CBSE Class 12 Physics Question Paper 2011 with Solution PDF-Set 55/2/1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours **Maximum Marks: 70**

General Instructions

(i) All questions are compulsory.

(ii) There are **30** questions in total. Questions **1 to 8** are very short answer type questions and carry **one** mark each.

(iii) Questions **9 to 18** carry **two** marks each, questions **19 to 27** carry **three** marks each and questions **28 to 30** carry **five** marks each.

(iv) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all three questions of five marks each. You have to attempt only one of the choices in such questions.

(v) Use of calculators is **not** permitted. However, you may use log tables if necessary.

You may use the following values of physical constants wherever necessary :

$c = 3 \times 10^8 \text{ m/s}$

$h = 6.63 \times 10^{-34} \text{ J s}$

$e = 1.6 \times 10^{-19} \text{ C}$

$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

$m_e = 9.1 \times 10^{-31} \text{ kg}$

1. Define the term 'threshold frequency' in relation to photoelectric effect. [1 mark]

Solution:

Concept:

- In photoelectric emission, an incident photon gives its energy $h\nu$ to an electron, where h is Planck's constant and ν is the radiation frequency.
- The work function ϕ is the minimum energy needed to liberate an electron from the surface of a given material.

Step 1: Identify the minimum frequency Emission can occur only if the photon energy is at least equal to the work function. At the limiting frequency ν_0, the electron just escapes without excess kinetic energy, so $h\nu_0 = \phi$.

Step 2: State the condition below threshold If $\nu < \nu_0$, each photon has insufficient energy to liberate an electron. Increasing the light intensity at such a frequency does not cause photoelectric emission.

Step 3: Conclusion Threshold frequency ν_0 is the minimum frequency of incident radiation required to cause photoelectric emission from a given material; $\nu_0 = \phi/h$.

Quick Tip to Solve: In a definition, state both the minimum frequency and the given material; threshold depends on its work function.

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2. The peak value of e.m.f. in a.c. is E_0. Write its (i) rms and (ii) average value over a complete cycle. [1 mark]

Solution:

Concept:

- For sinusoidal alternating emf, $E(t) = E_0 \sin \omega t$, where E_0 is the peak emf and ω is the angular frequency.
- RMS is the square root of the mean square over a cycle: $E_{\text{rms}} = \sqrt{\langle E^2 \rangle}$. The ordinary mean retains the sign of the emf.

Step 1: Evaluate the rms value Over a complete sinusoidal cycle, $\langle \sin^2 \omega t \rangle = 1/2$. Thus

$$E_{\text{rms}} = \sqrt{E_0^2/2} = \frac{E_0}{\sqrt{2}}.$$

Step 2: Average over the complete cycle The positive and negative half-cycles have equal areas with opposite signs. Their contributions cancel, giving

$$E_{\text{av}} = \frac{1}{T}\int_0^T E_0 \sin \omega t \, dt = 0.$$

This signed full-cycle average differs from a half-cycle average or the mean of the magnitude.

Step 3: Conclusion (i) $E_{\text{rms}} = E_0/\sqrt{2}$. (ii) The average emf over a complete cycle is zero.

Quick Tip to Solve: Square first for RMS; retain signs for a full-cycle average, where opposite half-cycles cancel.

3. Two insulated charged copper spheres A and B of identical size have charges q_A and q_B respectively. A third sphere C of the same size but uncharged is brought in contact with the first and then in contact with the second and finally removed from both. What are the new charges on A and B? [1 mark]

Solution:

Concept:

- Charge is conserved when isolated conducting spheres touch. Identical spheres share the combined charge equally because they have equal capacitances and equal final potentials.
- Apply charge sharing separately at each contact. A sphere removed after the first contact retains its charge during the second contact.

Step 1: Share charge between A and C Initially C is uncharged. On contact, the total charge of A and C is q_A, so each receives $q_A/2$. After C is removed, A retains this charge and C carries $q_A/2$.

Step 2: Share charge between B and C C now touches B, whose charge is still q_B. Their total charge is $q_B + q_A/2$, and each sphere receives half:

$$q'_B = q'_C = \frac{q_B + q_A/2}{2} = \frac{q_A + 2q_B}{4}.$$

A is isolated from this second contact, so its charge does not change further.

Step 3: Conclusion The new charges are $q'_A = q_A/2$ and $q'_B = (q_A + 2q_B)/4$.

Quick Tip to Solve: Carry C's charge forward after its first contact; do not treat C as uncharged again when it touches B.

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4. A narrow beam of protons and deuterons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to their direction of momentum. What would be the ratio of the radii of the circular paths described by them? [1 mark]

Solution:

Concept:

- For motion perpendicular to a uniform magnetic field, the magnetic force $|q|vB$ provides the centripetal force mv^2/r.
- Hence $r = mv/(|q|B) = p/(|q|B)$, where p is the particle's momentum and $|q|$ is its charge magnitude.

Step 1: Compare the particles' charges A proton and a deuteron each carry one positive elementary charge, so $q_p = q_d = e$. A deuteron has greater mass, but its charge is the same as a proton's.

Step 2: Use the given equal momenta Both particles enter the same field with equal momentum and perpendicular motion. Therefore

$$\frac{r_p}{r_d} = \frac{p_p/(eB)}{p_d/(eB)} = \frac{p_p}{p_d} = 1.$$

The larger mass of the deuteron is compensated by its lower speed at the given equal momentum.

Step 3: Conclusion The ratio of the radii of the proton and deuteron paths is $r_p : r_d = 1 : 1$.

Quick Tip to Solve: When momentum is specified, use $r = p/(qB)$; equal momentum and equal charge immediately give equal radii.

5. What is the function of a 'Repeater' in a communication system? [1 mark]

Solution:

Concept:

- A signal becomes weaker as it propagates through a communication channel; this loss of signal strength is called attenuation.
- A repeater combines a receiver and a transmitter at an intermediate point along the communication path.

Step 1: Receive the weakened signal The receiver in a repeater picks up the signal arriving from the previous transmitter. As the received signal has been attenuated during propagation, it needs restoration before travelling a further distance.

Step 2: Amplify and retransmit The repeater amplifies the received signal and retransmits it towards the next receiver or repeater. This compensates for attenuation and increases the effective communication range; it relays the same information onward.

Step 3: Conclusion A repeater receives a weakened signal, amplifies it and retransmits it, thereby extending the range of a communication system.

Quick Tip to Solve: Remember the repeater's sequence: receive, amplify, retransmit; connect its purpose directly to overcoming attenuation.

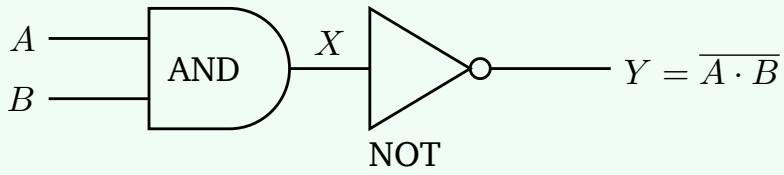
6. Draw the logic circuit of a NAND gate and write its truth table. [1 mark]

Solution:

Concept:

- A NAND operation is an AND operation followed by inversion: $Y = \overline{A \cdot B}$, where A and B are binary inputs and Y is the output.
- An AND gate produces 1 only when both inputs are 1. The NOT gate reverses that intermediate output, exchanging 0 and 1.

Step 1: Connect AND to NOT Connect inputs A and B to an AND gate and feed its output $X = A \cdot B$ into a NOT gate. The final output is $Y = \overline{X}$. The circuit below explicitly shows these two operations in sequence.



NAND circuit formed by an AND gate followed by a NOT gate

Quick Tip to Solve: For NAND, fill the AND column first and reverse every entry; the sole zero occurs for two high inputs.

Step 2: List every input combination Evaluate AND first, then invert its output:

A	B	$X = A \cdot B$	$Y = \overline{A \cdot B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Thus the output is high when either input is low; only the input pair $(1, 1)$ produces a low output.

Step 3: Conclusion A NAND circuit is an AND gate followed by a NOT gate, with output $Y = \overline{A \cdot B}$. Its truth-table outputs for $00, 01, 10, 11$ are $1, 1, 1, 0$, respectively.

Quick Tip to Solve: For NAND, fill the AND column first and reverse every entry; the sole zero occurs for two high inputs.

7. How is the mean life of a radioactive sample related to its half life? [1 mark]

Solution:

Concept:

- The mean life is $\tau = 1/\lambda$, where λ is the radioactive decay constant.
- The half life $T_{1/2}$ is the time for the number of undecayed nuclei to decrease to half its initial value.

Step 1: Apply the decay law at half life From $N = N_0 e^{-\lambda t}$, putting $N = N_0/2$ gives $e^{-\lambda T_{1/2}} = 1/2$. Taking natural logarithms yields $T_{1/2} = \ln 2/\lambda$.

Step 2: Eliminate the decay constant Substitute $1/\lambda = \tau$ to obtain $T_{1/2} = \tau \ln 2$. Since $\ln 2 = 0.693$, the mean life is larger than the half life.

Step 3: Conclusion

$$\boxed{\tau = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693} \simeq 1.443 T_{1/2}}$$

Quick Tip to Solve: Start from the two expressions in terms of λ; divide the half life by 0.693 to obtain the mean life.

8. Write two uses of microwaves. [1 mark]

Solution:

Concept:

- Microwaves are electromagnetic waves with short wavelengths, so they can be sent as narrow directional beams.
- Absorption of microwave energy by water-containing food produces heating.

Step 1: State a use in detection Microwaves are used in radar to detect aircraft and determine their

Quick Tip to Solve: Recall one use of a narrow microwave beam (radar) and one use of absorption of microwave energy (cooking).

distance. A transmitted pulse reflects from the target, and its return time gives the range.

Step 2: State a use in heating Microwaves are used in microwave ovens to cook or heat food. Water molecules in the food absorb energy from the alternating electric field, converting it into heat.

Step 3: Conclusion Two uses of microwaves are radar detection of aircraft and cooking or heating food in microwave ovens.

Quick Tip to Solve: Recall one use of a narrow microwave beam (radar) and one use of absorption of microwave energy (cooking).

9. Calculate the amount of work done in rotating a dipole, of dipole moment 3×10^{-8} C m, from its position of stable equilibrium to the position of unstable equilibrium, in a uniform electric field of intensity 10^4 N/C. [2 marks]

Solution:

Concept:

- The potential energy of an electric dipole in a uniform field is $U = -pE\cos\theta$, where p is its dipole moment and θ is the angle with the field.
- In a slow rotation, the work done by an external agent equals the increase in potential energy: $W_{\text{ext}} = U_f - U_i$.

Step 1: Identify the initial and final orientations At stable equilibrium the dipole is parallel to the electric field, so $\theta_i = 0$ and $U_i = -pE$. At unstable equilibrium it is antiparallel, so $\theta_f = \pi$ and $U_f = +pE$. Both orientations have zero torque, but the parallel orientation has minimum potential energy and the antiparallel orientation has maximum potential energy.

Step 2: Calculate the increase in potential energy The external agent must supply energy to rotate the dipole against the electric torque. Using $p = 3 \times 10^{-8}$ C m and $E = 10^4$ N C^{-1},

$$\begin{aligned} W_{\text{ext}} &= (+pE) - (-pE) = 2pE, \\ &= 2(3 \times 10^{-8})(10^4)\,\text{J}. \end{aligned}$$

The unit is joule because (C m)(N C^{-1}) = N m. The work done by the electric field itself has the opposite sign.

Step 3: Conclusion The work required to rotate the dipole is $\boxed{6 \times 10^{-4}\,\text{J}}$, supplied by the external agent.

Quick Tip to Solve: For rotation from stable to unstable equilibrium, use $W_{\text{ext}} = 2pE$; the energy rises from $-pE$ to $+pE$.

