CBSE Class 12 Mathematics 2015 - Set 65/1/A

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.
(ii) Please check that this question paper contains 26 questions.
(iii) Questions 1 – 6 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Questions 7 – 19 in Section B are long-answer I type questions carrying 4 marks each.
(v) Questions 20 – 26 in Section C are long-answer II type questions carrying 6 marks each.
(vi) Please write down the serial number of the question before attempting it.

Section A

1. If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, then for any natural number n, find the value of $\det(A^n)$.

2. Find the sum of the *order* and the *degree* of the following differential equation: $y = x\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2}$.

3. Find the solution of the following differential equation: $x\sqrt{1+y^2}\,dx + y\sqrt{1+x^2}\,dy = 0$.

4. In a triangle OAC, if B is the mid-point of side AC and $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, then what is $\overrightarrow{OC}$?

5. Find a vector of magnitude $\sqrt{171}$ which is perpendicular to both of the vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$.

6. Find the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$.

Section B

7. There are 2 families A and B. There are 4 men, 6 women and 2 children in family A, and 2 men, 2 women and 4 children in family B. The recommended daily amount of calories is 2400 for men, 1900 for women, 1800 for children and 45 grams of proteins for men, 55 grams for women and 33 grams for children. Represent the above information using matrices. Using matrix multiplication, calculate the total requirement of calories and proteins for each of the 2 families. What awareness can you create among people about the balanced diet from this question?

8. Evaluate: $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right)+\frac{\pi}{4}\right\}$.

9. Using properties of determinants, prove that $\begin{vmatrix} a^3 & 2 & a \\ b^3 & 2 & b \\ c^3 & 2 & c \end{vmatrix} = 2(a-b)(b-c)(c-a)(a+b+c)$.

10(a). Using elementary row operations (transformations), find the inverse of the following matrix: $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{pmatrix}$.

OR

10(b). If $A=\begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, $C=\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, then calculate AC, BC and $(A+B)C$. Also verify that $(A+B)C = AC+BC$.

11. Discuss the continuity and differentiability of the function $f(x)=|x|+|x-1|$ in the interval $(-1,2)$.

12. If $x=a(\cos 2t+2t\sin 2t)$ and $y=a(\sin 2t-2t\cos 2t)$, then find $\frac{d^2y}{dx^2}$.

13. If $(ax+b)e^{y/x}=x$, then show that $x^3\left(\frac{d^2y}{dx^2}\right)=\left(x\frac{dy}{dx}-y\right)^2$.

14(a). Evaluate: $\int \frac{\sin x - x\cos x}{x(x+\sin x)}\,dx$.

OR

14(b). Evaluate: $\int \frac{x^3}{(x-1)(x^2+1)}\,dx$.

15. Evaluate: $\int_0^{\pi/2} \frac{\cos^2 x\,dx}{1+3\sin^2 x}$.

16. Evaluate: $\int_0^{\pi/4}\left(\frac{\sin x+\cos x}{3+\sin 2x}\right)dx$.

17. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}\cdot\vec{d}=27$.

18(a). Find the shortest distance between the following lines: $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$, $\vec{r}=(2\hat{i}+4\hat{j}+5\hat{k})+\mu(4\hat{i}+6\hat{j}+8\hat{k})$.

OR

18(b). Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$ and $5x-3y+4z+9=0$ and is parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{5-z}{-5}$.

19(a). A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of 5 steps, he is one step away from the starting point.

OR

19(b). Suppose a girl throws a die. If she gets a 1 or 2, she tosses a coin three times and notes the number of 'tails'. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die?

Section C

20(a). Determine whether the relation R defined on the set $\mathbb{R}$ of all real numbers as $R = \{(a, b) : a, b \in \mathbb{R}$ and $a - b + \sqrt{3} \in S\}$, where S is the set of all irrational numbers, is reflexive, symmetric and transitive.

OR

20(b). Let $A = \mathbb{R} \times \mathbb{R}$ and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Prove that $*$ is commutative and associative. Find the identity element for $*$ on A. Also write the inverse element of the element $(3, -5)$ in A.

21. Tangent to the circle $x^2 + y^2 = 4$ at any point on it in the first quadrant makes intercepts OA and OB on the x and y axes respectively, O being the centre of the circle. Find the minimum value of $(OA + OB)$.

22. If the area bounded by the parabola $y^2 = 16ax$ and the line $y = 4mx$ is $\frac{a^2}{12}$ sq. units, then using integration, find the value of m.

23(a). Show that the differential equation $(x - y)\frac{dy}{dx} = x + 2y$ is homogeneous and solve it also.

OR

23(b). Find the differential equation of the family of curves $(x - h)^2 + (y - k)^2 = r^2$, where h and k are arbitrary constants.

24. Find the equation of a plane passing through the point P(6, 5, 9) and parallel to the plane determined by the points A(3, -1, 2), B(5, 2, 4) and C(-1, -1, 6). Also find the distance of this plane from the point A.

25. An urn contains 5 red and 2 black balls. Two balls are randomly drawn, without replacement. Let X represent the number of black balls drawn. What are the possible values of X? Is X a random variable? If yes, find the mean and variance of X.

26. Solve the following linear programming problem graphically. Minimise $z = 3x + 5y$ subject to the constraints $x + 2y \geq 10$, $x + y \geq 6$, $3x + y \geq 8$, $x, y \geq 0$.

- End of Question Paper -