CBSE Class 12 Physics Question Paper 2015 with Solution PDF-Set 1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 70**

General Instructions

(i) This question paper contains 26 questions. All questions are compulsory.

(ii) There is no overall choice. An internal choice is provided in 5 questions (Q9, Q14, Q24, Q25, Q26); attempt only one of the alternatives in such questions.

(iii) Marks for each question are indicated against it.

1. Define capacitor reactance. Write its S.I. units. [1 mark]

Solution:

Concept:

- Capacitive reactance measures the opposition offered by a capacitor to alternating current.
- For capacitance C and angular frequency $\omega = 2\pi f$, the capacitive reactance is $X_C = 1/(\omega C)$.

Step 1: Define the opposition to alternating current For an ideal capacitor connected to a sinusoidal supply, the ratio of voltage amplitude to current amplitude is called its capacitive reactance:

$$X_C = \frac{V_0}{I_0} = \frac{1}{2\pi f C}.$$

Here f is the supply frequency and C is the capacitance. This opposition decreases as frequency increases.

Step 2: Identify the unit Since reactance is a ratio of voltage to current, its SI unit is volt per ampere, which is the ohm:

$$[X_C] = \frac{\text{V}}{\text{A}} = \Omega.$$

Reactance describes opposition to AC without the energy dissipation associated with an ideal resistance.

Step 3: Conclusion Capacitive reactance is the opposition offered by a capacitor to alternating current, $X_C = 1/(2\pi f C)$. Its SI unit is the ohm (Ω).

Quick Tip to Solve: Remember that capacitive reactance is inversely proportional to both frequency and capacitance; its unit follows directly from voltage divided by current.

2. What is the electric flux through a cube of side 1 cm which encloses an electric dipole? [1 mark]

Solution:

Concept:

- Gauss's law gives the net outward electric flux through a closed surface: $\Phi_E = Q_{\text{enclosed}}/\varepsilon_0$.
- An electric dipole consists of equal and opposite charges $+q$ and $-q$, so its total charge is zero.

Step 1: Calculate the enclosed charge The cube encloses the electric dipole, hence both dipole charges lie inside its closed surface. Their algebraic sum is

$$Q_{\text{enclosed}} = (+q) + (-q) = 0.$$

The dipole has a nonzero electric field, but that does not imply a nonzero net flux.

Step 2: Apply Gauss's law to the whole cube For all six faces taken together,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{0}{\varepsilon_0}.$$

Flux entering the cube cancels the flux leaving it. The side length 1 cm does not affect this result,

Quick Tip to Solve: For any closed surface enclosing a complete dipole, add its two charges before using Gauss's law; the size and shape of the surface are irrelevant.

because Gauss's law depends on enclosed charge.

Step 3: Conclusion The net electric flux through the cube is $\boxed{\Phi_E = 0\,\text{N m}^2\text{C}^{-1}}$.

Quick Tip to Solve: For any closed surface enclosing a complete dipole, add its two charges before using Gauss's law; the size and shape of the surface are irrelevant.

3. A concave lens of refractive index 1.5 is immersed in a medium of refractive index 1.65. What is the nature of the lens? [1 mark]

Solution:

Concept:

- The power of a thin lens in a medium is $P = (n_\ell/n_m - 1)(1/R_1 - 1/R_2)$, where n_ℓ and n_m are the refractive indices of the lens and surrounding medium.
- A concave lens has a negative curvature factor $(1/R_1 - 1/R_2)$ with the usual Cartesian sign convention.

Step 1: Compare the refractive indices The relative refractive index of the lens is

$$n_{\text{rel}} = \frac{1.5}{1.65} = \frac{10}{11} < 1.$$

Consequently, $n_{\text{rel}} - 1 = -1/11$. The lens material is optically rarer than the surrounding medium.

Step 2: Determine the sign of the power The curvature factor for the concave lens is negative. Multiplying it by the negative factor $(n_{\text{rel}} - 1)$ gives positive power, so the focal length in this medium is positive. Thus parallel incident rays are brought towards a real focus, reversing its behaviour in air.

Step 3: Conclusion The concave lens behaves as a **converging lens** in the medium of refractive index 1.65.

Quick Tip to Solve: Compare the refractive index of the lens with its surroundings first: when the medium has the larger index, the usual converging or diverging behaviour reverses.

4. How are side bands produced? [1 mark]

Solution:

Concept:

- Amplitude modulation combines a high-frequency carrier with a lower-frequency information signal by varying the carrier amplitude.
- For carrier frequency f_c and a single modulating frequency f_m, the sideband frequencies are $f_c + f_m$ and $f_c - f_m$.

Step 1: Express the modulated wave If the carrier is $A_c \cos \omega_c t$, sinusoidal amplitude modulation gives

$$s(t) = A_c[1 + \mu \cos \omega_m t] \cos \omega_c t,$$

Quick Tip to Solve: For single-tone amplitude modulation, add and subtract the signal frequency from the carrier frequency to locate the two sidebands.

where μ is the modulation index and ω_m is the angular frequency of the information signal.

Step 2: Identify the new frequency components Using the product-to-sum identity,

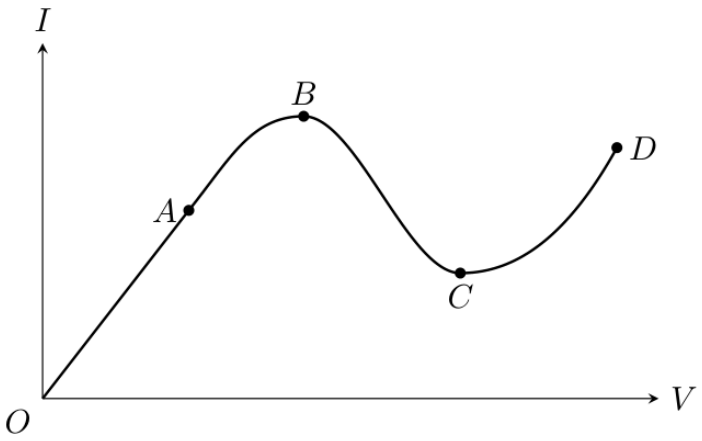
$$\begin{aligned} s(t) &= A_c \cos \omega_c t \\ &+ \frac{\mu A_c}{2} \cos(\omega_c + \omega_m)t \\ &+ \frac{\mu A_c}{2} \cos(\omega_c - \omega_m)t. \end{aligned}$$

The modulation term therefore produces components on either side of the carrier frequency; they carry the information.

Step 3: Conclusion Sidebands are produced by modulation of the carrier with the information signal. In amplitude modulation they occur at $f_c + f_m$ (upper sideband) and $f_c - f_m$ (lower sideband).

Quick Tip to Solve: For single-tone amplitude modulation, add and subtract the signal frequency from the carrier frequency to locate the two sidebands.

5. Graph showing the variation of current versus voltage for a material GaAs is shown in the figure. Identify the region of (i) negative resistance (ii) where Ohm's law is obeyed. [1 mark]



Solution:

Concept:

- On an I-versus-V graph, negative differential resistance means $dI/dV < 0$, or equivalently $dV/dI < 0$.
- Ohm's law is obeyed when $I \propto V$: the graph is a straight line through the origin with constant positive slope.

Step 1: Locate the negative-resistance region Between B and C, the plotted current decreases while the applied voltage increases. The tangent has negative slope, $dI/dV < 0$. This is the negative differential resistance region. It is the slope, rather than the positive ratio V/I, that identifies this behaviour.

Quick Tip to Solve: On an I–V characteristic, look for a descending segment for negative differential resistance and a straight segment through the origin for Ohm's law.

Step 2: Locate the ohmic region The segment OA is a straight line through the origin. Its slope I/V is constant, so the resistance $R = V/I$ is constant there and current is directly proportional to voltage. The curved rising portions do not obey Ohm's law merely because their slope is positive.

Step 3: Conclusion (i) Negative differential resistance occurs in region BC. (ii) Ohm's law is obeyed in region OA.

Quick Tip to Solve: On an I–V characteristic, look for a descending segment for negative differential resistance and a straight segment through the origin for Ohm's law.

6. A proton and an α-particle have the same de-Broglie wavelength. Determine the ratio of (i) their accelerating potentials (ii) their speeds. [2 marks]

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$; equal wavelengths imply equal momentum p.
- For a particle accelerated from rest through potential V, $qV = p^2/(2m)$. A proton has mass m_p and charge e; an alpha particle has approximately mass $4m_p$ and charge $2e$.

Step 1: Relate the momenta Since $\lambda_p = \lambda_\alpha$,

$$\frac{h}{p_p} = \frac{h}{p_\alpha} \quad \Rightarrow \quad p_p = p_\alpha = p.$$

Thus the comparison must be made at equal momentum, rather than at equal kinetic energy or equal accelerating potential.

Step 2: Find the ratio of accelerating potentials For nonrelativistic particles accelerated from rest,

$$V = \frac{p^2}{2mq}.$$

Therefore

$$\frac{V_p}{V_\alpha} = \frac{m_\alpha q_\alpha}{m_p q_p} = \frac{(4m_p)(2e)}{m_p e} = 8.$$

Both the larger mass and the double charge of the alpha particle reduce the potential needed to reach the same momentum.

Step 3: Find the ratio of speeds Using $p = mv$ for the two particles,

$$m_p v_p = m_\alpha v_\alpha \quad \Rightarrow \quad \frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = 4.$$

A heavier particle moves more slowly at the same momentum. The charge enters the potential comparison, but it does not enter this speed ratio.

Step 4: Conclusion (i) The accelerating potentials are in the ratio $\boxed{V_p : V_\alpha = 8 : 1}$. (ii) Their speeds are in the ratio $\boxed{v_p : v_\alpha = 4 : 1}$.

Quick Tip to Solve: Equal de Broglie wavelengths mean equal momentum: use $V \propto 1/(mq)$ for potentials and $v \propto 1/m$ for speeds.

7. Show that the radius of the orbit in hydrogen atom varies as n^2, where n is the principal quantum number of the atom. [2 marks]

Solution:

Concept:

- In Bohr's model, electrostatic attraction supplies the centripetal force for the electron's circular orbit. Here m, e, v and r are the electron's mass, charge magnitude, speed and orbit radius.
- Angular momentum is quantised: $mvr = n\hbar = nh/(2\pi)$, where $n = 1, 2, 3, \ldots$ is the principal quantum number.

Step 1: Write the force equation For hydrogen the nucleus has charge $+e$. Equating the magnitudes of electrostatic and centripetal forces gives

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}, \qquad mv^2 r = \frac{e^2}{4\pi\varepsilon_0}.$$

The radius is measured from the nucleus, and ε_0 is the permittivity of free space.

Step 2: Eliminate the orbital speed From angular-momentum quantisation, $v = n\hbar/(mr)$. Substitute this into the force equation:

$$\frac{mn^2\hbar^2 r}{m^2 r^2} = \frac{e^2}{4\pi\varepsilon_0}, \qquad r_n = \frac{4\pi\varepsilon_0\hbar^2}{me^2}\, n^2 = \frac{\varepsilon_0 h^2}{\pi m e^2}\, n^2.$$

The coefficient contains only constants for hydrogen and is independent of n.

Step 3: Conclusion Hence $r_n = a_0 n^2$, where $a_0 = 4\pi\varepsilon_0\hbar^2/(me^2)$ is the Bohr radius. Therefore the orbit radius varies as n^2; the second orbit has four times the first-orbit radius.

Quick Tip to Solve: Eliminate speed using angular-momentum quantisation; the squared factor of n then directly gives the radius dependence.

8. Distinguish between 'intrinsic' and 'extrinsic' semiconductors. [2 marks]

Solution:

Concept:

- A semiconductor conducts through two mobile charge carriers: electrons in the conduction band and holes in the valence band. Their number densities are denoted by n_e and n_h.
- Doping means adding a small, controlled amount of suitable impurity to a pure semiconductor. It changes carrier concentrations without changing the semiconductor's basic crystal structure.

Step 1: Compare composition and the origin of carriers An intrinsic semiconductor is chemically pure silicon or germanium, with no intentional doping. Its mobile carriers arise through thermal generation of electron-hole pairs. An extrinsic semiconductor is silicon or germanium doped with a suitable impurity. A pentavalent donor produces an n-type material, whereas a trivalent acceptor produces a p-type material; the impurities supply additional mobile carriers.

Step 2: Compare carrier concentrations and conductivity In an intrinsic semiconductor, each thermally

Quick Tip to Solve: Compare purity first and carrier concentrations second: pure means equal electron-hole densities; doping creates a majority carrier type.

generated electron leaves one hole, so $n_e = n_h = n_i$, where n_i is the intrinsic carrier concentration. Both types of carriers contribute to conduction. In an extrinsic semiconductor under normal operating conditions, their concentrations are unequal: electrons are majority carriers in n-type material ($n_e > n_h$), and holes are majority carriers in p-type material ($n_h > n_e$). Doping generally increases conductivity compared with the corresponding pure material at the same temperature, because it increases the concentration of mobile majority carriers.

Step 3: Conclusion Intrinsic semiconductors are pure and have equal electron and hole concentrations. Extrinsic semiconductors are doped, have majority and minority carriers, and generally have higher conductivity than the pure semiconductor at the same temperature.