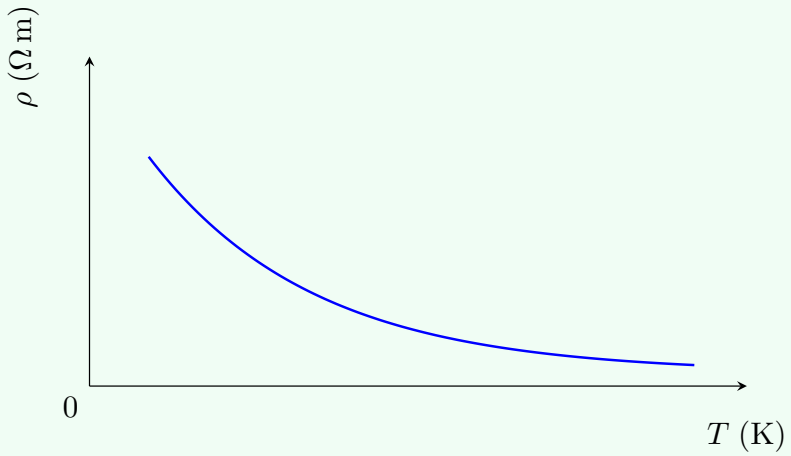
10. Plot a graph showing temperature dependence of resistivity for a typical semiconductor. How is this behaviour explained? [2 marks]

Solution:

Concept:

- Semiconductor conductivity is $\sigma = e(n\mu_e + p\mu_h)$, where n, p are electron and hole concentrations, μ_e, μ_h their mobilities, and e is the elementary charge.
- Resistivity is $\rho = 1/\sigma$. A typical semiconductor has a negative temperature coefficient of resistivity.

Step 1: Plot resistivity against temperature Take absolute temperature T on the horizontal axis and resistivity ρ on the vertical axis. Draw a smooth falling curve: the resistivity is large at low temperature and decreases as temperature rises. The curve is schematic; it shows the qualitative trend rather than measured values for a particular material.



Schematic temperature dependence of semiconductor resistivity

Step 2: Explain the decrease in resistivity Heating supplies energy to excite electrons from the valence band into the conduction band, leaving holes behind. Thus the concentrations of mobile electrons and holes increase substantially. Although collisions can reduce their mobilities as temperature rises, the increase in carrier concentration dominates in the usual semiconductor behaviour. Consequently conductivity increases and its reciprocal, resistivity, decreases. This accounts for the negative temperature coefficient shown in the graph.

Step 3: Conclusion The resistivity of a typical semiconductor decreases with increasing temperature because thermal excitation creates more electron-hole pairs and increases conductivity.

Quick Tip to Solve: For a semiconductor, link heating to more electron-hole pairs, then use $\rho = 1/\sigma$ to determine the downward trend.

11. When four hydrogen nuclei combine to form a helium nucleus, estimate the amount of energy in MeV released in this process of fusion. (Neglect the masses of electrons and neutrinos)

Given:

(i) mass of $^1_1$H = 1.007825 u

(ii) mass of helium nucleus = 4.002603 u, 1 u = 931 MeV/c^2 [2 marks]

Solution:

Concept:

- Energy released in fusion is the decrease in rest-mass energy: $Q = \Delta m c^2$, where Δm is the initial mass minus the final mass.
- With the given conversion $1\,\text{u} = 931\,\text{MeV}/c^2$, a mass decrease of Δm in atomic mass units releases $931\Delta m$ MeV.

Step 1: Find the mass decrease in the fusion process Four hydrogen nuclei combine to produce one helium nucleus. Use the supplied masses and neglect the masses of electrons and neutrinos as instructed. The initial mass is four times the given hydrogen mass, while the final mass is the given helium mass:

$$m_i = 4(1.007825)\,\text{u} = 4.031300\,\text{u},$$
$$m_f = 4.002603\,\text{u},$$
$$\Delta m = 4.031300 - 4.002603 = 0.028697\,\text{u}.$$

The positive mass decrease means the products have less rest-mass energy than the reacting nuclei.

Step 2: Convert the mass decrease into released energy Substitute the mass decrease into Einstein's mass-energy relation. The c^2 in the energy relation cancels the c^{-2} in the supplied mass conversion:

$$Q = (0.028697)(931)\,\text{MeV} = 26.717907\,\text{MeV}.$$

This energy appears as the energy carried by the reaction products, rather than remaining as their rest mass.

Step 3: Conclusion The energy released in forming one helium nucleus is approximately $\boxed{26.7\,\text{MeV}}$.

Quick Tip to Solve: Multiply the hydrogen mass by four before subtracting the helium mass; then multiply the mass decrease in u by 931.

12. For an amplitude modulated wave, the maximum amplitude is found to be 10 V while the minimum amplitude is 2 V. Calculate the modulation index. Why is modulation index generally kept less than one? [2 marks]

Solution:

Concept:

- In amplitude modulation the carrier amplitude varies with the message: the modulation index is $\mu = A_m/A_c$, where A_m is message amplitude and A_c is unmodulated carrier amplitude.
- For an undistorted AM envelope, $A_{\max} = A_c + A_m$ and $A_{\min} = A_c - A_m$. Hence $\mu = (A_{\max} -$

Quick Tip to Solve: Subtract the envelope extremes for the numerator and add them for the denominator; do not divide the maximum amplitude by the minimum amplitude.

$A_{\min})/(A_{\max} + A_{\min})$.

Step 1: Calculate the modulation index The given maximum and minimum amplitudes are $10\,\text{V}$ and $2\,\text{V}$. Their sum is twice the carrier amplitude and their difference is twice the message amplitude:

$$A_c = \frac{10+2}{2} = 6\,\text{V}, \qquad A_m = \frac{10-2}{2} = 4\,\text{V}.$$

Therefore the ratio is

$$\mu = \frac{4}{6} = \frac{10-2}{10+2} = \frac{2}{3} \simeq 0.667.$$

The ratio has no unit because both amplitudes are voltages.

Step 2: Explain why overmodulation is avoided When $\mu > 1$, the message amplitude exceeds the carrier amplitude. The factor $1 + \mu\cos\omega_m t$ changes sign during part of a message cycle, so the carrier undergoes phase reversals and its magnitude envelope no longer follows the original message faithfully. An ordinary envelope detector then produces a distorted recovered signal. Keeping $\mu < 1$ prevents overmodulation and allows faithful recovery of the message.

Step 3: Conclusion The modulation index is $\boxed{\mu = 2/3 \simeq 0.667}$, or about 66.7%. It is generally kept below unity to avoid overmodulation and distortion of the recovered signal.

Quick Tip to Solve: Subtract the envelope extremes for the numerator and add them for the denominator; do not divide the maximum amplitude by the minimum amplitude.

13. Draw a block diagram showing the important components in a communication system. What is the function of a transducer? [2 marks]

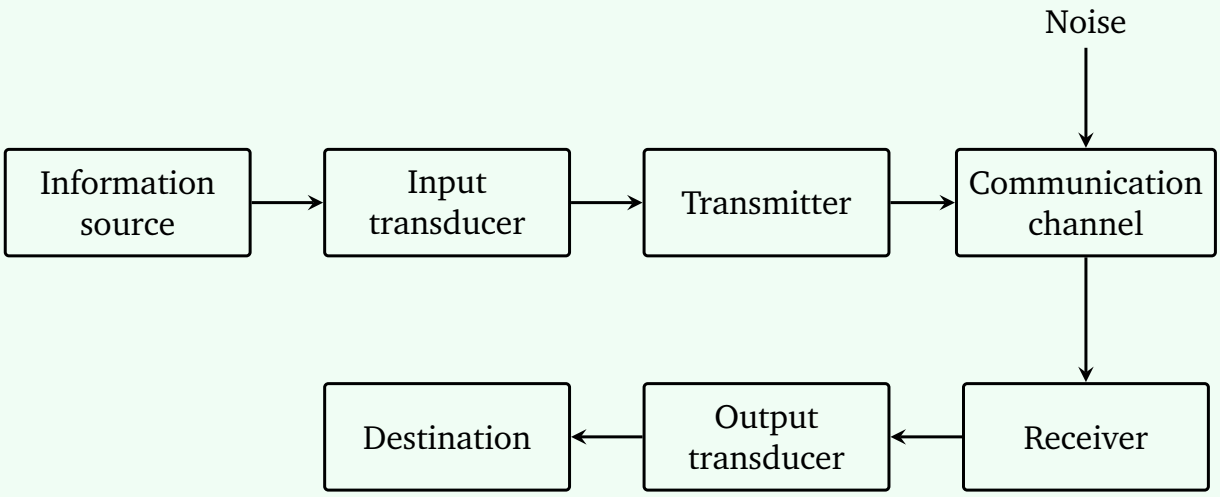
Solution:

Concept:

- A communication system transfers information from a source to a destination through a transmitter, a communication channel and a receiver.
- A transducer converts energy from one form into another. An input transducer converts a physical message, such as sound, into a corresponding electrical signal.

Step 1: Draw the communication chain The block diagram shows the direction in which the message travels. The input transducer supplies an electrical message signal to the transmitter, which processes it and produces a signal suitable for the channel. The channel is the transmission medium, for example a wire, optical fibre or free space. Noise can enter while the signal travels through this medium. The receiver extracts the message from the received signal, and the output transducer presents it to the user in a suitable physical form.

Quick Tip to Solve: Place the transmitter before the channel and the receiver after it; show noise entering the channel and the transducer at the physical-to-electrical interface.



Important components of a communication system; arrows show signal flow.

Step 2: State the function of a transducer A transducer converts the physical form of a signal into another form of energy. For example, a microphone converts sound-pressure variations into electrical variations so that the transmitter can process the message. At the receiving end, a loudspeaker performs the reverse conversion, changing the recovered electrical signal into sound. Thus transducers connect the physical information source and destination to the electrical communication equipment.

Step 3: Conclusion The essential chain is source, input transducer, transmitter, channel, receiver, output transducer and destination, as drawn. A transducer converts one form of energy into another; the input transducer converts the message into an electrical signal.

Quick Tip to Solve: Place the transmitter before the channel and the receiver after it; show noise entering the channel and the transducer at the physical-to-electrical interface.

14. Explain the following:

(i) Why do magnetic lines of force form continuous closed loops?

(ii) Why are the field lines repelled (expelled) when a diamagnetic material is placed in an external uniform magnetic field? [2 marks]

Solution:

Concept:

- Magnetic field lines have no sources or sinks: isolated magnetic poles do not occur, and the net magnetic flux through any closed surface is zero.
- A diamagnetic material has negative magnetic susceptibility χ_m. Its induced magnetisation $\mathbf{M} = \chi_m \mathbf{H}$ opposes the applied magnetic field $\mathbf{H}$.

Step 1: Explain the closed magnetic field lines Magnetic poles always occur in pairs, so a magnetic line cannot begin at an isolated north pole or terminate at an isolated south pole. For a bar magnet the field lines run from north to south outside the magnet and return from south to north inside it. The internal and external portions join continuously to form closed loops. This is consistent with

Quick Tip to Solve: For closed loops follow the line both outside and inside a magnet; for diamagnetism remember negative susceptibility and induced magnetisation opposite to the applied field.

Gauss's law for magnetism, $\oint \mathbf{B} \cdot d\mathbf{S} = 0$: there is no net magnetic flux emerging from a closed surface.

Step 2: Explain the expulsion in a diamagnetic material The applied field induces atomic magnetic moments in the diamagnetic material in a direction opposite to the external field. Their resultant magnetisation therefore weakens the field inside the specimen. Since $\chi_m < 0$, its relative permeability $\mu_r = 1 + \chi_m$ is less than unity. The field lines consequently tend to avoid the material, and their density inside it is lower than in the surrounding region. This is the meaning of expulsion here; an ordinary diamagnet generally weakens the field rather than excluding it completely.

Step 3: Conclusion Magnetic field lines form closed loops because there are no isolated magnetic poles. A diamagnetic material pushes field lines outward because its induced magnetisation opposes the applied field and reduces the field within it.

Quick Tip to Solve: For closed loops follow the line both outside and inside a magnet; for diamagnetism remember negative susceptibility and induced magnetisation opposite to the applied field.

OR

14. (i) Name the three elements of the Earth's magnetic field.

(ii) Where on the surface of the Earth is the vertical component of the Earth's magnetic field zero?

[2 marks]

Solution:

Concept:

- Earth's magnetic field at a place is specified by magnetic declination, magnetic inclination (dip) and the horizontal component of the magnetic field.
- If B is the magnitude of Earth's field and I is the angle of dip, its horizontal and vertical components are $B_H = B\cos I$ and $B_V = B\sin I$.

Step 1: Name and define the three elements The first element is magnetic declination D, the angle between the geographic meridian and the magnetic meridian at the place. The second is magnetic inclination or dip I, the angle the Earth's magnetic field makes with the horizontal in the magnetic meridian. The third is the horizontal component B_H of the Earth's magnetic field, which is its projection onto the horizontal plane. Together these determine the local direction and strength of the field; for example, $B_V = B_H \tan I$.

Step 2: Locate the zero vertical component At the magnetic equator the Earth's field is horizontal, so the angle of dip is zero. Resolve the field vertically:

$$B_V = B\sin I = B\sin 0^\circ = 0.$$

Thus there is no vertical component at the magnetic equator. The magnetic equator is defined by zero dip; it need not coincide with the geographic equator. At a magnetic pole, in contrast, the field is vertical and the horizontal component vanishes instead.

