CBSE Class 12 Mathematics 2014 - Set 65/1 - Compartment

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section A

1. Let $* : R \times R \to R$ given by $(a, b) \to a + 4b^2$ is a binary operation. Compute $(-5) * (2 * 0)$.

2. The elements a_{ij} of a 3×3 matrix are given by $a_{ij} = \frac{1}{2}|-3i + j|$. Write the value of element a_{32}.

3. Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

4. If $\begin{pmatrix} 2x & 4 \end{pmatrix}\begin{pmatrix} x \\ -8 \end{pmatrix} = 0$, find the positive value of x.

5. Write the value of $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$.

6. Find $\int \frac{\sin^6 x}{\cos^8 x}\, dx$.

7. Evaluate: $\int_0^1 \frac{\tan^{-1} x}{1 + x^2}\, dx$.

8. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, find the angle between $\vec{a}$ and $\vec{b}$.

9. Find the angle between x-axis and the vector $\hat{i} + \hat{j} + \hat{k}$.

10. Write the equation of the straight line through the point (α, β, γ) and parallel to z-axis.

Section B

11. Let $f, g : \mathbb{R} \to \mathbb{R}$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, for all $x \in \mathbb{R}$. Then find $f \circ g$ and $g \circ f$.

12(a). Prove that: $\cos^{-1}(x) + \cos^{-1}\left\{\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right\} = \frac{\pi}{3}$.

OR

12(b). Solve for x: $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$.

13. Using properties of determinants, prove the following: $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$.

14. Find the value of the constant k so that the function f, defined below, is continuous at $x = 0$, where $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$.

15. If $y = \tan^{-1}\left(\frac{a}{x}\right) + \log\sqrt{\frac{x-a}{x+a}}$, prove that $\frac{dy}{dx} = \frac{2a^3}{x^4 - a^4}$.

16(a). Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is (a) strictly increasing (b) strictly decreasing.

OR

16(b). The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which the area increases, when the side is 10 cm.

17(a). Show that: $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\, dx = \frac{1}{\sqrt{2}} \log\left(\sqrt{2}+1\right)$.

OR

17(b). Find: $\int \frac{x^3}{x^4 + 3x^2 + 2}\, dx$.

18. Find the particular solution of the differential equation $x\frac{dy}{dx} - y + x\,\text{cosec}\left(\frac{y}{x}\right) = 0$; given that $y = 0$ when $x = 1$.

19. Solve the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given $y\left(\frac{\pi}{2}\right) = 1$.

20. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

21(a). Find the vector $\vec{p}$ which is perpendicular to both $\vec{\alpha} = 4\hat{i} + 5\hat{j} - \hat{k}$ and $\vec{\beta} = \hat{i} - 4\hat{j} + 5\hat{k}$, and $\vec{p} \cdot \vec{q} = 21$, where $\vec{q} = 3\hat{i} + \hat{j} - \hat{k}$.

OR

21(b). Find the unit vector perpendicular to the plane ABC, where the position vectors of A, B and C are $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} + 3\hat{k}$ respectively.

22. A class has 15 students whose ages are $14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19$ and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find the mean of X.

Section C

23. AB is a diameter of a circle and C is any point on the circle. Show that the area of $\triangle ABC$ is maximum, when it is isosceles.

24. Using integration, find the area of the triangle PQR, coordinates of whose vertices are $P(2, 0)$, $Q(4, 5)$ and $R(6, 3)$.

25(a). Find:
$\int \frac{\sqrt{x^2+1}\left(\log(x^2+1) - 2\log x\right)}{x^4}\, dx.$

OR

25(b). Find: $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\, dx,\ x \in [0, 1]$
.

26. Find the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis.

27(a). An urn contains 4 balls. Two balls are drawn at random from the urn (without replacement) and are found to be white. What is the probability that all the four balls in the urn are white?

OR

27(b). In a game, a man wins rupees five for a six and loses rupee one for any other number, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses.

28. Two schools, P and Q, want to award their selected students for the values of sincerity, truthfulness and hard work at the rate of ₹ x, ₹ y and ₹ z for each respective value per student. School P awards its 2, 3 and 4 students on the above respective values with a total prize money of ₹ $4,600$. School Q wants to award its 3, 2 and 3 students on the respective values with a total award money of ₹ $4,100$. If the total amount of award money for one prize on each value is ₹ $1,500$, using matrices find the award money for each value. Suggest one other value which the school can consider for awarding the students.

29. One kind of cake requires 200 g of flour and 25 g of fat, another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it an LPP and solve it graphically.

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