CBSE Class 12 Physics Question Paper 2013 with Solution PDF-Delhi Set 1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours **Maximum Marks: 70**

General Instructions

(i) **All questions are compulsory.**

(ii) There are **29** questions in total. Question Nos. **1 to 8** are very short answer type questions and carry **one** mark each.

(iii) Question Nos. **9 to 16** carry **two** marks each, Question Nos. **17 to 25** carry **three** marks each and Question Nos. **27 to 29** carry **five** marks each.

(iv) There is no overall choice. However, an internal choice has been provided in **one** question of **two** marks, **one** question of **three** marks and **all three** questions of **five** marks each. You have to attempt only one of the choices in such questions.

(v) Question No. **26** is value based question carries **four** marks.

(vi) Use of calculators is not permitted. However, you may use log tables if necessary.

You may use the following values of physical constants wherever necessary:

$c = 3 \times 10^8$ m/s

$h = 6.63 \times 10^{-34}$ J s

$e = 1.6 \times 10^{-19}$ C

$\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2 C^{-2}

$m_e = 9.1 \times 10^{-31}$ kg

Mass of the Neutron $= 1.675 \times 10^{-27}$ kg

Mass of the Proton $= 1.673 \times 10^{-27}$ kg

1. What are permanent magnets? Give one example. [1 mark]

Solution:

Concept:

- A permanent magnet retains its magnetisation for a long time after the external magnetising field is removed.
- Materials with high retentivity and high coercivity retain a strong magnetisation and resist demagnetisation.

Step 1: State the defining property A material is called a permanent magnet when, once magnetised, it continues to produce its own magnetic field without a continuing external magnetising field or electric current. Its magnetisation persists under ordinary conditions.

Step 2: Give a suitable example A magnetised steel bar is a familiar example. Steel retains substantial magnetisation after the magnetising field is switched off, unlike a soft-iron core used for a temporary electromagnet.

Step 3: Conclusion Permanent magnets retain their magnetism for a long time; a magnetised steel bar is one example.

Quick Tip to Solve: Link the word permanent to retained magnetisation after removal of the magnetising field, and name a steel bar magnet.

2. What is the geometrical shape of equipotential surfaces due to a single isolated charge? [1 mark]

Solution:

Concept:

- An equipotential surface consists of points having the same electric potential.
- For an isolated point charge q, the potential at distance r is $V = kq/r$, where $k = 1/(4\pi\varepsilon_0)$ and the potential at infinity is zero.

Step 1: Identify what fixes the potential For a given isolated charge, k and q are fixed. The potential depends only on the distance r from the charge, not on the direction in which the observation point lies.

Step 2: Find the constant-potential locus Setting V equal to a constant fixes r. In three-dimensional space, all points at a fixed distance from a point form a spherical surface. Different fixed values of r give surfaces with the same centre.

Step 3: Conclusion The equipotential surfaces are concentric spheres centred on the isolated charge.

Quick Tip to Solve: Whenever potential depends only on radial distance, set that distance constant to identify the equipotential surface.

3. Which of the following waves can be polarized (i) Heat waves (ii) Sound waves? Give reason to support your answer. [1 mark]

Solution:

Concept:

- Polarisation restricts a transverse wave's oscillations to a specified direction perpendicular to propagation.
- Longitudinal waves oscillate along their direction of propagation and cannot show transverse polarisation.

Step 1: Examine heat waves Heat waves are infrared electromagnetic waves. Their electric and magnetic fields oscillate perpendicular to the direction of propagation. They therefore have transverse oscillations that can be restricted to one orientation by a suitable polariser.

Step 2: Examine sound waves Ordinary sound waves in air are longitudinal: particles vibrate parallel to the propagation direction, producing compressions and rarefactions. There are no alternative transverse vibration directions to select, so such sound waves cannot be polarised.

Step 3: Conclusion Heat waves can be polarised because they are transverse; ordinary sound waves cannot be polarised because they are longitudinal.

Quick Tip to Solve: First classify the vibration direction: transverse waves can be polarised, whereas longitudinal sound in air cannot.

4. A capacitor has been charged by a dc source. What are the magnitudes of conduction and displacement currents, when it is fully charged? [1 mark]

Solution:

Concept:

- Conduction current is the rate of flow of free charge, $I_c = dQ/dt$.
- Displacement current is associated with changing electric flux: $I_d = \varepsilon_0 \, d\Phi_E/dt$, where Φ_E is the electric flux between the capacitor plates.

Step 1: Examine the steady conduction current When an ideal capacitor is fully charged from a dc source, its potential difference equals the source voltage. No further charge is transferred to its plates, so Q is constant and $dQ/dt = 0$. The charging current in the connecting wires therefore ceases.

Step 2: Examine the electric flux The final electric field between the plates is nonzero but constant in time. Hence the electric flux is also constant, giving $d\Phi_E/dt = 0$. A static electric field does not produce displacement current.

Step 3: Conclusion When the capacitor is fully charged, both currents vanish: $I_c = 0$ and $I_d = 0$.

Quick Tip to Solve: Displacement current depends on the rate of change of electric field, so a fully charged capacitor's static field gives zero current.

5. Write the relationship between angle of incidence i, angle of prism A and angle of minimum deviation for a triangular prism. [1 mark]

Solution:

Concept:

- For a triangular prism, the deviation is $\delta = i + e - A$, where i and e are the angles of incidence and emergence, and A is the prism angle.
- At minimum deviation, the light path through the prism is symmetric, so $i = e$ and the internal refraction angles are both $A/2$.

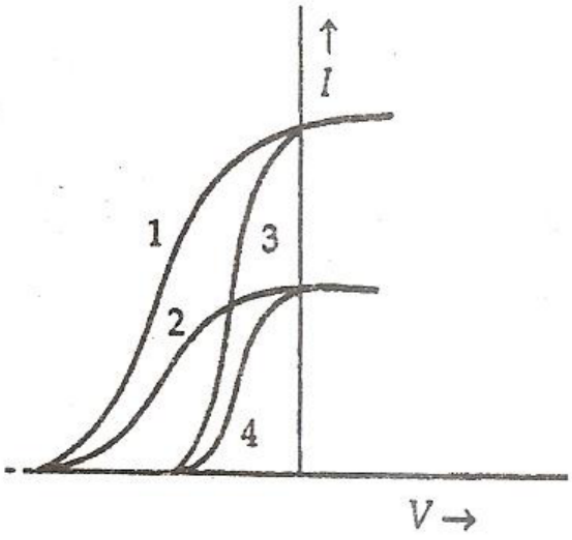
Step 1: Use the prism deviation relation The total deviation at the two refracting faces is $(i - r_1) + (e - r_2)$. Since the prism geometry gives $r_1 + r_2 = A$, this becomes $\delta = i + e - A$. All the angles are measured relative to the appropriate normals.

Step 2: Apply the minimum-deviation condition Denote the minimum deviation by δ_m. For the symmetric ray path at this position, the emergence angle equals the incidence angle. Substituting $e = i$ into the general relation makes the sum $i + e$ equal to $2i$.

Step 3: Conclusion The required relationship is $\boxed{\delta_m = 2i - A}$, or equivalently $\boxed{i = (A + \delta_m)/2}$.

Quick Tip to Solve: Start from deviation equals incidence plus emergence minus prism angle, and replace emergence by incidence only at minimum deviation.

6. The given graph shows the variation of photo-electric current (I) versus applied voltage (V) for two different photosensitive materials and for two different intensities of the incident radiation. Identify the pairs of curves that correspond to different materials but same intensity of incident radiation. [1 mark]



Solution:

Concept:

- In a photoelectric current graph, the saturation current indicates the intensity of the incident radiation under the conditions represented.
- The stopping potential obeys $eV_0 = h\nu - \phi$, where ϕ is the work function; at a fixed incident frequency, different stopping potentials indicate different photosensitive materials.

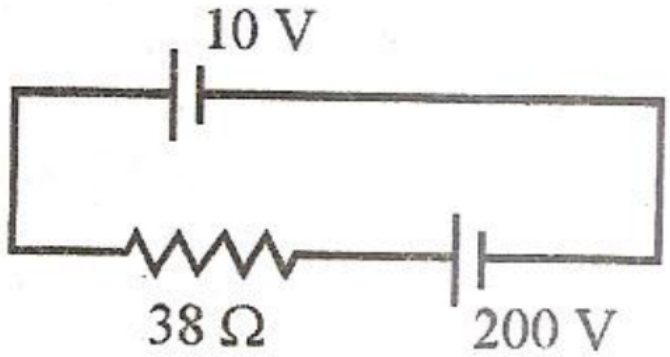
Step 1: Match the saturation-current levels Read the horizontal levels approached by the curves at positive applied voltage. Curves 1 and 3 reach the same higher saturation current. Curves 2 and 4 reach the same lower saturation current. Each such pair therefore represents the same incident intensity in the given graph.

Step 2: Compare the stopping potentials within each pair Curves 1 and 3 meet the voltage axis at different negative voltages, as do curves 2 and 4. Their stopping potentials differ, indicating different work functions and hence different materials for the same incident frequency. Pairing curves solely by a common voltage intercept would instead select the same material.

Step 3: Conclusion The required pairs are $\boxed{(1,3) \text{ and } (2,4)}$: each pair has equal saturation current but different stopping potentials.

Quick Tip to Solve: Match the horizontal saturation levels for equal intensity, then check that the negative voltage intercepts differ for different materials.

7. A 10 V battery of negligible internal resistance is connected across a 200 V battery and a resistance of $38\,\Omega$ as shown in the figure. Find the value of the current in circuit. [1 mark]



Solution:

Concept:

- Kirchhoff's loop rule requires the algebraic sum of potential changes around a closed circuit to be zero.
- For opposing ideal batteries in series with resistance R, the driving emf is their difference, and $I = (E_{\text{larger}} - E_{\text{smaller}})/R$.

Step 1: Read the battery polarities The longer plate denotes the positive terminal. In the printed diagram, both batteries have their positive terminals on the left, so their emfs oppose each other

Quick Tip to Solve: Mark the longer battery plates positive before applying the loop rule; opposing sources subtract even when drawn on opposite sides of the loop.

around the loop. The $200\,\text{V}$ battery drives current against the $10\,\text{V}$ battery, giving a net driving emf of $200 - 10 = 190\,\text{V}$.

Step 2: Apply the loop rule Take clockwise current: it passes right to left through the lower resistor and enters the positive terminal of the upper battery. With negligible internal resistance, the only resistive drop is across $38\,\Omega$:

$$200 - 38I - 10 = 0, \qquad 38I = 190.$$

The positive solution confirms the assumed current direction.

Step 3: Conclusion The current is $\boxed{I = 190/38 = 5\,\text{A}}$, clockwise in the printed diagram, and charges the $10\,\text{V}$ battery.

Quick Tip to Solve: Mark the longer battery plates positive before applying the loop rule; opposing sources subtract even when drawn on opposite sides of the loop.

8. The emf of a cell is always greater than its terminal voltage. Why? Give reason. [1 mark]

Solution:

Concept:

- The emf E is the energy supplied by a cell per unit charge; its terminal voltage V is the energy delivered per unit charge to the external circuit.
- A cell supplying current I has an internal voltage drop Ir, where r is its internal resistance: $V = E - Ir$.

Step 1: Account for the internal voltage drop When the cell supplies current, some of its energy is dissipated as heat in its internal resistance. The corresponding potential drop inside the cell is Ir.

Step 2: Compare emf and terminal voltage The remaining energy per unit charge is available at the terminals. Thus $E - V = Ir > 0$ for a discharging cell with nonzero current and internal resistance. In an open circuit, $I = 0$ and $V = E$; the stated comparison refers to a cell supplying current.

Step 3: Conclusion The emf exceeds the terminal voltage by the internal drop Ir while the cell supplies current: $E = V + Ir$.

Quick Tip to Solve: For a cell delivering current, subtract the internal drop from its emf to obtain the terminal voltage.

9. (a) Write the necessary conditions for the phenomenon of total internal reflection to occur.

(b) Write the relation between the refractive index and critical angle for a given pair of optical media. [2 marks]

Solution:

Concept:

- Total internal reflection is complete reflection at an interface when light travels from an optically denser medium to a rarer medium and the incidence angle exceeds the critical angle.
- Snell's law is $n_d \sin i = n_r \sin r$, where $n_d > n_r$ are the refractive indices of the denser and rarer media, and angles are measured from the normal.

Step 1: State both necessary conditions (a) The ray must travel from the optically denser medium towards the optically rarer medium. Its angle of incidence i, measured in the denser medium, must be greater than the critical angle C for that pair of media. At $i = C$, the refracted ray just grazes the interface; total internal reflection requires $i > C$.

Step 2: Apply Snell's law at the critical angle (b) At the critical angle, the angle of refraction is 90°. Hence

$$n_d \sin C = n_r \sin 90^\circ = n_r.$$

If $\mu = n_d/n_r$ is the refractive index of the denser medium relative to the rarer medium, rearrangement gives $\sin C = 1/\mu$. For a medium-air interface, $n_r \simeq 1$.

Step 3: Conclusion The two conditions are denser-to-rarer propagation and $i > C$. The required relation is $\mu = 1/\sin C$, with $\mu = n_d/n_r$.

Quick Tip to Solve: Set the refracted angle to 90° in Snell's law; the critical angle is always measured in the denser medium.

