CBSE Class 12 Mathematics 2014 - Set 65/2/1

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section A

1. Let $R = \{(a, a^3) :$ a is a prime number less than $5\}$ be a relation. Find the range of R.

2. Write the value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

3. Use elementary column operations $C_2 \to C_2 - 2C_1$ in the matrix equation $\begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$.

4. If $\begin{pmatrix} a+4 & 3b \\ 8 & -6 \end{pmatrix} = \begin{pmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{pmatrix}$, write the value of $a - 2b$.

5. If A is a 3×3 matrix, $|A| \neq 0$ and $|3A| = k|A|$, then write the value of k.

6. Evaluate: $\int \frac{dx}{\sin^2 x \cos^2 x}$.

7. Evaluate: $\int_0^{\pi/4} \tan x \, dx$.

8. Write the projection of vector $\hat{i} + \hat{j} + \hat{k}$ along the vector $\hat{j}$.

9. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units.

10. Find the angle between the lines $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$.

Section B

11. Let $f : W \to W$, be defined as $f(x) = x - 1$, if x is odd and $f(x) = x + 1$, if x is even, where W is the set of all whole numbers. Show that f is invertible. Find the inverse of f.

12(a). Solve for x: $\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$.

OR

12(b). Prove that: $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$.

13. Using properties of determinants, prove that $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a + x + y + z)$.

14. If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

15. If $x^m y^n = (x + y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

16(a). Find the approximate value of $f(3.02)$, upto 2 places of decimal, where $f(x) = 3x^2 + 5x + 3$.

OR

16(b). Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is (a) strictly increasing, (b) strictly decreasing.

17(a). Evaluate: $\int \frac{x\cos^{-1} x}{\sqrt{1 - x^2}}\,dx$.

OR

17(b). Evaluate: $\int (3x - 2)\sqrt{x^2 + x + 1}\,dx$.

18. Solve the differential equation $(x^2 - yx^2)\,dy + (y^2 + x^2y^2)\,dx = 0$, given that $y = 1$, when $x = 1$.

19. Solve the differential equation $\frac{dy}{dx} + y\cot x = 2\cos x$, given that $y = 0$ when $x = \frac{\pi}{2}$.

20(a). Show that the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if and only if $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are coplanar.

OR

20(b). Find a unit vector perpendicular to both of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

21. Find the shortest distance between the lines whose vector equations are $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$.

22. Three cards are drawn at random (without replacement) from a well shuffled pack of 52 playing cards. Find the probability distribution of number of red cards. Hence find the mean of the distribution.

Section C

23. Two schools P and Q want to award their selected students on the values of Tolerance, Kindness and Leadership. The school P wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with a total award money of ₹ 2,200. School Q wants to spend ₹ 3,100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is ₹ 1,200, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.

24. Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of its base.

25. Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$.

26. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

27(a). Find the equation of the plane that contains the point $(1, -1, 2)$ and is perpendicular to both the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$. Hence find the distance of point $P(-2, 5, 5)$ from the plane obtained above.

OR

27(b). Find the distance of the point $P(-1, -5, -10)$ from the point of intersection of the line joining the points $A(2, -1, 2)$ and $B(5, 3, 4)$ with the plane $x - y + z = 5$.

28. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on the grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is ₹ 25 and that from a shade is ₹ 15. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit. Formulate an LPP and solve it graphically.

29(a). An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident for them are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver or a car driver?

OR

29(b). Five cards are drawn one by one, with replacement, from a well shuffled deck of 52 cards. Find the probability that (i) all the five cards are diamonds, (ii) only 3 cards are diamonds, (iii) none is a diamond.

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