# CBSE Class 12 Mathematics 2015 - Set 65/2/G

## Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

**Time Allowed: 3 Hours** **Maximum Marks: 100**

### General Instructions

(i) All questions are compulsory.
(ii) Please check that this question paper contains 26 questions.
(iii) Questions 1 – 6 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Questions 7 – 19 in Section B are long-answer I type questions carrying 4 marks each.
(v) Questions 20 – 26 in Section C are long-answer II type questions carrying 6 marks each.
(vi) Please write down the serial number of the question before attempting it.

## Section A

**1.** Find the value of $a + b$, if the points $(2, a, 3)$, $(3, -5, b)$ and $(-1, 11, 9)$ are collinear.

**2.** Find the value of $\vec{a} \cdot \vec{b}$, if $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $|\vec{a} \times \vec{b}| = 16$.

**3.** Find the distance between the parallel planes $\vec{r} \cdot (2\hat{i} - \hat{j} - 2\hat{k}) = 6$ and $\vec{r} \cdot (6\hat{i} - 3\hat{j} - 6\hat{k}) = 27$.

**4.** If $A = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 5 \\ 3 & 4 \end{pmatrix}$, then find $(3A - B)$.

**5.** Find the differential equation representing the curve $y = e^{-x} + ax + b$, where $a$ and $b$ are arbitrary constants.

**6.** Write the sum of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right)^3 = y^3$.

## Section B

**7.** Examine the following function $f(x)$ for continuity at $x = 1$ and differentiability at $x = 2$:

$$f(x) = \begin{cases} 5x - 4, & 0 < x < 1 \\ 4x^2 - 3x, & 1 < x < 2 \\ 3x + 4, & x \geq 2 \end{cases}.$$

**8(a).** If $y = x^3 \log\left(\frac{1}{x}\right)$, then prove that $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$.

OR

**8(b).** Verify the mean value theorem for the function $f(x) = (x - 4)(x - 6)(x - 8)$ on the interval $[4, 10]$.

**9.** If $\frac{x}{x - y} = \log\left(\frac{a}{x - y}\right)$, then prove that $\frac{dy}{dx} = 2 - \frac{x}{y}$.

**10.** Find: $\int \frac{dx}{x^3(x^5 + 1)^{3/5}}$.

**11(a).** Evaluate: $\int_2^4 \{|x - 2| + |x - 3| + |x - 4|\}\, dx$.

OR

**11(b).** Evaluate: $\int_0^{\pi/4} \frac{\sec x}{1 + 2\sin^2 x}\, dx$.

**12(a).** Prove the following: $\sin\left[\tan^{-1}\left(\frac{1 - x^2}{2x}\right) + \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)\right] = 1, 0 < x < 1$.

OR

**12(b).** If $\tan^{-1}\left(\frac{x - 5}{x - 6}\right) + \tan^{-1}\left(\frac{x + 5}{x + 6}\right) = \frac{\pi}{4}$, then find the value of $x$.

**13.** Using the properties of determinants, prove that: $\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$.

**14(a).** For the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, show that $A^2 - 5A + 4I = O$. Hence find $A^{-1}$.

OR

**14(b).** Using elementary transformations, find the inverse of the following matrix: $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{pmatrix}$.

**15.** Bag I contains 4 red and 5 black balls and bag II contains 3 red and 4 black balls. One ball is transferred from bag I to bag II and then two balls are drawn at random (without replacement) from bag II. The balls so drawn are both found to be black. Find the probability that the transferred ball is black.

**16.** Find the vector and cartesian equations of a line through the point $(1, -1, 1)$ and perpendicular to the lines joining the points $(4, 3, 2)$, $(1, -1, 0)$ and $(1, 2, -1)$, $(2, 1, 1)$.

**17.** Three schools A, B and C want to award their selected students for the values of Honesty, Regularity and Hard work. Each school decided to award a sum of ₹2,500, ₹3,100 and ₹5,100 per student for the respective values. The number of students to be awarded by the three schools is given below in the table:

| Values \\ School | A | B | C |
|---|---|---|---|
| Honesty | 3 | 4 | 6 |
| Regularity | 4 | 5 | 2 |
| Hard work | 6 | 3 | 4 |

Find the total money given in awards by the three schools separately, using matrices. Apart from the above given values, suggest one more value which should be considered for giving award.

**18.** Evaluate: $\int_{\pi/4}^{\pi/2} e^{2x} \left( \frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$.

**19.** Show that the four points with position vectors $4\hat{i} + 8\hat{j} + 12\hat{k}$, $2\hat{i} + 4\hat{j} + 6\hat{k}$, $3\hat{i} + 5\hat{j} + 4\hat{k}$ and $5\hat{i} + 8\hat{j} + 5\hat{k}$ are coplanar.

## Section C

**20.** Using integration, find the area of the region bounded by the line $y - 1 = x$, the x-axis and the ordinates $x = -2$ and $x = 3$.

**21.** Find the particular solution of the differential equation $(y - \sin x)\,dx + (\tan x)\,dy = 0$ satisfying the condition that $y = 0$ when $x = 0$.

**22.** Find the value of $k$ for which the following lines are perpendicular to each other: $\frac{x+3}{k-5} = \frac{y-1}{1} = \frac{5-z}{-2k-1}$; $\frac{x+2}{-1} = \frac{2-y}{-k} = \frac{z}{5}$. Hence find the equation of the plane containing the above lines.

**23(a).** Check whether the operation $*$ defined on the set $A = R \times R$ as $(a, b) * (c, d) = (a + c, b + d)$ is a binary operation or not, where $R$ is the set of all real numbers. If it is a binary operation, is it commutative and associative too? Also find the identity element of $*$.

OR

**23(b).** Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g : A \to B$ be functions defined by $f(x) = x^2 - x$, $x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1$, $x \in A$. Find gof$(x)$ and hence show that $f = g =$ gof.

**24(a).** Find the equations of the tangent and the normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.

OR

**24(b).** Find the absolute maximum and absolute minimum values of the function $f$ given by $f(x) = \cos^2 x + \sin x$, $x \in [0, \pi]$.

**25.** An unbiased coin is tossed 'n' times. Let the random variable $X$ denote the number of times the head occurs. If $P(X = 1)$, $P(X = 2)$ and $P(X = 3)$ are in AP, find the value of $n$.

**26.** The standard weight of a special purpose brick is 5 kg and it must contain two basic ingredients $B_1$ and $B_2$. $B_1$ costs ₹5 per kg and $B_2$ costs ₹8 per kg. Strength considerations dictate that the brick should contain not more than 4 kg of $B_1$ and minimum 2 kg of $B_2$. Since the demand for the product is likely to be related to the price of the brick, find the minimum cost of brick satisfying the above conditions. Formulate this situation as an LPP and solve it graphically.

**- End of Question Paper -**