10. State Lenz's Law.

A metallic rod held horizontally along east-west direction, is allowed to fall under gravity. Will there be an emf induced at its ends? Justify your answer. [2 marks]

Solution:

Concept:

- Lenz's law states that the direction of induced current is such that its magnetic effect opposes the change in magnetic flux that produces it.
- Moving charges experience the magnetic force $\vec{F} = q(\vec{v} \times \vec{B})$. A rod of length l moving perpendicular to both its length and a magnetic field component B_H develops emf $\varepsilon = B_H l v$.

Step 1: State the opposition described by Lenz's law The induced current, when a conducting circuit is closed, opposes the change in flux responsible for induction. This opposition to the cause of induction expresses conservation of energy; it does not mean that the induced field always opposes the existing field.

Step 2: Apply the magnetic force to the falling rod Yes, an emf is induced between the rod's ends.

Quick Tip to Solve: Use Earth's horizontal field for a vertically falling rod; apply $\vec{v} \times \vec{B}_H$ to find which end becomes positive.

The east-west rod falls vertically and cuts the horizontal component B_H of Earth's magnetic field, which points approximately north. The vectors $\vec{v}$ and $\vec{B}_H$ are perpendicular, and $\vec{v} \times \vec{B}_H$ points east, along the rod. Positive charge is driven towards its eastern end, separating charge and establishing an emf of magnitude $B_H l v$. The vertical magnetic component is parallel to the falling motion and gives no contribution. An emf can exist even without a closed circuit.

Step 3: Conclusion Lenz's law gives the direction that opposes the flux change. The falling rod develops $\varepsilon = B_H l v$, with the east end at higher potential for a northward horizontal field.

Quick Tip to Solve: Use Earth's horizontal field for a vertically falling rod; apply $\vec{v} \times \vec{B}_H$ to find which end becomes positive.

11. A convex lens of focal length 25 cm is placed coaxially in contact with a concave lens of focal length 20 cm. Determine the power of the combination. Will the system be converging or diverging in nature? [2 marks]

Solution:

Concept:

- The power of a thin lens is $P = 1/f$, where focal length f is expressed in metres and power is in dioptres (D).
- For thin lenses in contact, powers add: $P = P_1 + P_2$. A convex lens has positive focal length and a concave lens has negative focal length.

Step 1: Assign signs and convert focal lengths The convex lens converges parallel light, so $f_1 = +25\,\text{cm} = +0.25\,\text{m}$. The concave lens diverges parallel light, so $f_2 = -20\,\text{cm} = -0.20\,\text{m}$. The negative sign for the concave lens must be retained when adding powers.

Step 2: Add the individual powers Since the lenses are coaxial and in contact, no separation correction is required. Their powers are

$$P_1 = \frac{1}{0.25} = +4.0\,\text{D}, \qquad P_2 = \frac{1}{-0.20} = -5.0\,\text{D}.$$

Therefore $P = P_1 + P_2 = 4.0 - 5.0 = -1.0\,\text{D}$. The negative power corresponds to an effective focal length $f = 1/P = -1.0\,\text{m}$. The concave lens has the larger power magnitude and determines the net behaviour.

Step 3: Conclusion The power of the combination is $-1.0\,\text{D}$; the system is diverging in nature.

Quick Tip to Solve: Convert centimetres to metres and attach the concave lens's minus sign before adding the two lens powers.

12. An ammeter of resistance $0.80\,\Omega$ can measure current upto 1.0 A.

(i) What must be the value of shunt resistance to enable the ammeter to measure current upto 5.0 A?

(ii) What is the combined resistance of the ammeter and the shunt? [2 marks]

Solution:

Concept:

- A shunt is connected in parallel with an ammeter to bypass excess current. At the extended full-scale reading, the original ammeter must still carry only its rated current I_A.
- Parallel branches have equal potential difference: $I_A R_A = I_s R_s$, where $I_s = I - I_A$. Their equivalent resistance is $R_{\text{eq}} = R_A R_s/(R_A + R_s)$.

Step 1: Calculate the shunt current and resistance (i) The ammeter resistance is $R_A = 0.80\,\Omega$, and its maximum safe current is $I_A = 1.0\,\text{A}$. For total current $I = 5.0\,\text{A}$, the shunt must carry

$$I_s = 5.0 - 1.0 = 4.0\,\text{A}.$$

The common branch voltage at full scale is $I_A R_A = 1.0 \times 0.80 = 0.80\,\text{V}$. Consequently,

$$R_s = \frac{I_A R_A}{I_s} = \frac{1.0 \times 0.80}{4.0} = 0.20\,\Omega.$$

Step 2: Determine the combined resistance (ii) The original ammeter and shunt form a parallel combination, so

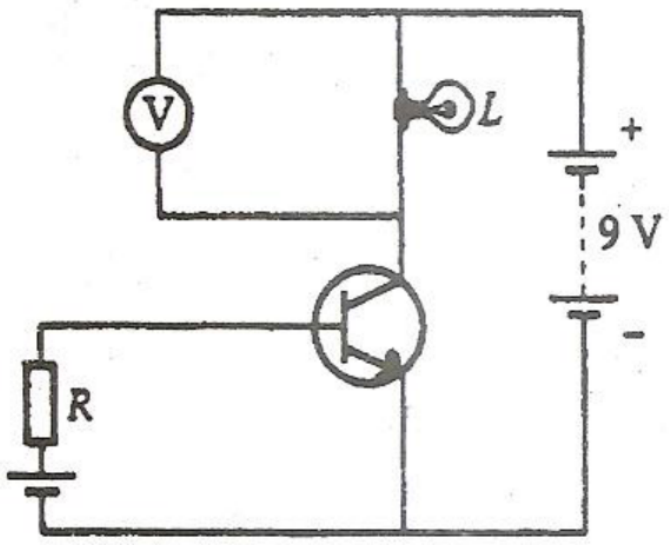
$$R_{\text{eq}} = \frac{0.80 \times 0.20}{0.80 + 0.20} = 0.16\,\Omega.$$

This also follows from the common voltage divided by the total current: $R_{\text{eq}} = 0.80/5.0 = 0.16\,\Omega$. As expected for parallel resistors, the result is less than either branch resistance.

Step 3: Conclusion (i) Connect a $0.20\,\Omega$ shunt in parallel. (ii) The combined resistance is $0.16\,\Omega$.

Quick Tip to Solve: The shunt carries only the excess current $5.0 - 1.0\,\text{A}$; use equal branch voltages to find its resistance.

13. In the given circuit diagram, a voltmeter 'V' is connected across a lamp 'L'. How would (i) the brightness of the lamp and (ii) voltmeter reading 'V' be affected, if the value of resistance 'R' is decreased? Justify your answer. [2 marks]



Solution:

Concept:

- In the given transistor circuit, the resistance R controls the base current I_B, while the lamp lies in the collector circuit.
- In the active region, the collector current is $I_C = \beta I_B$, where β is the current gain. A lamp glows more brightly when its electrical power increases.

Step 1: Determine the change in base and collector currents With the base supply voltage fixed, decreasing R increases the base current. Approximately, $I_B = (V_{BB} - V_{BE})/R$, where V_{BB} is the base supply voltage and V_{BE} is the forward base-emitter drop. The increased base current produces a larger collector current, so a larger current flows through the lamp.

Step 2: Relate the lamp current to its brightness and voltage The larger lamp current raises the power dissipated in its filament and makes it brighter. If its resistance is denoted by R_L, the relations $P_L = I_C^2 R_L$ and $V_L = I_C R_L$ explain the increases. The voltmeter is across the lamp, so it reads V_L, which increases as the collector-emitter voltage falls. Although the collector supply remains $9\,\text{V}$, this voltage is shared by the lamp and transistor; the voltmeter is not across the whole supply. These increases continue until the transistor approaches saturation.

Step 3: Conclusion (i) The lamp becomes brighter. (ii) The voltmeter reading increases, because decreasing R increases the base current and hence the collector current through the lamp.

Quick Tip to Solve: First locate the adjustable resistor: here it controls the base current, which controls the lamp current in the collector branch.

14. (a) An em wave is travelling in a medium with a velocity $\vec{v} = v\,\hat{i}$. Draw a sketch showing the propagation of the em wave, indicating the direction of the oscillating electric and magnetic fields.

(b) How are the magnitudes of the electric and magnetic fields related to the velocity of the em wave? [2 marks]

Solution:

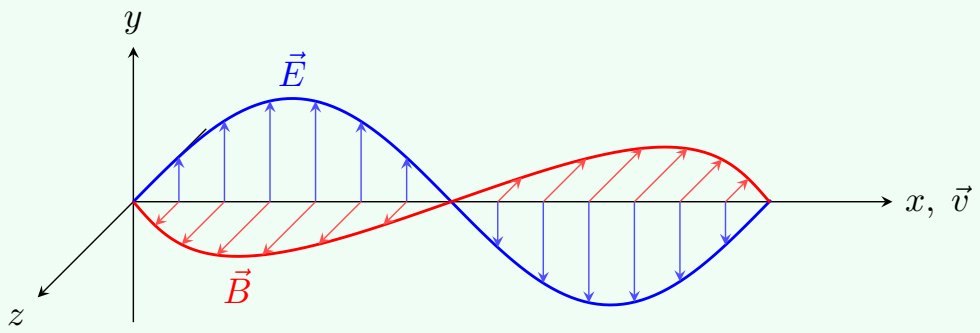
Concept:

- An electromagnetic wave is transverse: its electric field $\vec{E}$, magnetic field $\vec{B}$, and propagation direction are mutually perpendicular.
- The wave travels in the direction of $\vec{E} \times \vec{B}$. In a medium of wave speed v, the field amplitudes satisfy $E_0/B_0 = v$.

Step 1: Choose the field directions for propagation along the positive x-axis Since $\vec{v} = v\hat{i}$, choose the electric field to oscillate along the y-axis and the magnetic field along the z-axis. Then $\hat{j} \times \hat{k} = \hat{i}$, as required. One representation is

$$\vec{E} = E_0 \sin(kx - \omega t)\hat{j}, \qquad \vec{B} = B_0 \sin(kx - \omega t)\hat{k}.$$

Both fields oscillate in phase and reverse together. The sketch shows these mutually perpendicular oscillations; the field scales are arbitrary.



In-phase electric and magnetic oscillations transverse to propagation along $+x$.

Step 2: Relate the field magnitudes to the wave speed For the travelling plane wave, Faraday's law gives $kE_0 = \omega B_0$. Since the wave speed is $v = \omega/k$, the amplitude ratio is

$$\frac{E_0}{B_0} = \frac{\omega}{k} = v.$$

Thus the instantaneous magnitudes also satisfy $E = vB$, because the two fields share the same phase. The speed here is the speed in the specified medium, not necessarily the vacuum speed c.

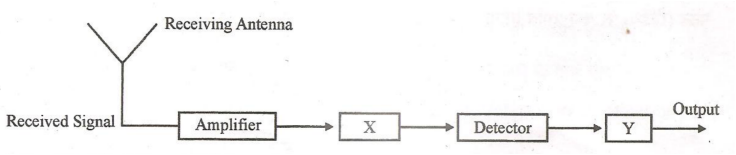
Step 3: Conclusion A valid sketch has $\vec{E} \parallel \hat{j}$, $\vec{B} \parallel \hat{k}$, and propagation along $+\hat{i}$. Their magnitudes obey $E = vB$, or $E_0 = vB_0$.

Quick Tip to Solve: Check the right-hand rule: electric field cross magnetic field must point along the given wave velocity.

15. Block diagram of a receiver is shown in the figure:

(a) Identify 'X' and 'Y'.

(b) Write their functions. [2 marks]



Solution:

Concept:

- A receiver strengthens the received modulated signal, processes it at an intermediate frequency, and detects the original information signal.
- Detection (demodulation) recovers the message from the carrier. The recovered message is usually weak and requires amplification before use at the output.

Step 1: Identify X and state its function X is the **intermediate-frequency (IF) stage**. It acts on the amplified incoming radio-frequency signal before detection. In this stage the carrier frequency is changed to a lower, fixed intermediate frequency, while the modulation carrying the information is preserved. Processing and amplification at a fixed intermediate frequency make it easier to obtain effective amplification and selectivity before the signal reaches the detector.

Step 2: Identify Y and state its function Y is the **amplifier after detection**, called an audio-frequency amplifier when the message is sound. The detector just before it has already separated the original message signal from the carrier. This recovered signal is generally weak, so Y increases its amplitude or power to a useful level for the output device, such as a loudspeaker. Thus the amplifier before X works on the received modulated signal, whereas Y works on the recovered message.

Step 3: Conclusion X: IF stage, which changes the carrier to a lower intermediate frequency for subsequent processing. Y: amplifier, which strengthens the detected message signal for the output.

Quick Tip to Solve: Locate the detector first: the IF stage is before it, while the message or audio amplifier is after it.

