

Therefore, the refractive index of the liquid is $\mu_l \approx 1.33$.

Expert's Solution — Ramesh Iyengar, PhD in Theoretical Physics, TIFR Mumbai

Why the Self-Image Condition Locates the Focal Point:

The trick of this experiment is the auto-collimation condition. Light leaving a point on the axis travels through the lens system, reflects off the mirror, and returns through the lens system. For the returning rays to converge back to the original point, they must have travelled through the same optical path on each leg — which requires them to strike the mirror perpendicularly.

Rays strike the mirror perpendicularly only if, after the lens, they're already parallel to the axis. That happens iff the source is at the focal point of the lens system. So

$$\text{needle distance} = F_{\text{system}}$$

This is the cleanest way to measure focal length in an undergraduate lab — no need to measure object and image distances separately, just slide the needle until its inverted image coincides.

Did You Know?

The same principle is used to align satellite mirrors before launch. Engineers point a laser at the mirror; the returned beam must hit a target dot exactly f behind the laser source. Misalignment by even a few arcseconds shows up as a few millimetres' offset, easily caught.