CBSE Class 12 Mathematics 2009 - Set 65/1 - Outside Delhi

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

1. Find the value of x, if $\begin{pmatrix} 3x+y & -y \\ 2y-x & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -5 & 3 \end{pmatrix}$.

2. Let $*$ be a binary operation on $\mathbb{N}$ given by $a * b = \text{HCF}(a, b)$, $a, b \in \mathbb{N}$. Write the value of $22 * 4$.

3. Evaluate: $\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1-x^2}}\, dx$.

4. Evaluate: $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$.

5. Write the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

6. Write the value of the following determinant: $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$.

7. Find the value of x from the following: $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$.

8. Find the value of p if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}$.

9. Write the direction cosines of a line equally inclined to the three coordinate axes.

10. If $\vec{p}$ is a unit vector and $(\vec{x} - \vec{p}) \cdot (\vec{x} + \vec{p}) = 80$, then find $|\vec{x}|$.

Section B

11(a). The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When $x = 8$ cm and $y = 6$ cm, find the rate of change of (a) the perimeter, (b) the area of the rectangle.

OR

11(b). Find the intervals in which the function f given by $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$, is strictly increasing or strictly decreasing.

12(a). If $\sin y = x \sin(a + y)$, prove that $\dfrac{dy}{dx} = \dfrac{\sin^2(a+y)}{\sin a}$.

OR

12(b). If $(\cos x)^y = (\sin y)^x$, find $\dfrac{dy}{dx}$.

13. Let $f : \mathbb{N} \to \mathbb{N}$ be defined by $f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in \mathbb{N}$. Find whether the function f is bijective.

14(a). Evaluate: $\displaystyle\int \frac{dx}{\sqrt{5 - 4x - 2x^2}}$.

OR

14(b). Evaluate: $\displaystyle\int x \sin^{-1} x \, dx$.

15. If $y = \dfrac{\sin^{-1} x}{\sqrt{1 - x^2}}$, show that $(1 - x^2)\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} - y = 0$.

16. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

17. Using properties of determinants, prove the following: $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$.

18. Solve the following differential equation: $x\dfrac{dy}{dx} = y - x\tan\left(\dfrac{y}{x}\right)$.

19. Solve the following differential equation: $\cos^2 x \dfrac{dy}{dx} + y = \tan x$.

20. Find the shortest distance between the following two lines: $\vec{r} = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (\lambda+1)\hat{k}$; $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$.

21(a). Prove the following: $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right] = \dfrac{x}{2}$, $x \in \left(0, \dfrac{\pi}{4}\right)$.

OR

21(b). Solve for x: $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$.

22. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

Section C

23. Find the equation of the plane determined by the points $A(3, -1, 2)$, $B(5, 2, 4)$ and $C(-1, -1, 6)$. Also find the distance of the point $P(6, 5, 9)$ from the plane.

24. Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.

25. Evaluate: $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2 \sin^2 x}$.

26(a). Using matrices, solve the following system of equations: $x + y + z = 6$; $x + 2z = 7$; $3x + y + z = 12$.

OR

26(b). Obtain the inverse of the following matrix using elementary operations: $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$.

27. Coloured balls are distributed in three bags as shown in the following table:

Bag	Black	White	Red
I	1	2	3
II	2	4	1
III	4	5	3

A bag is selected at random and then two balls are randomly drawn from the selected bag. They happen to be black and red. What is the probability that they came from bag I?

28. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5,760 to invest and has a space for at most 20 items. A fan costs him Rs. 360 and a sewing machine Rs. 240. His expectation is that he can sell a fan at a profit of Rs. 22 and a sewing machine at a profit of Rs. 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximise the profit? Formulate this as a linear programming problem and solve it graphically.

29(a). If the sum of the lengths of the hypotenuse and a side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

OR

29(b). A manufacturer can sell x items at a price of Rs. $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5} + 500\right)$. Find the number of items he should sell to earn maximum profit.

- End of Question Paper -