Quick Tip to Solve: Compare purity first and carrier concentrations second: pure means equal electron-hole densities; doping creates a majority carrier type.

9. Use the mirror equation to show that an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$. [2 marks]

Solution:

Concept:

- The mirror equation is $1/f = 1/v + 1/u$, where f, u and v are signed focal length, object distance and image distance measured from the mirror's pole.
- In the Cartesian convention with incident light travelling towards the mirror, distances in front of a concave mirror are negative. Thus a negative image distance represents a real image in front of the mirror.

Step 1: Express the given object position using positive magnitudes Let $F = |f|$ be the focal-length magnitude and $U = |u|$ the object-distance magnitude. For the concave mirror, $f = -F$ and $u = -U$. Since the object lies strictly between the focus and the centre of curvature,

$$F < U < 2F.$$

This notation avoids comparing negative signed distances as if they were positive lengths.

Step 2: Obtain the image distance and compare it with twice the focal length Substituting in the mirror equation gives

$$-\frac{1}{F} = \frac{1}{v} - \frac{1}{U}, \qquad \frac{1}{v} = \frac{F-U}{FU}, \qquad v = -\frac{FU}{U-F}.$$

Since $U > F$, the denominator $U - F$ is positive and hence $v < 0$. The image-distance magnitude is $V = |v| = FU/(U - F)$. To locate the image, calculate

$$V - 2F = \frac{FU - 2F(U-F)}{U-F} = \frac{F(2F-U)}{U-F} > 0,$$

because both $2F - U$ and $U - F$ are positive for the stated object position.

Step 3: Conclusion Therefore $v < 0$ and $|v| > 2|f|$: the image is real and lies beyond the centre of

Quick Tip to Solve: Use positive magnitudes F and U for the inequality, then restore signs in the mirror equation to identify a real image.

curvature, that is, beyond $2f$ in terms of distance magnitude.

Quick Tip to Solve: Use positive magnitudes F and U for the inequality, then restore signs in the mirror equation to identify a real image.

OR

9. Find an expression for intensity of transmitted light when a polaroid sheet is rotated between two crossed polaroids. In which position of the polaroid sheet will the transmitted intensity be maximum? [2 marks]

Solution:

Concept:

- Malus' law states that linearly polarised light of intensity I, incident on an ideal polaroid at angle ϕ to its transmission axis, emerges with intensity $I\cos^2\phi$.
- Crossed polaroids have perpendicular transmission axes. Light emerging from each sheet is polarised along that sheet's transmission axis, so Malus' law must be applied successively to the middle and final sheets.

Step 1: Find the intensity after the rotating middle sheet Let I_1 be the intensity emerging from the first fixed polaroid, and let θ be the angle between its axis and the rotating middle sheet's axis. Malus' law gives the intensity after the middle sheet as

$$I_2 = I_1\cos^2\theta.$$

This light is now polarised along the middle sheet's axis. If the original incident light is unpolarised with intensity I_0, an ideal first polaroid transmits half of it, so $I_1 = I_0/2$.

Step 2: Apply Malus' law at the last crossed polaroid and maximise The final sheet's axis is at 90° to the first, and therefore at $90^\circ - \theta$ to the middle sheet for $0 \le \theta \le 90^\circ$. Hence

$$\begin{aligned} I_{\text{out}} &= I_2\cos^2(90^\circ - \theta) = I_1\cos^2\theta\sin^2\theta \\ &= \frac{I_1}{4}\sin^2(2\theta). \end{aligned}$$

The factor $\sin^2(2\theta)$ has maximum value one when $2\theta = 90^\circ$, so the middle sheet's axis must bisect the angle between the two fixed axes. More generally, equivalent maxima recur every 90° of rotation.

Step 3: Conclusion The transmitted intensity is $I_{\text{out}} = (I_1/4)\sin^2(2\theta)$, with maximum $I_1/4$ when the middle axis is at 45° to each fixed axis. For unpolarised incident intensity I_0, this becomes $I_{\text{out}} = (I_0/8)\sin^2(2\theta)$, with maximum $I_0/8$.

Quick Tip to Solve: Apply Malus' law twice: the middle sheet contributes cosine squared and the crossed final sheet contributes sine squared.

10. Use Kirchhoff's rules to obtain conditions for the balance condition in a Wheatstone bridge.

[2 marks]

Solution:

Concept:

- Kirchhoff's junction rule expresses charge conservation: the sum of currents entering a junction equals the sum leaving it.
- Kirchhoff's loop rule expresses energy conservation: the algebraic sum of potential changes around a closed loop is zero. A balanced bridge has zero galvanometer current and equal potentials at its connected junctions.

Step 1: Define the bridge arms and the balance condition Let the four resistances be $AB = P$, $BC = Q$, $AD = R$ and $DC = S$. Connect the source across A and C, and the galvanometer between B and D. At balance, $I_g = 0$, so $V_B = V_D$. By the junction rule, the same current I_1 flows through P and Q, and the same current I_2 flows through R and S.

Step 2: Apply the loop rule to the two galvanometer loops Traverse $A \to B \to D \to A$. The galvanometer has zero potential drop at balance; traversing P along its current gives a drop and traversing R against its current gives a rise. Thus

$$-I_1P + I_2R = 0, \qquad I_1P = I_2R.$$

Similarly, traversing $B \to C \to D \to B$ gives

$$-I_1Q + I_2S = 0, \qquad I_1Q = I_2S.$$

Divide the first equality by the second to eliminate the branch currents:

$$\frac{I_1P}{I_1Q} = \frac{I_2R}{I_2S}.$$

Step 3: Conclusion The Wheatstone bridge is balanced when $I_g = 0$, equivalently $V_B = V_D$, and its arm resistances satisfy $\boxed{P/Q = R/S}$, or $PS = QR$.

Quick Tip to Solve: Label each source-to-source branch in the same order before writing the ratio: upper first/upper second equals lower first/lower second.

11. Name the parts of the electromagnetic spectrum which is

(a) suitable for radar systems used in aircraft navigation.

(b) used to treat muscular strain.

(c) used as a diagnostic tool in medicine.

Write in brief, how these waves can be produced. [3 marks]

Solution:

Concept:

- Electromagnetic waves are produced by accelerated charges. The frequency and wavelength determine how a wave interacts with matter and hence its practical application.
- Microwaves are useful in radar, infrared radiation supplies heat, and X-rays penetrate soft tissue much more readily than bone. These three regions have progressively increasing frequencies.

Step 1: Identify the waves used in aircraft radar (a) **Microwaves** are suitable for radar systems used in aircraft navigation. Their short wavelengths allow a narrow, directed beam; reflected pulses reveal the position and distance of objects. They are produced by special electronic devices such as a klystron or magnetron, in which electron motion generates high-frequency electromagnetic oscillations.

Step 2: Identify the waves used to treat muscular strain (b) **Infrared waves** are used to treat muscular strain. Their absorption produces a heating effect in the affected region, helping relieve muscular discomfort. Infrared radiation is emitted by hot bodies through thermal motion of their charges and molecular vibrations; an electrically heated filament or an infrared lamp provides a practical source.

Step 3: Identify the waves used for medical diagnosis (c) **X-rays** are used as a diagnostic tool in medicine, for example to obtain images of bones. Differences in absorption by bone and soft tissue produce the image contrast. In an X-ray tube, electrons emitted from a heated cathode are accelerated through a high potential difference and strike a heavy-metal target. Their rapid deceleration produces X-rays; transitions following the removal of inner-shell electrons also produce characteristic X-rays.

Step 4: Conclusion The required regions are (a) microwaves, produced by devices such as magnetrons or klystrons; (b) infrared waves, emitted by hot bodies; and (c) X-rays, produced when fast electrons strike a metal target.

Quick Tip to Solve: Link each application to its interaction: radar needs directed reflected pulses, muscular treatment needs heating, and bone imaging needs differential penetration.

12. (i) A giant refracting telescope has an objective lens of focal length 15 m. If an eye piece of focal length 1.0 cm is used, what is the angular magnification of the telescope?

(ii) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^6 m and the radius of lunar orbit is 3.8×10^8 m. [3 marks]

Solution:

Concept:

- An astronomical refracting telescope in normal adjustment forms its final image at infinity. Its angular magnification is $M = -f_o/f_e$, where f_o and f_e are the objective and eyepiece focal lengths. The negative sign denotes an inverted final image.
- A distant object of diameter D at distance L subtends the small angle $\theta \simeq D/L$. Its real image lies approximately in the objective's focal plane and has diameter $d \simeq f_o\theta = f_oD/L$.

Step 1: Convert the eyepiece focal length and find magnification (i) With no other viewing adjustment specified, use normal adjustment. Express both focal lengths in the same unit:

$$f_o = 15\,\text{m}, \qquad f_e = 1.0\,\text{cm} = 0.010\,\text{m}.$$

The objective forms a real inverted intermediate image, which the eyepiece magnifies. Substituting into the angular magnification relation gives

$$M = -\frac{15}{0.010} = -1.5 \times 10^3.$$

Angular magnification is a ratio of angles and therefore has no unit. Its magnitude is the factor by which the apparent angular size increases.

Step 2: Find the Moon's angular diameter (ii) Use the lunar-orbit radius as the given viewing distance. Since this distance is much larger than both the Moon's diameter and the objective focal length, the small-angle approximation and focal-plane image approximation apply:

$$\theta \simeq \frac{D}{L} = \frac{3.48 \times 10^6}{3.8 \times 10^8} = 9.16 \times 10^{-3}\,\text{rad}.$$

Step 3: Calculate the image diameter at the objective's focus The objective alone forms the Moon's real image. Its diameter depends on the objective focal length and the Moon's angular diameter, rather than on the eyepiece focal length:

$$d = f_o\theta = 15(9.16 \times 10^{-3})\,\text{m} = 0.1374\,\text{m} = 13.74\,\text{cm}.$$

To two significant figures, this is approximately 0.14 m, or 14 cm.

Step 4: Conclusion (i) The angular magnification has magnitude $\boxed{1500}$; its signed value is -1500, indicating inversion. (ii) The image diameter is approximately $\boxed{0.137\,\text{m}}$, or 13.7 cm before rounding to two significant figures.

Quick Tip to Solve: Convert centimetres to metres before using the focal-length ratio. For the Moon's objective image, use $d = f_oD/L$; the eyepiece focal length affects magnification, not that image diameter.

13. Write Einstein's photoelectric equation and mention which important features in photoelectric effect can be explained with the help of this equation.

The maximum kinetic energy of the photoelectrons gets doubled when the wavelength of light incident on the surface changes from λ_1 to λ_2. Derive the expressions for the threshold wavelength λ_0 and work function for the metal surface. [3 marks]

Solution:

Concept:

- A photon of frequency ν carries energy $h\nu$, where h is Planck's constant. One photon transfers its energy to one electron.
- The work function ϕ is the minimum energy needed to release an electron from the surface; $\phi = h\nu_0 = hc/\lambda_0$, where ν_0 and λ_0 are the threshold frequency and wavelength.
- The maximum kinetic energy is $K_{\max} = \frac{1}{2}mv_{\max}^2 = eV_s$, where V_s is the magnitude of the stopping potential and e is the elementary charge.

Step 1: State Einstein's equation and explain the observations Conservation of energy for the fastest emitted electron gives

$$h\nu = \phi + K_{\max}, \qquad K_{\max} = h\nu - \phi.$$

Emission is possible only for $h\nu \geq \phi$; hence a threshold frequency exists, regardless of intensity. Above threshold, $K_{\max}$ and V_s increase linearly with frequency. At a fixed frequency, increasing intensity increases the number of incident photons and hence the photocurrent, but does not increase $K_{\max}$. The one-photon energy transfer also explains prompt emission without an appreciable time lag: an electron need not accumulate energy from a continuous wave. The equation applies to the maximum energy because electrons that lose energy inside the metal emerge with smaller kinetic energies.

Step 2: Apply the doubling condition Let the initial maximum kinetic energy be K_1. For the two incident wavelengths,

$$K_1 = \frac{hc}{\lambda_1} - \phi, \qquad K_2 = \frac{hc}{\lambda_2} - \phi = 2K_1.$$

Substitution and rearrangement give

$$\frac{hc}{\lambda_2} - \phi = \frac{2hc}{\lambda_1} - 2\phi, \qquad \phi = hc\left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right).$$

Step 3: Determine the threshold wavelength At threshold the maximum kinetic energy is zero, so $hc/\lambda_0 = \phi$. Therefore

$$\frac{1}{\lambda_0} = \frac{2}{\lambda_1} - \frac{1}{\lambda_2}, \qquad \lambda_0 = \frac{\lambda_1\lambda_2}{2\lambda_2 - \lambda_1}.$$

For a positive work function and nonzero initial emission, the physically consistent wavelengths satisfy $\lambda_1/2 < \lambda_2 < \lambda_1$. The shorter second wavelength supplies the extra photon energy needed to double the kinetic energy.

Step 4: Conclusion Einstein's equation is $h\nu = \phi + K_{\max}$. The required expressions are $\lambda_0 = \dfrac{\lambda_1\lambda_2}{2\lambda_2 - \lambda_1}$ and $\phi = \dfrac{hc(2\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}$.