Step 3: Conclusion The three elements are magnetic declination, magnetic dip (inclination) and the horizontal component of Earth's magnetic field. The vertical component is zero at the

Quick Tip to Solve: Use $B_V = B\sin I$: zero dip means zero vertical field at the magnetic equator; do not confuse this with the zero horizontal field at the magnetic poles.

magnetic equator.

Quick Tip to Solve: Use $B_V = B \sin I$: zero dip means zero vertical field at the magnetic equator; do not confuse this with the zero horizontal field at the magnetic poles.

15. Show how the equation for Ampere's circuital law, viz.
$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$
is modified in the presence of displacement current. [2 marks]

Solution:

Concept:

- Ampere's circuital law for steady currents is $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_c$, where I_c is the conduction current enclosed by the loop.
- A changing electric flux $\Phi_E = \int \vec{E} \cdot d\vec{A}$ produces a magnetic field. Its contribution is expressed by the displacement current $I_d = \varepsilon_0 \, d\Phi_E/dt$.

Step 1: Identify the missing current contribution Consider a capacitor being charged by a conduction current I_c. A surface bounded by an Amperian loop and cutting the connecting wire encloses I_c; a surface with the same boundary passing through the gap between the plates encloses no conduction current. The original law would therefore give different magnetic circulations for the same loop. The changing electric field in the gap must supply the missing contribution.

Step 2: Add Maxwell's displacement current For parallel plates of area A, neglecting edge effects, $E = Q/(\varepsilon_0 A)$, so $\Phi_E = EA = Q/\varepsilon_0$. Hence

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} = I_c.$$

Thus the effective current enclosed by a chosen surface is the sum of conduction and displacement currents through that surface. Adding the flux-change term makes the magnetic circulation independent of the chosen surface.

Step 3: Conclusion The modified Ampere-Maxwell law is

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_c + I_d) = \mu_0 I_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}.$$

For a steady electric field, $d\Phi_E/dt = 0$, and the original law is recovered.

Quick Tip to Solve: Add the term $\mu_0 \varepsilon_0 \, d\Phi_E/dt$ to the conduction-current law whenever the electric flux changes.

16. Current in a circuit falls steadily from 5.0 A to 0.0 A in 100 ms. If an average e.m.f. of 200 V is induced, calculate the self-inductance of the circuit. [2 marks]

Solution:

Concept:

- A changing current in a circuit produces a self-induced emf $\mathcal{E} = -L\,dI/dt$, where L is the self-inductance.
- For a steady change in current, $|\mathcal{E}| = L|\Delta I|/\Delta t$. The negative sign expresses Lenz's law: the induced emf opposes the current change.

Step 1: Express the rate of current change in SI units The current decreases from $I_i = 5.0\,\text{A}$ to $I_f = 0.0\,\text{A}$, so $\Delta I = I_f - I_i = -5.0\,\text{A}$. The duration is

$$\Delta t = 100\,\text{ms} = 100 \times 10^{-3}\,\text{s} = 0.100\,\text{s}.$$

Since the fall is steady, its rate is constant:

$$\frac{\Delta I}{\Delta t} = \frac{-5.0}{0.100} = -50\,\text{A s}^{-1}.$$

Step 2: Substitute in the self-induction relation The stated average emf is its magnitude, $|\mathcal{E}| = 200\,\text{V}$. A constant rate of change also gives a constant induced-emf magnitude, so

$$L = \frac{|\mathcal{E}|}{|\Delta I|/\Delta t} = \frac{200}{50}\,\frac{\text{V s}}{\text{A}}.$$

The unit V s/A is the henry. The negative current slope does not make the inductance negative; it specifies that the induced emf acts to maintain the original current direction during the fall.

Step 3: Conclusion The self-inductance of the circuit is $\boxed{L = 4.0\,\text{H}}$.

Quick Tip to Solve: Convert milliseconds to seconds first, then divide the emf magnitude by the current-change rate; inductance is positive.

17. (a) You are required to select a carbon resistor of resistance $47\,\text{k}\Omega \pm 10\%$ from a large collection. What should be the sequence of colour bands used to code it?

(b) Write two characteristics of manganin which make it suitable for making standard resistances.

[2 marks]

Solution:

Concept:

- In a four-band resistor code, the first two bands give the significant digits, the third gives the power-of-ten multiplier, and the fourth gives the tolerance.
- A standard resistance should have high resistivity and a very small temperature coefficient of resistance, so a practical wire length gives a stable resistance.

Quick Tip to Solve: Express the resistance as two digits times a power of ten in ohms; keep the silver tolerance band separate from the multiplier.

Step 1: (a) Determine the colour-band sequence Write the required value in the form used by the colour code:

$$47\,\text{k}\Omega = 47 \times 10^3\,\Omega.$$

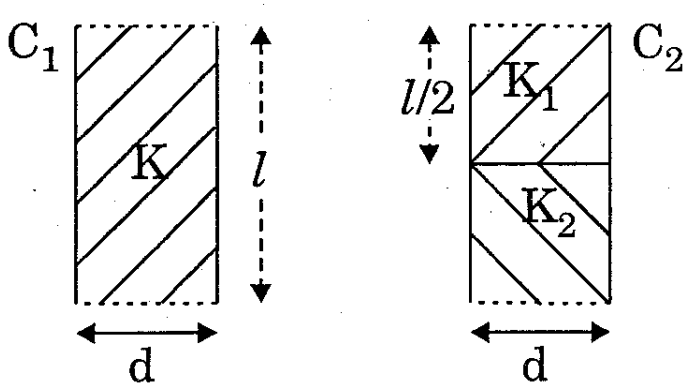
The first significant digit is 4, coded by yellow; the second is 7, coded by violet. The multiplier is 10^3, coded by orange. A tolerance of $\pm 10\%$ is coded by silver. Read from the end with the significant-digit bands towards the separated tolerance band. These bands specify a nominal resistance of $47\,\text{k}\Omega$, with an allowed variation of $4.7\,\text{k}\Omega$ on either side.

Step 2: (b) Explain manganin's suitability Manganin has high resistivity, so a useful resistance can be obtained using a reasonably short length of wire. It also has a very small temperature coefficient of resistance: over ordinary operating temperature changes, its resistance changes very little. The first property makes the standard compact and practical; the second keeps its calibrated value nearly constant when the room temperature changes or small heating occurs during use.

Step 3: Conclusion (a) The colour sequence is $\boxed{\text{Yellow–Violet–Orange–Silver}}$.
(b) Manganin is suitable because it has high resistivity and a very small temperature coefficient of resistance.

Quick Tip to Solve: Express the resistance as two digits times a power of ten in ohms; keep the silver tolerance band separate from the multiplier.

18. Two identical parallel plate (air) capacitors C_1 and C_2 have capacitances C each. The space between their plates is now filled with dielectrics as shown. If the two capacitors still have equal capacitance, obtain the relation between dielectric constants K, K_1 and K_2. [2 marks]



Solution:

Concept:

- A parallel-plate air capacitor has $C = \varepsilon_0 A/d$, where A is plate area and d is separation. Complete filling with dielectric constant K gives KC.
- Dielectrics covering different parts of plate area, each spanning the full gap, form parallel capacitors because they share the same voltage.

Step 1: Find the first capacitor's capacitance Its whole plate area A is filled with dielectric K, so its

Quick Tip to Solve: Equal half-areas spanning the full plate gap act in parallel, giving the arithmetic mean of the dielectric constants.

new capacitance is

$$C_1' = \frac{K\varepsilon_0 A}{d} = KC.$$

Step 2: Add the two parallel dielectric regions The figure divides the plate length l into equal halves. Each dielectric therefore covers area $A/2$ and spans the full separation d. The two regions share the same plates and voltage; they are in parallel, not in series. Neglecting edge effects,

$$C_2' = \frac{K_1\varepsilon_0(A/2)}{d} + \frac{K_2\varepsilon_0(A/2)}{d} = \frac{C}{2}(K_1 + K_2).$$

Step 3: Equate the final capacitances The stated equality gives $KC = C(K_1 + K_2)/2$. Cancel the nonzero common original capacitance C; the equal geometrical factors thus disappear from the required relation.

Step 4: Conclusion The required relation is $\boxed{K = \frac{K_1 + K_2}{2}}$, or $\boxed{2K = K_1 + K_2}$.

Quick Tip to Solve: Equal half-areas spanning the full plate gap act in parallel, giving the arithmetic mean of the dielectric constants.

19. State the principle of the device that can build up high voltages of the order of a few million volts. Draw its labelled diagram. A stage reaches in this device when the potential at the outer sphere cannot be increased further by piling up more charge on it. Explain why. [3 marks]

Solution:

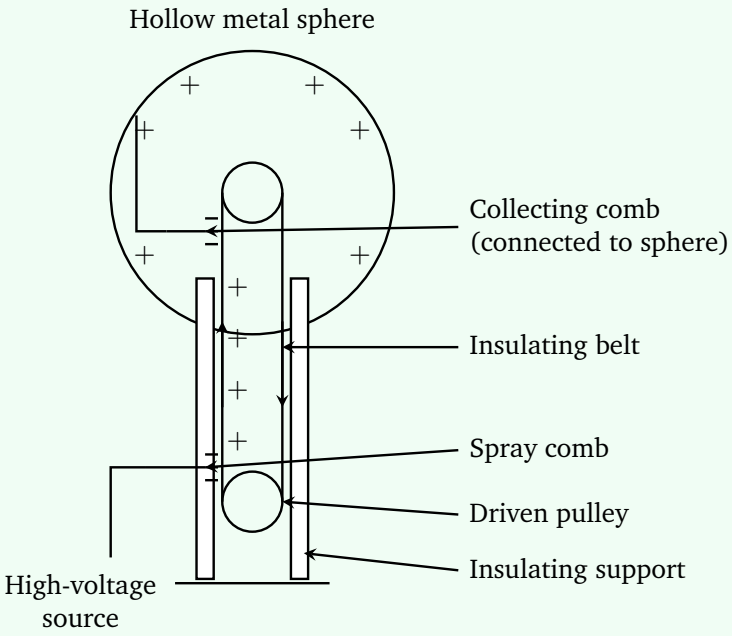
Concept:

- A Van de Graaff generator uses action of points to spray charge onto an insulating belt and electrostatic induction to transfer it to a hollow conducting sphere.
- Charge given to the inside of a hollow conductor moves to its outer surface. For a sphere of radius R, $V = Q/(4\pi\varepsilon_0 R)$ and the surface field is $E = V/R$.

Step 1: State the principle and identify the device The device is a Van de Graaff generator. Its sharply pointed combs produce strong local electric fields, enabling charge transfer through ionised air. Charge carried into a hollow conducting sphere is transferred to its outer surface, irrespective of the charge already present on that surface. Thus repeated delivery of charge can raise the sphere to a potential of several million volts.

Step 2: Explain the construction and charge transfer An insulating belt passes over two pulleys, of which the lower one is motor driven. The lower spray comb, connected to a high-voltage source, deposits positive charge on the upward-moving belt. Near the upper pulley inside the metal sphere, the collecting comb removes this charge from the belt and transfers it to the sphere. The upper comb is electrically connected to the sphere; the sphere and belt assembly stand on insulating supports. The diagram shows the positive-charging arrangement; the returning belt has been discharged.

Quick Tip to Solve: Use $E = V/R$ to explain the voltage limit: the surrounding air breaks down when the sphere's surface field becomes too large.



Van de Graaff generator with spray and collecting combs

Step 3: Explain why the voltage cannot increase indefinitely As charge Q accumulates, both the potential and the electric field at the sphere's surface rise. When the surface field reaches the breakdown strength of the surrounding air, the air becomes conducting and charge leaks from the sphere by corona discharge or sparking. At the limiting stage, the leakage rate balances the rate at which the belt supplies charge. Therefore additional charge delivered by the belt does not produce a sustained rise in potential. A larger smooth sphere reduces the surface field for a given potential and raises this limit.

Step 4: Conclusion The Van de Graaff generator builds high voltage by point discharge and transfer of charge to a hollow conductor's outer surface. Air breakdown and the resulting leakage set its maximum potential.

Quick Tip to Solve: Use $E = V/R$ to explain the voltage limit: the surrounding air breaks down when the sphere's surface field becomes too large.

20. Light of wavelength 2000 Å falls on a metal surface of work function 4.2 eV. What is the kinetic energy (in eV) of the fastest electrons emitted from the surface?

(i) What will be the change in the energy of the emitted electrons if the intensity of light with same wavelength is doubled?

