

CBSE Class 12 Mathematics 2015 - Set 65/1/1 - Compartment



Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours

Maximum Marks: 100

General Instructions

- (i) All questions are compulsory.
- (ii) Please check that this Question Paper contains 26 Questions.
- (iii) Questions 1 – 6 in Section-A are Very Short Answer Type Questions carrying 1 mark each.
- (iv) Questions 7 – 19 in Section-B are Long Answer I Type Questions carrying 4 marks each.
- (v) Questions 20 – 26 in Section-C are Long Answer II Type Questions carrying 6 marks each.
- (vi) Please write down the serial number of the Question before attempting it.

Section A

1. Find a vector in the direction of $\vec{a} = \hat{i} - 2\hat{j}$ that has magnitude 7 units.

2. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ so that $\sqrt{2}\vec{a} - \vec{b}$ is a unit vector?

3. If a line makes angles α, β, γ with the positive direction of coordinate axes, then write the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$.

4. If $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ -1 & 1 \end{pmatrix}$, write the value of $|AB|$.

5. Write the differential equation obtained by eliminating the arbitrary constant C in the equation representing the family of curves $xy = C \cos x$.

6. Write the sum of the order and degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^4 = 7 \left(\frac{d^2y}{dx^2}\right)^3$.

Section B

7. Two schools A and B decided to award prizes to their students for three values, team spirit, truthfulness and tolerance at the rate of ₹ x , ₹ y and ₹ z per student respectively. School A , decided to award a total of ₹ 1,100 for the three values to 3, 1 and 2 students respectively while school B decided to award ₹ 1,400 for the three values to 1, 2 and 3 students respectively. If one prize for all the three values together amount to ₹ 600 then

- (i) Represent the above situation by a matrix equation after forming linear equations.
- (ii) Is it possible to solve the system of equations so obtained using matrices?
- (iii) Which value you prefer to be rewarded most and why?

8(a). If $\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = O$, find x .

OR

8(b). Find the inverse of the matrix $A =$

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}.$$

9. Using properties of determinants, prove that

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2.$$

10. Evaluate: $\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}.$

11(a). Find: $\int \frac{x}{(x-1)^2(x+2)} dx.$

OR

11(b). Find: $\int \frac{x+2}{2x^2+6x+5} dx.$

12(a). From a lot of 15 bulbs which include 5 defectives, a sample of 2 bulbs is drawn at random (without replacement). Find the probability distribution of the number of defective bulbs.

OR

12(b). Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

- (i) the problem is solved
- (ii) exactly one of them solves the problem.

13. Find the value of λ so that the four points A, B, C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + \lambda\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ respectively are coplanar.

14. Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$.

15(a). Solve: $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$.

OR

15(b). Prove that: $\tan^{-1} \frac{63}{16} = \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$.

16. If $y = \frac{x \cos^{-1} x}{\sqrt{1-x^2}} - \log \sqrt{1-x^2}$, then prove that $\frac{dy}{dx} = \frac{\cos^{-1} x}{(1-x^2)^{3/2}}$.

17. Find the derivative of $(\sin x)^x + \sin^{-1} \sqrt{x}$ w.r.t. x .

18. If $x = a \sec^3 \theta$, $y = a \tan^3 \theta$, find $\frac{d^2 y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

19. Find $\int \frac{(x^2 + 1)e^x}{(x + 1)^2} dx$.

Section C

20. Let $A = \mathbb{R} \times \mathbb{R}$ and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that $*$ is commutative and associative. Find the identity element for $*$ on A .

21. Find the area of the region in the first quadrant enclosed by the y -axis, the line $y = x$ and the circle $x^2 + y^2 = 32$, using integration.

22(a). Find the particular solution of the differential equation $x \frac{dy}{dx} + y - x + xy \cot x = 0$; $x \neq 0$, given that when $x = \frac{\pi}{2}$, $y = 0$.

OR

22(b). Solve the differential equation $x^2 dy + (xy + y^2) dx = 0$ given $y = 1$, when $x = 1$.

23. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. Then find the distance of plane thus obtained from the point $A(1, 3, 6)$.

24. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random without replacement from the bag and are found to be both red. Find the probability that the balls are drawn from the first bag.

25. One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Make an L.P.P. of the above and solve it graphically.

26(a). A tank with rectangular base and rectangular sides open at the top is to be constructed so that its depth is 3 m and volume is 75 m^3 . If building of tank costs ₹ 100 per square metre for the base and ₹ 50 per square metres for the sides, find the cost of least expensive tank.

OR

26(b). A point on the hypotenuse of a right triangle is at distances 'a' and 'b' from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$.

- End of Question Paper -