16. Explain, with the help of a circuit diagram, the working of a photo-diode. Write briefly how it is used to detect the optical signals. [2 marks]

Solution:

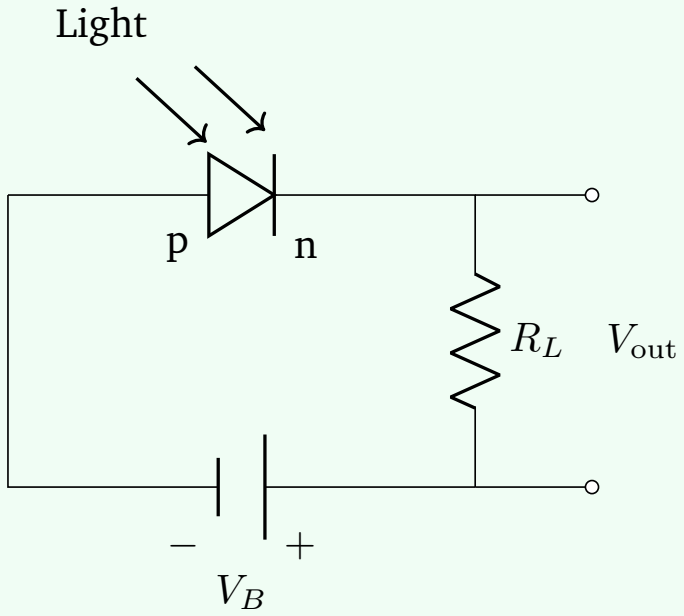
Concept:

- A photodiode is a p-n junction operated in reverse bias, with a transparent window that allows light to fall on its depletion region.
- A photon of energy $h\nu \geq E_g$, where E_g is the semiconductor band gap, can generate an electron-hole pair. The reverse electric field separates the carriers and produces photocurrent.

Step 1: Connect the reverse-biased detector Connect the p-side (anode) to the negative terminal and the n-side (cathode) to the positive terminal of a battery, through a load resistor R_L. In darkness only

Quick Tip to Solve: In a detector diagram put the n-side towards battery positive and show light arrows pointing into the diode.

a small reverse dark current flows. The reverse bias widens the depletion region and helps collect the light-generated carriers rapidly.



Reverse-biased photodiode with output measured across the load

Step 2: Convert the optical signal into an electrical signal When light reaches the junction, electron-hole pairs are created in or near the depletion region. Its electric field sweeps electrons towards the n-side and holes towards the p-side. The resulting reverse photocurrent increases with incident light intensity. Hence an intensity-varying optical signal gives a corresponding change in current and in the voltage across the load, $V_{\text{out}} = IR_L$. Measuring these voltage changes detects the optical signal.

Step 3: Conclusion A reverse-biased photodiode detects light by converting changes in light intensity into changes in reverse current, and therefore into an electrical output across the load resistor.

Quick Tip to Solve: In a detector diagram put the n-side towards battery positive and show light arrows pointing into the diode.

OR

16. Mention the important considerations required while fabricating a p-n junction diode to be used as a Light Emitting Diode (LED). What should be the order of band gap of an LED if it is required to emit light in the visible range? [2 marks]

Solution:

Concept:

- An LED emits light by radiative recombination of electrons and holes injected across a forward-biased p-n junction.
- The photon energy is approximately the band gap: $E_g \simeq h\nu = hc/\lambda$. The semiconductor composition therefore determines the colour emitted.

Quick Tip to Solve: Use $E_g(\text{eV}) \simeq 1240/\lambda(\text{nm})$: shorter-wavelength LED colours require a larger band gap.

Step 1: Choose the material and junction construction Use a direct-band-gap semiconductor, such as a suitable III-V compound, so electron-hole recombination efficiently releases energy as photons. Ordinary silicon or germanium junctions are unsuitable for efficient visible LEDs because most recombination energy is released as heat. The junction is heavily doped to provide a large supply of injected carriers under forward bias. Make the light-emitting junction accessible through a transparent window or transparent encapsulation, so the emitted radiation can escape without strong absorption.

Step 2: Estimate the required band gap For visible wavelengths of roughly 400–$700\,\text{nm}$, use $hc \simeq 1240\,\text{eV nm}$:

$$E_g \simeq \frac{1240}{\lambda\,(\text{nm})}\,\text{eV} \quad \Rightarrow \quad E_g \simeq 1.8 \text{ to } 3.1\,\text{eV}.$$

Thus a band gap of the order of a few electron-volts is required; about $2\,\text{eV}$ produces visible red light. The device is operated in forward bias with current limited to prevent excessive heating.

Step 3: Conclusion An LED requires an efficient radiative, heavily doped p-n junction with a transparent path for emitted light. For visible emission its band gap should be of order $2\,\text{eV}$, approximately 1.8–$3.1\,\text{eV}$ across the visible range.

Quick Tip to Solve: Use $E_g(\text{eV}) \simeq 1240/\lambda(\text{nm})$: shorter-wavelength LED colours require a larger band gap.

17. Write three important factors which justify the need of modulating a message signal. Show diagrammatically how an amplitude modulated wave is obtained when a modulating signal is superimposed on a carrier wave. [3 marks]

Solution:

Concept:

- Modulation makes a parameter of a high-frequency carrier vary with the instantaneous value of a low-frequency message signal.
- For amplitude modulation, $s(t) = A_c[1 + \mu\cos(\omega_m t)]\cos(\omega_c t)$, where A_c is carrier amplitude, μ is modulation index and $\omega_c \gg \omega_m$.

Step 1: Obtain a practical antenna size A transmitting antenna typically has length $\lambda/4$. At $1\,\text{kHz}$, $\lambda = c/f = 300\,\text{km}$, requiring an impractical $75\,\text{km}$ antenna. A high-frequency carrier reduces this length to a practical value.

Step 2: Increase radiated power and separate messages Second, for a short antenna of length l at fixed current, radiated power scales as $(l/\lambda)^2$. A shorter carrier wavelength improves radiation efficiency. Third, audio messages occupy overlapping frequency bands and would mix if sent directly. Distinct carrier frequencies separate the channels, allowing a tuned receiver to select one station.

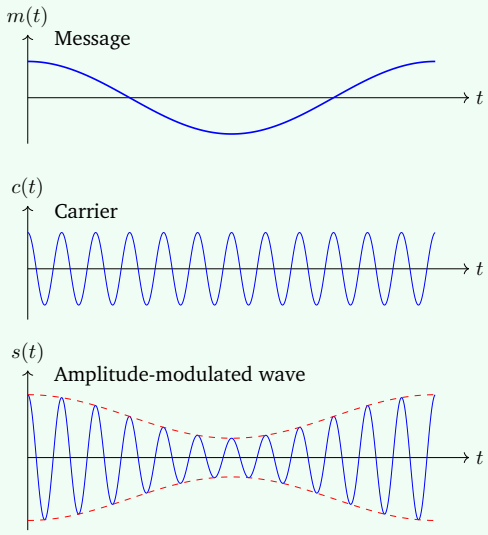
Step 3: Form the amplitude-modulated waveform For message $m(t) = A_m\cos\omega_m t$, vary the carrier amplitude proportionally while keeping carrier frequency and phase fixed:

$$s(t) = \left[A_c + k_a A_m\cos\omega_m t\right]\cos\omega_c t, \qquad \mu = \frac{k_a A_m}{A_c}.$$

Here k_a is amplitude sensitivity. The dashed envelopes $\pm A_c[1 + \mu\cos\omega_m t]$ follow the message. Use

Quick Tip to Solve: For an AM sketch, first draw the message-shaped upper and lower envelopes, then fit uniformly spaced rapid carrier oscillations inside them.

$0 \leq \mu \leq 1$ to avoid overmodulation. Simply adding message and carrier does not produce AM.



Message, carrier and AM signal plotted against time (arbitrary amplitude units)

Step 4: Conclusion Modulation is needed for practical antenna length, efficient radiation and separation of different messages. In AM the carrier amplitude follows the message, while the carrier frequency remains constant.

Quick Tip to Solve: For an AM sketch, first draw the message-shaped upper and lower envelopes, then fit uniformly spaced rapid carrier oscillations inside them.

18. A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is $360\,\mu$C. When potential across the capacitor is reduced by 120 V, the charge stored in it becomes $120\,\mu$C.

Calculate:

(i) The potential V and the unknown capacitance C.

(ii) What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V?

[3 marks]

Solution:

Concept:

- For a capacitor, $Q = CV$, where Q is the magnitude of charge on either plate, C is its capacitance and V is the potential difference.
- The same capacitor has unchanged geometry and dielectric, so C is constant. Hence charge changes in direct proportion to voltage: $\Delta Q = C\Delta V$.

Step 1: Form the two charge-voltage equations Initially the potential difference is V and the stored charge is $360\,\mu$C. After reducing the voltage by $120\,$V, the potential is $V - 120\,$V, and the charge is

Quick Tip to Solve: First use $C = \Delta Q/\Delta V$ to avoid simultaneous equations. Equal increases and decreases of voltage produce equal changes of charge for the same capacitor.

$120\,\mu$C. Thus,

$$CV = 360 \times 10^{-6}\,\text{C},$$
$$C(V - 120\,\text{V}) = 120 \times 10^{-6}\,\text{C}.$$

The voltage reduction is a difference; it is not the final voltage itself.

Step 2: Determine the capacitance and initial potential Subtract the second equation from the first to eliminate the unknown initial potential:

$$C(120\,\text{V}) = (360 - 120) \times 10^{-6}\,\text{C}.$$

Therefore,

$$C = \frac{240 \times 10^{-6}}{120}\,\text{F} = 2.0\,\mu\text{F}.$$

Substitute this in the initial charge equation:

$$V = \frac{360 \times 10^{-6}\,\text{C}}{2.0 \times 10^{-6}\,\text{F}} = 180\,\text{V}.$$

As a check, the reduced potential is $60\,$V, giving $Q = 2.0 \times 60 = 120\,\mu$C, as stated.

Step 3: Calculate the charge for the increased voltage For part (ii), increase the original voltage by $120\,$V. The new potential difference is

$$V' = 180 + 120 = 300\,\text{V}.$$

Using the same capacitance,

$$Q' = CV' = (2.0 \times 10^{-6})(300)\,\text{C} = 600\,\mu\text{C}.$$

Step 4: Conclusion (i) The original potential is $\boxed{V = 180\,\text{V}}$ and the capacitance is $\boxed{C = 2.0\,\mu\text{F}}$. (ii) On increasing the original voltage by $120\,$V, the charge becomes $\boxed{600\,\mu\text{C}}$.

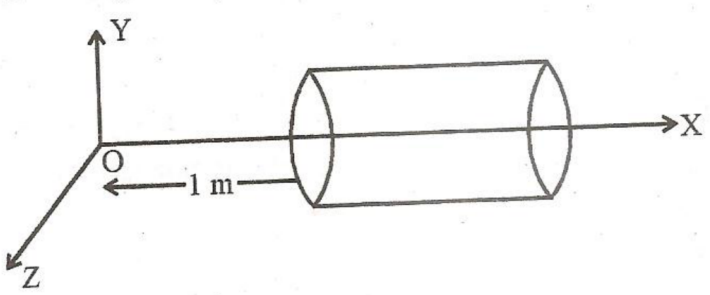
Quick Tip to Solve: First use $C = \Delta Q/\Delta V$ to avoid simultaneous equations. Equal increases and decreases of voltage produce equal changes of charge for the same capacitor.

OR

18. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in the figure. The electric field in the region is given by $\vec{E} = 50\,x\,\hat{i}$, where E is in N C^{-1} and x is in metres. Find

(i) Net flux through the cylinder.

(ii) Charge enclosed by the cylinder. [3 marks]



Solution:

Concept:

- Electric flux through a closed surface is $\Phi = \oint \vec{E} \cdot d\vec{A}$, with each area vector directed outward.
- Gauss's law gives $\Phi = Q_{\text{enc}}/\varepsilon_0$, where Q_{enc} is the net enclosed charge and $\varepsilon_0 = 8.854 \times 10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}$.
- For a cylinder parallel to a field directed along its axis, the curved surface contributes no flux; the two end faces contribute with opposite signs.

Step 1: Identify the end faces and convert the area The diagram places the left face at $x_1 = 1\,\text{m}$. Since the cylinder is 1 m long, the right face is at $x_2 = 2\,\text{m}$. Its cross-sectional area is

$$A = 25\,\text{cm}^2 = 25 \times 10^{-4}\,\text{m}^2 = 2.5 \times 10^{-3}\,\text{m}^2.$$

The field $\vec{E} = 50x\hat{i}$ is along the positive x-axis. It is tangent to the curved surface, so $\vec{E} \cdot d\vec{A} = 0$ there.

Step 2: Add the signed fluxes through the end faces At the left and right faces, respectively,

$$E_1 = 50(1) = 50\,\text{N C}^{-1}, \qquad E_2 = 50(2) = 100\,\text{N C}^{-1}.$$

The outward normal is $-\hat{i}$ on the left face and $+\hat{i}$ on the right face. The inward field flux through the left face must therefore be subtracted:

$$\begin{aligned}\Phi &= -E_1 A + E_2 A \\ &= (100 - 50)(2.5 \times 10^{-3}) \\ &= 0.125\,\text{N m}^2\text{C}^{-1}.\end{aligned}$$

The positive result means that more field flux leaves the cylinder than enters it.

Step 3: Apply Gauss's law to find the enclosed charge Use the net flux through the entire closed

Quick Tip to Solve: For $E_x = kx$, the two end faces give $\Phi = kAL$. Their position cancels; remember to convert cm^2 to m^2 using 10^{-4}.

cylindrical box:

$$\begin{aligned} Q_{\text{enc}} &= \varepsilon_0 \Phi \\ &= (8.854 \times 10^{-12})(0.125)\,\text{C} \\ &= 1.10675 \times 10^{-12}\,\text{C}. \end{aligned}$$

Since ε_0 is positive, the enclosed charge has the same positive sign as the net outward flux.