(ii) If the same light falls on another surface of work function 6.5 eV, what will be the energy of emitted electrons? [3 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = hc/\lambda - \phi$, where ϕ is the metal's work function and $K_{\max}$ is the maximum kinetic energy.
- One photon transfers its energy to one electron. Increasing intensity at a fixed wavelength increases the incident photon flux, while the energy hc/λ of each photon stays fixed.
- Photoemission is possible only when $hc/\lambda \geq \phi$. A negative value obtained by subtraction means no photoelectrons are emitted, rather than negative kinetic energy.

Step 1: Convert the wavelength and calculate photon energy Since $1\,\text{Å} = 10^{-10}\,\text{m}$, the incident wavelength is

$$\lambda = 2000 \times 10^{-10} = 2.00 \times 10^{-7}\,\text{m} = 200\,\text{nm}.$$

Using $h = 6.63 \times 10^{-34}\,\text{J s}$, $c = 3.00 \times 10^{8}\,\text{m s}^{-1}$ and $1\,\text{eV} = 1.60 \times 10^{-19}\,\text{J}$,

$$E_\gamma = \frac{hc}{\lambda} = 9.945 \times 10^{-19}\,\text{J} \simeq 6.22\,\text{eV}.$$

The convenient rounded value $hc \simeq 1240\,\text{eV nm}$ gives $E_\gamma \simeq 6.2\,\text{eV}$.

Step 2: Find the kinetic energy for the first metal Of the incident photon's energy, $4.2\,\text{eV}$ is needed to release an electron. The remaining energy is the maximum kinetic energy of the fastest emitted electron:

$$K_{\max} = E_\gamma - \phi \simeq 6.2 - 4.2 = 2.0\,\text{eV}.$$

Electrons that lose energy before escaping can have lower kinetic energies, which is why the equation gives the fastest electrons' energy.

Step 3: Answer part (i): double the intensity The wavelength and therefore the photon energy remain unchanged. The work function of the same metal also remains unchanged, so the maximum kinetic energy remains approximately $2.0\,\text{eV}$, with change $\Delta K_{\max} = 0$. Doubling intensity increases the number of emitted electrons per second, and thus the saturation photocurrent, rather than doubling their maximum energy.

Step 4: Answer part (ii): test the second metal's threshold For the second surface, $\phi' = 6.5\,\text{eV} > E_\gamma \simeq 6.2\,\text{eV}$. Each incident photon has insufficient energy to overcome the work function. Thus there is no photoelectric emission. The subtraction $6.2 - 6.5 = -0.3\,\text{eV}$ signifies that the emission condition is not met; it is not a physical kinetic energy.

Step 5: Conclusion The fastest electrons have $K_{\max} \simeq 2.0\,\text{eV}$. Doubling intensity at the same wavelength changes this energy by zero. The surface with work function $6.5\,\text{eV}$ emits no photoelectrons.

Quick Tip to Solve: First use $E_\gamma(\text{eV}) \simeq 1240/\lambda(\text{nm})$, then compare it with each work function before subtracting.

21. Name the important processes that occur during the formation of a p-n junction. Explain briefly, with the help of a suitable diagram, how a p-n junction is formed. Define the term 'barrier potential'.

[3 marks]

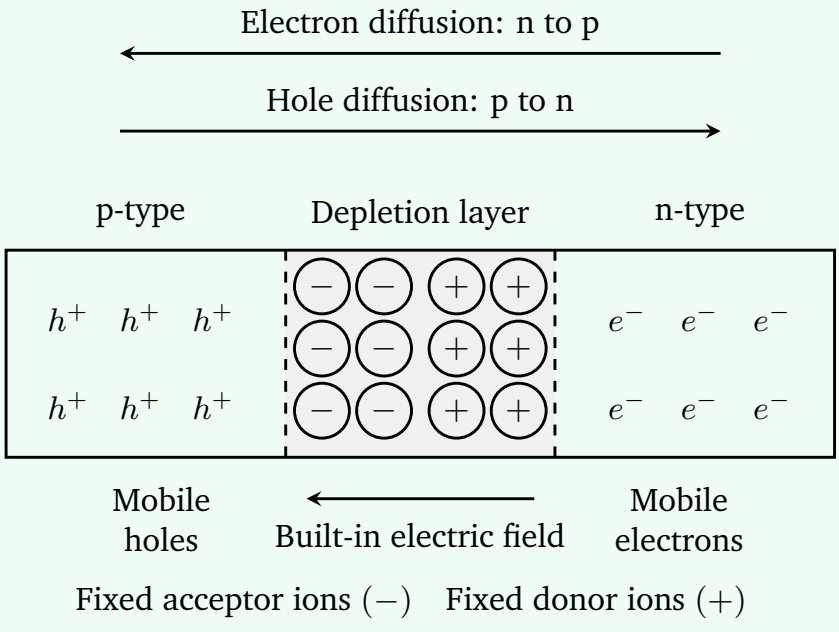
Solution:

Concept:

- The important processes in formation of a p-n junction are diffusion and drift. Diffusion results from a carrier concentration gradient; drift results from an electric field.
- The depletion layer contains fixed negative acceptor ions on the p side and fixed positive donor ions on the n side, but is depleted of mobile majority carriers.
- The junction's built-in field points from the n side to the p side. At equilibrium, drift and diffusion currents are equal in magnitude and opposite in direction.

Step 1: Explain initial diffusion and recombination A p-n junction is formed by creating p-type and n-type regions in the same semiconductor crystal. Electrons are majority carriers on the n side and holes are majority carriers on the p side. Because their concentrations differ across the junction, electrons diffuse from n to p and holes diffuse from p to n. Close to the junction they recombine, removing mobile electrons and holes from that region.

Step 2: Show the depletion layer and the resulting electric field When electrons leave the n region, immobile positively charged donor ions remain. When holes leave the p region, immobile negatively charged acceptor ions remain. These uncovered ions form a space-charge or depletion layer on the two sides of the junction. The ion charges produce an electric field directed from the positive n-side ions towards the negative p-side ions. The sketch distinguishes these fixed charges from the mobile carriers outside the depletion region.



Unbiased p-n junction: fixed ions, depletion region and carrier diffusion

Step 3: Explain drift and the establishment of equilibrium The junction field opposes further diffusion of the majority carriers. Under this field, electrons experience a force towards the n side and holes

Quick Tip to Solve: In a junction sketch put fixed minus ions on the p side and plus ions on the n side; the electric-field arrow then points from n to p.

towards the p side, producing a drift current opposite to the diffusion current. As the space-charge layer grows, its field strengthens until the drift and diffusion currents balance. The net current through an unbiased junction is then zero, although both microscopic processes continue.

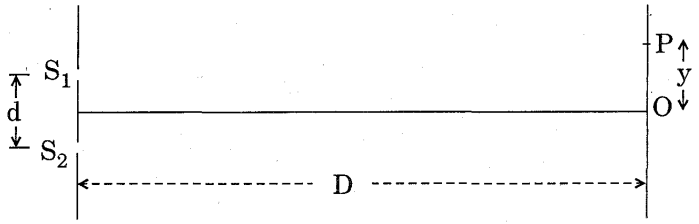
Step 4: Define barrier potential Barrier potential is the built-in potential difference across the depletion region which opposes further diffusion of majority charge carriers across the junction. If its magnitude is V_B, a charge of magnitude e must overcome an energy barrier eV_B. The potential is higher on the n side than on the p side. Thus the barrier is an electrostatic potential difference, whereas the depletion layer is the region of fixed ion charge which produces it.

Step 5: Conclusion Diffusion and drift form and stabilise the p-n junction. Recombination exposes fixed ions and creates the depletion layer; its electric field establishes the barrier potential that opposes further majority-carrier diffusion.

Quick Tip to Solve: In a junction sketch put fixed minus ions on the p side and plus ions on the n side; the electric-field arrow then points from n to p.

22. The intensity at the central maxima (O) in a Young's double slit experiment is I_0. If the distance OP equals one-third of the fringe width of the pattern, show that the intensity at point P would be $\frac{I_0}{4}$.

[3 marks]



Solution:

Concept:

- For equal coherent slits, the resultant intensity is $I = 4I_s \cos^2(\phi/2)$, where I_s is the intensity due to either slit alone and ϕ is the phase difference.
- In Young's experiment, the fringe width is $\beta = \lambda D/d$; at transverse distance y, the path difference is $\Delta = dy/D$. Here d is slit separation, D is screen distance, and λ is wavelength.

Step 1: Relate the central intensity to a single slit At O, the two waves travel equal distances, so their path and phase differences vanish. Their amplitudes add constructively. If each wave has amplitude a, the resultant amplitude is $2a$. Since intensity is proportional to amplitude squared, the intensity at O is $I_0 = 4I_s$. We can therefore express the intensity at any point using the measured central value:

$$I(y) = I_0 \cos^2\left(\frac{\phi}{2}\right).$$

Step 2: Calculate the path and phase differences at P The displacement is $y = OP = \beta/3$. Substituting

Quick Tip to Solve: Write $I(y)/I_0 = \cos^2(\pi y/\beta)$: a displacement of one-third of a fringe gives the cosine argument $\pi/3$.

the fringe-width expression into the path difference gives

$$\Delta = \frac{dy}{D} = \frac{d}{D}\frac{\beta}{3} = \frac{d}{3D}\frac{\lambda D}{d} = \frac{\lambda}{3}.$$

A full wavelength of path difference corresponds to phase 2π, so

$$\phi = \frac{2\pi\Delta}{\lambda} = \frac{2\pi}{3}.$$

In particular, the half-phase in the intensity formula is $\pi/3$, rather than $2\pi/3$.

Step 3: Substitute in the interference intensity formula Using the phase found above,

$$I_P = I_0 \cos^2\left(\frac{\pi}{3}\right) = I_0\left(\frac{1}{2}\right)^2.$$

The point is between the central bright fringe and the first dark fringe, so partial destructive interference reduces its intensity below the central value.

Step 4: Conclusion Thus the intensity at P is $\boxed{I_P = I_0/4}$, as required.

Quick Tip to Solve: Write $I(y)/I_0 = \cos^2(\pi y/\beta)$: a displacement of one-third of a fringe gives the cosine argument $\pi/3$.

OR

22. In the experiment on diffraction due to a single slit, show that

(i) the intensity of diffraction fringes decreases as the order (n) increases.

(ii) angular width of the central maximum is twice that of the first order secondary maximum.

[3 marks]

Solution:

Concept:

- For a uniformly illuminated single slit of width a, diffraction intensity is $I(\theta) = I_c(\sin\alpha/\alpha)^2$, where $\alpha = \pi a \sin\theta/\lambda$, λ is wavelength and I_c is central peak intensity.
- Minima satisfy $a\sin\theta_m = m\lambda$, with $m = \pm1, \pm2, \ldots$. A maximum lies between successive minima. Angular widths use the usual small-angle diffraction approximation $\sin\theta \simeq \theta$.

Step 1: (i) Locate the secondary maxima Differentiating the intensity factor away from its zeros, the condition for a secondary maximum is

$$\alpha\cos\alpha - \sin\alpha = 0, \qquad \tan\alpha = \alpha.$$

Let $\alpha_n > 0$ denote the root for the secondary maximum of order n. These roots occur in successive side lobes, so α_n increases with n. The negative-angle side has the same intensities because the pattern is symmetric about the central axis.

Quick Tip to Solve: Compare the bounding minima: the central maximum spans orders -1 to $+1$, whereas the first secondary maximum spans $+1$ to $+2$.

Step 2: (i) Show that successive peak intensities decrease At a secondary maximum, $\sin\alpha_n = \alpha_n \cos\alpha_n$. Therefore

$$\frac{I_n}{I_c} = \left(\frac{\sin\alpha_n}{\alpha_n}\right)^2 = \cos^2\alpha_n = \frac{1}{1+\tan^2\alpha_n} = \frac{1}{1+\alpha_n^2}.$$

Since α_n increases with order, the denominator increases and the secondary peak intensity decreases. Approximately $\alpha_n \simeq (n+\frac{1}{2})\pi$, giving $I_n/I_c \simeq 4/[(2n+1)^2\pi^2]$. This approximation also displays the decreasing intensity directly.