Quick Tip to Solve: Write two separate photoelectric equations before imposing the doubling condition; the work function is subtracted in each equation.

14. In the study of Geiger-Marsden experiment on scattering of α particles by a thin foil of gold, draw the trajectory of α-particles in the coulomb field of target nucleus. Explain briefly how one gets the information on the size of the nucleus from this study.

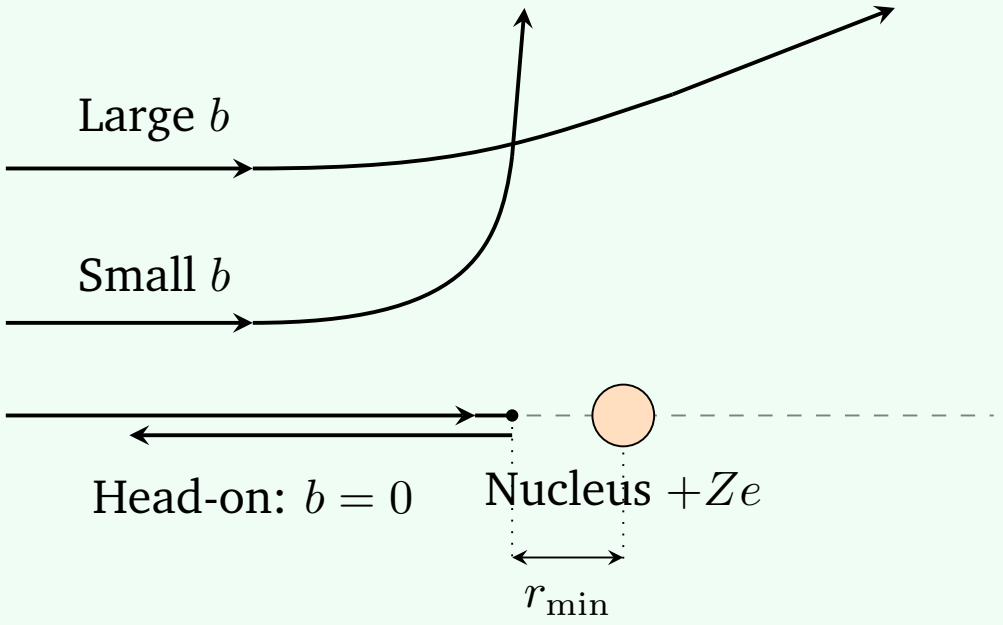
From the relation $R = R_0 A^{1/3}$, where R_0 is constant and A is the mass number of the nucleus, show that nuclear matter density is independent of A. [3 marks]

Solution:

Concept:

- An alpha particle has charge $+2e$; a nucleus of atomic number Z has charge $+Ze$. Their Coulomb interaction is repulsive.
- The impact parameter b is the perpendicular distance between the incident straight-line direction and the nucleus. Smaller b gives a larger scattering angle.
- The nuclear radius is $R = R_0 A^{1/3}$. For a spherical nucleus, $V = 4\pi R^3/3$, and its mass is approximately Am_u, where m_u is the atomic mass unit.

Step 1: Describe the trajectories in the Coulomb field Alpha particles approaching the positive nucleus bend away from it along hyperbolic trajectories. Those passing far away suffer small deflections; a close approach produces a large deflection. In a head-on encounter, $b = 0$, the alpha particle slows to rest at its closest point and returns along its incident line. The sketch shows representative trajectories; the nucleus is drawn enlarged for clarity. In the experiment, most particles pass with very little deflection, while only a small fraction are strongly scattered, showing that the positive charge is concentrated in a very small central region.



Representative repulsive Coulomb trajectories; arrows show the direction of motion.

Step 2: Obtain a bound on nuclear size from closest approach For a head-on collision with a heavy gold nucleus, the initial kinetic energy K is converted into electrostatic potential energy at the distance

Quick Tip to Solve: Cube the radius law before calculating volume: $(A^{1/3})^3 = A$, so the mass-number factors cancel immediately.

of closest approach $r_{\min}$:

$$K = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{r_{\min}}, \qquad r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{K}.$$

Here the initial potential energy is taken as zero far from the nucleus and recoil of the heavy target is neglected. If Rutherford scattering remains valid down to this distance, the alpha particle has turned back without reaching the nuclear interior. Thus $r_{\min}$ provides an upper estimate of the nuclear radius, rather than an exact radius. The closest-approach distances inferred in this way are of order 10^{-14} m, demonstrating the tiny nuclear size compared with an atomic radius of order 10^{-10} m.

Step 3: Show that nuclear density does not depend on mass number Using the mass and radius relations,

$$\rho = \frac{\text{nuclear mass}}{\text{nuclear volume}} = \frac{Am_u}{\frac{4}{3}\pi(R_0A^{1/3})^3}$$
$$= \frac{Am_u}{\frac{4}{3}\pi R_0^3 A} = \frac{3m_u}{4\pi R_0^3}.$$

The factor A cancels because nuclear mass and nuclear volume both grow in proportion to A. Since m_u and R_0 are constants, nuclear matter has approximately the same density for all nuclei within this model.

Step 4: Conclusion Coulomb repulsion bends alpha-particle trajectories away from the nucleus; head-on scattering gives $r_{\min} = 2Ze^2/(4\pi\varepsilon_0 K)$ as an upper estimate of its radius. Nuclear density is $\rho = 3m_u/(4\pi R_0^3)$, independent of A.

Quick Tip to Solve: Cube the radius law before calculating volume: $(A^{1/3})^3 = A$, so the mass-number factors cancel immediately.

OR

14. Distinguish between nuclear fission and fusion. Show how in both these processes energy is released.

Calculate the energy release in MeV in the deuterium-tritium fusion reaction:

${}^2_1H + {}^3_1H \rightarrow {}^4_2He + n$

Using the data:

$m({}^2_1H) = 2.014102$ **u**

$m({}^3_1H) = 3.016049$ **u**

$m({}^4_2He) = 4.002603$ **u**

$m_n = 1.008665$ **u**

$1u = 931.5$ **MeV**/c^2 [3 marks]

Solution:

Concept:

- Nuclear fission splits a heavy nucleus into lighter nuclei, whereas nuclear fusion combines light nuclei to form a heavier nucleus.

Quick Tip to Solve: Add all reactant masses and all product masses separately before subtracting; include the emitted neutron in the product total.

- Binding energy is the energy required to separate a nucleus into free nucleons. A larger binding energy per nucleon means greater stability.
- The reaction energy is $Q = (m_{\text{initial}} - m_{\text{final}})c^2$. With masses in atomic mass units, multiply the mass difference by $931.5\,\text{MeV}$ per atomic mass unit.

Step 1: Distinguish the two nuclear processes In fission, a heavy nucleus such as uranium splits into two medium-mass fragments, usually with emission of neutrons. Neutron-induced fission can support a chain reaction because the released neutrons may cause further fissions. In fusion, light nuclei such as deuterium and tritium combine to form a more tightly bound nucleus. Fusion requires sufficiently high temperature, or another means of overcoming the electrostatic repulsion between the positively charged nuclei; it is the source of energy in stars.

Step 2: Explain why energy is released in both The binding-energy-per-nucleon curve rises rapidly for light nuclei and reaches a maximum near iron and nickel, then decreases gradually for heavy nuclei. Fission of a heavy nucleus produces medium-mass fragments with greater binding energy per nucleon. Fusion of suitable light nuclei produces a nucleus with greater binding energy per nucleon. In each energy-releasing process the total binding energy increases, so the products have a smaller total rest mass than the reactants. The lost rest mass appears as kinetic energy of the products and possibly radiation, in accordance with $Q = \Delta m c^2$. Energy is released for reactions with positive Q; fusion of arbitrarily heavy nuclei need not release energy.

Step 3: Calculate the mass difference for deuterium-tritium fusion For ${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n}$,

$$\begin{aligned} m_{\text{initial}} &= 2.014102 + 3.016049 = 5.030151\,\text{u}, \\ m_{\text{final}} &= 4.002603 + 1.008665 = 5.011268\,\text{u}, \\ \Delta m &= 5.030151 - 5.011268 = 0.018883\,\text{u}. \end{aligned}$$

The supplied hydrogen and helium values can be used as atomic masses: the two hydrogen atoms contain two electrons in total, matching the two electrons in the helium atom. Thus electron rest masses cancel in the difference.

Step 4: Convert the mass difference to released energy Using the supplied mass-energy conversion,

$$Q = 0.018883 \times 931.5\,\text{MeV} = 17.5895145\,\text{MeV}.$$

The positive result confirms that the products have lower rest energy. Most of this energy appears as kinetic energy of the helium nucleus and neutron; the total energy release is approximately $17.6\,\text{MeV}$ per fusion reaction.

Step 5: Conclusion Fission divides a heavy nucleus; fusion combines light nuclei. Both release energy when the products have greater total binding energy and lower rest mass. The given deuterium-tritium reaction releases $Q \approx 17.59\,\text{MeV}$, or $17.6\,\text{MeV}$.

Quick Tip to Solve: Add all reactant masses and all product masses separately before subtracting; include the emitted neutron in the product total.

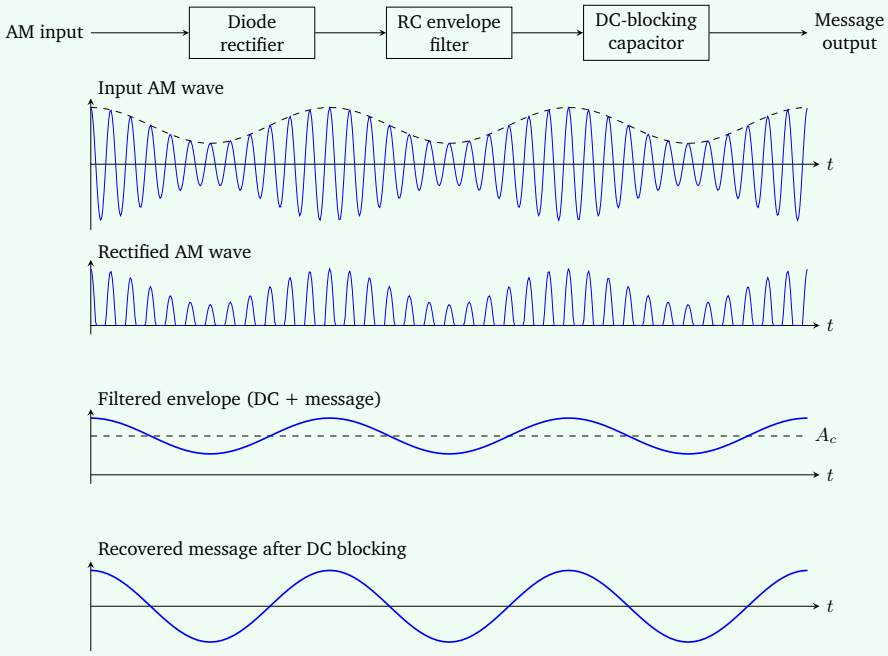
15. Draw a block diagram of a detector for AM signal and show, using necessary processes and the waveforms, how the original message signal is detected from the input AM wave. [3 marks]

Solution:

Concept:

- Detection (demodulation) recovers the low-frequency message from a high-frequency amplitude-modulated carrier.
- For a sinusoidal message, the AM wave is $s(t) = A_c[1 + \mu\cos(\omega_m t)]\cos(\omega_c t)$, where A_c is the carrier amplitude, μ the modulation index, and $\omega_m \ll \omega_c$. Its envelope follows the message when $0 \le \mu \le 1$.
- A diode rectifier followed by an envelope filter extracts this envelope. Removing the constant component gives the original message waveform.

Step 1: Rectify the incoming AM signal Apply the received AM wave to a diode rectifier. The diode conducts during positive portions of the carrier and blocks negative portions, producing a succession of positive pulses. The heights of these pulses follow the slowly varying upper envelope of the AM wave. Rectification therefore makes the envelope available without cancellation by the negative carrier half-cycles.



Detector blocks and the associated waveforms; the envelope filter suppresses the carrier, and DC blocking leaves the message.

Step 2: Extract the envelope and remove the constant component Pass the rectified signal through an RC envelope filter. The capacitor charges near a positive peak and discharges through the resistor between successive peaks, smoothing the rapid carrier pulsations while following the slowly varying envelope. Choose the time constant so that $T_c \ll RC \ll T_m$, where T_c and T_m are the carrier and message periods. The filter output is approximately $A_c[1 + \mu\cos(\omega_m t)]$. A coupling capacitor blocks its DC component A_c, leaving $A_c\mu\cos(\omega_m t)$, which reproduces the message apart from an overall amplitude factor.

Quick Tip to Solve: Draw the envelope first: rectification keeps its positive peaks, filtering traces it, and DC removal centres it about zero.

Step 3: Conclusion The detector consists of a rectifier, an envelope filter and DC blocking. Rectification preserves the positive envelope, filtering removes carrier oscillations, and DC blocking recovers a signal proportional to the original message.

Quick Tip to Solve: Draw the envelope first: rectification keeps its positive peaks, filtering traces it, and DC removal centres it about zero.

16. A cell of emf 'E' and internal resistance 'r' is connected across a variable load resistor R. Draw the plots of the terminal voltage V versus (i) R and (ii) the current I.

