CBSE Class 12 Mathematics 2014 - Set 65/1/2 - Compartment

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section A

1. If $\vec{a}$ and $\vec{b}$ are unit vectors, then find the angle between $\vec{a}$ and $\vec{b}$, given that $(\sqrt{3}\,\vec{a} - \vec{b})$ is a unit vector.

2. Find:

$$\int \frac{1}{\sin^2 x \cdot \cos^2 x}\, dx$$

3. Write the value of the determinant $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$.

4. Write the derivative of $\sin x$ w.r.t. $\cos x$.

5. Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) :$ 2 divides $(a - b)\}$. Write the equivalence class $[0]$.

6. Write the projection of the vector $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$.

7. Write the principal value of $\cos^{-1}[\cos(680^\circ)]$.

8. If $\begin{bmatrix} x \cdot y & 4 \\ z + 6 & x + y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, write the value of $(x + y + z)$.

9. Write the distance of a point $P(a, b, c)$ from x-axis.

10. Write a 2×2 matrix which is both symmetric and skew-symmetric.

Section B

11. Let R be a relation defined on the set of natural numbers N as follows:

$R = \{(x, y) : x \in N, y \in N \text{ and } 2x + y = 24\}$

Find the domain and range of the relation R. Also, find if R is an equivalence relation or not.

12. If $(x - y) \cdot e^{\frac{x}{x-y}} = a$, prove that $y\dfrac{dy}{dx} + x = 2y$.

13(a). The sum of the perimeters of a circle and a square is k, where k is some constant. Prove that the sum of their areas is least when the side of the square is double the radius of the circle.

OR

13(b). Using differentials, find the approximate value of $(3.968)^{3/2}$.

14. If $x = a\left(\cos t + \log \tan \dfrac{t}{2}\right)$, $y = a \sin t$, evaluate $\dfrac{d^2y}{dx^2}$ at $t = \dfrac{\pi}{3}$.

15(a). Evaluate:

$$\int_0^{\pi/2} x^2 \sin x \, dx$$

OR

15(b). Find:

$$\int (x + 3)\sqrt{3 - 4x - x^2}\, dx$$

16. Find the general solution of the differential equation $(x - y)\dfrac{dy}{dx} = x + 2y$.

17. If $y(x)$ is a solution of the differential equation $\left(\dfrac{2 + \sin x}{1 + y}\right)\dfrac{dy}{dx} = -\cos x$ and $y(0) = 1$, then find the value of $y\left(\dfrac{\pi}{2}\right)$.

18. A plane makes intercepts -6, 3, 4 respectively on the coordinate axes. Find the length of the perpendicular from the origin on it.

19. Using properties of determinants, prove the following:

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a - b)(b - c)(c - a)(bc + ca + ab)$$

20(a). If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = 2\hat{j} - \hat{k}$ are three vectors, find the area of the parallelogram having diagonals $(\vec{a} + \vec{b})$ and $(\vec{b} + \vec{c})$.

OR

20(b). If the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar, prove that the vectors $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are also coplanar.

21(a). Show that:

$$\cot^{-1}\left(\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right) = \frac{x}{2},\ x \in \left(0, \frac{\pi}{4}\right)$$

OR

21(b). Solve for x:

$2\tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

22. A couple has 2 children. Find the probability that both are boys, if it is known that

(i) one of them is a boy,

(ii) the older child is a boy.

Section C

23. Prove that the function f defined by $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in $(-1, 1)$. Hence, find the intervals in which $f(x)$ is

(i) strictly increasing,

(ii) strictly decreasing.

24(a). Consider the experiment of tossing a coin. If the coin shows head, toss it again, but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'.

OR

24(b). A man is known to speak the truth 3 out of 5 times. He throws a die and reports that it is '1'. Find the probability that it is actually 1.

25. Using integration, find the area of the region bounded by the curves:

$y = |x + 1| + 1$, $x = -3$, $x = 3$, $y = 0$

26. Find the vector and cartesian forms of the equation of the plane passing through the point $(1, 2, -4)$ and parallel to the lines

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and

$\vec{r} = \hat{i} - 3\hat{j} + 5\hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$.

Also, find the distance of the point $(9, -8, -10)$ from the plane thus obtained.

27(a). Evaluate $\int_1^3 (3x^2 + 1)\, dx$ by the method of limit of sums.

OR

27(b). Find:

$$\int \frac{x^2 + x + 1}{(x + 1)^2(x + 2)}\, dx$$

28. A housewife wishes to mix together two kinds of food, X and Y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg of food are given below:

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	1

One kg of food X costs ₹6 and one kg of food Y costs ₹10. Formulate the above problem as a linear programming problem and find the least cost of the mixture which will produce the diet graphically. What value will you like to attach to this problem?

29. A total amount of ₹7,000 is deposited in three different savings bank accounts, with annual interest rates of 5%, 8% and $8\frac{1}{2}$% respectively. The total annual interest from these three accounts is ₹550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices.

- End of Question Paper -