Step 3: (ii) Compare the angular intervals between minima The central maximum extends from the first negative minimum to the first positive minimum. For small angles these lie at $-\lambda/a$ and $+\lambda/a$, giving

$$\Delta\theta_c = \frac{2\lambda}{a}.$$

The first secondary maximum on the positive side lies between the first and second minima, at λ/a and $2\lambda/a$. Hence

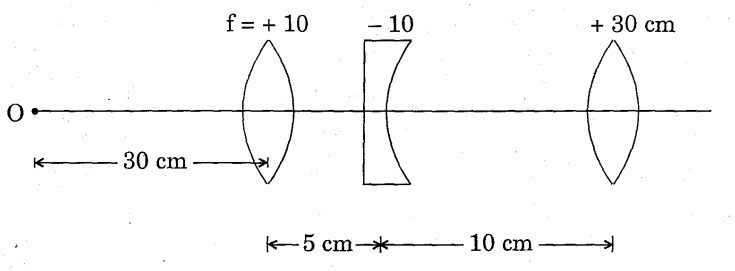
$$\Delta\theta_1 = \frac{2\lambda}{a} - \frac{\lambda}{a} = \frac{\lambda}{a}.$$

The negative-side secondary maximum has the same width by symmetry. Width means the interval bounded by adjacent dark minima, not the angular position of the peak.

Step 4: Conclusion The secondary peak intensities decrease as their order increases. In the usual small-angle diffraction limit, $\boxed{\Delta\theta_c = 2\Delta\theta_1}$: the central maximum is twice as wide as the first secondary maximum.

Quick Tip to Solve: Compare the bounding minima: the central maximum spans orders -1 to $+1$, whereas the first secondary maximum spans $+1$ to $+2$.

23. Find the position of the image formed of the object 'O' by the lens combination given in the figure. [3 marks]



Solution:

Concept:

- For a thin lens, the Cartesian lens formula is $1/v - 1/u = 1/f$. Light travels left to right; distances measured rightward from the current lens are positive and those measured leftward are negative.
- The image that one lens would form becomes the object for the next lens. Converging rays directed toward a point on the right constitute a virtual object with $u > 0$; parallel incident rays focus at

Quick Tip to Solve: After the first lens, measure the convergence point from the second lens: a virtual object at $+10$ cm cancels its -10 cm focal power.

the principal focus of a convex lens.

Step 1: Find the image due to the first convex lens Let the three lenses, in order, be L_1, L_2, L_3. The printed focal lengths are $+10\,\text{cm}$, $-10\,\text{cm}$, and $+30\,\text{cm}$. For L_1, the object is $30\,\text{cm}$ to its left, so $u_1 = -30\,\text{cm}$. Applying the lens formula,

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15}\,\text{cm}^{-1}.$$

Thus rays leaving L_1 converge toward a point $15\,\text{cm}$ to its right, if the other lenses are absent.

Step 2: Treat this converging beam at the concave lens The concave lens L_2 is $5\,\text{cm}$ to the right of L_1. The convergence point is therefore $15 - 5 = 10\,\text{cm}$ to the right of L_2. This is a virtual object, giving $u_2 = +10\,\text{cm}$. With $f_2 = -10\,\text{cm}$,

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10} = 0.$$

Hence the rays emerging from L_2 are parallel. Using a negative object distance here would incorrectly treat an already converging beam as a diverging beam.

Step 3: Focus the parallel rays through the last convex lens The final lens L_3 lies $10\,\text{cm}$ to the right of L_2. Since the intermediate beam is parallel, this separation does not change its vergence. The object distance for L_3 is infinite, so

$$\frac{1}{v_3} = \frac{1}{f_3} = \frac{1}{30}\,\text{cm}^{-1}.$$

Its outgoing rays converge at its right-hand principal focus. Relative to L_1, the final position is $5 + 10 + 30 = 45\,\text{cm}$ to the right.

Step 4: Conclusion The final image is real and forms $\boxed{30\,\text{cm}}$ to the right of the third lens, or $45\,\text{cm}$ to the right of the first lens.

Quick Tip to Solve: After the first lens, measure the convergence point from the second lens: a virtual object at $+10\,\text{cm}$ cancels its $-10\,\text{cm}$ focal power.

24. Draw transfer characteristics of a common emitter n-p-n transistor. Point out the region in which the transistor operates as an amplifier.

Define the following terms used in transistor amplifiers:

(i) Input resistance

(ii) Output resistance

(iii) Current amplification factor [3 marks]

Solution:

Concept:

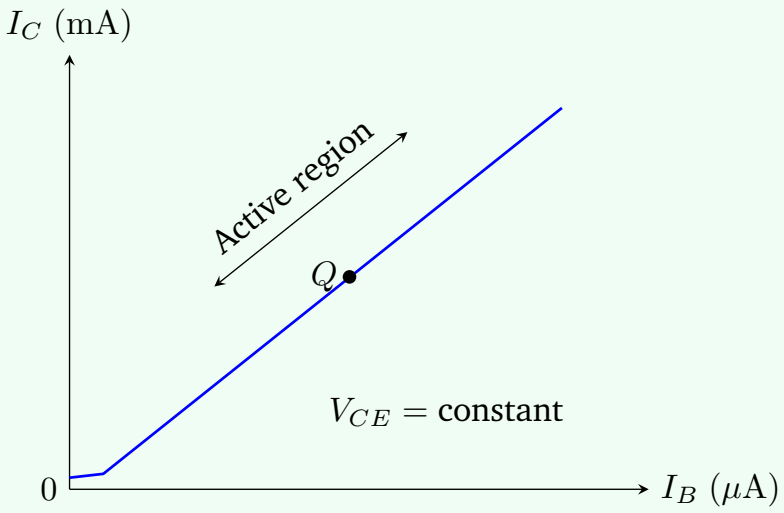
- In the common-emitter configuration, the input is the base-emitter circuit and the output is the collector-emitter circuit; the emitter is common to both.
- Transfer characteristics show collector current I_C against base current I_B, with collector-emitter

Quick Tip to Solve: For each definition, identify the voltage-current pair first, then state which bias variable is held constant.

voltage V_{CE} held constant. The slope in the active region is the small-signal current gain β_{ac}.

- An amplifier operates about a bias point in the active region, where the base-emitter junction is forward biased and the collector-base junction is reverse biased.

Step 1: Draw the transfer characteristic and identify amplification Plot I_B horizontally and I_C vertically at a fixed V_{CE}. Over the useful active region, the characteristic is approximately a straight line: a small increase in base current produces a much larger increase in collector current. Choose the operating point Q on this portion so that signal variations remain in the active region. Operation near cutoff, where collector current is negligible, or at saturation in a practical amplifier gives distortion instead of faithful amplification.



Common-emitter transfer characteristic at constant collector-emitter voltage; the active portion is used for amplification.

Step 2: Define input and output resistance Input resistance is the ratio of a small change in base-emitter voltage to the corresponding change in base current, keeping collector-emitter voltage constant:

$$r_i = \left.\frac{\Delta V_{BE}}{\Delta I_B}\right|_{V_{CE}\text{ constant}}.$$

Output resistance is the ratio of a small change in collector-emitter voltage to the corresponding change in collector current, keeping base current constant:

$$r_o = \left.\frac{\Delta V_{CE}}{\Delta I_C}\right|_{I_B\text{ constant}}.$$

Both are differential resistances evaluated near the operating point and have the unit ohm.

Step 3: Define the current amplification factor The common-emitter current amplification factor is the ratio of the change in collector current to the change in base current at constant collector-emitter voltage:

$$\beta_{\text{ac}} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE}\text{ constant}}.$$

It is dimensionless and equals the slope of the active portion of the transfer characteristic when

Quick Tip to Solve: For each definition, identify the voltage-current pair first, then state which bias variable is held constant.

both currents are expressed in the same unit. For dc currents, the corresponding dc current gain is $\beta_{\text{dc}} = I_C/I_B$.

Step 4: Conclusion The transistor acts as an amplifier in the approximately linear active region of its transfer characteristic. Its input resistance, output resistance and ac current gain are respectively $\Delta V_{BE}/\Delta I_B$, $\Delta V_{CE}/\Delta I_C$ and $\Delta I_C/\Delta I_B$, with the stated quantities held constant.

Quick Tip to Solve: For each definition, identify the voltage-current pair first, then state which bias variable is held constant.

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25. (i) Light passes through two polaroids P_1 and P_2 with pass axis of P_2 making an angle θ with the pass axis of P_1. For what value of θ is the intensity of emergent light zero?
(ii) A third polaroid is placed between P_1 and P_2 with its pass axis making an angle β with the pass axis of P_1. Find a value of β for which the intensity of light emerging from P_2 is $\frac{I_0}{8}$, where I_0 is the intensity of light on the polaroid P_1. [3 marks]

Solution:

Concept:

- Malus's law states that an analyser transmits intensity $I = I_{\text{in}} \cos^2 \phi$, where ϕ is the angle between its pass axis and the incident light's direction of polarization.
- An ideal polaroid transmits half the intensity of incident unpolarized light. Thus, after P_1, the intensity is $I_1 = I_0/2$, polarized along the pass axis of P_1.
- Each subsequent polaroid sets the polarization direction of the transmitted light along its own pass axis, so the angle in Malus's law must be recalculated at each stage.

Step 1: Find the extinction angle for the two polaroids The light emerging from P_1 is linearly polarized. For the second polaroid,

$$I_2 = I_1 \cos^2 \theta.$$

For nonzero incident intensity, extinction requires $\cos\theta = 0$. Thus the two pass axes must be perpendicular: $\theta = 90^\circ$, or any equivalent odd multiple of 90°. These are called crossed polaroids.

Step 2: Apply Malus's law at the inserted polaroid Keep P_1 and P_2 crossed as in part (i). With the third polaroid between them, its axis is at angle β to P_1, so its transmitted intensity is

$$I_3 = \frac{I_0}{2} \cos^2 \beta.$$

This light is polarized along the third axis. The angle between that axis and P_2 is $90^\circ - \beta$, not 90°. Therefore,

$$\begin{aligned} I_{\text{out}} &= \frac{I_0}{2} \cos^2 \beta \cos^2(90^\circ - \beta) \\ &= \frac{I_0}{2} \cos^2 \beta \sin^2 \beta = \frac{I_0}{8} \sin^2(2\beta). \end{aligned}$$

Step 3: Determine the required intermediate angle Setting the final intensity equal to the specified

Quick Tip to Solve: Start with the factor one-half for unpolarized incident light, then multiply by one cosine-squared factor for each later polaroid.

value gives

$$\frac{I_0}{8}\sin^2(2\beta) = \frac{I_0}{8}, \qquad \sin^2(2\beta) = 1.$$

In the range $0^\circ \le \beta \le 90^\circ$, this requires $2\beta = 90^\circ$. The intermediate axis therefore bisects the right angle between the original crossed axes. Each of the two later transmissions contributes a factor $1/2$, giving $(I_0/2)(1/2)(1/2) = I_0/8$, which checks the result.

Step 4: Conclusion (i) The emergent intensity vanishes at $\boxed{\theta = 90^\circ}$. (ii) A suitable value is $\boxed{\beta = 45^\circ}$, giving emergent intensity $I_0/8$.

Quick Tip to Solve: Start with the factor one-half for unpolarized incident light, then multiply by one cosine-squared factor for each later polaroid.

26. Using the postulates of Bohr's model of hydrogen atom, obtain an expression for the frequency of radiation emitted when the atom makes a transition from the higher energy state with quantum number n_i to the lower energy state with quantum number n_f ($n_f < n_i$). [3 marks]

Solution:

Concept:

- An electron in a stationary orbit does not radiate. Bohr's angular-momentum condition is $m_e v_n r_n = nh/(2\pi)$.
- Radiation is emitted during a transition between stationary states. The photon energy is the decrease in atomic energy: $h\nu = E_{n_i} - E_{n_f}$, where $n_f < n_i$ and h is Planck's constant.
- Coulomb attraction supplies the centripetal force. Here e is electron charge magnitude, m_e its mass and ε_0 vacuum permittivity; the nucleus is treated as fixed.

Step 1: Obtain the radius of an allowed orbit Equate Coulomb attraction to centripetal force and impose angular-momentum quantization:

$$\frac{m_e v_n^2}{r_n} = \frac{e^2}{4\pi\varepsilon_0 r_n^2}, \qquad m_e v_n r_n = \frac{nh}{2\pi}.$$

Substituting $v_n = nh/(2\pi m_e r_n)$ into the force equation and simplifying yields

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m_e e^2}.$$

Step 2: Derive the energy of the stationary state With zero potential energy at infinity, the kinetic and potential energies are

$$K = \frac{1}{2}m_e v_n^2 = \frac{e^2}{8\pi\varepsilon_0 r_n}, \qquad U = -\frac{e^2}{4\pi\varepsilon_0 r_n}.$$

Inserting the allowed radius into their sum,

$$E_n = K + U = -\frac{e^2}{8\pi\varepsilon_0 r_n} = -\frac{m_e e^4}{8\varepsilon_0^2 h^2 n^2}.$$

Quick Tip to Solve: Write the lower-state reciprocal square first in the emission formula; this keeps the photon energy and frequency positive.

