CBSE Class 12 Mathematics 2015 - Set 65/3/B

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.
(ii) Please check that this question paper contains 26 questions.
(iii) Questions 1 – 6 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Questions 7 – 19 in Section B are long-answer I type questions carrying 4 marks each.
(v) Questions 20 – 26 in Section C are long-answer II type questions carrying 6 marks each.
(vi) Please write down the serial number of the question before attempting it.

Section A

1. Find the sum of the degree and the order for the following differential equation:
$\frac{d}{dx}\left[\left(\frac{d^2y}{dx^2}\right)^4\right] = 0.$

2. Find the integrating factor of the following differential equation: $x \log x \frac{dy}{dx} + y = 2 \log x$.

3. Find the value of $(x + y)$ from the following matrix equation: $2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$.

4. If $|\vec{a}| = a$, then find the value of the following: $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$.

5. Find the angle θ, between the line $\frac{x-2}{3} = \frac{y-3}{5} = \frac{z-4}{4}$ and the plane $2x - 2y + z - 5 = 0$.

6. The vectors $\vec{a} = 3\hat{i} + x\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ are mutually perpendicular. If $|\vec{a}| = |\vec{b}|$, then find the value of y.

Section B

7. Find: $\int \frac{x\,dx}{1+x\tan x}$.

8. Find: $\int \frac{x^4}{(x-1)(x^2+1)}\,dx$.

9. Find a unit vector perpendicular to the plane of triangle ABC, where the coordinates of its vertices are $A(3,-1,2)$, $B(1,-1,-3)$ and $C(4,-3,1)$.

10(a). Find the shortest distance between the lines $x+1=2y=-12z$ and $x=y+2=6z-6$.

OR

10(b). From the point $P(a,b,c)$, perpendiculars PL and PM are drawn to YZ and ZX planes respectively. Find the equation of the plane OLM.

11(a). In 3 trials of a binomial distribution, the probability of exactly 2 successes is 9 times the probability of 3 successes. Find the probability of success in each trial.

OR

11(b). An urn contains 3 red and 5 black balls. A ball is drawn at random, its colour is noted and returned to the urn. Moreover, 2 additional balls of the colour noted down, are put in the urn and then two balls are drawn at random (without replacement) from the urn. Find the probability that both the balls drawn are of red colour.

12. If $A=\begin{pmatrix}1 & -1 & 0\\ 2 & 5 & 3\\ 0 & 2 & 1\end{pmatrix}$, find A^{-1} using elementary row transformations.

13. Let $f(x)=x-|x-x^2|$, $x\in[-1,1]$. Find the point of discontinuity, (if any), of this function on $[-1,1]$.

14. If $y=\log\left(\frac{x}{a+bx}\right)^x$, prove that $x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2$.

15. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x=\frac{1}{2}$.

16(a). Evaluate: $\int_0^{\pi/2}\left(\frac{5\sin x+3\cos x}{\sin x+\cos x}\right)dx$.

OR

16(b). Find: $\int\left[\log\log x+\frac{1}{(\log x)^2}\right]dx$.

17. A trust caring for handicapped children gets ₹ 30,000 every month from its donors. The trust spends half of the funds received for medical and educational care of the children and for that it charges 2% of the spent amount from them, and deposits the balance amount in a private bank to get the money multiplied so that in future the trust goes on functioning regularly. What percent of interest should the trust get from the bank to get a total of ₹ 1,800 every month? Use matrix method, to find the rate of interest. Do you think people should donate to such trusts?

18(a). Find the value of x, if $\sin\left[\cot^{-1}(x+1)\right]=\cos\left(\tan^{-1}x\right)$.

OR

18(b). Prove the following: $2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$.

19. If $a+b+c\neq 0$ and $\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}=0$, then using properties of determinants, prove that $a=b=c$.

Section C

20(a). Suppose a boy throws a die. If he gets a 1 or 2, he tosses a coin three times and notes down the number of heads. If he gets 3, 4, 5 or 6 he tosses the coin once and notes down whether a head or a tail is obtained. If he obtains exactly one head, what is the probability that he obtained 3, 4, 5, or 6 with the die?

OR

20(b). A man is known to speak truth 3 out of 5 times. He throws a die and reports that it is 4. Find the probability that it is actually a 4.

21. A dealer in a rural area wishes to purchase some sewing machines. He has only ₹ 57,600 to invest and has space for at most 20 items. An electronic machine costs him ₹ 3,600 and a manually operated machine costs ₹ 2,400. He can sell an electronic machine at a profit of ₹ 220 and a manually operated machine at a profit of ₹ 180. Assuming that he can sell all the machines that he buys, how should he invest his money in order to maximize his profit? Make it as a LPP and solve it graphically.

22. On the set $\{0, 1, 2, 3, 4, 5, 6\}$, a binary operation $*$ is defined as: $a * b = \begin{cases} a+b, & \text{if } a+b<7 \\ a+b-7, & \text{if } a+b \geq 7 \end{cases}$. Write the operation table of the operation $*$ and prove that zero is the identity for this operation and each element $a \neq 0$ of the set is invertible with $7-a$ being the inverse of a.

23. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ with its vertex at one end of the major axis.

24(a). $(x^2+y^2)\,dy = xy\,dx$. If $y(1)=1$ and $y(x_0)=e$, then find the value of x_0.

OR

24(b). Find the particular solution of the differential equation $\frac{dy}{dx}+y\tan x = 3x^2 + x^3\tan x$, $x \neq \frac{\pi}{2}$, given that $y=0$ when $x=\frac{\pi}{3}$.

25. Find the coordinates of the point where the line through the points $A(3,4,1)$ and $B(5,1,6)$ crosses the plane determined by the points $P(2,1,2)$, $Q(3,1,0)$ and $R(4,-2,1)$.

26. Find the area of the region $\{(x,y) : x^2+y^2 \leq 4, x+y \geq 2\}$, using the method of integration.

- End of Question Paper -