It is found that when $R = 4\,\Omega$, the current is 1 A and when R is increased to $9\,\Omega$, the current reduces to 0.5 A. Find the values of the emf E and internal resistance r. [3 marks]

Solution:

Concept:

- For a cell of emf E, internal resistance r, and external load R, the circuit current is $I = E/(R + r)$.
- The terminal voltage is the voltage across the load: $V = IR = E - Ir$. Thus internal resistance causes a voltage loss Ir inside a cell delivering current.

Step 1: Obtain the terminal voltage versus load resistance Substituting the current into $V = IR$ gives

$$V = \frac{ER}{R + r}.$$

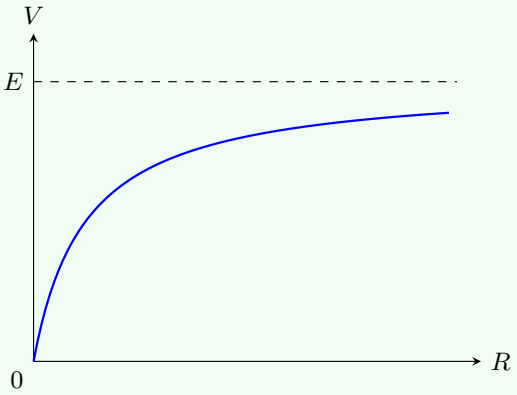
At $R = 0$, the terminals are short-circuited and $V = 0$. As R increases, the current and internal voltage loss decrease, so V rises. For a very large load resistance, V approaches E. The curve rises from the origin, becomes progressively flatter, and has the horizontal asymptote $V = E$; it is not a straight line.

Step 2: Obtain the terminal voltage versus current Kirchhoff's loop rule gives $E = IR + Ir$, hence

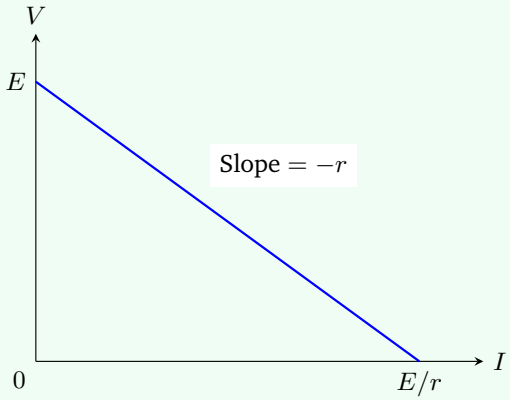
$$V = E - rI.$$

The V-versus-I graph is a straight line with vertical intercept E and slope $-r$. At open circuit, $I = 0$ and $V = E$. At short circuit, $V = 0$ and $I = E/r$. The plotted segment corresponds to a cell supplying current to a non-negative load resistance.

Quick Tip to Solve: Write E = I(R + r) for each load condition; equating the two emf expressions eliminates E immediately.



(i) $V = ER/(R+r)$



(ii) $V = E - rI$

Terminal voltage rises asymptotically with load resistance and falls linearly with the current supplied by the cell.

Step 3: Use the two load conditions to determine the cell parameters For the first condition, $R = 4\,\Omega$ and $I = 1\,\text{A}$, so $E = I(R+r)$. For the second, $R = 9\,\Omega$ and $I = 0.5\,\text{A}$. Expressing E in volts and r in ohms, these equations are

$$E = 4 + r,$$
$$E = 0.5(9 + r) = 4.5 + 0.5r.$$

Equating the right-hand sides gives $0.5r = 0.5$, so $r = 1\,\Omega$, and substitution gives $E = 5\,\text{V}$. As a check, the two terminal voltages are $IR = 4\,\text{V}$ and $4.5\,\text{V}$, respectively; adding their internal drops of $1\,\text{V}$ and $0.5\,\text{V}$ yields the same emf.

Step 4: Conclusion The required plots are an increasing curve approaching $V = E$ for V versus R, and a straight line of slope $-r$ for V versus I. The cell has emf $\boxed{E = 5\,\text{V}}$ and internal resistance $\boxed{r = 1\,\Omega}$.

Quick Tip to Solve: Write E = I(R + r) for each load condition; equating the two emf expressions eliminates E immediately.

17. Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the value of C_1 and C_2. Also calculate the charge on each capacitor in parallel combination. [3 marks]

Solution:

Concept:

- For a capacitor of capacitance C across a potential difference V, the stored energy is $U = \frac{1}{2}CV^2$ and the charge is $Q = CV$.
- The equivalent capacitances are $C_s = C_1C_2/(C_1 + C_2)$ in series and $C_p = C_1 + C_2$ in parallel. In parallel, each capacitor has the full battery voltage.

Step 1: Determine the two equivalent capacitances Both combinations are connected across the same

Quick Tip to Solve: Use the parallel energy to find the sum, then multiply that sum by the series capacitance to find the product.

100 V battery. Hence the energy formula gives

$$C_s = \frac{2(0.045)}{100^2} = 9.0 \times 10^{-6}\,\text{F} = 9\,\mu\text{F},$$
$$C_p = \frac{2(0.25)}{100^2} = 5.0 \times 10^{-5}\,\text{F} = 50\,\mu\text{F}.$$

These are the capacitances of the combinations, rather than those of the individual capacitors.

Step 2: Recover the individual capacitances The sum of the unknown capacitances is $50\,\mu$F. Their product follows from the series relation:

$$C_1 + C_2 = 50\,\mu\text{F}, \qquad C_1 C_2 = C_s C_p = 450\,(\mu\text{F})^2.$$

Writing a capacitance as $x\,\mu$F, the two values are the roots of

$$x^2 - 50x + 450 = 0, \qquad x = \frac{50 \pm \sqrt{2500 - 1800}}{2} = 25 \pm 5\sqrt{7}.$$

Thus the two capacitors can be interchanged without changing either measured energy. Their series capacitance indeed equals $450/50 = 9\,\mu$F.

Step 3: Calculate the parallel charges Choose C_1 as the larger capacitance. Each capacitor is across 100 V in parallel, so

$$Q_1 = (25 + 5\sqrt{7}) \times 100\,\mu\text{C} \approx 3822.9\,\mu\text{C},$$
$$Q_2 = (25 - 5\sqrt{7}) \times 100\,\mu\text{C} \approx 1177.1\,\mu\text{C}.$$

The unequal charges have the sum $5000\,\mu$C, consistent with $C_p V$. Each quantity is the magnitude of charge on either plate of that capacitor.

Step 4: Conclusion The capacitances are $\boxed{38.23\,\mu\text{F and } 11.77\,\mu\text{F}}$. Their corresponding parallel charges are $\boxed{3.823 \times 10^{-3}\,\text{C and } 1.177 \times 10^{-3}\,\text{C}}$. Either capacitor may be labelled C_1.

Quick Tip to Solve: Use the parallel energy to find the sum, then multiply that sum by the series capacitance to find the product.

18. State the principle of working of a galvanometer.

A galvanometer of resistance G is converted into a voltmeter to measure upto V volts by connecting a resistance R_1 in series with the coil. If a resistance R_2 is connected in series with it, then it can measure upto V/2 volts. Find the resistance, in terms of R_1 and R_2, required to be connected to convert it into a voltmeter that can read upto 2 V. Also find the resistance G of the galvanometer in terms of R_1 and R_2. [3 marks]

Solution:

Concept:

- A moving-coil galvanometer works because a current-carrying coil experiences a torque in a magnetic field. In a radial field, magnetic torque is $NIAB$, where N, A and B denote turns, coil area and magnetic field.
- A series resistance limits the galvanometer current. If its full-scale current is I_g and coil resistance is G, the voltage range with a series resistance R is $V_{\max} = I_g(G + R)$.

Step 1: State the operating principle The magnetic torque turns the coil against the restoring torque of its spring. If the spring's torsional constant is k, equilibrium gives

$$NIAB = k\theta, \qquad \theta = \frac{NAB}{k} I.$$

Thus the angular deflection θ is directly proportional to the current I. The radial magnetic field keeps the torque proportional to current throughout the useful angular range. This allows the deflection to indicate current and, after adding a suitable series resistance, voltage.

Step 2: Use the two stated voltage ranges The two series resistances are used separately with the same coil and therefore with the same full-scale current I_g. The stated ranges give

$$V = I_g(G + R_1), \qquad \frac{V}{2} = I_g(G + R_2).$$

Eliminating V and I_g,

$$G + R_1 = 2(G + R_2), \qquad G = R_1 - 2R_2.$$

Also, subtraction of the original equations gives $V/2 = I_g(R_1 - R_2)$. For a physical galvanometer with positive coil resistance, the stated values must obey $R_1 > 2R_2$.

Step 3: Find the resistance for twice the range Let R be the required new resistance connected in series with the coil. To read up to twice the original range, the total resistance must be twice its value for range V:

$$2V = I_g(G + R) = 2I_g(G + R_1).$$

Consequently,

$$G + R = 2G + 2R_1, \qquad R = G + 2R_1 = (R_1 - 2R_2) + 2R_1.$$

This resistance is the added series resistance alone; the coil's own resistance G is already present in the voltmeter.

Step 4: Conclusion The galvanometer resistance is $\boxed{G = R_1 - 2R_2}$, and the resistance required in series for range $2V$ is $\boxed{R = 3R_1 - 2R_2}$.

Quick Tip to Solve: The full-scale current stays fixed: double the voltage range by doubling the total resistance, including the coil.

19. With what considerations in view, a photodiode is fabricated? State its working with the help of a suitable diagram.

Eventhough the current in the forward bias is known to be more than in the reverse bias, yet the photodiode works in reverse bias. What is the reason? [3 marks]

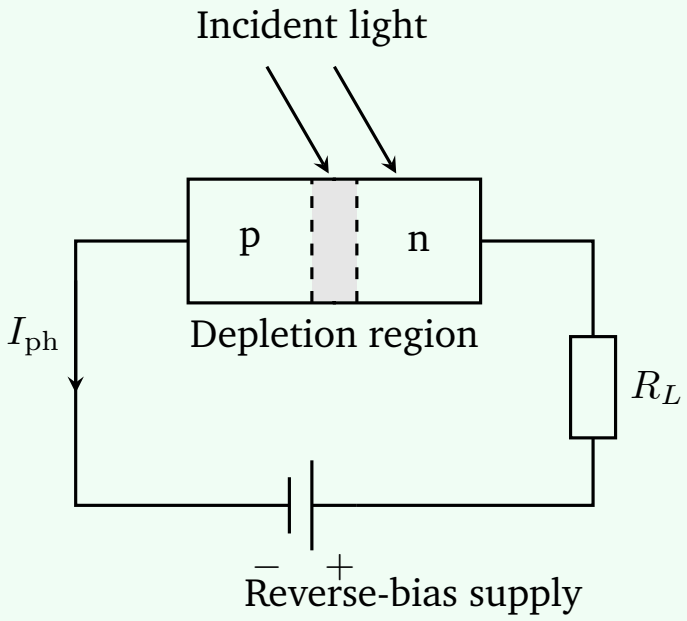
Solution:

Concept:

- A photodiode is a light-sensitive p-n junction. Incident photons with energy $h\nu \geq E_g$, where E_g is the semiconductor band gap, can create electron-hole pairs.
- Reverse bias widens the depletion region and establishes a strong electric field that separates photogenerated carriers. The resulting photocurrent increases with incident light intensity.

Step 1: Explain the fabrication considerations A photodiode is fabricated so that light can reach the junction's depletion region through a transparent window. The photosensitive junction is exposed over a suitable area, rather than being hidden by an opaque package. The semiconductor band gap is selected for the wavelength to be detected, so that the incident photons have sufficient energy to generate electron-hole pairs. Efficient illumination of the depletion region allows the electric field to collect those carriers before they recombine.

Step 2: Describe operation in the reverse-biased circuit In the diagram the n-side is connected towards the positive supply terminal, and the p-side towards the negative terminal. Thus the junction is reverse biased. When light falls on or near the depletion region, it generates electron-hole pairs. The junction field drives electrons towards the n-side and holes towards the p-side, producing a reverse photocurrent in the external circuit. Increasing the illumination generates more pairs per second, so the photocurrent increases. The load resistance R_L converts the change in current into a measurable voltage.



Illuminated p-n junction connected in reverse bias

Quick Tip to Solve: For a photodiode, compare the light-induced change with the dark current, rather than comparing total forward and reverse currents.

Step 3: Explain the choice of reverse bias Although forward bias produces a much larger total current, that current is mainly due to carrier injection by the applied voltage. The small additional current caused by light is difficult to distinguish from this large background. Under reverse bias, the current in darkness is small; illumination therefore produces a large fractional change in current. The photocurrent can be detected more sensitively and is approximately proportional to the incident intensity for fixed wavelength and operating conditions. The stronger depletion-region field also promotes rapid separation and collection of the photogenerated carriers.

Step 4: Conclusion A photodiode has an optically accessible junction and a suitable band gap. It detects light through the reverse photocurrent; reverse bias provides a small dark-current background and a clearly measurable change with illumination.

Quick Tip to Solve: For a photodiode, compare the light-induced change with the dark current, rather than comparing total forward and reverse currents.

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20. Draw a circuit diagram of a transistor amplifier in CE configuration.