Step 3: Use the frequency postulate for the transition Apply Bohr's frequency postulate to the emitted photon:

$$\begin{aligned} h\nu &= E_{n_i} - E_{n_f} \\ &= \frac{m_e e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right). \end{aligned}$$

Since $n_f < n_i$, the bracket is positive. Divide by h to obtain the frequency.

Step 4: Conclusion The emitted frequency is

$$\boxed{\nu = \frac{m_e e^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right).}$$

Quick Tip to Solve: Write the lower-state reciprocal square first in the emission formula; this keeps the photon energy and frequency positive.

27. State the underlying principle of a potentiometer.

Describe briefly, giving the necessary circuit diagram, how a potentiometer is used to measure the internal resistance of a given cell. [3 marks]

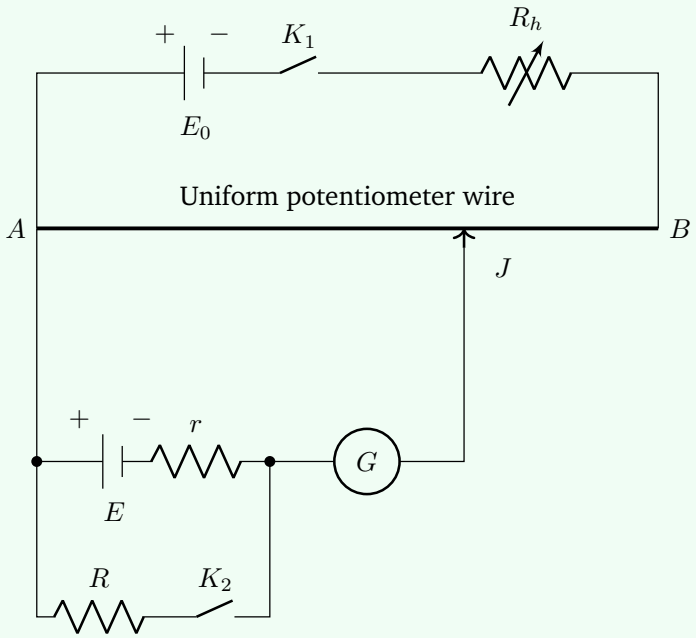
Solution:

Concept:

- For a uniform potentiometer wire carrying a steady current, the potential drop is proportional to length: $V = kl$, where k is the constant potential gradient.
- At the balance point the galvanometer current is zero. Thus an unloaded cell's emf E, or its loaded terminal voltage V, can be measured without drawing current through the measuring branch.
- For a cell of internal resistance r supplying a known external resistance R, $E = I(R + r)$ and $V = IR$.

Step 1: Arrange the potentiometer circuit Connect a driving battery E_0, key K_1 and rheostat R_h in series with the uniform wire AB. The positive terminal of the driving battery goes to A, so potential falls from A to B. Connect the positive terminal of the test cell E to A and its negative terminal through galvanometer G to jockey J. Connect a known resistance R, controlled by key K_2, across the test cell. The driving battery must produce a wire voltage greater than the test-cell emf so that a balance point lies on AB.

Quick Tip to Solve: Take the emf balance with the load key open first; keep the wire current fixed, then close the load key and use the shorter length in the denominator.



Potentiometer circuit: K2 connects the known load across the test cell; J is the sliding contact.

Step 2: Measure the unloaded emf Close K_1, keep K_2 open, and adjust the jockey until G shows no deflection. Let the balance length from A be l_1. With no current drawn from the test cell at balance, its terminal voltage equals its emf, giving

$$E = kl_1.$$

Keep the primary current and rheostat setting unchanged for the next measurement.

Step 3: Measure the loaded terminal voltage and derive the resistance Close K_2, allowing the cell to supply current through R. Find the new null point at length l_2, so $V = kl_2$. The cell now loses a potential Ir internally. Using the external current $I = V/R$,

$$\begin{aligned} E &= V + Ir, \\ \frac{E}{V} &= 1 + \frac{r}{R}, \\ \frac{l_1}{l_2} &= 1 + \frac{r}{R}. \end{aligned}$$

Since the potential gradient is unchanged, it cancels in the ratio. A cell with positive internal resistance has $l_2 < l_1$.

Step 4: Conclusion The principle is $V \propto l$ for a uniform wire with steady current. The cell's internal resistance is

$$\boxed{r = R\left(\frac{l_1}{l_2} - 1\right) = R\frac{l_1 - l_2}{l_2}}.$$

Quick Tip to Solve: Take the emf balance with the load key open first; keep the wire current fixed, then close the load key and use the shorter length in the denominator.

28. (a) Show that a planar loop carrying a current I, having N closely wound turns and area of cross-section A, possesses a magnetic moment $\vec{m} = NI\vec{A}$.

(b) When this loop is placed in a magnetic field $\vec{B}$, find out the expression for the torque acting on it.

(c) A galvanometer coil of $50\,\Omega$ resistance shows full scale deflection for a current of 5 mA. How will you convert this galvanometer into a voltmeter of range 0 to 15 V? [5 marks]

Solution:

Concept:

- A current loop behaves as a magnetic dipole. Its area vector $\vec{A} = A\hat{n}$ points along the normal given by the right-hand rule: curled fingers follow the current and the thumb gives $\hat{n}$.
- The force on a straight segment of current-carrying wire in a uniform field is $\vec{F} = I\vec{L} \times \vec{B}$. Opposite sides of a closed loop form a couple, with zero net force but generally nonzero torque.
- The torque of a magnetic dipole is $\vec{\tau} = \vec{m} \times \vec{B}$. A galvanometer becomes a voltmeter when a sufficiently large resistance is connected in series to limit its current.

Step 1: (a) Find the magnetic moment of one turn First consider a rectangular loop with sides a and b, carrying current I. Let its normal make an angle θ with a uniform field $\vec{B}$, with the two sides of length b perpendicular to the field. Each of these sides experiences force $F = IbB$. The forces are equal and opposite, and their perpendicular separation is $a\sin\theta$. Their torque is therefore

$$\tau = (IbB)(a\sin\theta) = IAB\sin\theta, \qquad A = ab.$$

The other pair of sides contributes no torque about the axis of this couple. Comparing with the dipole relation $\tau = mB\sin\theta$ gives $m = IA$. The force couple fixes the direction of the moment to be the current's right-hand normal: $\vec{m}_1 = I\vec{A}$. This result holds for any planar shape: divide the enclosed area into small rectangular current loops. Currents along shared edges cancel, leaving the original boundary current, while the moments of the small areas add to $I\vec{A}$.

Step 2: (a) Add the moments of N closely wound turns Closely wound turns have the same area, normal and current direction. Their magnetic moments add vectorially, so

$$\vec{m} = \sum_{j=1}^{N} I\vec{A} = NI\vec{A}.$$

Here A is the area enclosed by each turn, not the cross-sectional area of the wire. The magnitude is NIA, measured in $\mathrm{A\,m^2}$.

Step 3: (b) Obtain the torque on the complete loop The torque of every turn has the same direction, so the force-couple calculation also multiplies by N:

$$\tau = NIAB\sin\theta = mB\sin\theta.$$

Its vector form is

$$\boxed{\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{m} \times \vec{B}}.$$

The angle θ is between the normal to the loop and the field, not between the plane and the field. The torque tends to align $\vec{m}$ with $\vec{B}$. Its magnitude is zero for parallel or antiparallel moment and field,

Quick Tip to Solve: For torque use the angle with the area normal; for a voltmeter first calculate V/Ig, then subtract the galvanometer's own resistance.

and is maximum, $NIAB$, when they are perpendicular.

Step 4: (c) Calculate the series resistance for the voltmeter The galvanometer resistance is $G = 50\,\Omega$, and its full-scale current is $I_g = 5\,\text{mA} = 0.005\,\text{A}$. Connect a resistance R_s in series with the coil. At the required maximum voltage $V = 15\,\text{V}$, the same full-scale current flows through both components, so

$$\begin{aligned} V &= I_g(G + R_s), \\ R_s &= \frac{V}{I_g} - G \\ &= \frac{15}{0.005} - 50 \\ &= 3000 - 50 = 2950\,\Omega. \end{aligned}$$

The coil itself drops $I_g G = 0.25\,\text{V}$; the added resistor drops the remaining $14.75\,\text{V}$. The series combination has total resistance $3000\,\Omega$ and is connected across the points whose voltage is measured. Calibrate the scale from zero to $15\,\text{V}$.

Step 5: Conclusion (a) The magnetic moment is $\boxed{\vec{m} = NI\vec{A}}$. (b) The torque is $\boxed{\vec{\tau} = \vec{m} \times \vec{B}}$, with magnitude $NIAB\sin\theta$. (c) Connect $\boxed{2950\,\Omega = 2.95\,\text{k}\Omega}$ in series with the galvanometer to obtain the 0–15 V voltmeter.

Quick Tip to Solve: For torque use the angle with the area normal; for a voltmeter first calculate V/Ig, then subtract the galvanometer's own resistance.

OR

28. (a) Draw a schematic sketch of a cyclotron, explain its working principle and deduce the expression for the kinetic energy of the ions accelerated.

(b) Two long and parallel straight wires carrying currents of 2 A and 5 A in the opposite directions are separated by a distance of 1 cm. Find the nature and magnitude of the magnetic force between them. [5 marks]

Solution:

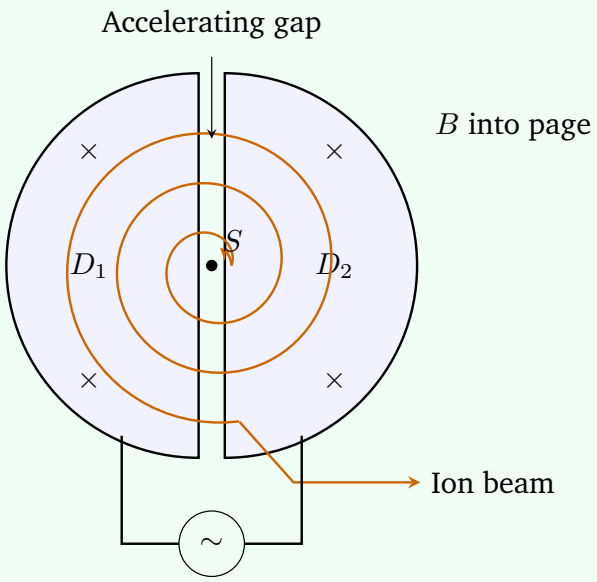
Concept:

- In a cyclotron, a uniform magnetic field perpendicular to the ion velocity provides the centripetal force: $qvB = mv^2/r$, where q is the positive charge magnitude and m is the ion mass. The magnetic force changes direction but does no work.
- An alternating electric field across the gap between the dees supplies energy. For non-relativistic motion, the cyclotron frequency $f = qB/(2\pi m)$ is independent of radius and speed.
- The magnetic force per unit length between parallel wires is $F/L = \mu_0 I_1 I_2/(2\pi d)$, with separation d. Currents in opposite directions repel.

Step 1: (a) Construction and schematic sketch Two hollow D-shaped metal electrodes D_1 and D_2, called dees, are separated by a narrow gap in an evacuated chamber. They are connected to a

Quick Tip to Solve: Eliminate the ion speed using $v = qBr/m$ to obtain both the cyclotron period and kinetic energy. For unspecified wire length, calculate F/L, and use the current directions to decide attraction or repulsion.

high-frequency alternating voltage source and placed between the poles of a strong electromagnet. Its uniform field B is perpendicular to the plane of the dees. Positive ions are released from a source S near the centre. An extraction electrode directs the accelerated beam out near the outer edge. The sketch shows the top view; crosses represent the field into the page.



Cyclotron schematic: accelerating gap, hollow dees, alternating source and outward ion path.

Step 2: (a) Working and resonance condition The electric field in the gap accelerates an ion towards the dee at lower potential. Inside a hollow dee the electric field is screened, so the ion moves at constant speed along a semicircular path under the magnetic force. Equating the magnetic and centripetal forces gives

$$qvB = \frac{mv^2}{r}, \qquad r = \frac{mv}{qB}.$$

The time for a semicircle is

$$t_{1/2} = \frac{\pi r}{v} = \frac{\pi m}{qB}.$$

It is independent of speed and radius. Set the alternating voltage period to $T = 2\pi m/(qB)$, or frequency $f = qB/(2\pi m)$. The polarity then reverses during each semicircle, so the ion is accelerated again at its next crossing of the gap. Its speed and orbital radius increase after each crossing, producing an outward spiral. The magnetic field only bends the path; all kinetic energy is supplied by the alternating electric field. This fixed-frequency synchronism assumes non-relativistic ions, for which the mass remains effectively constant.

