CBSE Class 12 Mathematics 2009 - Set 65/1/1 - Delhi

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

1. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a} \cdot \vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$.

2. Write a unit vector in the direction of $\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}$.

3. Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors.

4. If matrix $A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$, write AA', where A' is the transpose of matrix A.

5. Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$.

6. Using principal value, evaluate the following: $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.

7. Evaluate: $\int \frac{\sec^2 x}{3 + \tan x}\, dx$.

8. If $\int_0^1 (3x^2 + 2x + k)\, dx = 0$, find the value of k.

9. If the binary operation $*$ on the set of integers $\mathbb{Z}$, is defined by $a * b = a + 3b^2$, then find the value of $2 * 4$.

10. If A is an invertible matrix of order 3 and $|A| = 5$, then find the value of $|\text{adj } A|$.

Section B

11. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

12(a). Prove that: $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right) = \frac{\pi}{2}$.

OR

12(b). Solve for x: $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$.

13. Find the value of λ so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.

14. Solve the following differential equation: $\frac{dy}{dx} + y = \cos x - \sin x$.

15. Find the particular solution, satisfying the given condition, for the following differential equation: $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$; $y = 0$ when $x = 1$.

16. By using properties of determinants, prove the following: $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x + 4)(4 - x)^2$.

17. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

18. Differentiate the following function with respect to x: $x^{\sin x} + (\sin x)^{\cos x}$.

19(a). Evaluate: $\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}\, dx$.

OR

19(b). Evaluate: $\int \frac{(x-4)e^x}{(x-2)^3}\, dx$.

20. Prove that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$, is an equivalence relation.

21(a). Find $\frac{dy}{dx}$ if $(x^2 + y^2)^2 = xy$.

OR

21(b). If $y = 3\cos(\log x) + 4\sin(\log x)$, then show that $x^2 \cdot \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

22(a). Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$.

OR

22(b). Find the intervals in which the function f given by $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is (i) increasing (ii) decreasing.

Section C

23(a). Find the volume of the largest cylinder that can be inscribed in a sphere of radius r.

OR

23(b). A tank with rectangular base and rectangular sides, open at the top, is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs ₹70 per sq. metre for the base and ₹45 per sq. metre for sides, what is the cost of the least expensive tank?

24. A diet is to contain at least 80 units of Vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs ₹4 per unit and F_2 costs ₹6 per unit. One unit of food F_1 contains 3 units of Vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of Vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consists of a mixture of these two foods and also meets the minimal nutritional requirements.

25. Three bags contain balls as shown in the following table:

Bag	White	Black	Red
I	1	2	3
II	2	1	1
III	4	3	2

A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they came from bag III?

26. Using matrices, solve the following system of equations: $2x - 3y + 5z = 11$; $3x + 2y - 4z = -5$; $x + y - 2z = -3$.

27(a). Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\, dx$.

OR

27(b). Evaluate: $\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x)\, dx$.

28. Using the method of integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

29. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

- End of Question Paper -