CBSE Class 12 Mathematics 2014 - Set 65/1/3 - Compartment

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section A

1. Write the value of cosine of the angle which the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ makes with y-axis.

2. If $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$, find $(\text{adj } A)$.

3. Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) :$ 2 divides $(a - b)\}$. Write the equivalence class $[0]$.

4. Write the value of the determinant $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$.

5. Write the distance of a point $P(a, b, c)$ from x-axis.

6. Write a vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 units.

7. Write the derivative of $\sin x$ w.r.t. $\cos x$.

8. Find:

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\, dx$$

9. Write the principal value of $\cos^{-1}[\cos(680^\circ)]$.

10. If $\begin{bmatrix} x \cdot y & 4 \\ z + 6 & x + y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, write the value of $(x + y + z)$.

Section B

11. Let $A = R - \{3\}$, $B = R - \{1\}$. Let $f : A \to B$ be defined by $f(x) = \left(\frac{x-2}{x-3}\right)$, for all $x \in A$. Then show that f is bijective. Hence find $f^{-1}(x)$.

12. A bag contains 3 red and 7 black balls. Two balls are selected at random one-by-one (without replacement). If the second selected ball happens to be red, what is the probability that the first selected ball is also red?

13(a). If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = 2\hat{j} - \hat{k}$ are three vectors, find the area of the parallelogram having diagonals $(\vec{a} + \vec{b})$ and $(\vec{b} + \vec{c})$.

OR

13(b). If the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar, prove that the vectors $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are also coplanar.

14(a). Show that:

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2},\ x \in \left(0, \frac{\pi}{4}\right)$$

OR

14(b). Solve for x:

$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

15. Using the properties of determinants, prove that

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$$

16. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

17. If $(x - y) \cdot e^{\frac{x}{x-y}} = a$, prove that $y\frac{dy}{dx} + x = 2y$.

18. Find the general solution of the differential equation $(x - y)\frac{dy}{dx} = x + 2y$.

19(a). Find:

$$\int \frac{5x - 2}{1 + 2x + 3x^2}\, dx$$

OR

19(b). Evaluate:

$$\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16\sin 2x}\, dx$$

20(a). The sum of the perimeters of a circle and a square is k, where k is some constant. Prove that the sum of their areas is least when the side of the square is double the radius of the circle.

OR

20(b). Using differentials, find the approximate value of $(3.968)^{3/2}$.

21. If $y(x)$ is a solution of the differential equation $\left(\frac{2 + \sin x}{1 + y}\right)\frac{dy}{dx} = -\cos x$ and $y(0) = 1$, then find the value of $y\left(\frac{\pi}{2}\right)$.

22. Find the shortest distance between the two lines whose vector equations are

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$; and

$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

Section C

23. Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

24. A total amount of ₹7,000 is deposited in three different savings bank accounts, with annual interest rates of 5%, 8% and $8\frac{1}{2}\%$ respectively. The total annual interest from these three accounts is ₹550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices.

25. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

(i) parallel to the line $2x - y + 9 = 0$,

(ii) perpendicular to the line $5y - 15x = 13$.

26(a). Find:

$$\int \frac{x^2}{(x^2+1)(x^2+4)}\,dx$$

OR

26(b). Evaluate:

$$\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\,dx$$

27. Find the vector and cartesian forms of the equation of the plane passing through the point $(1, 2, -4)$ and parallel to the lines

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and

$\vec{r} = \hat{i} - 3\hat{j} + 5\hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$.

Also, find the distance of the point $(9, -8, -10)$ from the plane thus obtained.

28. If a young man rides his motor-cycle at 25 km per hour, he has to spend ₹2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km per hour, the petrol cost increases to ₹5 per km and the rate of pollution also increases. He has ₹100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this problem as a linear programming problem. Solve it graphically to find the distance to be covered with different speeds. What value is indicated in this question?

29(a). Consider the experiment of tossing a coin. If the coin shows head, toss it again, but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'.

OR

29(b). A man is known to speak the truth 3 out of 5 times. He throws a die and reports that it is '1'. Find the probability that it is actually 1.

- End of Question Paper -