Step 3: (a) Maximum kinetic energy of the extracted ions If R is the largest usable orbit radius inside a dee, the speed just before extraction follows from the same force balance:

$$v_{\max} = \frac{qBR}{m}.$$

Quick Tip to Solve: Eliminate the ion speed using $v = qBr/m$ to obtain both the cyclotron period and kinetic energy. For unspecified wire length, calculate F/L, and use the current directions to decide attraction or repulsion.

Substituting into the kinetic-energy expression yields

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{q^2B^2R^2}{2m}.$$

More generally, at an orbit radius r, $K = q^2B^2r^2/(2m)$. Thus the limiting energy is set by the ion charge and mass, magnetic field and available radius. The voltage across the gap controls the energy gained at each crossing and hence the number of crossings needed to reach the outer edge.

Step 4: (b) Nature and magnitude of the force between the wires The currents are $I_1 = 2\,\text{A}$ and $I_2 = 5\,\text{A}$, and their separation is $d = 1\,\text{cm} = 10^{-2}\,\text{m}$. The field due to wire 1 at wire 2 is $B_1 = \mu_0 I_1/(2\pi d)$. Therefore the force on a length L of wire 2 is $F = I_2 L B_1$, giving

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{4\pi \times 10^{-7} \times 2 \times 5}{2\pi \times 10^{-2}} = 2.0 \times 10^{-4}\,\text{N m}^{-1}.$$

Applying the right-hand rule for the field and $\vec{F} = I\vec{L} \times \vec{B}$ shows that each wire experiences a force away from the other: the interaction is repulsive. The forces are equal in magnitude and opposite in direction. Since no wire length is specified, the numerical magnitude is expressed per unit length; a common length L, measured in metres, experiences $F = (2.0 \times 10^{-4}\,\text{N m}^{-1})L$.

Step 5: Conclusion (a) The cyclotron operates at $f = qB/(2\pi m)$, with an alternating gap field accelerating the ions each half revolution. At extraction radius R, $K_{\max} = q^2B^2R^2/(2m)$. (b) The wires repel, with force per unit length $F/L = 2.0 \times 10^{-4}\,\text{N m}^{-1}$.

Quick Tip to Solve: Eliminate the ion speed using $v = qBr/m$ to obtain both the cyclotron period and kinetic energy. For unspecified wire length, calculate F/L, and use the current directions to decide attraction or repulsion.

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29. (a) Derive the expression for the mutual inductance of two long coaxial solenoids of same length l having radii r_1 and r_2 ($r_2 > r_1$ and $l \gg r_2$).

(b) Show that mutual inductance of solenoid 1 due to solenoid 2, M_{12}, is the same as that of 2 due to 1 i.e. M_{21}.

(c) A power transmission line feeds power at 2200 V with a current of 5 A to a step down transformer with its primary winding having 4000 turns. Calculate the number of turns and the current in the secondary in order to get output power at 220 V. [5 marks]

Solution:

Concept:

- Mutual inductance is the flux linkage of one coil per unit current in the other. Write $M_{12} = N_1\Phi_{12}/I_2$, where Φ_{12} is the flux through one turn of solenoid 1 due to current I_2 in solenoid 2.
- A long air-core solenoid has uniform interior field $B = \mu_0 nI$, where $n = N/l$ is its number of turns per unit length; its exterior field is negligible.
- An ideal transformer satisfies $V_s/V_p = N_s/N_p$ and conserves power: $V_pI_p = V_sI_s$. Subscripts p and s denote primary and secondary quantities.

Quick Tip to Solve: Use the smaller solenoid's area in both linkage calculations. For an ideal transformer, a tenfold voltage reduction means tenfold fewer turns and tenfold greater current.

Step 1: (a) Calculate linkage in the inner solenoid Let the inner and outer solenoids have N_1 and N_2 turns respectively, with $n_1 = N_1/l$, $n_2 = N_2/l$. Their axes coincide. Since $l \gg r_2$, end effects may be neglected. Pass current I_2 through the outer solenoid. The field throughout the inner solenoid is

$$B_2 = \mu_0 n_2 I_2.$$

Each turn of the inner solenoid encloses area $A_1 = \pi r_1^2$, with its area vector parallel to the field. Hence the flux per inner turn and total linkage are

$$\Phi_{12} = \mu_0 n_2 I_2 A_1, \qquad N_1\Phi_{12} = \mu_0 N_1 n_2 A_1 I_2.$$

Dividing linkage by the source current gives

$$M_{12} = \mu_0 \frac{N_1 N_2}{l}\pi r_1^2 = \mu_0 n_1 n_2 l \pi r_1^2.$$

Step 2: (b) Reverse the current-carrying solenoid Now pass current I_1 through the inner solenoid. Its field is $B_1 = \mu_0 n_1 I_1$ inside its own cross-section and approximately zero outside. Although each outer turn encloses area πr_2^2, magnetic flux crosses only the common area A_1. Thus it would be incorrect to use the entire outer area in the flux calculation.

$$\Phi_{21} = B_1 A_1 = \mu_0 n_1 I_1 \pi r_1^2.$$

There are N_2 linked outer turns, so

$$M_{21} = \frac{N_2\Phi_{21}}{I_1} = \mu_0 \frac{N_1 N_2}{l}\pi r_1^2 = M_{12}.$$

The two values are therefore equal. The result is the mutual-inductance magnitude for the chosen consistent winding directions; reversing a winding reverses both corresponding signed linkages.

Step 3: (c) Find the secondary turns Take the transformer as ideal, since no losses or efficiency are specified. The primary has $V_p = 2200\,\text{V}$, $I_p = 5\,\text{A}$, $N_p = 4000$; the required secondary voltage is $V_s = 220\,\text{V}$. The turns ratio gives

$$N_s = N_p \frac{V_s}{V_p} = 4000 \frac{220}{2200} = 400.$$

Reducing voltage by a factor of ten requires the secondary to have one tenth as many turns as the primary.

Step 4: (c) Apply power conservation for the secondary current The input power is

$$P_p = V_p I_p = 2200 \times 5 = 11000\,\text{W}.$$

With no transformer losses, the output power equals this input power. Therefore

$$I_s = \frac{P_p}{V_s} = \frac{11000}{220} = 50\,\text{A}.$$

Quick Tip to Solve: Use the smaller solenoid's area in both linkage calculations. For an ideal transformer, a tenfold voltage reduction means tenfold fewer turns and tenfold greater current.

This also agrees with the inverse current ratio $I_s/I_p = N_p/N_s = 10$: the lower output voltage is accompanied by a higher current at unchanged power.

Step 5: Conclusion $M_{12} = M_{21} = \mu_0 N_1 N_2 \pi r_1^2/l = \mu_0 n_1 n_2 l \pi r_1^2$. The ideal step-down transformer needs 400 secondary turns and delivers secondary current $50\,\text{A}$ at $220\,\text{V}$.

Quick Tip to Solve: Use the smaller solenoid's area in both linkage calculations. For an ideal transformer, a tenfold voltage reduction means tenfold fewer turns and tenfold greater current.

OR

29. (a) An alternating voltage $v = v_m \sin \omega t$ applied to a series LCR circuit drives a current given by $i = i_m \sin(\omega t + \phi)$. Deduce an expression for the average power dissipated over a cycle.

(b) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

(c) Determine the current and quality factor at resonance for a series LCR circuit with $L = 1.00\,\text{mH}$, $C = 1.00\,\text{nF}$ and $R = 100\,\Omega$ connected to an a.c. source having peak voltage of 100 V. [5 marks]

Solution:

Concept:

- Instantaneous electrical power is $p(t) = v(t)i(t)$. Average power is obtained by averaging over a full period $T = 2\pi/\omega$.
- For sinusoidal signals, $V_{\text{rms}} = v_m/\sqrt{2}$ and $I_{\text{rms}} = i_m/\sqrt{2}$. Real power is $P = V_{\text{rms}} I_{\text{rms}} \cos\phi$, where $\cos\phi$ is the power factor.
- Series resonance occurs when $\omega_0 L = 1/(\omega_0 C)$. Then $Z = R$, and the quality factor is $Q = \omega_0 L/R = (1/R)\sqrt{L/C}$.

Step 1: (a) Form the instantaneous power Use the printed current convention, in which current has phase $+\phi$ relative to voltage. Substituting the given signals gives

$$p(t) = v_m i_m \sin\omega t \sin(\omega t + \phi).$$

The product identity $2\sin A \sin B = \cos(A - B) - \cos(A + B)$ separates a constant contribution from an oscillatory contribution:

$$p(t) = \frac{v_m i_m}{2}\big[\cos\phi - \cos(2\omega t + \phi)\big].$$

The result is unchanged if the current lags instead, because the average depends on the even function $\cos\phi$.

Step 2: (a) Average over a complete cycle The cosine at angular frequency 2ω completes two cycles in one supply period. Its average vanishes explicitly:

$$\frac{1}{T}\int_0^T \cos(2\omega t + \phi)\, dt = \frac{\sin(2\omega T + \phi) - \sin\phi}{2\omega T} = 0.$$

Quick Tip to Solve: At series resonance cancel the two reactances before finding current. Match peak voltage with peak current; use $Q = \sqrt{L/C}/R$ after converting mH and nF to SI units.

Hence the average dissipated power is

$$P_{\text{av}} = \frac{v_m i_m}{2}\cos\phi = V_{\text{rms}} I_{\text{rms}} \cos\phi.$$

In a series LCR circuit, $\cos\phi = R/Z$ and $V_{\text{rms}} = I_{\text{rms}} Z$, so this is also $P_{\text{av}} = I_{\text{rms}}^2 R$. An ideal inductor and capacitor store and return energy; the resistance accounts for the net dissipation over a full cycle.

Step 3: (b) Relate low power factor to line loss Suppose the same real power P is delivered at a fixed transmission rms voltage V. If the load power factor is $\cos\phi$, the required rms line current is

$$I = \frac{P}{V\cos\phi}.$$

For total line resistance R_{line}, the unwanted heating loss is therefore

$$P_{\text{loss}} = I^2 R_{\text{line}} = \frac{P^2 R_{\text{line}}}{V^2\cos^2\phi}.$$

A smaller power factor thus demands larger current for the same useful power, and the line loss rises as the square of that current. For example, halving the power factor at fixed P, V and line resistance multiplies the heating loss by four.

Step 4: (c) Find the current at resonance Convert the component values to SI units: $L = 1.00\times10^{-3}\,\text{H}$, $C = 1.00\times10^{-9}\,\text{F}$, and $R = 100\,\Omega$. At resonance the equal inductive and capacitive reactances cancel, so the impedance is purely resistive:

$$Z_0 = R = 100\,\Omega.$$

The source voltage is explicitly a peak value $v_m = 100\,\text{V}$, so the peak current and its rms value are

$$i_m = \frac{v_m}{R} = \frac{100}{100} = 1.00\,\text{A}, \qquad I_{\text{rms}} = \frac{1.00}{\sqrt{2}} = 0.707\,\text{A}.$$

The current is in phase with the source voltage; equivalently, $i(t) = 1.00\sin(\omega_0 t)\,\text{A}$.

Step 5: (c) Calculate the quality factor First calculate the resonant angular frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}\times10^{-9}}} = 1.00\times10^6\ \text{rad s}^{-1}.$$

Thus

$$Q = \frac{\omega_0 L}{R} = \frac{(1.00\times10^6)(1.00\times10^{-3})}{100} = 10.0.$$

As a check, $(1/R)\sqrt{L/C} = \sqrt{10^6}/100 = 10.0$. The quality factor is dimensionless because both $\omega_0 L$ and R have the dimensions of resistance.

Step 6: Conclusion $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}}\cos\phi$. A low power factor increases current and hence transmission heating for fixed delivered power and voltage. At resonance, the peak current is $1.00\,\text{A}$ (rms current $0.707\,\text{A}$) and $Q = 10.0$.

Quick Tip to Solve: At series resonance cancel the two reactances before finding current. Match peak voltage with peak current; use $Q = \sqrt{L/C}/R$ after converting mH and nF to SI units.

30. (i) A plane wavefront approaches a plane surface separating two media. If medium 'one' is optically denser and medium 'two' is optically rarer, using Huygens' principle, explain and show how a refracted wavefront is constructed.

(ii) Hence verify Snell's law.