Step 4: Conclusion (i) The net flux is $\boxed{0.125\,\text{N m}^2\text{C}^{-1}}$. (ii) The enclosed charge is $\boxed{+1.11 \times 10^{-12}\,\text{C}}$, approximately.

Quick Tip to Solve: For $E_x = kx$, the two end faces give $\Phi = kAL$. Their position cancels; remember to convert cm^2 to m^2 using 10^{-4}.

19. (a) In a typical nuclear reaction, e.g.

$$^2_1\text{H} + ^2_1\text{H} \longrightarrow ^3_2\text{He} + n + 3.27\ \text{MeV},$$

although number of nucleons is conserved, yet energy is released. How? Explain.

(b) Show that nuclear density in a given nucleus is independent of mass number A. [3 marks]

Solution:

Concept:

- Nucleon number is conserved in a nuclear reaction, but the sum of the rest masses can decrease. The released energy is $Q = (M_{\text{initial}} - M_{\text{final}})c^2$.
- Nuclear binding energy reduces the mass of a bound system relative to its separated nucleons. Total mass-energy, rather than rest mass alone, is conserved.
- A nucleus of mass number A has approximate mass $M = Am_n$ and radius $R = R_0 A^{1/3}$, where m_n is the mean nucleon mass and R_0 is a constant.

Step 1: Explain the energy released in part (a) Initially, the two deuterium nuclei contain $2 + 2 = 4$ nucleons. Finally, the helium-3 nucleus and the emitted neutron contain $3 + 1 = 4$ nucleons. Thus no nucleon disappears during the reaction. Nevertheless, the total rest mass of the products is less than that of the reactants because the products have greater total binding energy. This decrease in rest mass supplies the released energy:

$$\begin{aligned} \Delta m &= 2m(^2\text{H}) - m(^3\text{He}) - m(n), \\ Q &= \Delta m c^2 = 3.27\,\text{MeV}. \end{aligned}$$

The released energy appears primarily as the kinetic energy of the helium nucleus and neutron. The mass difference is approximately $3.27/931.5 = 0.00351$ u. Conservation of nucleon number is therefore fully compatible with energy release: the number of particles need not determine their bound system's total rest mass.

Step 2: Express the nuclear volume in terms of mass number For part (b), use the empirical radius

Quick Tip to Solve: Distinguish nucleon count from rest mass in energy-release questions. For density, cube $R_0 A^{1/3}$ before substituting; then the factor A cancels directly.

relation $R = R_0 A^{1/3}$ and approximate the nucleus as a sphere. Its volume is

$$V_{\text{nucleus}} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A.$$

Thus nuclear volume increases in direct proportion to the number of nucleons, just as the approximate nuclear mass does.

Step 3: Divide mass by volume and cancel the mass number Using $M \simeq Am_n$, the average nuclear density is

$$\rho = \frac{M}{V_{\text{nucleus}}} = \frac{Am_n}{(4/3)\pi R_0^3 A} = \frac{3m_n}{4\pi R_0^3}.$$

The factor A cancels. Since m_n and R_0 are constants in this nuclear model, the density is approximately the same for nuclei of different mass numbers. Small binding-energy differences do not alter this standard approximation.

Step 4: Conclusion (a) The $3.27\,\text{MeV}$ comes from a decrease in total rest mass, $Q = \Delta mc^2$, while all four nucleons are conserved. (b) $\boxed{\rho = 3m_n/(4\pi R_0^3)}$, which is independent of A.

Quick Tip to Solve: Distinguish nucleon count from rest mass in energy-release questions. For density, cube $R_0 A^{1/3}$ before substituting; then the factor A cancels directly.

20. (a) Why photoelectric effect can not be explained on the basis of wave nature of light? Give reasons.

(b) Write the basic features of photon picture of electromagnetic radiation on which Einstein's photoelectric equation is based. [3 marks]

Solution:

Concept:

- In classical wave theory, light energy is continuously distributed over the wavefront and its intensity determines the energy delivered per unit area per unit time.
- A photon of frequency ν has energy $E = h\nu$ and momentum $p = h\nu/c$, where h is Planck's constant and c is the speed of light.
- Einstein's energy balance is $h\nu = \phi + K_{\max}$, where ϕ is the metal's work function and $K_{\max}$ is the maximum photoelectron kinetic energy.

Step 1: (a) Compare the threshold and energy observations with wave theory Wave theory allows an electron to accumulate energy continuously. It would therefore predict emission at any frequency if sufficiently intense light acted for a sufficiently long time. Experimentally, however, there is a definite threshold frequency ν_0; below it no photoelectrons are emitted, whatever the intensity. Also, wave theory associates greater intensity with greater energy supplied to electrons, suggesting greater electron kinetic energy. Instead, the measured maximum kinetic energy depends on frequency and is independent of intensity at fixed frequency. Increased intensity above threshold principally increases the number of emitted electrons per second.

Step 2: (a) Explain the absence of an observable time delay At low intensity, continuous absorption

Quick Tip to Solve: At fixed frequency, change intensity to change the photon count; change frequency to change each photon's energy and hence the stopping potential.

would require an appreciable time for an electron to collect the minimum escape energy. Photoemission is nevertheless practically instantaneous once light above the threshold frequency falls on the surface, even at very low intensity. The threshold, frequency dependence of kinetic energy and absence of this predicted delay cannot be explained by the classical wave picture.

Step 3: (b) State the photon features and apply energy conservation Electromagnetic radiation consists of discrete photons. For monochromatic light, all photons have the same energy $h\nu$ and momentum $h\nu/c$, irrespective of intensity. Raising intensity at fixed frequency raises the number of photons incident per second, not the energy of each photon. In the photoelectric interaction, one photon transfers its entire energy to one electron in a single event. The electron uses energy ϕ to escape; the largest possible remaining energy becomes its kinetic energy:

$$K_{\max} = h\nu - \phi = h(\nu - \nu_0), \qquad \phi = h\nu_0.$$

Energy and momentum are conserved in photon interactions. The discrete, single-event energy transfer accounts for immediate emission when the photon energy reaches the work function.

Step 4: Conclusion Classical wave theory fails to explain threshold frequency, the frequency dependence of $K_{\max}$, and instantaneous emission. The photon picture, with $E = h\nu$ transferred to one electron, gives Einstein's equation $h\nu = \phi + K_{\max}$.

Quick Tip to Solve: At fixed frequency, change intensity to change the photon count; change frequency to change each photon's energy and hence the stopping potential.

21. A metallic rod of length 'l' is rotated with a frequency ν with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius r, about an axis passing through the centre and perpendicular to the plane of the ring. A constant uniform magnetic field B parallel to the axis is present every where. Using Lorentz force, explain how emf is induced between the centre and the metallic ring and hence obtain the expression for it. [3 marks]

Solution:

Concept:

- A charge q moving with velocity $\mathbf{v}$ in magnetic field $\mathbf{B}$ experiences the magnetic Lorentz force $\mathbf{F}_B = q(\mathbf{v} \times \mathbf{B})$.
- For rigid rotation, the speed at distance x from the axis is $v = \omega x$, where the angular speed is $\omega = 2\pi\nu$ for rotation frequency ν.
- The motional emf along a moving conductor is $\mathcal{E} = \int (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$. The radial rod extends from the centre to the ring, so its length l equals the ring radius r.

Step 1: Explain charge separation along the rotating rod Consider the sense of rotation for which the angular velocity is parallel to $\mathbf{B}$. At any point on the rod, the velocity is tangential while the magnetic field is perpendicular to the plane of rotation. Thus $\mathbf{v} \times \mathbf{B}$ is directed radially outwards. Positive charges would be pushed towards the rim, whereas the free electrons in the metal move towards the centre. The centre therefore becomes negative and the rim positive. The separation produces an electric field directed from rim to centre. In the open-circuit steady state the electric force balances

Quick Tip to Solve: For a rod rotating about one end, integrate $B\omega x\, dx$; using the rim speed for the whole rod incorrectly doubles the emf.

the magnetic force on the mobile charges, establishing a potential difference between the centre and ring. Reversing either the rotation or the magnetic field reverses this polarity.

Step 2: Find the emf of a small radial element Take an element of length dx at distance x from the centre. Since its speed is ωx, the magnetic force per unit charge along it has magnitude $B\omega x$. Its contribution to the emf is therefore

$$d\mathcal{E} = Bv\,dx = B\omega x\,dx.$$

The speed varies from zero at the hinge to ωl at the rim, so the full rod cannot be treated as moving everywhere at the rim speed.

Step 3: Integrate from the centre to the ring Adding the emf contributions of all the radial elements gives

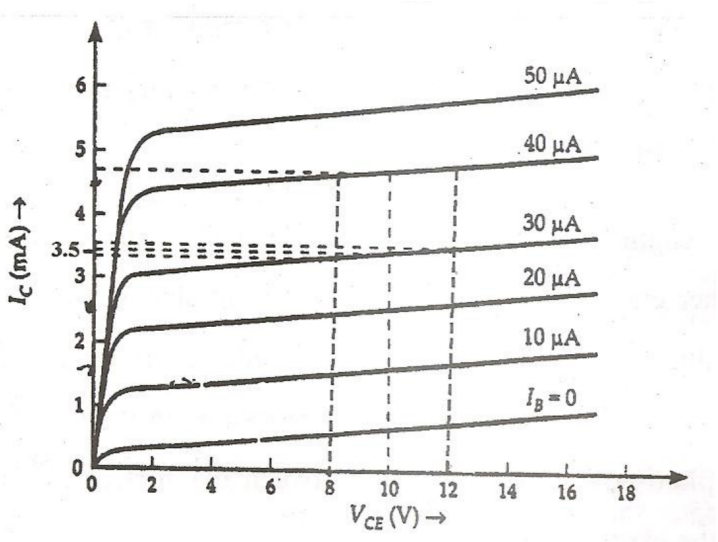
$$|\mathcal{E}| = \int_0^l B\omega x\,dx = \frac{1}{2}B\omega l^2,$$
$$\omega = 2\pi\nu, \qquad |\mathcal{E}| = \pi B\nu l^2.$$

Since the rod joins the centre to the circumference, $l = r$. The same result can consequently be expressed in terms of the ring's radius. The stationary ring transmits the rim potential; it does not add a motional contribution.

Step 4: Conclusion The induced emf has magnitude $\boxed{|\mathcal{E}| = \frac{1}{2}B\omega l^2 = \pi B\nu l^2 = \pi B\nu r^2}$. The ring is positive relative to the centre when $\boldsymbol{\omega}$ and $\mathbf{B}$ are parallel, with the polarity reversed when they are antiparallel.

Quick Tip to Solve: For a rod rotating about one end, integrate $B\omega x\,dx$; using the rim speed for the whole rod incorrectly doubles the emf.

22. Output characteristics of an n-p-n transistor in CE configuration is shown in the figure. Determine:
(i) dynamic output resistance
(ii) dc current gain and
(iii) ac current gain at an operating point $V_{CE} = 10$ V, when $I_B = 30\,\mu$A. [3 marks]



Solution:

Concept:

- Output resistance is $r_o = (\Delta V_{CE}/\Delta I_C)_{I_B}$; use two points on the same base-current curve.
- The dc gain is $\beta_{\text{dc}} = I_C/I_B$, whereas the ac gain is $\beta_{\text{ac}} = (\Delta I_C/\Delta I_B)_{V_{CE}}$. Both gains are dimensionless.

Step 1: (i) Find the dynamic output resistance On the $I_B = 30\,\mu$A curve, the indicated points at $V_{CE} = 8\,$V and $12\,$V give approximately $I_C = 3.4\,$mA and $3.6\,$mA. Thus, at constant base current,

$$r_o = \frac{12 - 8}{(3.6 - 3.4) \times 10^{-3}}\,\Omega = 2.0 \times 10^4\,\Omega.$$

This is the reciprocal slope near the operating point, not the ratio V_{CE}/I_C.

Step 2: (ii) Find the dc current gain At $V_{CE} = 10\,$V, the $I_B = 30\,\mu$A curve gives $I_C \simeq 3.5\,$mA. Converting both currents to amperes,

$$\beta_{\text{dc}} = \frac{3.5 \times 10^{-3}}{30 \times 10^{-6}} \simeq 116.7.$$

Step 3: (iii) Find the ac current gain Keep $V_{CE} = 10\,$V fixed. The neighbouring $I_B = 40\,\mu$A curve gives $I_C \simeq 4.7\,$mA. Hence a base-current change of $10\,\mu$A produces a collector-current change of about $1.2\,$mA:

$$\beta_{\text{ac}} = \frac{(4.7 - 3.5) \times 10^{-3}}{(40 - 30) \times 10^{-6}} \simeq 120.$$

This gain describes current changes about the operating point. All results are approximate because the currents are read from a graph.

Step 4: Conclusion $\boxed{r_o \simeq 20\,\text{k}\Omega}$, $\boxed{\beta_{\text{dc}} \simeq 117}$, and $\boxed{\beta_{\text{ac}} \simeq 120}$.

Quick Tip to Solve: Use one curve for output resistance, one point for dc gain, and neighbouring curves at fixed voltage for ac gain.

23. Using Bohr's postulates, obtain the expression for the total energy of the electron in the stationary states of the hydrogen atom. Hence draw the energy level diagram showing how the line spectra corresponding to Balmer series occur due to transition between energy levels. [3 marks]

Solution:

Concept:

- Bohr's stationary orbits do not radiate. Allowed angular momenta satisfy $mvr = n\hbar$, where m is the electron mass, v its speed and $\hbar = h/(2\pi)$.
- The electrostatic force supplies centripetal acceleration: $mv^2/r = ke^2/r^2$, where $k = 1/(4\pi\varepsilon_0)$ and e is the magnitude of electron charge.
- A transition from a higher energy E_i to a lower energy E_f emits a photon with $h\nu = E_i - E_f$. Balmer emission ends at $n = 2$.

