CBSE Class 12 Mathematics 2015 - Set 65/1/MT

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.
(ii) Please check that this question paper contains 26 questions.
(iii) Questions 1 – 6 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Questions 7 – 19 in Section B are long-answer I type questions carrying 4 marks each.
(v) Questions 20 – 26 in Section C are long-answer II type questions carrying 6 marks each.
(vi) Please write down the serial number of the question before attempting it.

Section A

1. Write a 3×3 skew symmetric matrix.

2. Find the product of the order and degree of the following differential equation : $x\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 + y^2 = 0$.

3. Write a differential equation for $y = A\cos\alpha x + B\sin\alpha x$, where A and B are arbitrary constants.

4. Write the projection of vector $2\hat{i} + 3\hat{j} - \hat{k}$ along the vector $\hat{i} + \hat{j}$.

5. Write the value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$.

6. Write the direction cosines of the normal to the plane $3x + 4y + 12z = 52$.

Section B

7. There are 3 families A, B and C. The number of men, women and children in these families are as under :

	Men	Women	Children
Family A	2	3	1
Family B	2	1	3
Family C	4	2	6

Daily expenses of men, women and children are ₹200, ₹150 and ₹200 respectively. Only men and women earn and children do not. Using matrix multiplication, calculate the daily expenses of each family. What impact does more children in the family create on the society?

8. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, $x, y, z > 0$, then find the value of $xy + yz + zx$.

9. If $a \neq b \neq c$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then using properties of determinants, prove that $a + b + c = 0$.

10(a). If $X \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$, then find the matrix X.

OR

10(b). Find the inverse of matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$ and hence show that $A^{-1} \cdot A = I$.

11. If function $f(x) = |x - 3| + |x - 4|$, then show that $f(x)$ is not differentiable at $x = 3$ and $x = 4$.

12(a). If $y = xe^{-x^2}$, find $\frac{dy}{dx}$.

OR

12(b). If $\log \sqrt{x^2 + y^2} = \tan^{-1}\left(\frac{x}{y}\right)$, then show that $\frac{dy}{dx} = \frac{y - x}{y + x}$.

13. If $y = \sqrt{x + 1} - \sqrt{x - 1}$, prove that $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - \frac{1}{4}y = 0$.

14. Find : $\int \frac{1 - \cos x}{\cos x(1 + \cos x)}\, dx$.

15. Evaluate : $\int x \cdot \sin^{-1} x\, dx$.

16(a). Find $\int_0^2 (x^2 + e^{2x+1})\, dx$ as the limit of a sum.

OR

16(b). Evaluate : $\int_0^{\pi} \frac{x \tan x}{\sec x \cdot \operatorname{cosec} x}\, dx$.

17. Show that the lines $\frac{x - 1}{3} = \frac{y - 1}{-1}, z + 1 = 0$ and $\frac{x - 4}{2} = \frac{z + 1}{3}, y = 0$ intersect each other. Also find their point of intersection.

18(a). Let P(3, 2, 6) be a point in the space and Q be a point on the line $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$, then find the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$.

OR

18(b). Find the vector and cartesian equations of the plane which bisects the line joining the points $(3, -2, 1)$ and $(1, 4, -3)$ at right angles.

19. From a set of 100 cards numbered 1 to 100, one card is drawn at random. Find the probability that the number on the card is divisible by 6 or 8, but not by 24.

Section C

20. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a - b| is divisible by 2} is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other, but no element of {1, 3, 5} is related to any element of {2, 4}.

21. Using integration, find the area bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$.

22. Find the point on the curve $y = \frac{x}{1+x^2}$, where the tangent to the curve has the greatest slope.

23(a). Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$.

OR

23(b). Solve the following differential equation, given that $y = 0$, when $x = \frac{\pi}{4}$: $\sin 2x \frac{dy}{dx} - y = \tan x$.

24. Find the vector and cartesian equations of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ and the point (1, 1, 1).

25. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduces the risk of heart attack by 30% and the prescription of a certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.

26(a). A dealer deals in two items only – item A and item B. He has ₹50,000 to invest and a space to store at most 60 items. An item A costs ₹2,500 and an item B costs ₹500. A net profit to him on item A is ₹500 and on item B ₹150. If he can sell all the items that he purchases, how should he invest his amount to have maximum profit? Formulate an LPP and solve it graphically.

OR

26(b). A dietician wants to develop a special diet using two foods X and Y. Each packet (contains 30 g) of food X contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Y contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. Make an LPP to find how many packets of each food should be used to minimise the amount of vitamin A in the diet, and solve it graphically.

- End of Question Paper -