CBSE Class 12 Mathematics 2015 - Set 65/3/C

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.
(ii) Please check that this question paper contains 26 questions.
(iii) Questions 1 – 6 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Questions 7 – 19 in Section B are long-answer I type questions carrying 4 marks each.
(v) Questions 20 – 26 in Section C are long-answer II type questions carrying 6 marks each.
(vi) Please write down the serial number of the question before attempting it.

Section A

1. Find the angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$.

2. Find the distance of a point $(2, 5, -3)$ from the plane $\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$.

3. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$, then find $|\vec{a} \times \vec{b}|$.

4. Find the differential equation of the family of lines passing through the origin.

5. Find the integrating factor for the following differential equation : $x \log x \frac{dy}{dx} + y = 2 \log x$.

6. Write the element a_{12} of the matrix $A = [a_{ij}]_{2\times 2}$, whose elements a_{ij} are given by $a_{ij} = e^{2ix} \sin jx$.

Section B

7. If $y = e^{ax} \cdot \cos bx$, then prove that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$.

8. If $x^x + x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

9. If $x = a\sin 2t\,(1 + \cos 2t)$ and $y = b\cos 2t\,(1 - \cos 2t)$, then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

10. Evaluate : $\int \frac{(x+3)e^x}{(x+5)^3}\,dx$.

11. Three schools X, Y and Z organized a fete (mela) for collecting funds for flood victims in which they sold hand-held fans, mats and toys made from recycled material, the sale price of each being ₹25, ₹100 and ₹50 respectively. The following table shows the number of articles of each type sold :

Article \\ School	X	Y	Z
Hand-held fans	30	40	35
Mats	12	15	20
Toys	70	55	75

Using matrices, find the funds collected by each school by selling the above articles and the total funds collected. Also write any one value generated by the above situation.

12. Evaluate : $\int \frac{x^2}{x^4 + x^2 - 2}\,dx$.

13(a). In a set of 10 coins, 2 coins are with heads on both the sides. A coin is selected at random from this set and tossed five times. If all the five times, the result was heads, find the probability that the selected coin had heads on both the sides.

OR

13(b). How many times must a fair coin be tossed so that the probability of getting at least one head is more than 80% ?

14. Find x such that the four points $A(4, 1, 2)$, $B(5, x, 6)$, $C(5, 1, -1)$ and $D(7, 4, 0)$ are coplanar.

15. A line passing through the point A with position vector $\vec{a} = 4\hat{i} + 2\hat{j} + 2\hat{k}$ is parallel to the vector $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$. Find the length of the perpendicular drawn on this line from a point P with position vector $\vec{r_1} = \hat{i} + 2\hat{j} + 3\hat{k}$.

16(a). Solve the following for x : $\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$.

OR

16(b). Show that : $2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$.

17(a). If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then show that $A^2 - 4A - 5I = O$, and hence find A^{-1}.

OR

17(b). If $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, then find A^{-1} using elementary row operations.

18. Using the properties of determinants, solve the following for x : $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$.

19(a). Evaluate : $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\,dx$.

OR

19(b). Evaluate $\int_{-1}^{2} (e^{3x} + 7x - 5)\,dx$ as a limit of sums.

Section C

20. Minimise and maximise $z = 5x + 2y$ subject to the following constraints :

$x - 2y \leq 2$

$3x + 2y \leq 12$

$-3x + 2y \leq 3$

$x \geq 0, y \geq 0$

21. Two numbers are selected at random (without replacement) from first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of X. Find the mean and variance of this distribution.

22(a). Let $A = Q \times Q$, where Q is the set of all rational numbers, and $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Then find

(i) The identity element of $*$ in A.

(ii) Invertible elements of A, and hence write the inverse of elements $(5, 3)$ and $\left(\frac{1}{2}, 4\right)$.

OR

22(b). Let $f : W \to W$ be defined as $f(n) = \begin{cases} n - 1, & \text{if } n \text{ is odd} \\ n + 1, & \text{if } n \text{ is even} \end{cases}$. Show that f is invertible and find the inverse of f. Here, W is the set of all whole numbers.

23. Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area using integration.

24. Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \sin^2 x - \cos x$, $x \in [0, \pi]$.

25. Show that the lines :

$\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\vec{r} = 4\hat{j} + 2\hat{k} + \mu(2\hat{i} - \hat{j} + 3\hat{k})$ are coplanar.

Also, find the equation of the plane containing these lines.

26(a). Find the particular solution of the differential equation $x^2\, dy = (2xy + y^2)\, dx$, given that $y = 1$ when $x = 1$.

OR

26(b). Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} = (e^{m \tan^{-1} x} - y)$, given that $y = 1$ when $x = 0$.

- End of Question Paper -