Define the terms: (i) Input resistance and (ii) Current amplification factor. How are these determined using typical input and output characteristics? [3 marks]

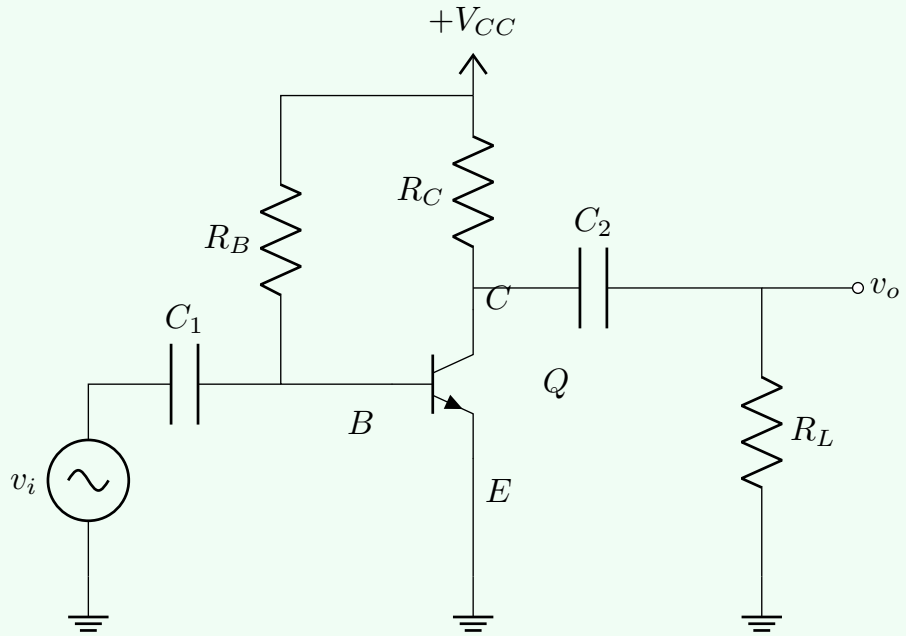
Solution:

Concept:

- In the common-emitter (CE) configuration, the emitter is common to the input base-emitter circuit and the output collector-emitter circuit. A small change in base current controls a larger change in collector current.
- The small-signal input resistance is $r_i = (\Delta V_{BE}/\Delta I_B)_{V_{CE}}$; the current amplification factor is $\beta_{ac} = (\Delta I_C/\Delta I_B)_{V_{CE}}$. Both are determined near the operating point in the active region.

Step 1: CE amplifier circuit The circuit below uses an npn transistor with its emitter grounded. The resistor R_B supplies the base bias and R_C is the collector load. The positive supply V_{CC} keeps the collector-base junction reverse biased while the base-emitter junction is forward biased. The coupling capacitors C_1 and C_2 pass the alternating input and output signals but block the dc bias from the signal source and load R_L. An increase in input voltage increases I_B and hence I_C; the increased drop across R_C decreases the collector voltage. Thus the output voltage is inverted relative to the input.

Quick Tip to Solve: For both CE parameters, keep V_{CE} fixed: use nearby points on one input curve for r_i, and nearby base-current curves for β_{ac}.



Fixed-bias npn common-emitter amplifier with coupling capacitors

Step 2: Input resistance from the input characteristic Input resistance is the ratio of a small change in base-emitter voltage to the resulting change in base current, keeping collector-emitter voltage constant:

$$r_i = \left.\frac{\Delta V_{BE}}{\Delta I_B}\right|_{V_{CE}\text{ constant}}.$$

On the typical input characteristic, I_B is plotted against V_{BE} for a fixed V_{CE}. Choose two nearby points about the operating point and divide their voltage difference by their current difference. Equivalently, take the reciprocal of the local slope dI_B/dV_{BE}. Its unit is the ohm; it is a differential resistance, rather than the ratio V_{BE}/I_B.

Step 3: Current amplification from the output characteristics The CE current amplification factor is the ratio of the change in collector current to the change in base current at fixed collector-emitter voltage:

$$\beta_{ac} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE}\text{ constant}}.$$

The output characteristics plot I_C against V_{CE} for different fixed values of I_B. Draw a vertical line at the operating V_{CE} in the active region. Read the collector currents I_{C1} and I_{C2} where this line cuts two nearby curves labelled I_{B1} and I_{B2}. Then $\beta_{ac} = (I_{C2} - I_{C1})/(I_{B2} - I_{B1})$, a dimensionless quantity. For dc current gain one instead uses $\beta_{dc} = I_C/I_B$ at the operating point.

Step 4: Conclusion The emitter is the common terminal in the shown amplifier. Input resistance is the inverse local slope of the input curve at fixed V_{CE}, while CE current gain is found from the separation of neighbouring output curves at the same V_{CE}.

Quick Tip to Solve: For both CE parameters, keep V_{CE} fixed: use nearby points on one input curve for r_i, and nearby base-current curves for β_{ac}.

21. Answer the following questions:

(a) In a double slit experiment using light of wavelength 600 nm, the angular width of the fringe formed on a distant screen is 0.1°. Find the spacing between the two slits.

(b) Light of wavelength 5000 Å propagating in air gets partly reflected from the surface of water. How will the wavelengths and frequencies of the reflected and refracted light be affected? [3 marks]

Solution:

Concept:

- In Young's double-slit experiment, the linear fringe width is $\beta = \lambda D/d$. For a distant screen, its angular width in radians is $\theta = \beta/D = \lambda/d$, where d is the slit spacing.
- At a stationary interface, the frequency of reflected and transmitted light equals the incident frequency. Wavelength follows $\lambda = v/f$, so a change of propagation speed changes wavelength.

Step 1: Convert the angular fringe width to radians The wavelength is $\lambda = 600\,\text{nm} = 6.00 \times 10^{-7}\,\text{m}$. The small-angle fringe formula requires radians, not degrees. Hence

$$\theta = 0.1\frac{\pi}{180} = 1.7453 \times 10^{-3}\,\text{rad}.$$

This angle is small enough for the approximation $\sin\theta \simeq \tan\theta \simeq \theta$. The distance of the screen is unnecessary because it cancels when linear fringe width is divided by D.

Step 2: Determine the slit spacing in part (a) For adjacent bright fringes, the change in path difference is one wavelength. Therefore the angular fringe separation is λ/d, giving

$$d = \frac{\lambda}{\theta} = \frac{6.00 \times 10^{-7}}{1.7453 \times 10^{-3}} = 3.44 \times 10^{-4}\,\text{m} = 0.344\,\text{mm}.$$

Increasing the slit spacing decreases the angular fringe width, consistent with the inverse dependence in this expression.

Step 3: Compare reflected and refracted light in part (b) The incident wavelength is $5000\,\text{Å} = 5.00 \times 10^{-7}\,\text{m} = 500\,\text{nm}$. Taking the speed in air as c, the incident frequency is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}}{5.00 \times 10^{-7}} = 6.00 \times 10^{14}\,\text{Hz}.$$

The reflected wave travels in air, so its speed, frequency and wavelength are the same as those of the incident wave: $f_r = f$ and $\lambda_r = 5000\,\text{Å}$. A phase change on reflection does not change either of these quantities. The refracted wave travels more slowly in water. Its frequency remains $6.00 \times 10^{14}\,\text{Hz}$, but its wavelength becomes $\lambda_t = \lambda/\mu$, where μ is the refractive index of water relative to air. Thus its wavelength decreases. For the usual approximation $\mu = 4/3$, it is $3750\,\text{Å} = 375\,\text{nm}$.

Step 4: Conclusion (a) The slit spacing is $\boxed{0.344\,\text{mm}}$. (b) Reflected light retains wavelength $5000\,\text{Å}$; refracted light has wavelength $5000/\mu\,\text{Å}$. Both retain the incident frequency $6.00 \times 10^{14}\,\text{Hz}$.

Quick Tip to Solve: Convert degrees to radians before using $d = \lambda/\theta$; at the water surface keep frequency fixed and change wavelength with speed.

22. An inductor L of inductance X_L is connected in series with a bulb B and an ac source. How would brightness of the bulb change when (i) number of turns in the inductor is reduced, (ii) an iron rod is inserted in the inductor and (iii) a capacitor of reactance $X_C = X_L$ is inserted in series in the circuit. Justify your answer in each case. [3 marks]

Solution:

Concept:

- At a fixed angular frequency ω, an inductor has reactance $X_L = \omega L$. For a given coil geometry its inductance is proportional to permeability and the square of its turn count: $L \propto \mu N^2$.
- For a series circuit, $I_{\text{rms}} = V_{\text{rms}}/Z$, with $Z = \sqrt{R^2 + (X_L - X_C)^2}$. The bulb dissipates real power $P = I_{\text{rms}}^2 R_B$, where R_B is the bulb resistance, so a larger current makes it brighter.

Step 1: Establish how reactance controls the brightness The ac supply voltage and frequency are held fixed. Let R denote the total series resistance, including the bulb and any winding resistance. Before adding a capacitor, the impedance is $Z = \sqrt{R^2 + X_L^2}$. An ideal inductor consumes no average power, but its reactance limits the current through the bulb. Accordingly, reducing the magnitude of the net reactance increases the current and the power dissipated in the bulb. The printed symbol X_L is used for inductive reactance; the corresponding inductance is L.

Step 2: (i) Reduce the number of turns For the same core and coil dimensions, $L \propto N^2$. Reducing the number of turns therefore reduces L, and hence reduces $X_L = \omega L$. The circuit impedance decreases, so the rms current increases. Since the bulb's heating power increases with the square of current, its brightness increases.

Step 3: (ii) Insert an iron rod Iron has a much higher magnetic permeability than air. Inserting it into the coil increases magnetic flux linkage per unit current and thus increases its inductance. At the same frequency, X_L increases. Consequently $Z = \sqrt{R^2 + X_L^2}$ increases and the rms current decreases. The bulb dissipates less power and becomes dimmer. This is the usual idealized inductive-reactance explanation of the stated experiment.

Step 4: (iii) Add a capacitor with equal reactance In a series circuit the capacitive and inductive voltage contributions oppose each other in phase. When the capacitor has $X_C = X_L$ at the supply frequency, their net reactance vanishes:

$$X_L - X_C = 0, \qquad Z = R, \qquad I_{\text{rms}} = \frac{V_{\text{rms}}}{R}.$$

This is series resonance. Compared with the initial inductive circuit, the impedance is smaller and current is larger. Therefore the bulb becomes brighter; for fixed supply voltage, resistance and frequency, the current and brightness are maximum when the two reactances exactly cancel. The reactances cancel in the net series voltage, although each component may separately have a nonzero voltage across it.

Step 5: Conclusion (i) The bulb becomes brighter because fewer turns reduce inductive reactance. (ii) It becomes dimmer because the iron core increases inductance and reactance. (iii) It becomes brighter, with maximum current at $X_C = X_L$, because series resonance reduces the impedance to the circuit resistance.

Quick Tip to Solve: Track Z, then current, then brightness: fewer turns lower X_L, iron raises X_L, and an equal series X_C cancels it.

23. A group of students while coming from the school noticed a box marked "Danger H.T. 2200 V" at a substation in the main street. They did not understand the utility of such a high voltage, while they argued, the supply was only 220 V. They asked their teacher this question the next day. The teacher thought it to be an important question and therefore explained to the whole class.

Answer the following questions:

(i) What device is used to bring the high voltage down to low voltage of a.c. current and what is the principle of its working?

(ii) Is it possible to use this device for bringing down the high dc voltage to the low voltage? Explain.

(iii) Write the values displayed by the students and the teacher. [4 marks]

Solution:

Concept:

- A transformer changes the voltage of an alternating supply by mutual induction between two insulated coils wound on a common magnetic core.
- Faraday's law gives $e_s = -N_s\, d\Phi/dt$, where N_s is the number of secondary turns and Φ is the magnetic flux through each turn. An induced secondary emf requires changing flux.
- For an ideal transformer, $V_s/V_p = N_s/N_p$. A step-down transformer has fewer secondary turns than primary turns, so its secondary voltage is smaller.

Step 1: (i) Identify the device and explain its working A step-down transformer is used to reduce the high a.c. voltage to a lower a.c. voltage. Its primary coil is connected to the high-voltage alternating supply. The alternating primary current creates a changing magnetic flux in the laminated soft-iron core. This flux links the secondary coil and induces an alternating emf in it. Thus the working principle is mutual induction, based on Faraday's law of electromagnetic induction. Choosing $N_s < N_p$ gives $V_s < V_p$. For the voltages stated in the question, the ideal turns ratio is

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{220}{2200} = \frac{1}{10}.$$

Hence the primary has ten times as many turns as the secondary. High voltage is useful in transmission because, for a given transmitted power, a higher voltage permits a smaller current and therefore smaller I^2R losses in the lines. The voltage is then reduced near consumers for domestic use.

Step 2: (ii) Examine operation with a direct-current supply An ordinary transformer cannot continuously bring a steady high d.c. voltage down to a low d.c. voltage. After the brief switching transient, direct current does not produce the alternating core flux needed for transformer action. The flux becomes steady, so $d\Phi/dt = 0$ and no sustained secondary emf is induced. A momentary emf at switching does not provide a continuous low-voltage output. Moreover, a primary designed for a.c. may draw an excessive current on d.c., limited mainly by its winding resistance, causing overheating and possible damage. Thus this device must be supplied with a.c. for its intended operation.