Quick Tip to Solve: Use $U = -2K$ for a Coulomb orbit, so $E = -K$; all Balmer arrows must finish on the $n = 2$ level.

Step 1: Obtain the allowed radii Squaring the angular-momentum condition gives $m^2v^2r^2 = n^2\hbar^2$. From the force equation, $mv^2 = ke^2/r$. Combining these relations gives

$$mke^2r_n = n^2\hbar^2, \qquad r_n = \frac{n^2\hbar^2}{mke^2} = \frac{\varepsilon_0 n^2h^2}{\pi me^2}.$$

Thus the allowed radii are discrete and increase as n^2, with $n = 1, 2, 3, \ldots$.

Step 2: Add kinetic and potential energies Taking the potential energy of an electron and proton infinitely far apart as zero, their attractive Coulomb interaction gives negative potential energy. Hence

$$K_n = \frac{1}{2}mv^2 = \frac{ke^2}{2r_n}, \qquad U_n = -\frac{ke^2}{r_n}.$$

The total energy is therefore

$$\begin{aligned} E_n &= K_n + U_n = -\frac{ke^2}{2r_n} \\ &= -\frac{mk^2e^4}{2\hbar^2n^2} = -\frac{me^4}{8\varepsilon_0^2h^2n^2} = -\frac{13.6}{n^2}\ \text{eV}. \end{aligned}$$

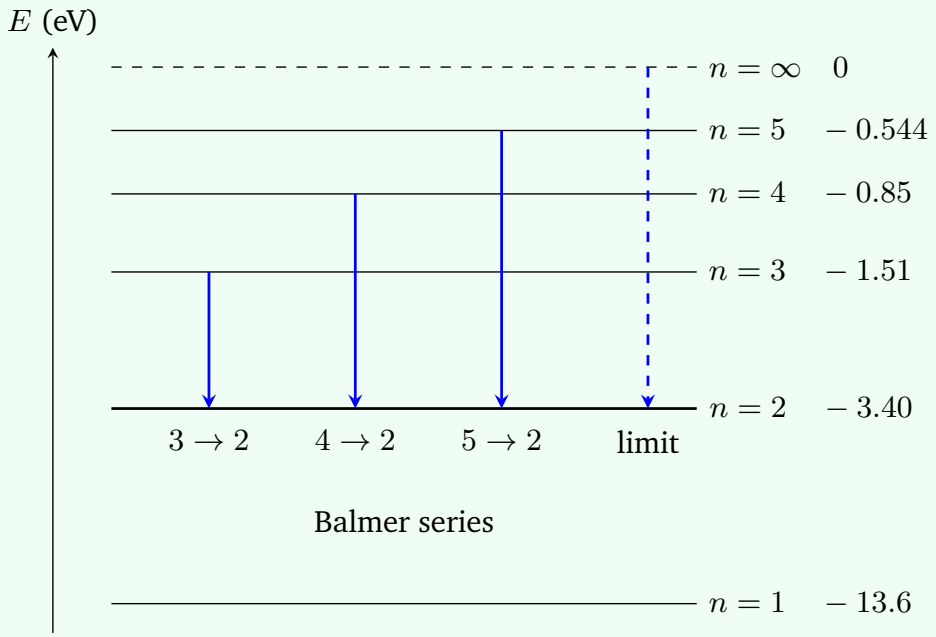
Its negative sign signifies a bound electron; progressively higher stationary states approach the ionisation limit $E = 0$.

Step 3: Identify the Balmer transitions Draw downward transitions from $n = 3, 4, 5, \ldots$ to the common final level $n = 2$. Their different photon energies produce discrete spectral lines:

$$h\nu = 13.6\left(\frac{1}{2^2} - \frac{1}{n^2}\right)\ \text{eV}, \qquad \frac{1}{\lambda} = R_H\left(\frac{1}{4} - \frac{1}{n^2}\right), \quad n \geq 3.$$

The first three lines are approximately 656 nm, 486 nm, and 434 nm. As the initial level increases, the lines converge towards the Balmer limit near 365 nm.

Quick Tip to Solve: Use $U = -2K$ for a Coulomb orbit, so $E = -K$; all Balmer arrows must finish on the $n = 2$ level.



Hydrogen energy levels and downward Balmer transitions (vertical spacings schematic).

Step 4: Conclusion The stationary-state energy is $E_n = -me^4/(8\varepsilon_0^2 h^2 n^2) = -13.6/n^2$ eV. Transitions $n \geq 3 \to 2$ generate the Balmer series.

Quick Tip to Solve: Use $U = -2K$ for a Coulomb orbit, so $E = -K$; all Balmer arrows must finish on the $n = 2$ level.

24. (a) In what way is diffraction from each slit related to the interference pattern in a double slit experiment?

(b) Two wavelengths of sodium light 590 nm and 596 nm are used, in turn, to study the diffraction taking place at a single slit of aperture 2×10^{-4} m. The distance between the slit and the screen is 1.5 m. Calculate the separation between the positions of the first maxima of the diffraction pattern obtained in the two cases. [3 marks]

Solution:

Concept:

- Each finite-width slit diffracts light; interference of the waves from two coherent slits produces fine fringes whose intensities lie under the single-slit diffraction envelope.
- For a slit of width a, diffraction minima satisfy $a \sin\theta = m\lambda$. The first secondary maximum lies approximately midway between the first and second minima, at $a \sin\theta \simeq 3\lambda/2$.
- For small diffraction angles on a screen at distance D, $y \simeq D \sin\theta$, so the first secondary maximum is approximately at $y_1 = 3D\lambda/(2a)$.

Step 1: Relate diffraction to the double-slit pattern (a) Diffraction at each slit spreads its light into the region where the two waves overlap and interfere. Because the slits have finite width, the amplitude reaching the screen varies with direction. Thus the interference bright fringes have unequal intensities:

Quick Tip to Solve: For the first secondary maximum use $3\lambda/2$, not the first-minimum condition λ; calculate the shift directly with $\Delta\lambda$.

they are brightest near the centre and become weaker towards the minima of the broad diffraction envelope. At a diffraction minimum the intensity vanishes, even if the two-slit interference condition predicts a bright fringe there. The slit separation controls the fine fringe spacing, whereas the width of each slit controls the broad envelope.

Step 2: Write the positions for the two wavelengths (b) Here the first maxima mean the first secondary maxima on the same side of the central maximum. The central maximum itself is at $y = 0$ for either wavelength. With

$$a = 2 \times 10^{-4}\,\text{m}, \quad D = 1.5\,\text{m}, \quad \lambda_1 = 590 \times 10^{-9}\,\text{m}, \quad \lambda_2 = 596 \times 10^{-9}\,\text{m},$$

the standard approximate secondary-maximum condition gives

$$y_1 = \frac{3D\lambda_1}{2a} = 6.6375 \times 10^{-3}\,\text{m}, \qquad y_2 = \frac{3D\lambda_2}{2a} = 6.705 \times 10^{-3}\,\text{m}.$$

The larger wavelength places the first secondary maximum slightly farther from the centre.

Step 3: Calculate their separation Subtracting the positions directly avoids rounding two nearby values:

$$\begin{aligned}\Delta y &= \frac{3D}{2a}(\lambda_2 - \lambda_1) \\ &= \frac{3(1.5)(6 \times 10^{-9})}{2(2 \times 10^{-4})} \\ &= 6.75 \times 10^{-5}\,\text{m} = 0.0675\,\text{mm}.\end{aligned}$$

The same separation is obtained on the other side by symmetry. This result uses the usual school-level approximation for secondary-maximum positions.

Step 4: Conclusion (a) Single-slit diffraction supplies the broad intensity envelope of the double-slit interference fringes. (b) The separation of the corresponding first secondary maxima is approximately $6.75 \times 10^{-5}\,\text{m}$, or $0.0675\,\text{mm}$.

Quick Tip to Solve: For the first secondary maximum use $3\lambda/2$, not the first-minimum condition λ; calculate the shift directly with $\Delta\lambda$.

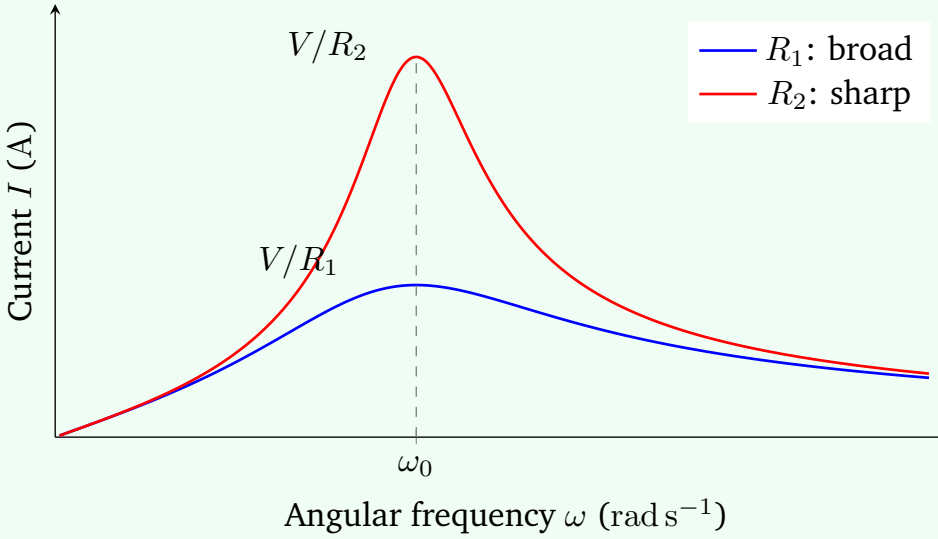
25. In a series LCR circuit connected to an ac source of variable frequency and voltage $v = v_m \sin \omega t$, draw a plot showing the variation of current (I) with angular frequency (ω) for two different values of resistance R_1 and R_2 $(R_1 > R_2)$. Write the condition under which the phenomenon of resonance occurs. For which value of the resistance out of the two curves, a sharper resonance is produced? Define Q-factor of the circuit and give its significance. [3 marks]

Solution:

Concept:

- In a series LCR circuit, the impedance is $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$, where L is inductance, C capacitance and R resistance.
- For fixed rms source voltage $V = v_m/\sqrt{2}$, the rms current is $I = V/Z$. Resonance occurs when the inductive and capacitive reactances cancel.
- The dimensionless quality factor measures the sharpness of resonance: $Q = \omega_0 L/R = \omega_0/\Delta\omega$, where $\Delta\omega$ is the half-power bandwidth.

Step 1: Plot the current response for the two resistances Keep L, C and the source-voltage amplitude unchanged while varying ω. At low frequency the capacitive reactance is large; at high frequency the inductive reactance is large. The current therefore rises to a maximum between these limits and then falls. Both curves peak at the same angular frequency ω_0. Since $R_1 > R_2$, the R_2 curve has the taller, narrower peak, whereas the R_1 curve has the lower, broader peak. The graph is schematic; current may equivalently be plotted as amplitude, with both vertical values scaled by $\sqrt{2}$.



Current response for the same L and C, with $R_1 > R_2$; the smaller resistance gives sharper resonance.

Step 2: State the resonance condition At resonance the net reactance is zero, so the impedance is purely resistive and is minimum:

$$\omega_0 L = \frac{1}{\omega_0 C}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}, \qquad Z_0 = R.$$

Hence the maximum rms current is $I_0 = V/R$, and current is in phase with the applied voltage. Reducing resistance increases this maximum without shifting ω_0.

Quick Tip to Solve: Put both peaks directly above the same ω_0; lower resistance means a taller peak and smaller bandwidth, not a different resonant frequency.

Step 3: Define the Q-factor and explain its significance The Q-factor is 2π times the ratio of energy stored in the circuit to energy dissipated per cycle at resonance. Equivalently, for a series LCR circuit,

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

It is also the ratio of the voltage across the inductor (or capacitor) to the applied voltage at resonance. The half-power bandwidth is $\Delta\omega = R/L$, giving $Q = \omega_0/\Delta\omega$. A larger Q means a narrower range of frequencies gives a large current: the circuit has sharper resonance and better frequency selectivity, as required in tuning circuits. Thus the smaller resistance R_2 has the larger Q-factor.

Step 4: Conclusion Resonance occurs at $X_L = X_C$, or $\omega_0 = 1/\sqrt{LC}$. The R_2 curve is sharper because $R_2 < R_1$. The quality factor $Q = (1/R)\sqrt{L/C}$ quantifies resonance sharpness and frequency selectivity.

Quick Tip to Solve: Put both peaks directly above the same ω_0; lower resistance means a taller peak and smaller bandwidth, not a different resonant frequency.

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26. While travelling back to his residence in the car, Dr. Pathak was caught up in a thunderstorm. It became very dark. He stopped driving the car and waited for thunderstorm to stop. Suddenly he noticed a child walking alone on the road. He asked the boy to come inside the car till the thunderstorm stopped. Dr. Pathak dropped the boy at his residence. The boy insisted that Dr. Pathak should meet his parents. The parents expressed their gratitude to Dr. Pathak for his concern for safety of the child.

Answer the following questions based on the above information:

(a) Why is it safer to sit inside a car during a thunderstorm?

