CBSE Class 12 Mathematics 2015 - Set 65/3/P

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.
(ii) Please check that this question paper contains 26 questions.
(iii) Questions 1 – 6 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Questions 7 – 19 in Section B are long-answer I type questions carrying 4 marks each.
(v) Questions 20 – 26 in Section C are long-answer II type questions carrying 6 marks each.
(vi) Please write down the serial number of the question before attempting it.

Section A

1. Find the differential equation representing the curve $y = cx + c^2$.

2. Write the integrating factor of the following differential equation: $(1 + y^2)\,dx - (\tan^{-1} y - x)\,dy = 0$.

3. If $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$, then write A^{-1}.

4. If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 5\hat{i} - 4\hat{j} + 3\hat{k}$, then find the value of $(\vec{a} + \vec{b}) \cdot \vec{c}$.

5. Write the direction ratios of the following line: $x = -3,\ \dfrac{y-4}{3} = \dfrac{2-z}{1}$.

6. Write the value of $\vec{a} \cdot (\vec{b} \times \vec{a})$.

Section B

7. Evaluate: $\int (\sqrt{\cot x} + \sqrt{\tan x})\, dx$.

8. Find: $\int \frac{x^3 - 1}{x^3 + x}\, dx$.

9. Show that four points A, B, C and D whose position vectors are $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ respectively are coplanar.

10(a). Show that the following two lines are coplanar: $\frac{x - a + d}{\alpha - \delta} = \frac{y - a}{\alpha} = \frac{z - a - d}{\alpha + \delta}$ and $\frac{x - b + c}{\beta - \gamma} = \frac{y - b}{\beta} = \frac{z - b - c}{\beta + \gamma}$.

OR

10(b). Find the acute angle between the plane $5x - 4y + 7z - 13 = 0$ and the y-axis.

11(a). A and B throw a die alternately till one of them gets a number greater than four and wins the game. If A starts the game, what is the probability of B winning?

OR

11(b). A die is thrown three times. Events A and B are defined as below:

A: 5 on the first and 6 on the second throw.

B: 3 or 4 on the third throw.

Find the probability of B, given that A has already occurred.

12. Using the properties of determinants, prove the following:
$\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(1-x) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix} = 6x^2(1 - x^2)$.

13. If $x = \alpha \sin 2t\,(1 + \cos 2t)$ and $y = \beta \cos 2t\,(1 - \cos 2t)$, show that $\frac{dy}{dx} = \frac{\beta}{\alpha} \tan t$.

14. Find: $\frac{d}{dx} \cos^{-1}\left(\frac{x - x^{-1}}{x + x^{-1}}\right)$.

15. Find the derivative of the following function $f(x)$ w.r.t. x, at $x = 1$: $\cos^{-1}\left[\sin\sqrt{\frac{1+x}{2}}\right] + x^x$.

16(a). Evaluate: $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}}\, dx$.

OR

16(b). Evaluate: $\int_0^{3/2} |x \cdot \cos(\pi x)|\, dx$.

17. To raise money for an orphanage, students of three schools A, B and C organised an exhibition in their locality, where they sold paper bags, scrap-books and pastel sheets made by them using recycled paper, at the rate of ₹ 20, ₹ 15 and ₹ 5 per unit respectively. School A sold 25 paper bags, 12 scrap-books and 34 pastel sheets. School B sold 22 paper bags, 15 scrap-books and 28 pastel sheets while School C sold 26 paper bags, 18 scrap-books and 36 pastel sheets. Using matrices, find the total amount raised by each school.

By such exhibition, which values are generated in the students?

18(a). Prove that: $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) = \cos^{-1}\left(\frac{a\cos x + b}{a + b\cos x}\right)$.

OR

18(b). Solve the following for x:
$\tan^{-1}\left(\frac{x-2}{x-3}\right) + \tan^{-1}\left(\frac{x+2}{x+3}\right) = \frac{\pi}{4}$, $|x| < 1$.

19. If $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$, find $A^2 - 5A + 16I$.

Section C

20. 40% students of a college reside in hostel and the remaining reside outside. At the end of the year, 50% of the hostelers got A grade while from outside students, only 30% got A grade in the examination. At the end of the year, a student of the college was chosen at random and was found to have gotten A grade. What is the probability that the selected student was a hosteler?

21. The postmaster of a local post office wishes to hire extra helpers during the Deepawali season, because of a large increase in the volume of mail handling and delivery. Because of the limited office space and the budgetary conditions, the number of temporary helpers must not exceed 10. According to past experience, a man can handle 300 letters and 80 packages per day, on the average, and a woman can handle 400 letters and 50 packets per day. The postmaster believes that the daily volume of extra mail and packages will be no less than 3400 and 680 respectively. A man receives ₹ 225 a day and a woman receives ₹ 200 a day. How many men and women helpers should be hired to keep the pay-roll at a minimum? Formulate an LPP and solve it graphically.

22(a). If the function $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x - 3$ and $g : \mathbb{R} \to \mathbb{R}$ by $g(x) = x^3 + 5$, then find the value of $(fog)^{-1}(x)$.

OR

22(b). Let $A = Q \times Q$, where Q is the set of all rational numbers, and $*$ be a binary operation defined on A by $(a, b) * (c, d) = (ac, b + ad)$, for all $(a, b), (c, d) \in A$. Find

(i) the identity element in A.

(ii) the invertible element of A.

23. If the function $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1$, where $m > 0$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then find the value of m.

24(a). Find the differential equation for all the straight lines, which are at a unit distance from the origin.

OR

24(b). Show that the differential equation $2xy\dfrac{dy}{dx} = x^2 + 3y^2$ is homogeneous and solve it.

25. Find the direction ratios of the normal to the plane, which passes through the points $(1, 0, 0)$ and $(0, 1, 0)$, and makes angle $\dfrac{\pi}{4}$ with the plane $x + y = 3$. Also find the equation of the plane.

26. Using integration, find the area of the region bounded by the lines $y = 2 + x$, $y = 2 - x$ and $x = 2$.

- End of Question Paper -