

CBSE Class 12 Mathematics 2015 - Set 65/2/1/F



Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours

Maximum Marks: 100

General Instructions

- (i) All questions are compulsory.
- (ii) Please check that this question paper contains 26 questions.
- (iii) Questions 1 – 6 in Section A are very short-answer type questions carrying 1 mark each.
- (iv) Questions 7 – 19 in Section B are long-answer I type questions carrying 4 marks each.
- (v) Questions 20 – 26 in Section C are long-answer II type questions carrying 6 marks each.
- (vi) Please write down the serial number of the question before attempting it.

Section A

1. Find the unit vector in the direction of the sum of the vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$.

2. Find the area of a parallelogram whose adjacent sides are represented by the vectors $2\hat{i} - 3\hat{k}$ and $4\hat{j} + 2\hat{k}$.

3. Find the sum of the intercepts cut off by the plane $2x + y - z = 5$, on the coordinate axes.

4. If $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then write the cofactor of the element a_{21} of its 2nd row.

5. Write the sum of the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$.

6. Write the solution of the differential equation $\frac{dy}{dx} = 2^{-y}$.

Section B

7(a). If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2 = 4A - 3I$. Hence find A^{-1} .

OR

7(b). If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then find the values of a and b .

8. Using properties of determinants, prove the following: $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (1 - a^3)^2$.

9(a). Evaluate: $\int \frac{\sin(x-a)}{\sin(x+a)} dx$.

OR

9(b). Evaluate: $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$.

10. Evaluate: $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$.

11(a). Three machines E_1 , E_2 and E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E_1 and E_2 are defective and that 5% of those produced by machine E_3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective.

OR

11(b). Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6, and 7. Let X denote the larger of the two numbers obtained. Find the mean and variance of the probability distribution of X .

12. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two side vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC . Find the length of the median through A .

13. Find the equation of a plane which passes through the point $(3, 2, 0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

14(a). If $2 \tan^{-1}(\cos \theta) = \tan^{-1}(2 \operatorname{cosec} \theta)$, ($\theta \neq 0$), then find the value of θ .

OR

14(b). If $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \dots + \tan^{-1}\left(\frac{1}{1+n.(n+1)}\right) = \tan^{-1} \theta$, then find the value of θ .

15. Find the point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts on the axes.

16. If $y = (x + \sqrt{1+x^2})^n$, then show that $(1+x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = n^2 y$.

17. Find whether the following function is differentiable at $x = 1$ and $x = 2$ or not: $f(x) =$

$$\begin{cases} x, & x < 1 \\ 2-x, & 1 \leq x \leq 2. \\ -2+3x-x^2, & x > 2 \end{cases}$$

18. In a parliament election, a political party hired a public relations firm to promote its candidates in three ways: telephone, house calls and letters. The cost per contact (in paise) is

given in matrix A as $A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix}$, the three rows

representing Telephone, House Call and Letters respectively. The number of contacts of each type made in two cities X and Y is given in the matrix

B as $B = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}$, where the

columns represent Telephone, House Call and Letters, and the rows represent City X and City Y respectively.

Find the total amount spent by the party in the two cities. What should one consider before casting his/her vote: party's promotional activity or their social activities?

19. Evaluate: $\int e^{2x} \cdot \sin(3x+1) dx$.

Section C

20. Let $f : N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \rightarrow S$, where S is the range of f , is invertible. Also find the inverse of f .

21. Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and y -axis.

22(a). If $xy = c^2$, then find the minimum value of $(ax + by)$.

OR

22(b). Find the coordinates of a point of the parabola $y = x^2 + 7x + 2$ which is closest to the straight line $y = 3x - 3$.

23. Maximise $z = 8x + 9y$ subject to the constraints given below: $2x + 3y \leq 6$, $3x - 2y \leq 6$, $y \leq 1$, $x, y \geq 0$.

24. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line whose direction cosines are proportional to $2, 3, -6$.

25(a). Solve the following differential equation:
$$\left[y - x \cos \left(\frac{y}{x} \right) \right] dy + \left[y \cos \left(\frac{y}{x} \right) - 2x \sin \left(\frac{y}{x} \right) \right] dx = 0.$$

OR

25(b). Solve the following differential equation:
$$\sqrt{1 + x^2 + y^2 + x^2 y^2} dx + xy dy = 0.$$

26. Find the probability distribution of the number of doublets in four throws of a pair of dice. Also find the mean and variance of this distribution.

- End of Question Paper -