(b) Which two values are displayed by Dr. Pathak in his actions?

(c) Which values are reflected in parents' response to Dr. Pathak?

(d) Give an example of a similar action on your part in the past from everyday life. [4 marks]

Solution:

Concept:

- Electrostatic shielding: the electric field in the charge-free cavity of a closed conductor is zero under electrostatic conditions; the conductor is an equipotential.
- The metal body of a closed car provides a conducting enclosure. During a lightning strike the current is carried mainly around the exterior, protecting occupants inside.
- Responsible use of scientific knowledge includes protecting others from danger; compassion, social responsibility and gratitude are values expressed by the actions in this passage.

Step 1: (a) Explain why the car provides protection A closed car with a metal roof and body acts approximately as a conducting enclosure, often called a Faraday cage. Charges redistribute on its outer surface so that an external electric field does not produce an electrostatic field in its empty interior. In the ideal electrostatic model, $E = 0$ inside the cavity. If lightning strikes, the conducting shell offers a path for the large current around the passenger compartment, rather than through a

Quick Tip to Solve: Link car safety to its conducting metal shell, then connect each named value to a particular action described in the passage.

person sitting inside. This is why remaining inside such a car is safer during a thunderstorm. The protection is due to the metal enclosure, not primarily to the rubber tyres; occupants should avoid contact with exposed metal connected to the exterior.

Step 2: (b) Identify two values shown by Dr. Pathak Dr. Pathak shows **compassion**, because he notices a child in difficulty and offers shelter, and **social responsibility**, because he takes practical steps to protect the child and then brings him safely home. These values are supported by specific actions in the passage, rather than merely by a statement of concern.

Step 3: (c) Identify the values shown by the parents The parents show **gratitude and appreciation** for the help received. Their respectful response recognises Dr. Pathak's concern for their child's safety and acknowledges a considerate act by another member of society.

Step 4: (d) Give a suitable everyday example One possible student response is: "On a rainy day I noticed a younger pupil waiting outside the school without an umbrella. I took the pupil to the supervised waiting area and helped contact the pupil's parents. I waited there until a parent arrived." This example describes a practical action that protects someone vulnerable and shows the same compassion and responsibility as in the passage.

Step 5: Conclusion The conducting metal body of a closed car provides shielding. Dr. Pathak demonstrates compassion and social responsibility; the parents demonstrate gratitude and appreciation. Helping a younger pupil reach supervised shelter and contact a parent is a comparable caring action.

Quick Tip to Solve: Link car safety to its conducting metal shell, then connect each named value to a particular action described in the passage.

27. (a) Draw a ray diagram showing the image formation by a compound microscope. Hence obtain expression for total magnification when the image is formed at infinity.

(b) Distinguish between myopia and hypermetropia. Show diagrammatically how these defects can be corrected. [5 marks]

Solution:

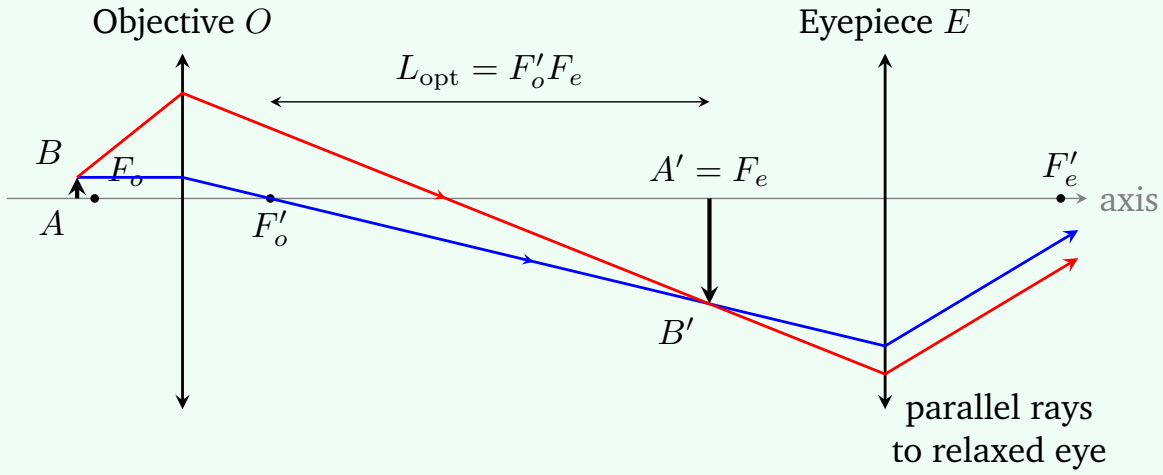
Concept:

- A compound microscope uses a short-focal-length objective to form a real, enlarged intermediate image, and an eyepiece to magnify its angular size.
- Total magnifying power is the product $M = m_o m_e$, where m_o is the objective's linear magnification and m_e is the eyepiece's angular magnification.
- Myopia is corrected by a diverging lens; hypermetropia is corrected by a converging lens. A correcting lens forms a virtual image at a distance that the defective eye can see clearly.

Step 1: (a) Form the intermediate image and obtain parallel emergent rays Place the small object AB just beyond the first focal point F_o of the objective O. The objective forms a real, inverted and enlarged image $A'B'$. For the final image to be at infinity (normal adjustment), place $A'B'$ in the first focal plane of the eyepiece E, so that $A' = F_e$. Rays from B', after refraction through the eyepiece, emerge parallel. The final image is virtual, highly magnified and inverted relative to the original object; the eye views it without accommodation. The two solid ray paths in the diagram pass through

Quick Tip to Solve: For infinity adjustment, place the intermediate image at the eyepiece focus and use D/f_e, without the extra 1 used for a near-point final image.

the same intermediate image point B'.



Compound microscope in normal adjustment: $A' = F_e$, with parallel emergent rays.

Step 2: Derive the magnifying power in normal adjustment Let f_o, f_e be the positive focal lengths of objective and eyepiece, $u_o < 0$ and $v_o > 0$ the objective's signed object and image distances, and $D = 25\,\text{cm}$ the conventional least distance of distinct vision. The objective's linear magnification is

$$m_o = \frac{A'B'}{AB} = \frac{v_o}{u_o}.$$

If the intermediate image has height magnitude h', its angular size through the eyepiece is approximately h'/f_e. Seen directly at distance D, the same height subtends h'/D; hence $m_e = D/f_e$. Therefore

$$M = \frac{v_o}{u_o}\frac{D}{f_e}, \qquad |M| = \frac{v_o}{|u_o|}\frac{D}{f_e}.$$

Since the object is very close to F_o, $|u_o| \simeq f_o$; taking $v_o \simeq L$, where L is the microscope tube length in the usual long-tube approximation, gives $M \simeq -LD/(f_o f_e)$. The negative sign denotes inversion. More precisely, if optical tube length L_{opt} means the separation $F_o' F_e$, then $L_{\text{opt}} = v_o - f_o$. The lens equation gives $m_o = 1 - v_o/f_o = -L_{\text{opt}}/f_o$, so $M = -L_{\text{opt}} D/(f_o f_e)$ within paraxial optics.

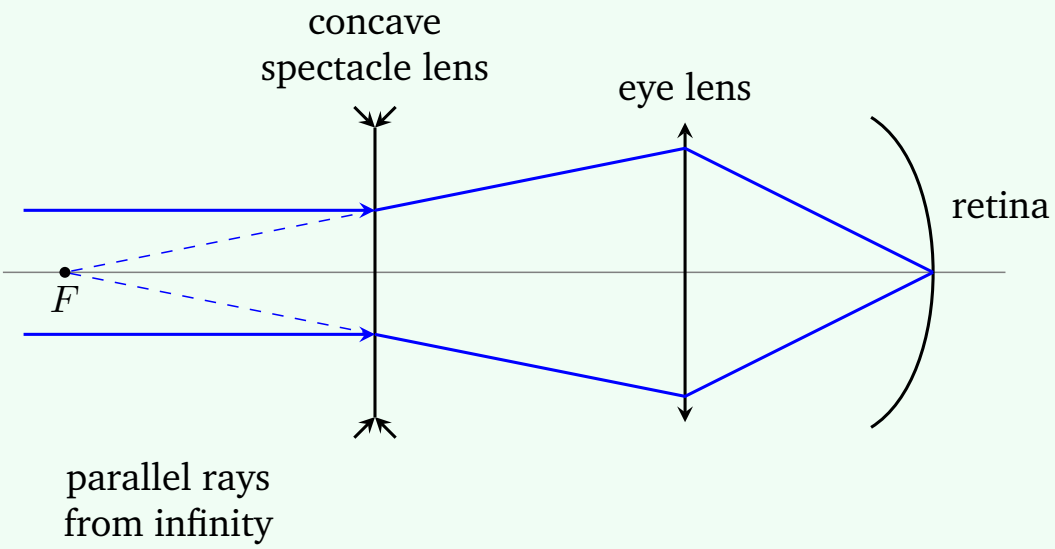
Step 3: (b) Distinguish the two defects **Myopia (short-sightedness):** near objects are seen clearly but distant objects appear blurred. Its far point is at a finite distance instead of infinity. With the eye relaxed, parallel rays from a distant object converge *in front of the retina*. Causes include excessive converging power of the eye lens or an elongated eyeball.

Hypermetropia (long-sightedness): distant objects are comparatively clear but nearby objects are blurred. Its near point lies farther away than the normal $25\,\text{cm}$. Rays from an object at the normal near point would converge *behind the retina*, even when the eye attempts to accommodate sufficiently. Causes include inadequate converging power of the eye lens or a shortened eyeball.

Step 4: Correct myopia with a concave lens A concave spectacle lens diverges parallel incident rays so that they appear to originate from the myopic eye's far point F. The eye then focuses these rays on the retina, as shown. If the far point is distance x from a correcting lens close to the eye, $u = -\infty$, $v = -x$, and the lens formula $1/f = 1/v - 1/u$ gives $f = -x$. Thus the required lens has negative

Quick Tip to Solve: For infinity adjustment, place the intermediate image at the eyepiece focus and use D/f_e, without the extra 1 used for a near-point final image.

focal length and negative power. Dashed lines are backward extensions, not actual light paths.

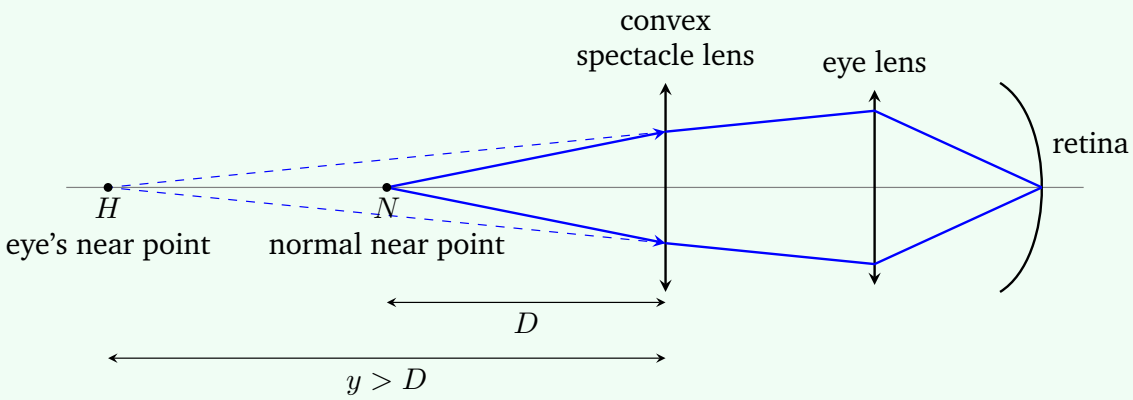


Myopia correction: a concave lens makes distant rays appear to come from the finite far point F.

Step 5: Correct hypermetropia with a convex lens A convex spectacle lens makes rays from an object at the normal near point N, distance D, less divergent. It forms a virtual, erect image at the hypermetropic eye's own near point H, at distance $y > D$. The eye can then focus the rays on its retina. With the lens close to the eye, $u = -D$ and $v = -y$, giving

$$\frac{1}{f} = \frac{1}{D} - \frac{1}{y} > 0.$$

The correcting lens therefore has positive focal length and positive power. The schematic shows solid rays from N reaching the retina and dashed backward extensions meeting at H.



Hypermetropia correction: the convex lens forms a virtual image at H of the object at N; the eye focuses on the retina.

Step 6: Conclusion For a compound microscope with the final image at infinity, the magnifying-power magnitude is $\boxed{|M| \simeq (L/f_o)(D/f_e)}$; the final image is inverted. Myopia requires a **concave** correcting lens, whereas hypermetropia requires a **convex** correcting lens.

Quick Tip to Solve: For infinity adjustment, place the intermediate image at the eyepiece focus and use D/f_e, without the extra 1 used for a near-point final image.

OR

27. (a) State Huygen's principle. Using this principle draw a diagram to show how a plane wave front incident at the interface of the two media gets refracted when it propagates from a rarer to a denser medium. Hence verify Snell's law of refraction.

(b) When monochromatic light travels from a rarer to a denser medium, explain the following, giving reasons:

(i) Is the frequency of reflected and refracted light same as the frequency of incident light?