Step 3: (iii) State the values shown in the incident The students show curiosity, scientific inquisitiveness, observation of their surroundings and a willingness to seek a reasoned explanation from their teacher. They connect a real-life observation with classroom learning instead of accepting an unexplained fact. The teacher shows helpfulness, patience, concern for students' understanding and a sense of responsibility. By explaining the issue to the whole class, the teacher also displays a

Quick Tip to Solve: Check whether the supply changes the core flux: a transformer needs changing flux, and a lower secondary voltage requires fewer secondary turns.

willingness to share knowledge and encourage scientific thinking.

Step 4: Conclusion (i) A step-down transformer lowers the a.c. voltage by mutual induction; here its ideal secondary-to-primary turns ratio is $1 : 10$. (ii) It cannot provide a steady reduced d.c. output because steady flux induces no secondary emf. (iii) The students display curiosity and a scientific attitude, while the teacher displays helpfulness, responsibility and a commitment to sharing knowledge.

Quick Tip to Solve: Check whether the supply changes the core flux: a transformer needs changing flux, and a lower secondary voltage requires fewer secondary turns.

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24. (a) State Ampere's circuital law. Use this law to obtain the expression for the magnetic field inside an air cored toroid of average radius 'r', having 'n' turns per unit length and carrying a steady current I.

(b) An observer to the left of a solenoid of N turns each of cross section area 'A' observes that a steady current I in it flows in the clockwise direction. Depict the magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic moment $m = NIA$. [5 marks]

Solution:

Concept:

- Ampere's circuital law for steady currents states that $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$, where I_{enc} is the algebraic current passing through a surface bounded by the closed path C.
- In an air-cored toroid, symmetry makes the magnetic field tangential to a circular path concentric with the toroid. The permeability of air is approximately μ_0.
- A current loop has magnetic moment $\vec{m} = IA\,\hat{n}$; $\hat{n}$ is fixed by the right-hand rule. Magnetic moments of identical coaxial turns add vectorially.

Step 1: (a) State the law and choose the Amperian path For any closed path surrounding steady conduction currents, the line integral of the magnetic field equals μ_0 times the net enclosed current:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}.$$

Let the total number of turns of the toroid be N_t. Choose a circular Amperian path of radius ρ lying within the winding and concentric with the toroid. By rotational symmetry, B has the same magnitude everywhere on this circle and is parallel to its tangent. A surface bounded by this path is linked by all N_t turns, so the enclosed current is $N_t I$.

Step 2: (a) Obtain the field inside the air-cored toroid Applying the circuital law gives

$$B(\rho)(2\pi\rho) = \mu_0 N_t I, \qquad B(\rho) = \frac{\mu_0 N_t I}{2\pi\rho}.$$

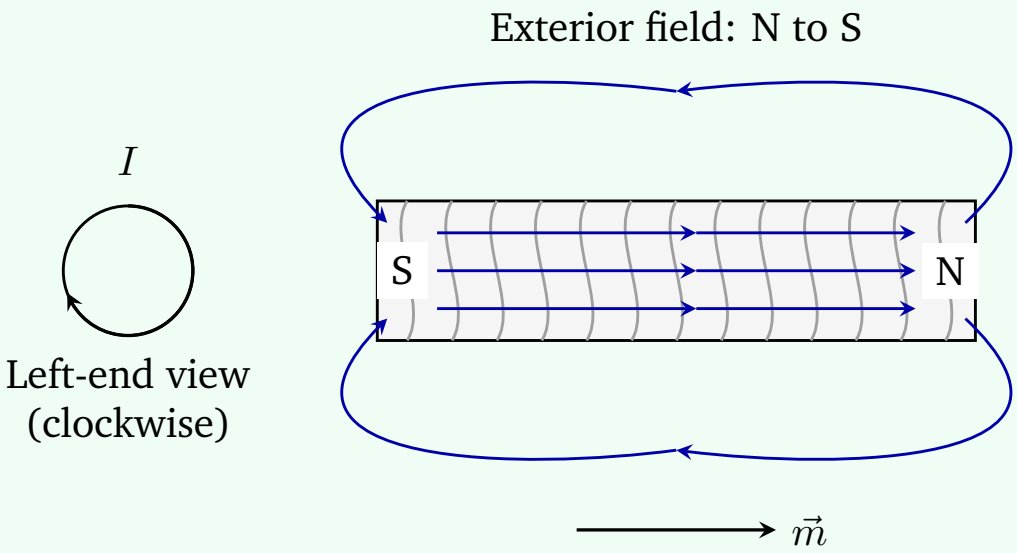
The circumference at the average radius r is $2\pi r$. Since there are n turns per unit length along this circumference,

$$N_t = n(2\pi r), \qquad B(r) = \frac{\mu_0 (2\pi r n) I}{2\pi r} = \mu_0 n I.$$

Quick Tip to Solve: Read the current from the specified observer's end: clockwise means south at that end. For a toroid, replace total turns by $N_t = 2\pi r n$ only after applying Ampere's law.

Thus the usual expression for a thin toroid is $B = \mu_0 n I$, directed around its circular axis according to the right-hand grip rule. For a toroid with appreciable radial thickness, the exact field varies as $1/\rho$; the result at the mean radius remains as above. The expression has units of tesla because n is measured in m^{-1}.

Step 3: (b) Determine the solenoid's polarity and depict its field Viewed from the left, the current circulates clockwise. Curl the fingers of the right hand in that direction: the thumb points away from the left observer, towards the right. Hence the field inside the solenoid points from left to right. The left end is the south pole and the right end is the north pole. Outside, the field lines leave the right-hand north pole and enter the left-hand south pole. Inside, they return from south to north, forming closed loops. The almost parallel interior lines represent the nearly uniform field of a long solenoid; the curved exterior lines show the same pattern as a bar magnet.



Solenoid field: left end S, right end N; the left observer sees clockwise current.

Step 4: (b) Derive the magnetic moment Each of the N turns is a current loop of area A, carrying current I. Its magnetic moment is $\vec{m}_1 = IA\,\hat{n}$, directed along the axis towards the right by the right-hand rule. Since all turns have the same area and their normals point in the same direction, their moments add:

$$\vec{m} = \sum_{j=1}^{N} IA\,\hat{n} = NIA\,\hat{n}.$$

Therefore the magnitude is $m = NIA$, in A m^2, with the moment directed from the south pole to the north pole. The solenoid has two opposite end polarities, the external field pattern of a bar magnet, and a magnetic dipole moment; it therefore acts as a bar magnet.

Step 5: Conclusion The toroidal field at its average radius is $\boxed{B = \mu_0 n I}$. The solenoid has $\boxed{S \text{ on the left, } N \text{ on the right}}$, with internal field and magnetic moment towards the right, and $\boxed{m = NIA}$.

Quick Tip to Solve: Read the current from the specified observer's end: clockwise means south at that end. For a toroid, replace total turns by $N_t = 2\pi r n$ only after applying Ampere's law.

OR

24. (a) Define mutual inductance and write its S.I. units.

(b) Derive an expression for the mutual inductance of two long co-axial solenoids of same length wound one over the other.

(c) In an experiment, two coils c_1 and c_2 are placed close to each other. Find out the expression for the emf induced in the coil c_1 due to a change in the current through the coil c_2. [5 marks]

Solution:

Concept:

- Mutual inductance measures the flux linkage of one coil produced by unit current in the other: $N_1\Phi_{12} = M_{12}I_2$, where Φ_{12} is the flux through each turn of coil 1 due to coil 2.
- Inside a long air-core solenoid, $B = \mu_0 nI$, where $n = N/l$ is the number of turns per unit length. The external field is negligible and the internal field is approximately uniform.
- Faraday's law gives $\varepsilon_1 = -d(N_1\Phi_{12})/dt$. Lenz's law supplies the negative sign: the induced effect opposes the change in linked magnetic flux.

Step 1: (a) Define mutual inductance and its unit The mutual inductance of two coils is the magnetic flux linkage with either coil per unit current flowing through the other coil, with their relative positions fixed. Equivalently, it is the magnitude of the emf induced in one coil when the current in the other changes at unit rate.

$$M = \frac{N_1\Phi_{12}}{I_2} = \frac{|\varepsilon_1|}{|dI_2/dt|}.$$

Its S.I. unit is the henry, written H. Two coils have mutual inductance of one henry if a current change of $1\,\text{A s}^{-1}$ in one induces an emf of magnitude $1\,\text{V}$ in the other.

$$1\,\text{H} = 1\,\text{Wb A}^{-1} = 1\,\text{V s A}^{-1}.$$

Step 2: (b) Calculate the magnetic field and linked flux Let the two long coaxial solenoids have common length l, turn counts N_1 and N_2, and turn densities $n_1 = N_1/l$ and $n_2 = N_2/l$. Let A be their common cross-sectional area; if their radii differ, A is the area of the inner solenoid. Neglect end effects and take the intervening medium as air. Pass a current I_2 through solenoid 2. Its field in the common enclosed region is

$$B_2 = \mu_0 \frac{N_2}{l} I_2.$$

The field is along the common axis and hence normal to the area of each turn. Thus the flux through one turn of solenoid 1 and its total flux linkage are

$$\Phi_{12} = B_2 A = \mu_0 \frac{N_2}{l} I_2 A,$$
$$N_1\Phi_{12} = \mu_0 \frac{N_1 N_2 A}{l} I_2.$$

Only the common flux area contributes: the field outside a long inner solenoid is negligible even when the outer coil encloses a larger area.

Step 3: (b) Identify the mutual inductance By definition, $N_1\Phi_{12} = M_{12}I_2$. Comparing its coefficient

Quick Tip to Solve: First calculate the field from the current-carrying solenoid, multiply by the common area and the receiving coil's turn count, then divide by the source current to obtain M.

of I_2 with the expression just derived gives

$$M_{12} = \mu_0 \frac{N_1 N_2 A}{l} = \mu_0 n_1 n_2 A l.$$

Repeating the calculation with current in solenoid 1 gives the same expression for M_{21}. Thus $M_{12} = M_{21} = M$, as expected for these fixed coils in a linear medium. The result depends on their geometry and magnetic medium, rather than on the particular value of current. For a uniform magnetic core, replace μ_0 by its permeability μ.

Step 4: (c) Apply Faraday's law to coil 1 Denote the current through the nearby coil c_2 by $I_2(t)$. If the coils remain fixed, their mutual inductance M is constant. The flux linkage of c_1 due to this current is $N_1\Phi_{12} = MI_2$. Therefore

$$\begin{aligned} \varepsilon_1 &= -\frac{d(N_1\Phi_{12})}{dt} \\ &= -\frac{d(MI_2)}{dt} = -M\frac{dI_2}{dt}. \end{aligned}$$

The magnitude is $M|dI_2/dt|$. For a finite interval, the average induced emf is $-M\Delta I_2/\Delta t$. Its polarity tends to oppose the change of magnetic flux through c_1; reversing whether I_2 increases or decreases reverses that polarity. A steady current in c_2 gives no induced emf in c_1.

Step 5: Conclusion Mutual inductance is flux linkage per unit current and is measured in henry. For two long coaxial air-core solenoids, $M = \mu_0 N_1 N_2 A/l = \mu_0 n_1 n_2 A l$, with A the common flux area. A changing current in c_2 induces $\varepsilon_1 = -M\, dI_2/dt$ in c_1.

Quick Tip to Solve: First calculate the field from the current-carrying solenoid, multiply by the common area and the receiving coil's turn count, then divide by the source current to obtain M.

25. (a) Using Huygens's construction of secondary wavelets explain how a diffraction pattern is obtained on a screen due to a narrow slit on which a monochromatic beam of light is incident normally.

(b) Show that the angular width of the first diffraction fringe is half that of the central fringe.

(c) Explain why the maxima at $\theta = \left(n + \frac{1}{2}\right)\frac{\lambda}{a}$ become weaker and weaker with increasing n.

[5 marks]

Solution:

Concept:

- Huygens's principle: every point of the illuminated slit acts as a source of coherent secondary wavelets. Their superposition determines the amplitude at each point of the screen.
- For slit width a, wavelength λ and observation angle θ, the path difference between wavelets from the two edges is $a\sin\theta$. In the small-angle limit, $\sin\theta \simeq \theta$.
- Destructive interference gives minima at $a\sin\theta = m\lambda$, where $m = \pm1, \pm2, \ldots$. The intensity is $I(\theta) = I_0(\sin\beta/\beta)^2$, where $\beta = \pi a \sin\theta/\lambda$ and I_0 is the central intensity.

Step 1: (a) Construct the diffracted wavefront and central maximum Let a parallel monochromatic

Quick Tip to Solve: Measure a bright fringe between its adjacent dark minima: the central fringe spans the negative and positive first minima, whereas each secondary fringe spans consecutive minima on one side.

beam fall normally on a narrow slit of width a. All points across the slit receive the same incident phase and emit secondary wavelets. In the usual far-field arrangement, a distant screen, or a screen in the focal plane of a convex lens, receives contributions travelling in a common direction θ. The resultant at a screen point is found by adding these wavelets with their different phases. Along the slit normal, $\theta = 0$, their optical paths are equal. All contributions therefore reinforce, producing the bright central maximum. Away from the normal, a point a distance y from one slit edge has an additional path $y \sin\theta$, so the contributions progressively differ in phase.