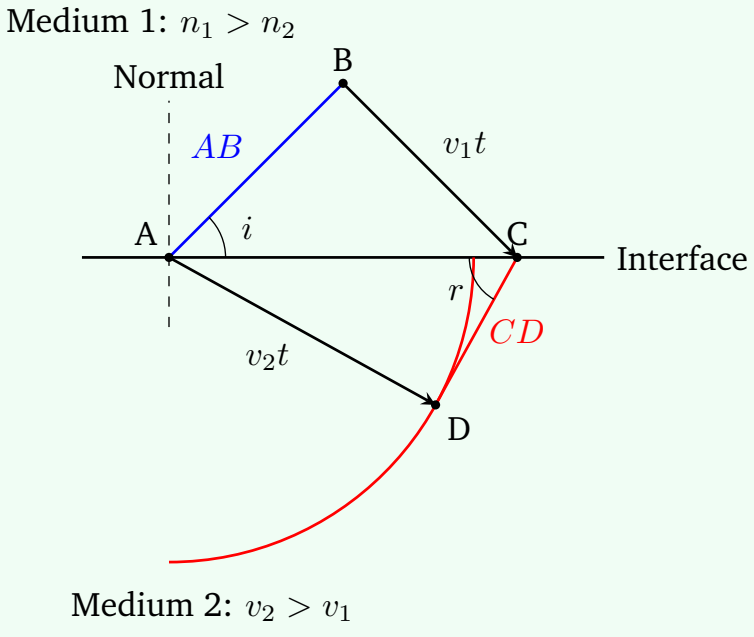
(iii) When a light wave travels from a rarer to a denser medium, the speed decreases. Does it imply reduction in its energy? Explain. [5 marks]

Solution:

Concept:

- Huygens' principle: every point on a wavefront emits secondary wavelets; their forward envelope gives the later wavefront. Rays are perpendicular to wavefronts.
- The speed in medium j is $v_j = c/n_j$, where n_j is its refractive index. Here $n_1 > n_2$, so $v_1 < v_2$.
- At a stationary interface, light retains its frequency ν. Photon energy is $E = h\nu$, with h Planck's constant; wavelength changes according to $\lambda = v/\nu$.

Step 1: (i) Construct the refracted wavefront Let AB be the incident plane wavefront in denser medium 1. Its end A reaches the plane interface first, while B is still in medium 1. In time t, the disturbance from B travels along the incident ray to C on the interface, giving $BC = v_1 t$. Meanwhile, the secondary wavelet from A spreads into rarer medium 2 with radius $AD = v_2 t$. Draw this wavelet and the tangent CD from C touching it at D. The tangent CD is the refracted plane wavefront: it is the common forward envelope of the secondary wavelets emitted successively by the intervening interface points. The refracted ray AD is perpendicular to CD. Since $v_2 > v_1$, the ray bends away from the normal. The construction assumes an incidence below the critical angle, so a propagating refracted wavefront exists.



Huygens construction from denser medium 1 to rarer medium 2: BC equals v1 t and AD equals v2 t.

Step 2: (ii) Obtain Snell's law from the two triangles Let i and r denote the angles that the incident and refracted rays make with the interface normal. Since rays are normal to their respective wavefronts,

Quick Tip to Solve: Use the common hypotenuse AC in both right triangles; the speed ratio follows immediately from their sine ratios.

$\angle BAC = i$ and $\angle DCA = r$. Triangles ABC and ADC are right-angled at B and D, respectively, and share hypotenuse AC. Thus

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC},$$
$$\sin r = \frac{AD}{AC} = \frac{v_2 t}{AC},$$
$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}.$$

Rearranging gives $n_1 \sin i = n_2 \sin r$, the usual Snell relation. The ratio $\sin i / \sin r$ is constant for these two media at a fixed wavelength.

Step 3: (iii) Distinguish photon energy from propagation speed A decrease in speed on entering a denser medium does not imply a decrease in photon energy. The light's frequency remains fixed by the source and is continuous across the stationary boundary; otherwise successive wavefronts could not remain matched there. Consequently $E = h\nu$ remains unchanged, although the wavelength becomes smaller because $\lambda = v/\nu$. Reflection or absorption can reduce the energy carried by the transmitted beam, but that reduction concerns the number of transmitted photons or absorption, not an automatic reduction of each photon's energy caused by the lower speed.

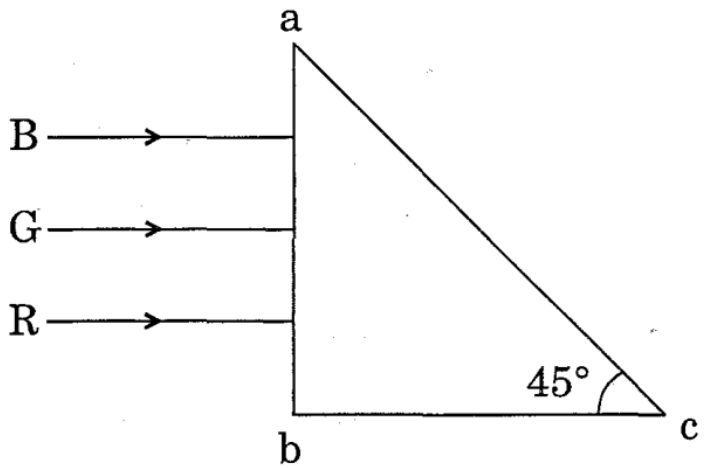
Step 4: Conclusion The tangent CD to the secondary wavelet gives the refracted wavefront and yields $\boxed{n_1 \sin i = n_2 \sin r}$. On entering a denser medium, speed and wavelength decrease while frequency and photon energy $\boxed{E = h\nu}$ remain unchanged.

Quick Tip to Solve: Use the common hypotenuse AC in both right triangles; the speed ratio follows immediately from their sine ratios.

OR

30. (i) A ray of monochromatic light is incident on one of the faces of an equilateral triangular prism of refracting angle A. Trace the path of ray passing through the prism. Hence derive an expression for the refractive index of the material of the prism in terms of the angle of minimum deviation and its refracting angle.

(ii) Three light rays red (R), green (G) and blue (B) are incident on the right angled prism abc at face ab. The refractive indices of the material of the prism for red, green and blue wavelengths are respectively 1.39, 1.44 and 1.47. Trace the paths of these rays reasoning out the difference in their behaviour. [5 marks]

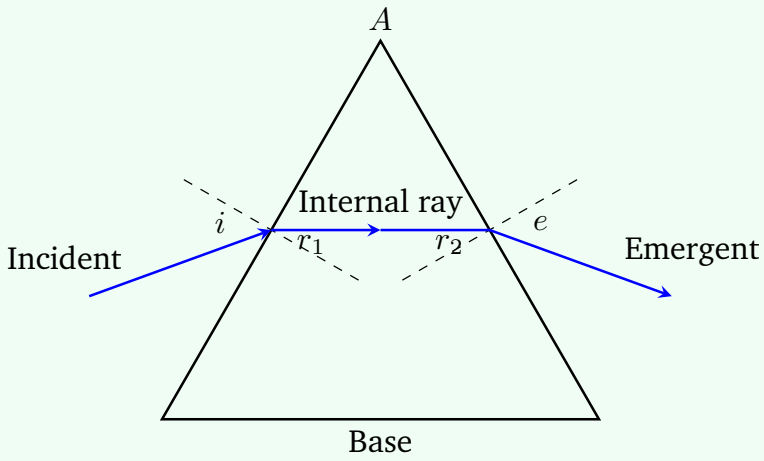


Solution:

Concept:

- Snell's law at a prism in air is $\sin i = \mu \sin r_1$ and $\mu \sin r_2 = \sin e$. Here i, e are the external angles of incidence and emergence, r_1, r_2 the internal refraction angles, and μ the prism's refractive index relative to air.
- Prism geometry gives $r_1 + r_2 = A$ and total deviation $\delta = i + e - A$, where A is the refracting angle. At minimum deviation the path is symmetric: $i = e$ and $r_1 = r_2$.
- For a prism-to-air boundary, the critical angle C obeys $\sin C = 1/\mu$. Total internal reflection requires incidence in the denser medium greater than C.

Step 1: (i) Trace the monochromatic ray through the prism The incident ray meets the first inclined face, refracts towards its normal, and travels across the prism. At the second inclined face it refracts away from that face's normal and emerges into air. These two refractions cause a net deviation towards the prism's base. In the diagram, dashed lines are the face normals, the horizontal internal segment represents the symmetric minimum-deviation path, and arrowheads show the direction of propagation. For an equilateral prism $A = 60°$, but retaining A gives the general result.



Symmetric ray path at minimum deviation in an equilateral prism; dashed lines are normals.

Step 2: Derive the refractive index at minimum deviation Each face changes the ray direction by $i - r_1$ and $e - r_2$, respectively. The angle between the face normals, together with the internal ray geometry, gives $r_1 + r_2 = A$. Therefore

$$\delta = (i - r_1) + (e - r_2) = i + e - A.$$

As the prism is turned, deviation decreases to a minimum and then increases. Reversibility makes the minimum-deviation path symmetric: the incoming and outgoing rays make equal angles with their face normals. For $\delta = \delta_m$,

$$i = e = \frac{A + \delta_m}{2},$$

$$r_1 = r_2 = \frac{A}{2},$$

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin\left[(A + \delta_m)/2\right]}{\sin(A/2)}.$$

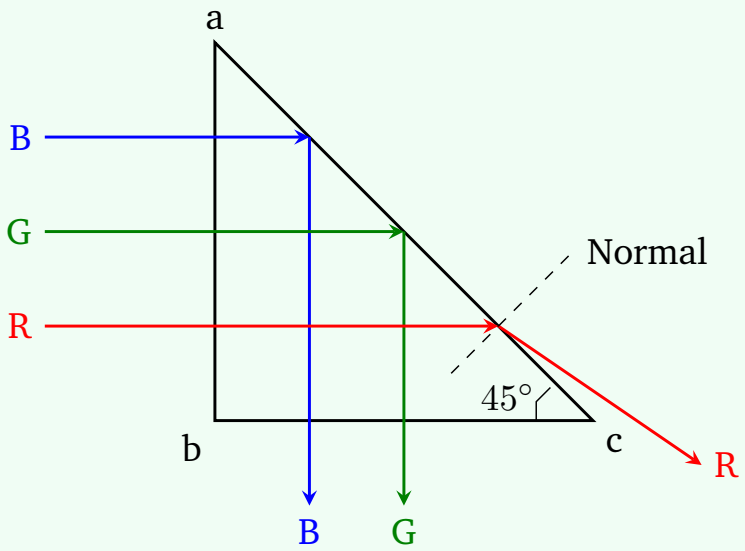
Quick Tip to Solve: At 45 degrees compare each refractive index with sqrt(2): below it a transmitted ray exists, above it total internal reflection occurs.

For this equilateral prism, the denominator is $\sin 30^\circ = 1/2$, so $\mu = 2\sin[(60^\circ + \delta_m)/2]$. The formula assumes the surrounding medium is air.

Step 3: (ii) Compare the three rays with their critical angles All three rays enter face ab normally, so their directions remain horizontal inside the prism. The face ac is at 45° to the horizontal; consequently the incidence at this glass-to-air face is 45° for each ray. The critical angles are

$$C_R = \sin^{-1}(1/1.39) \simeq 46.0^\circ,$$
$$C_G = \sin^{-1}(1/1.44) \simeq 44.0^\circ,$$
$$C_B = \sin^{-1}(1/1.47) \simeq 42.9^\circ.$$

Thus red has $45^\circ < C_R$ and emerges by refraction from ac, bending away from its normal. Its emergence angle is obtained from $\sin e_R = 1.39 \sin 45^\circ$, giving $e_R \simeq 79.4^\circ$ to the outward normal. The transmitted red ray travels to the right and downwards, nearly along ac. Green and blue have $45^\circ > C_G, C_B$, so both undergo total internal reflection at ac. The law of reflection turns each of them vertically downwards. They then strike face bc normally and emerge through bc without further deviation. The red boundary also has a reflected component; the diagram traces its transmitted refracted path and the totally reflected green and blue paths.



Red transmits through ac; blue and green reflect internally and emerge normally through bc.

Step 4: Conclusion At minimum deviation, $\boxed{\mu = \sin[(A + \delta_m)/2]/\sin(A/2)}$. In the right-angled prism, red refracts out through ac, whereas green and blue undergo total internal reflection at ac and emerge normally through bc.

Quick Tip to Solve: At 45 degrees compare each refractive index with sqrt(2): below it a transmitted ray exists, above it total internal reflection occurs.