(ii) Does the decrease in speed imply a reduction in the energy carried by light wave? [5 marks]

Solution:

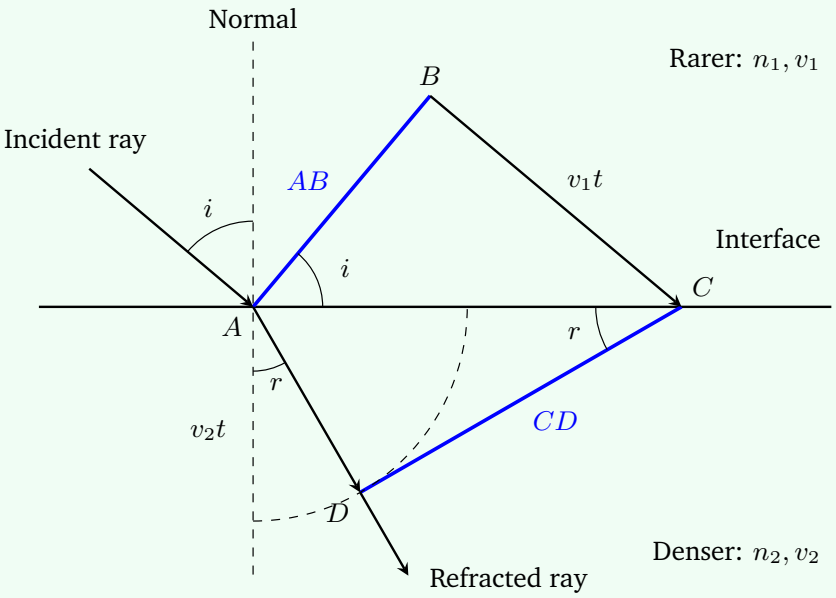
Concept:

- A wavefront is a surface of constant phase. Huygens' principle constructs the next wavefront as the forward envelope of secondary wavelets.
- In a medium of refractive index n, the wave speed is $v = c/n$; a ray is normal to the wavefront.
- For monochromatic light, $v = f\lambda$ and each photon has energy $E = hf$, where f is frequency and h is Planck's constant.

Step 1: (a) State Huygens' principle Every point on a wavefront acts as a source of secondary spherical wavelets that spread with the speed of light in that medium. The forward common tangent surface, or envelope, of these wavelets at a later instant is the new wavefront. For a plane wavefront incident obliquely on a plane interface, different points reach the interface at different times.

Step 2: Construct the refracted wavefront Let AB be the incident plane wavefront, with A just reaching the interface. Medium 1 is rarer and medium 2 is denser, so $v_1 > v_2$. During time t, the disturbance at B reaches C, giving $BC = v_1 t$. In the same time the secondary wavelet from A has radius $AD = v_2 t$ in medium 2. Draw the tangent CD from C to this wavelet. The common envelope CD is the refracted wavefront, and AD is normal to it. The ray bends towards the normal because the disturbance advances a smaller distance in the denser medium.

Quick Tip to Solve: Use the same time interval for $BC = v_1 t$ and $AD = v_2 t$; divide their sine relations to obtain Snell's law. At an interface, keep frequency fixed and change wavelength.



Huygens' construction: incident wavefront AB, refracted wavefront CD, and secondary wavelet of radius $AD = v_2t$.

Step 3: Derive Snell's law from the two right triangles Since $AB \perp BC$ and $AD \perp CD$, triangles ABC and ADC are right-angled. The angle between the incident wavefront AB and the interface AC equals the incidence angle i; similarly, the angle between CD and AC equals the refraction angle r. Hence

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}, \qquad \sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}.$$

Dividing eliminates the same time interval and the common hypotenuse:

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}.$$

Thus $n_1 \sin i = n_2 \sin r$, which verifies Snell's law. For $n_2 > n_1$, the construction gives $r < i$.

Step 4: (b)(i) Compare the frequencies The frequencies of both reflected and refracted light are equal to the incident frequency. The stationary interface is driven by the incident field at frequency f, so the reflected and transmitted fields oscillate at that same frequency. A change of medium changes propagation speed and wavelength, not the source frequency:

$$f_{\text{incident}} = f_{\text{reflected}} = f_{\text{refracted}}, \qquad \frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} < 1.$$

Step 5: (b)(ii) Relate energy to frequency A decrease in speed does not itself imply a decrease in the energy of a photon. The photon energy is $E = hf$, and f remains unchanged on refraction. Thus each transmitted photon has the same energy as an incident photon, although its wavelength in the denser medium is shorter. The transmitted beam can have less total energy than the incident beam because some light is reflected, or absorbed in an absorbing medium; that is not a consequence of the lower propagation speed alone.

Step 6: Conclusion Huygens' construction gives $\sin i / \sin r = n_2/n_1$, with bending towards the normal

Quick Tip to Solve: Use the same time interval for $BC = v_1t$ and $AD = v_2t$; divide their sine relations to obtain Snell's law. At an interface, keep frequency fixed and change wavelength.

in the denser medium. Reflected and refracted light retain the incident frequency, so the decrease in speed does not reduce the photon energy hf.

Quick Tip to Solve: Use the same time interval for $BC = v_1 t$ and $AD = v_2 t$; divide their sine relations to obtain Snell's law. At an interface, keep frequency fixed and change wavelength.

28. (a) State the working principle of a potentiometer. With the help of the circuit diagram, explain how a potentiometer is used to compare the emf's of two primary cells. Obtain the required expression used for comparing the emfs.

(b) Write two possible causes for one sided deflection in a potentiometer experiment. [5 marks]

Solution:

Concept:

- For a uniform wire carrying a steady current at constant temperature, the potential drop across a length l is $V = kl$, where k is the constant potential gradient.
- At the potentiometer balance point, the galvanometer current is zero. The cell supplies no current, so its emf $\mathcal{E}$ equals the balancing potential drop kl, independently of its internal resistance.

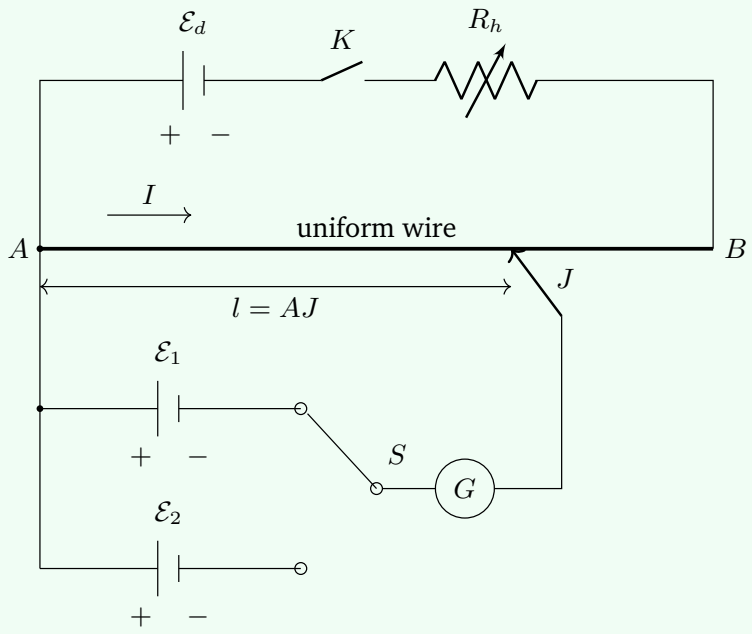
Step 1: (a) State and derive the working principle Let the potentiometer wire have resistivity ρ, uniform cross-sectional area S, and steady current I. The resistance of a length l is $R_l = \rho l/S$. By Ohm's law,

$$V_l = IR_l = \frac{I\rho}{S} l = kl, \qquad k = \frac{I\rho}{S}.$$

Thus the potential difference across any portion of a uniform wire is directly proportional to its length, provided the current and temperature remain constant. This is the working principle of the potentiometer.

Step 2: Connect the comparison circuit Connect the driver battery $\mathcal{E}_d$, key K, rheostat R_h, and uniform wire AB in series. The driver's positive terminal faces A, so the potential decreases from A to B. Connect the positive terminals of both primary cells $\mathcal{E}_1$ and $\mathcal{E}_2$ to A. A two-way selector S connects the negative terminal of either cell to the galvanometer G, whose other terminal goes to the jockey J. Close K and adjust R_h so that the total potential drop V_{AB} exceeds each cell's emf, allowing both balance points to lie on the wire. Select only one cell at a time; keep the rheostat setting and primary current unchanged during comparison.

Quick Tip to Solve: Measure both balance lengths from the common positive end and leave the rheostat unchanged; if no null occurs, check cell polarity and whether V_{AB} exceeds the cell emf.



Potentiometer comparison circuit; selector S connects one cell at a time and J is the movable jockey.

Step 3: Locate the two null points and compare the emfs Select cell $\mathcal{E}_1$ and slide the jockey until the galvanometer shows zero deflection. Measure the balancing length $AJ_1 = l_1$ from the common positive end A. No current flows through the selected cell at balance; hence there is no voltage drop across its internal resistance and

$$\mathcal{E}_1 = V_{AJ_1} = kl_1.$$

Now select cell $\mathcal{E}_2$, keeping the primary circuit unchanged, and find its balancing length $AJ_2 = l_2$. At its null point,

$$\mathcal{E}_2 = kl_2.$$

Dividing the two equations eliminates the same potential gradient k. The method compares the emfs rather than the terminal voltages of current-supplying cells, because each measurement is made at zero secondary current.

Step 4: (b) Explain two causes of one-sided deflection **Reversed cell polarity:** If the positive terminal of the test cell is connected towards the lower-potential end instead of the common positive end A, its emf and the wire's potential drop assist instead of opposing each other in the secondary loop. They cannot cancel, so moving the jockey along the wire does not reverse the deflection.

Insufficient potential drop along the wire: If the test cell's emf exceeds the total drop $V_{AB} = kL$, even the full length L cannot balance it. The quantity $\mathcal{E} - kl$ has the same sign for every accessible jockey position. A weak driver battery or too large a rheostat resistance can produce this condition.

Step 5: Conclusion The potentiometer works on $V \propto l$ for a uniform wire at steady current, and the comparison formula is

$$\boxed{\frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{l_1}{l_2}}.$$

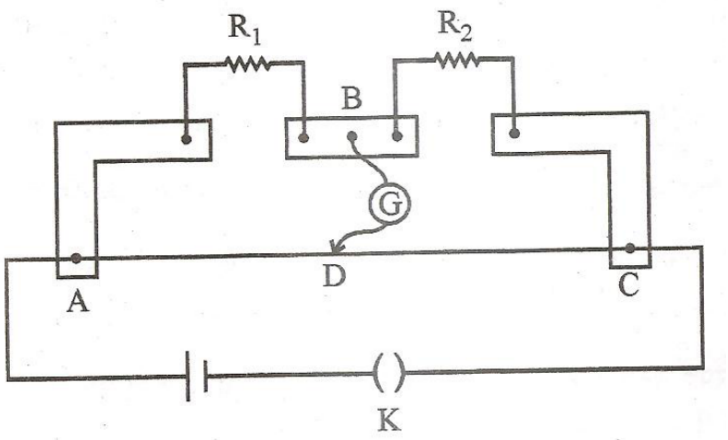
One-sided deflection can result from reversed test-cell polarity or from $V_{AB} < \mathcal{E}$.

Quick Tip to Solve: Measure both balance lengths from the common positive end and leave the rheostat unchanged; if no null occurs, check cell polarity and whether V_{AB} exceeds the cell emf.

OR

28. (a) State Kirchhoff's rules for an electric network. Using Kirchhoff's rules, obtain the balance condition in terms of the resistances of four arms of Wheatstone bridge.

(b) In the meterbridge experimental set up, shown in the figure, the null point 'D' is obtained at a distance of 40 cm from end A of the meterbridge wire. If a resistance of $10\,\Omega$ is connected in series with R_1, null point is obtained at $AD = 60$ cm. Calculate the values of R_1 and R_2. [5 marks]



Solution:

Concept:

- Kirchhoff's junction rule expresses conservation of charge, while the loop rule expresses conservation of energy in a steady electric circuit.
- A Wheatstone bridge balances when the galvanometer current is zero, so its two connecting junctions have equal potentials.
- For a uniform meterbridge wire, resistance is proportional to length. With left and right gap resistances R_1 and R_2, balance at length l centimetres from the left end gives $R_1/R_2 = l/(100-l)$.

Step 1: (a) State Kirchhoff's two rules **Junction rule:** At any junction, the sum of currents entering equals the sum leaving. Equivalently, the algebraic sum of currents at a junction is zero:

$$\sum I = 0.$$

Charge does not accumulate at a junction in a steady circuit. **Loop rule:** Around any closed circuit loop, the algebraic sum of potential changes is zero:

$$\sum \Delta V = 0, \qquad \sum \mathcal{E} - \sum IR = 0.$$

Traversing a resistor along its current gives a change $-IR$, while going against the current gives $+IR$. Across a cell, going from its negative to positive terminal gives $+\mathcal{E}$; reversing this traversal gives $-\mathcal{E}$.

Step 2: Label the four arms of the bridge Let the four successive vertices of a Wheatstone bridge be A, B, C, D, with resistances P, Q, S, R in arms AB, BC, CD, DA, respectively. Connect a cell between A and C and a galvanometer between B and D. At balance, $I_g = 0$ and $V_B = V_D$. By the junction rule, the same current I_1 flows through P and Q, while current I_2 flows through R and S. Take both

Quick Tip to Solve: Put the left gap resistance over the right gap resistance and equate it to $l/(100-l)$. Adding series resistance to the left gap must move the balance point towards the right, as it does here.

branch currents from A towards C.