Step 2: (a) Obtain dark bands and secondary bright bands For $a \sin\theta = \lambda$, divide the slit into two equal halves. Pair every point in one half with the corresponding point in the other half, separated by $a/2$. Each pair has path difference $(a/2)\sin\theta = \lambda/2$; its wavelets are opposite in phase and cancel. This gives the first minimum on either side. For $a \sin\theta = m\lambda$, divide the slit into $2m$ equal strips and pair adjacent strips. Every pair again differs in path by $\lambda/2$, producing the successive minima. Between consecutive minima the cancellation is incomplete, giving secondary maxima. Hence a broad central bright band is flanked symmetrically by alternating dark bands and narrower, less intense bright bands.

Step 3: (b) Compare the angular widths of the fringes The central maximum lies between the first minima, at $\theta \simeq -\lambda/a$ and $\theta \simeq +\lambda/a$. Its angular width is therefore

$$\Delta\theta_{\text{central}} = \frac{2\lambda}{a}.$$

The first secondary bright fringe on the positive side lies between the first and second minima, $\theta_1 \simeq \lambda/a$ and $\theta_2 \simeq 2\lambda/a$. Its angular width is

$$\Delta\theta_{\text{first}} = \theta_2 - \theta_1 = \frac{\lambda}{a} = \frac{1}{2}\Delta\theta_{\text{central}}.$$

The negative side gives the same result. This usual width relation uses the small-angle approximation for a slit much wider than the wavelength; the exact minima are at $\theta_m = \sin^{-1}(m\lambda/a)$.

Step 4: (c) Explain the decreasing intensity of successive maxima The secondary maxima lie approximately midway between successive minima, at the angles quoted in the question,

$$\theta \simeq \left(n + \frac{1}{2}\right)\frac{\lambda}{a}, \qquad n = 1, 2, 3, \ldots.$$

At these small angles, $\beta \simeq (n + 1/2)\pi$, and $\sin^2\beta = 1$. The intensity formula consequently gives

$$\frac{I_n}{I_0} \simeq \frac{1}{\pi^2(n + 1/2)^2}.$$

Thus the intensity falls approximately as the inverse square of the fringe order. Physically, as the angle increases, the wavelets across the slit span more phase cycles and cancel more extensively; their remaining resultant amplitude is smaller. The quoted midpoint positions are approximate: the exact noncentral maxima satisfy $\tan\beta = \beta$, but their intensities also decrease with increasing order.

Step 5: Conclusion Huygens's wavelets produce a central maximum, minima at $a \sin\theta = m\lambda$, and weaker secondary maxima between the minima. For small angles, the central angular width is

Quick Tip to Solve: Measure a bright fringe between its adjacent dark minima: the central fringe spans the negative and positive first minima, whereas each secondary fringe spans consecutive minima on one side.

$2\lambda/a$ and the first secondary-fringe width is λ/a, exactly half in this approximation. Successive secondary maxima weaken because the wavelets increasingly cancel, with the approximate intensities proportional to $1/(n+1/2)^2$.

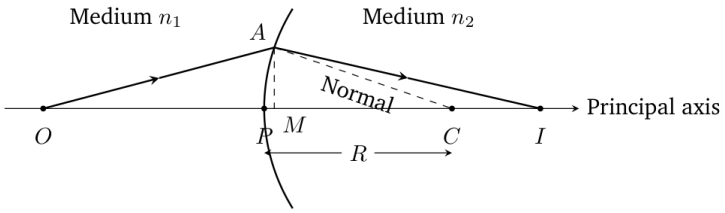
Quick Tip to Solve: Measure a bright fringe between its adjacent dark minima: the central fringe spans the negative and positive first minima, whereas each secondary fringe spans consecutive minima on one side.

OR

25. (a) A point object 'O' is kept in a medium of refractive index n_1 in front of a convex spherical surface of radius of curvature R which separates the second medium of refractive index n_2 from the first one, as shown in the figure.

Draw the ray diagram showing the image formation and deduce the relationship between the object distance and the image distance in terms of n_1, n_2 and R.

(b) When the image formed above acts as a virtual object for a concave spherical surface separating the medium n_2 from n_1 ($n_2 > n_1$), draw this ray diagram and write the similar (similar to (a)) relation. Hence obtain the expression for the lens maker's formula. [5 marks]



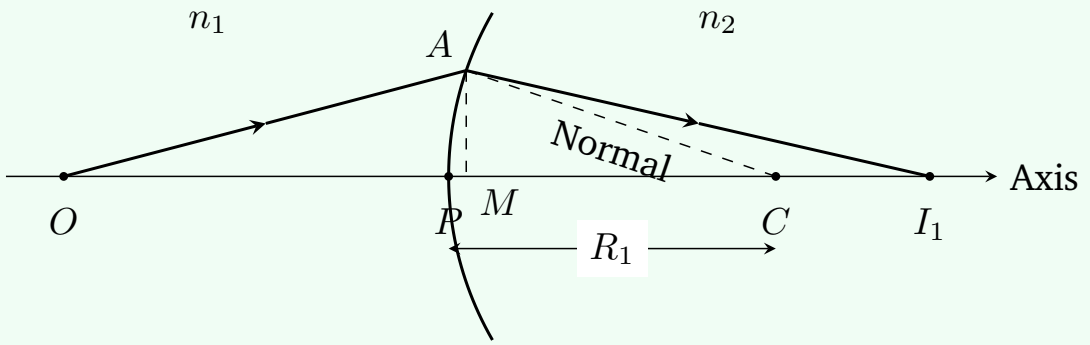
Solution:

Concept:

- For paraxial rays, Snell's law reduces to $n_1 i = n_2 r$, because $\sin\theta \simeq \theta$ for small angles in radians.
- Use the Cartesian sign convention: measure all axial distances from the surface pole, with the direction of incident light positive. Thus a real object on the left has $u < 0$, and a centre on the right gives $R > 0$.
- A thin lens is treated as two spherical refracting surfaces with negligible separation. Its focal length f is the image distance for an object at infinity, in the same external medium on both sides.

Step 1: (a) Draw the rays and define the small angles Let P be the pole, C the centre of curvature and A a point near the axis on the convex surface. The normal at A is AC. An axial ray from O passes through P undeviated; the oblique ray OA refracts towards the normal and meets it at I. Write $h = AM$, where M is the perpendicular projection of A on the principal axis. Let the incident ray, refracted ray and normal make acute angles α, β, γ with the axis. For the illustrated real-image case, $i = \alpha + \gamma$ and $r = \gamma - \beta$. The signed formula derived from this construction also applies to virtual images.

Quick Tip to Solve: Write the incident and emergent indices afresh at each surface. The intermediate image is a positive-distance virtual object at the second surface; its two terms cancel only after using the thin-lens approximation.



Refraction at a convex spherical surface: AC is the normal and I_1 is the intermediate image.

Step 2: Apply Snell's law at the first surface Small angles allow us to replace their tangents by the angles. Neglect the small axial displacement of A from P, giving

$$\alpha \simeq \frac{h}{-u}, \qquad \beta \simeq \frac{h}{v_1}, \qquad \gamma \simeq \frac{h}{R_1}.$$

Here $u = PO < 0$, $v_1 = PI$ and $R_1 = PC$. Consequently,

$$\begin{aligned} n_1(\alpha + \gamma) &= n_2(\gamma - \beta), \\ n_1\alpha + n_2\beta &= (n_2 - n_1)\gamma, \\ -\frac{n_1}{u} + \frac{n_2}{v_1} &= \frac{n_2 - n_1}{R_1}. \end{aligned}$$

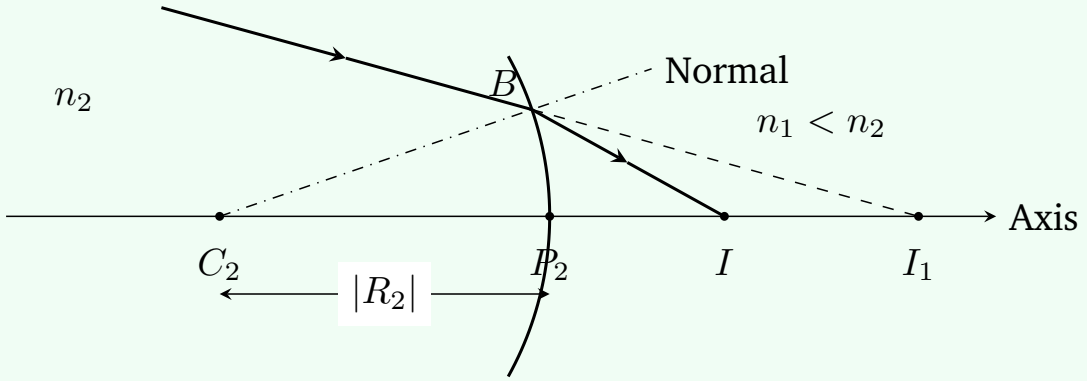
Therefore, for the single convex surface of part (a), set $R_1 = R$. This result follows from refraction and the paraxial approximation, with all distances signed.

Step 3: (b) Refract the converging rays at the second surface Place the concave second surface before the intermediate image I_1 is reached. Within the medium n_2, the incident rays converge towards I_1, so I_1 is a virtual object for this surface. Its object distance u_2 is positive. At the second surface the incident index is n_2, the emergent index is n_1, and the centre lies to the left, giving $R_2 < 0$. Because $n_2 > n_1$, the emergent ray bends away from the normal and converges at the final image I, as shown.

$$\frac{n_1}{v} - \frac{n_2}{u_2} = \frac{n_1 - n_2}{R_2}.$$

Here v is measured from the second pole. For surface separation t, the common intermediate image gives $u_2 = v_1 - t$. This identifies the virtual object correctly; it is not assigned a negative distance merely because it is virtual.

Quick Tip to Solve: Write the incident and emergent indices afresh at each surface. The intermediate image is a positive-distance virtual object at the second surface; its two terms cancel only after using the thin-lens approximation.



Second-surface refraction: dashed continuation shows the virtual object I_1; the solid emergent ray reaches I.

Step 4: Add the two surface relations for a thin lens For a thin lens, neglect t, so $u_2 \simeq v_1$, and the two poles can be treated as coincident. Add the first and second surface equations. The intermediate-image terms cancel:

$$\frac{n_2}{v_1} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1},$$
$$\frac{n_1}{v} - \frac{n_2}{v_1} = \frac{n_1 - n_2}{R_2},$$
$$n_1\left(\frac{1}{v} - \frac{1}{u}\right) = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

Divide by n_1. Then let the object recede to infinity, so $1/u \to 0$ and $v = f$. This gives the lens maker's formula in an external medium of refractive index n_1. In air, use $n_1 \simeq 1$. Since a biconvex lens has $R_1 > 0$ and $R_2 < 0$, its focal length is positive when $n_2 > n_1$, in agreement with its converging action.

Step 5: Conclusion The spherical-surface relation is $\frac{n_2}{v_1} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$. At the second surface, $\frac{n_1}{v} - \frac{n_2}{u_2} = \frac{n_1 - n_2}{R_2}$. For a thin lens,

$$\boxed{\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}.$$

In air this becomes $\frac{1}{f} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

Quick Tip to Solve: Write the incident and emergent indices afresh at each surface. The intermediate image is a positive-distance virtual object at the second surface; its two terms cancel only after using the thin-lens approximation.

26. (a) An electric dipole of dipole moment $\vec{p}$ consists of point charges $+q$ and $-q$ separated by a distance $2a$ apart. Deduce the expression for the electric field $\vec{E}$ due to the dipole at a distance x from the centre of the dipole on its axial line in terms of the dipole moment $\vec{p}$. Hence show that in the limit $x \gg a$, $\vec{E} \to 2\vec{p}/(4\pi\varepsilon_0 x^3)$.

(b) Given the electric field in the region $\vec{E} = 2x\hat{i}$, find the net electric flux through the cube and the charge enclosed by it. [5 marks]

Solution:

Concept:

- The electric dipole moment is $\vec{p} = q(2a)\hat{p}$, where $\hat{p}$ points from the negative charge to the positive charge. Electric fields add vectorially according to the superposition principle.
- Coulomb's law gives the field magnitude of a point charge at distance r as $q/(4\pi\varepsilon_0 r^2)$. On the exterior axial line, the two charge fields are collinear and oppositely directed.
- Electric flux through a closed surface is $\Phi_E = \oint \vec{E} \cdot d\vec{A}$, with $d\vec{A}$ directed outward. Gauss's law relates the net flux to enclosed charge: $\Phi_E = Q_{\text{enc}}/\varepsilon_0$.