Step 3: Apply the loop rule and divide Around $A \to B \to D \to A$, there is no galvanometer voltage drop at balance. Kirchhoff's loop rule gives

$$-I_1 P + I_2 R = 0, \qquad I_1 P = I_2 R.$$

Around $B \to C \to D \to B$, the corresponding equation is

$$-I_1 Q + I_2 S = 0, \qquad I_1 Q = I_2 S.$$

Dividing eliminates the branch currents:

$$\frac{P}{Q} = \frac{R}{S}, \qquad PS = QR.$$

This is the balance condition in terms of the four arm resistances.

Step 4: (b) Use the first null-point position In the given meterbridge, R_1 is in the left gap and R_2 in the right gap. The wire length AC is $100\,\text{cm}$. For $AD = 40\,\text{cm}$, the remaining length is $DC = 60\,\text{cm}$. Hence

$$\frac{R_1}{R_2} = \frac{40}{60} = \frac{2}{3}, \qquad R_1 = \frac{2}{3} R_2.$$

The ratio uses lengths measured from the same end as the corresponding gap resistance.

Step 5: Use the second balance and solve simultaneously Adding $10\,\Omega$ in series changes the left resistance to $R_1 + 10\,\Omega$. The new lengths are $AD = 60\,\text{cm}$ and $DC = 40\,\text{cm}$, so

$$\frac{R_1 + 10\,\Omega}{R_2} = \frac{60}{40} = \frac{3}{2}.$$

Substituting the first balance relation gives

$$\begin{aligned} \frac{2}{3} R_2 + 10\,\Omega &= \frac{3}{2} R_2, \\ 10\,\Omega &= \frac{5}{6} R_2, \\ R_2 &= 12\,\Omega, \\ R_1 &= \frac{2}{3}(12\,\Omega) = 8\,\Omega. \end{aligned}$$

As a check, the resistance ratios are $8/12 = 40/60$ initially and $18/12 = 60/40$ after adding the series resistance, consistent with both observations.

Step 6: Conclusion Kirchhoff's rules give $\sum I = 0$ at a junction and $\sum \Delta V = 0$ around a closed loop. The Wheatstone bridge balance condition is $P/Q = R/S$ for the labelled arms. The meterbridge resistances are $\boxed{R_1 = 8\,\Omega}$ and $\boxed{R_2 = 12\,\Omega}$.

Quick Tip to Solve: Put the left gap resistance over the right gap resistance and equate it to $l/(100-l)$. Adding series resistance to the left gap must move the balance point towards the right, as it does here.

29. (a) Derive the expression for the torque on a rectangular current carrying loop suspended in a uniform magnetic field.

(b) A proton and a deuteron having equal momenta enter in a region of uniform magnetic field at right angle to the direction of the field. Depict their trajectories in the field. [5 marks]

Solution:

Concept:

- A straight wire carrying current I in a uniform magnetic field experiences $\mathbf{F} = I\boldsymbol{\ell} \times \mathbf{B}$, where $\boldsymbol{\ell}$ points along the current.
- A current loop has magnetic moment $\boldsymbol{\mu} = IA\hat{\mathbf{n}}$, where A is its area and $\hat{\mathbf{n}}$ is its normal given by the right-hand rule. A couple has torque equal to one force times the perpendicular distance between the force lines.
- For motion perpendicular to a uniform magnetic field, the magnetic force supplies centripetal force: $qvB = mv^2/r$. With momentum $p = mv$, the radius is $r = p/(|q|B)$.

Step 1: (a) Set up the rectangular loop Consider a single rectangular loop of sides ℓ and b, carrying current I, so its area is $A = \ell b$. Let θ be the angle between the loop's positive normal $\hat{\mathbf{n}}$ and the uniform field $\mathbf{B}$. Choose the pair of sides of length ℓ perpendicular to the field; the plane of the loop can be inclined by rotating about an axis parallel to these sides. This geometry represents an arbitrary angle between the normal and the field.

Step 2: (a) Find the force couple and its arm The currents in the two opposite ℓ-sides are opposite. Each side therefore experiences a force of magnitude

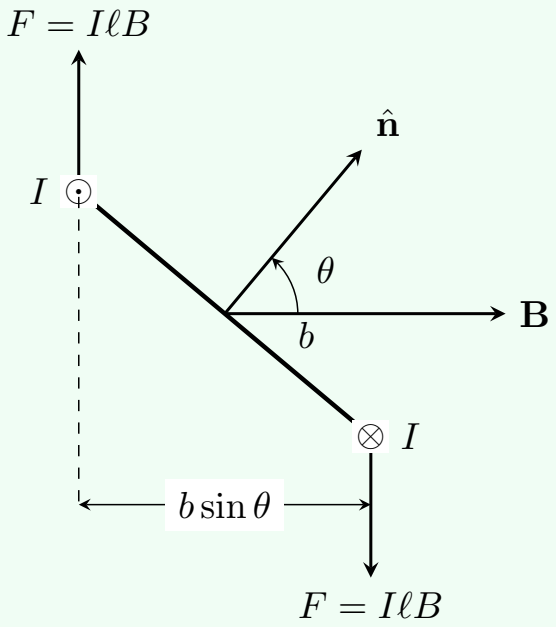
$$F = I\ell B,$$

and the two forces have opposite directions, perpendicular to $\mathbf{B}$ and to the sides. Their resultant force is zero, but their parallel lines of action are separated by the perpendicular distance

$$d = b\sin\theta.$$

The forces on the remaining two sides are equal and opposite and act along the line joining their midpoints, so they give no net torque. Thus only the first force pair contributes to the turning effect. In the edge view below, the two ℓ-sides run perpendicular to the page; dots and crosses denote current out of and into the page.

Quick Tip to Solve: Measure θ from the loop's normal, not its plane. For equal momenta in a magnetic field, compare p/q: the larger deuteron mass alone does not imply a larger radius.



Edge view of the loop: opposite forces form a couple with arm $b\sin\theta$.

Step 3: (a) Obtain the torque and its direction Taking moments about the centre, each force has lever arm $d/2$, and both turn the loop in the same sense. Hence

$$\begin{aligned}\tau &= F\frac{d}{2} + F\frac{d}{2} = Fd \\ &= I\ell B\,(b\sin\theta) \\ &= IAB\sin\theta.\end{aligned}$$

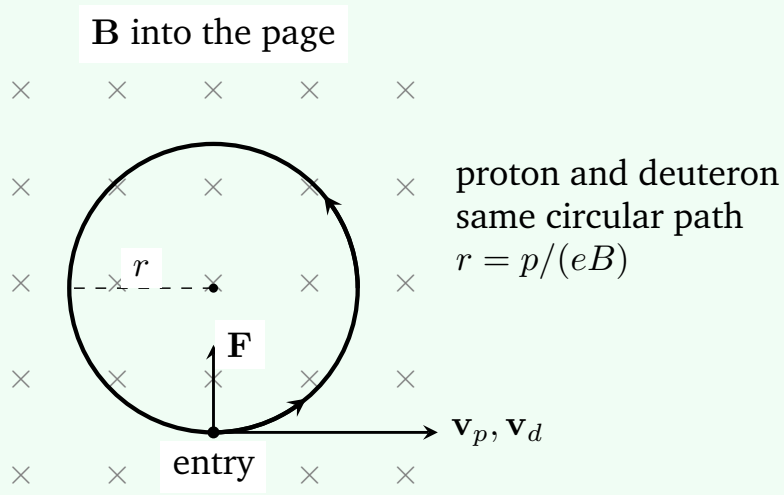
The direction is given by $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$. The torque tends to align the magnetic moment with the field. For N identical turns, the torques add, giving $\tau = NIAB\sin\theta$, with $\boldsymbol{\mu} = NIA\hat{\mathbf{n}}$. It is maximum when the normal is perpendicular to the field and zero when the normal is parallel or antiparallel to it; the parallel orientation is stable.

Step 4: (b) Compare the radii and depict the trajectories A proton and a deuteron both have charge $+e$. Their masses differ, but their momenta have the same magnitude p. Since each enters perpendicular to $\mathbf{B}$, the force is perpendicular to its velocity and does no work. Speed stays constant and each trajectory is a circle in the plane perpendicular to the field. From the centripetal-force equation,

$$evB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{eB} = \frac{p}{eB}.$$

Consequently $r_p = r_d = p/(eB)$. Both bend in the same sense for the same initial direction because both charges are positive. The sketch uses a field into the page and an initial velocity to the right, so the initial force is upward. If they enter at the same point in the same direction, their spatial paths coincide as shown. The deuteron travels more slowly, approximately $v_d = v_p/2$, because its mass is approximately twice the proton's mass; this changes the traversal time, not the radius.

Quick Tip to Solve: Measure θ from the loop's normal, not its plane. For equal momenta in a magnetic field, compare p/q: the larger deuteron mass alone does not imply a larger radius.



Equal-radius paths coincide for the same entry point and initial direction.

Step 5: Conclusion The torque on a single rectangular current loop is $\boxed{\tau = IAB\sin\theta}$, or $\boxed{\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}}$; for N turns its magnitude is $NIAB\sin\theta$. The proton and deuteron describe circular trajectories of equal radius $\boxed{r_p = r_d = p/(eB)}$, bending in the same sense for the same incident direction.

Quick Tip to Solve: Measure θ from the loop's normal, not its plane. For equal momenta in a magnetic field, compare p/q: the larger deuteron mass alone does not imply a larger radius.

OR

29. (a) A small compass needle of magnetic moment 'm' is free to turn about an axis perpendicular to the direction of uniform magnetic field 'B'. The moment of inertia of the needle about the axis is 'I'. The needle is slightly disturbed from its stable position and then released. Prove that it executes simple harmonic motion. Hence deduce the expression for its time period.

(b) A compass needle, free to turn in a vertical plane orients itself with its axis vertical at a certain place on the earth. Find out the values of (i) horizontal component of earth's magnetic field and (ii) angle of dip at the place. [5 marks]

Solution:

Concept:

- A magnetic dipole with magnetic moment $\vec{m}$ in a uniform magnetic field $\vec{B}$ experiences torque $\vec{\tau} = \vec{m} \times \vec{B}$. Its stable orientation has $\vec{m}$ parallel to $\vec{B}$.
- Rotational dynamics gives $\tau = I\ddot{\theta}$, where I is the moment of inertia about the rotation axis and θ is the angular displacement from equilibrium. Angular SHM has the form $\ddot{\theta} = -\omega^2\theta$.
- The angle of dip δ is the angle made by the earth's magnetic field with the horizontal. For field magnitude B_E, its horizontal and vertical components are $B_H = B_E\cos\delta$ and $B_V = B_E\sin\delta$.

Step 1: (a) Establish the restoring torque Initially the needle is at stable equilibrium, with its magnetic moment along the applied field. Let a small displacement θ rotate it away from this direction. The

Quick Tip to Solve: Write the restoring torque with its negative sign before using $\sin\theta \simeq \theta$; compare the coefficient of $-\theta$ with ω^2, not with ω.

field exerts a torque of magnitude $mB\sin\theta$ which turns it back towards equilibrium. Taking the direction of increasing θ as positive, the signed torque is

$$\tau = -mB\sin\theta.$$

The negative sign is essential: the torque opposes the displacement on either side of equilibrium. Since the axis is perpendicular to the field, this is the torque about the allowed rotation axis.

Step 2: Apply the small-angle approximation Using the rotational equation of motion and taking I to be constant gives

$$I\frac{d^2\theta}{dt^2} = -mB\sin\theta.$$

The needle is only slightly disturbed, so $\sin\theta \simeq \theta$, with θ measured in radians. Hence

$$\frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta.$$

Thus the angular acceleration is directly proportional to the angular displacement and is directed towards equilibrium. This is the defining condition for simple harmonic motion, valid for small oscillations when damping is neglected. Large angular displacements would not permit replacing $\sin\theta$ by θ.

Step 3: Deduce the time period Comparing the equation with $\ddot{\theta} = -\omega^2\theta$, identify

$$\omega^2 = \frac{mB}{I}, \qquad \omega = \sqrt{\frac{mB}{I}}.$$

If the needle is released from rest at displacement θ_0 at $t = 0$, its subsequent displacement is $\theta(t) = \theta_0\cos(\omega t)$. One complete oscillation corresponds to a phase change of 2π, so

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mB}}.$$

Increasing the moment of inertia slows the oscillation, whereas increasing the magnetic moment or field strengthens the restoring torque and reduces the period.

Step 4: (b) Determine the earth-field components and dip A freely turning dip needle aligns along the earth's magnetic field in the magnetic meridian. Its vertical equilibrium orientation at the stated place means that the earth's field is vertical there, as at a magnetic pole. Therefore the field makes a right angle with the horizontal. The horizontal component is

$$B_H = B_E\cos 90^\circ = 0,$$

and the magnitude of the angle of dip is 90°. If a signed dip convention is used, downward and upward field directions correspond respectively to $+90^\circ$ and -90°; the question does not specify which pole.

Step 5: Conclusion (a) For small angular displacements, $\ddot{\theta} = -(mB/I)\theta$, so the needle executes SHM with $T = 2\pi\sqrt{I/(mB)}$. (b) The horizontal component of the earth's magnetic field is $B_H = 0$, and the angle of dip has magnitude 90°.

Quick Tip to Solve: Write the restoring torque with its negative sign before using $\sin\theta \simeq \theta$; compare the coefficient of $-\theta$ with ω^2, not with ω.