Step 1: (a) Set up the axial field contributions Put the dipole centre at O, the charge $-q$ at coordinate $-a$, and $+q$ at $+a$, with the coordinate axis along $\hat{p}$. Consider an exterior axial point P at $x > a$. Its distances from $+q$ and $-q$ are $x - a$ and $x + a$, respectively. The positive charge produces a field along $\hat{p}$, whereas the negative charge produces a field opposite to $\hat{p}$. Hence

$$\vec{E}(P) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right]\hat{p}.$$

The nearer positive charge gives the larger contribution, so the resultant field points in the direction of the dipole moment. At the exterior point on the opposite side, the nearer negative charge also gives a resultant along $\hat{p}$; thus the same expression applies with x interpreted as the positive distance from the centre.

Step 2: (a) Express the exact field in terms of dipole moment Subtract the fractions using their common denominator:

$$\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2} = \frac{(x+a)^2 - (x-a)^2}{(x^2-a^2)^2} = \frac{4ax}{(x^2-a^2)^2}.$$

Since $\vec{p} = 2aq\hat{p}$, substitution converts the charge and separation factors into the dipole moment:

$$\vec{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\frac{2x\vec{p}}{(x^2-a^2)^2}, \qquad x > a.$$

This is the exact expression for an exterior point on the axial line of a finite dipole. The restriction excludes the charge positions, where the point-charge field is singular.

Step 3: (a) Take the far-field limit Factor x^4 from the denominator to obtain

$$\vec{E}_{\text{axial}} = \frac{2\vec{p}}{4\pi\varepsilon_0 x^3}\left(1 - \frac{a^2}{x^2}\right)^{-2}.$$

Quick Tip to Solve: For an axial dipole field, subtract the two inverse-square fields before substituting $p = 2aq$. For $\vec{E} = kx\hat{i}$, opposite-face fluxes immediately give kV, independent of the cube's position.

When $x \gg a$, the ratio a^2/x^2 is negligible and the factor in parentheses tends to unity. Thus the dipole field decreases as the inverse cube of distance, while its axial direction remains parallel to $\vec{p}$. This limiting approximation retains the dipole moment while neglecting the finite separation relative to the observation distance.

Step 4: (b) Evaluate the flux through the cube Let the cube have edge length ℓ, and first take its edges parallel to the coordinate axes. Let its two faces normal to the x-axis lie at $x = x_0$ and $x = x_0 + \ell$. Each has area ℓ^2. The field $\vec{E} = 2x\hat{i}$ is perpendicular to these faces and tangent to the remaining four faces. The outward area vector of the left face points along $-\hat{i}$, while that of the right face points along $+\hat{i}$. Therefore

$$\begin{aligned} \Phi_{\text{left}} &= -2x_0\ell^2, \\ \Phi_{\text{right}} &= 2(x_0+\ell)\ell^2, \\ \Phi_{\text{other four}} &= 0, \\ \Phi_E &= 2(x_0+\ell)\ell^2 - 2x_0\ell^2 = 2\ell^3. \end{aligned}$$

The position x_0 cancels. Equivalently, $\nabla \cdot \vec{E} = \partial(2x)/\partial x = 2$, so the divergence theorem gives $\Phi_E = 2V$, where $V = \ell^3$. This also shows that the net flux depends on the volume and is independent of the cube's orientation. In SI units the coefficient 2 in the field has units $\mathrm{N\,C^{-1}\,m^{-1}}$, ensuring that flux has units $\mathrm{N\,m^2\,C^{-1}}$.

Step 5: (b) Obtain the enclosed charge from Gauss's law Apply Gauss's law to the entire closed cube:

$$Q_{\text{enc}} = \varepsilon_0 \Phi_E = 2\varepsilon_0 \ell^3 = 2\varepsilon_0 V.$$

The enclosed charge is positive because the field has positive divergence: the outward flux through the face at the larger x-coordinate exceeds the inward flux through the opposite face. Expressing the result in terms of the cube's edge or volume gives the charge for any specified cube size.

Step 6: Conclusion (a) For an exterior axial point,

$$\boxed{\vec{E} = \frac{2x\vec{p}}{4\pi\varepsilon_0(x^2-a^2)^2}}, \qquad \boxed{\vec{E} \longrightarrow \frac{2\vec{p}}{4\pi\varepsilon_0 x^3} \quad (x \gg a)}.$$

(b) For a cube of edge ℓ and volume $V = \ell^3$,

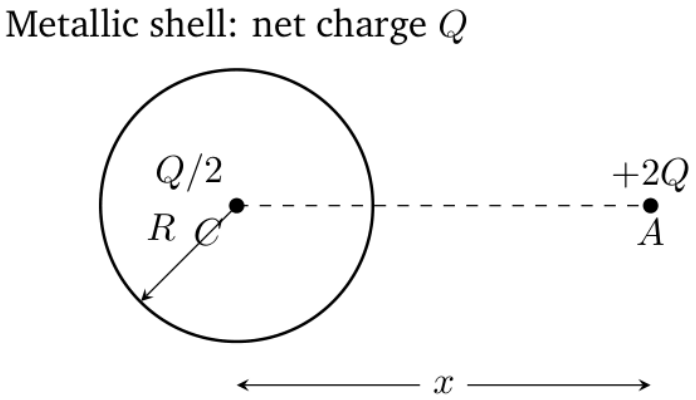
$$\boxed{\Phi_E = 2\ell^3 = 2V}, \qquad \boxed{Q_{\text{enc}} = 2\varepsilon_0\ell^3 = 2\varepsilon_0 V}.$$

Quick Tip to Solve: For an axial dipole field, subtract the two inverse-square fields before substituting $p = 2aq$. For $\vec{E} = kx\hat{i}$, opposite-face fluxes immediately give kV, independent of the cube's position.

OR

26. (a) Explain, using suitable diagrams, the difference in the behaviour of a (i) conductor and (ii) dielectric in the presence of external electric field. Define the terms polarization of a dielectric and write its relation with susceptibility.

(b) A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge $\frac{Q}{2}$ is placed at its centre C and an other charge $+2Q$ is placed outside the shell at a distance x from the centre as shown in the figure. Find (i) the force on the charge at the centre of shell and at the point A, (ii) the electric flux through the shell. [5 marks]



Solution:

Concept:

- Electrostatic equilibrium requires zero electric field in the material of a conductor. Free charges redistribute until their induced field cancels the applied field.
- In a dielectric, bound charges are displaced or permanent molecular dipoles orient. Polarization $\vec{P}$ is electric dipole moment per unit volume; for a linear isotropic dielectric, $\vec{P} = \varepsilon_0 \chi_e \vec{E}$, where $\vec{E}$ is the total internal field and χ_e is electric susceptibility.
- Gauss's law gives $\Phi_E = Q_{\text{enclosed}}/\varepsilon_0$. A closed conducting shell shields its cavity from external electrostatic fields.

Step 1: Explain charge redistribution in a conductor (a)(i) An applied electric field $\vec{E}_0$ drives free electrons opposite to the field. Negative charge accumulates on the left surface and an equal positive charge is exposed on the right. These surface charges produce an induced field directed left. Redistribution stops when

$$\vec{E}_{\text{induced}} = -\vec{E}_0, \qquad \vec{E}_{\text{inside}} = 0.$$

The conductor is then an equipotential. Charges can move through its bulk during redistribution; in equilibrium excess charge resides on its surface.

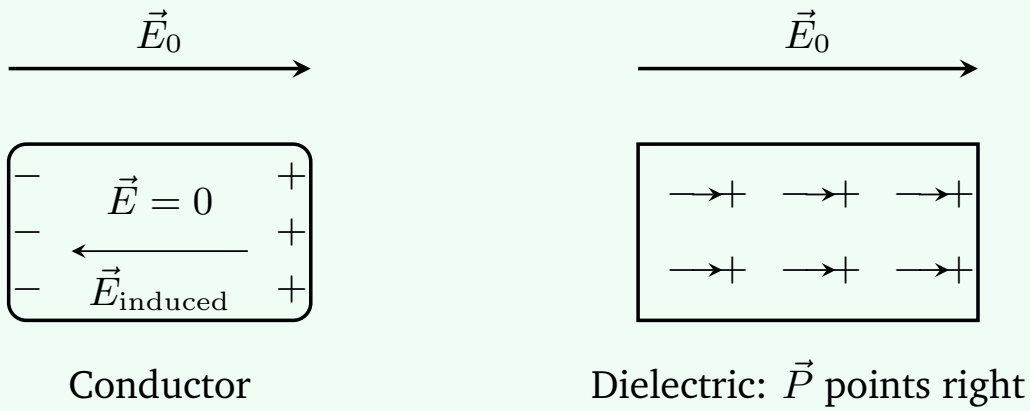
Step 2: Explain polarization in a dielectric (a)(ii) A dielectric has no freely moving charge carriers. The applied field slightly separates positive and negative bound charges in nonpolar molecules, or partially aligns pre-existing dipoles in polar molecules. Dipole moments point along the applied field. Negative bound charge appears on the left face and positive bound charge on the right face. The

Quick Tip to Solve: First use zero field in the metal to find the inner induced charge, then conserve the shell's net charge. Gauss's law fixes flux, but an external point charge destroys outer-surface symmetry, so it cannot alone determine the force at that point.

induced field opposes the applied field, but generally reduces rather than completely cancels it. Thus the internal field remains nonzero. The polarization vector is the net dipole moment per unit volume:

$$\vec{P} = \frac{\sum_i \vec{p}_i}{\Delta V}, \qquad \vec{P} = \varepsilon_0 \chi_e \vec{E}.$$

Its SI unit is C m^{-2}. For a linear isotropic medium, $\varepsilon_r = 1 + \chi_e$; the relation uses the actual field inside the dielectric.



Free-charge redistribution in a conductor and bound-dipole polarization in a dielectric; the applied field points right.

Step 3: Find the force at the centre and the induced surface charges (b)(i) A Gaussian surface lying within the metal has zero flux because the field in the metal is zero. Consequently the inner surface carries charge $-Q/2$. Since the shell itself retains its net charge Q, the outer surface carries

$$Q_{\text{outer}} = Q - (-Q/2) = 3Q/2.$$

The central position makes the inner surface charge uniform. Its electric field at C is zero. The external charge cannot produce a field in the shielded cavity. Excluding the charge's own field, the net force at the centre is therefore zero.

Step 4: Account for the external charge's effect on the outer surface The charge $+2Q$ at A also redistributes charge on the outer conducting surface. Consequently, treating the entire shell as a fixed point charge $3Q/2$ at C is not exact. To calculate the force for arbitrary $x > R$, let $\hat{r}$ point from C to A, and put $k = 1/(4\pi\varepsilon_0)$. The exact external field of the conductor can be represented by two mathematical image charges inside the sphere. The image of $2Q$ is

$$q' = -\frac{2QR}{x}, \qquad b = \frac{R^2}{x},$$

located a distance b from C towards A. For any point on the spherical surface, its distance to this image equals R/x times its distance to A. Their potentials therefore cancel on the surface. An additional central image preserves the required total enclosed charge:

$$q_0 + q' = \frac{3Q}{2}, \qquad q_0 = \frac{3Q}{2} + \frac{2QR}{x}.$$

Quick Tip to Solve: First use zero field in the metal to find the inner induced charge, then conserve the shell's net charge. Gauss's law fixes flux, but an external point charge destroys outer-surface symmetry, so it cannot alone determine the force at that point.

The central image adds a constant potential over the sphere, so this construction satisfies both the equipotential condition and the prescribed charge. The force at A, excluding its own field, follows from the two image fields:

$$\begin{aligned}\vec{F}_A &= 2kQ\left[\frac{q_0}{x^2}+\frac{q'}{(x-b)^2}\right]\hat{r}\\ &= kQ^2\left[\frac{3}{x^2}+\frac{4R}{x^3}-\frac{4Rx}{(x^2-R^2)^2}\right]\hat{r}.\end{aligned}$$

The sign of the bracket specifies attraction or repulsion; sufficiently close to the shell, the attractive induced-charge effect dominates. At large separation $x \gg R$, the induction correction is small and the usual monopole result is $\vec{F}_A \simeq 3kQ^2\hat{r}/x^2$.

Step 5: Apply Gauss's law to the closed shell (b)(ii) Take a Gaussian surface just outside the shell but inside the position of A. It encloses both the shell charge Q and the central charge $Q/2$. The charge $+2Q$ is outside and contributes zero net flux through this closed surface:

$$\Phi_E = \frac{Q+Q/2}{\varepsilon_0} = \frac{3Q}{2\varepsilon_0}.$$

Outer-surface redistribution changes the field locally, but cannot change this net flux.

Step 6: Conclusion A conductor cancels the applied field internally; a dielectric polarizes and generally retains a reduced internal field, with $\vec{P} = \varepsilon_0\chi_e\vec{E}$. The central charge experiences $\vec{F}_C = 0$, and the outward flux through the shell is $3Q/(2\varepsilon_0)$. The exact external force is

$$\vec{F}_A = \frac{Q^2}{4\pi\varepsilon_0}\left[\frac{3}{x^2}+\frac{4R}{x^3}-\frac{4Rx}{(x^2-R^2)^2}\right]\hat{r}.$$

For $x \gg R$, this reduces to $\vec{F}_A \simeq 3Q^2\hat{r}/(4\pi\varepsilon_0 x^2)$.

Quick Tip to Solve: First use zero field in the metal to find the inner induced charge, then conserve the shell's net charge. Gauss's law fixes flux, but an external point charge destroys outer-surface symmetry, so it cannot alone determine the force at that point.