CBSE Class 12 Mathematics 2015 - Set 65/3/RU

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.

(ii) Please check that this question paper contains 26 questions.

(iii) Questions 1 – 6 in Section A are very short-answer type questions carrying 1 mark each.

(iv) Questions 7 – 19 in Section B are long-answer I type questions carrying 4 marks each.

(v) Questions 20 – 26 in Section C are long-answer II type questions carrying 6 marks each.

(vi) Please write down the serial number of the question before attempting it.

Section A

1. Write the sum of the order and the degree of the following differential equation:
$\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0.$

2. Write the integrating factor of the following differential equation: $(1+y^2)+(2xy-\cot y)\frac{dy}{dx}=0.$

3. Write the value of $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$.

4. Write a unit vector which is perpendicular to both the vectors $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}+\hat{j}$.

5. The equations of a line are $5x-3 = 15y+7 = 3-10z$. Write the direction cosines of the line.

6. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2\hat{a}+\hat{b}+\hat{c}|$.

Section B

7. Find: $\int_0^{\pi/4} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$.

8. Find: $\int \frac{\log x}{(x+1)^2} dx$.

9. Find a unit vector perpendicular to both of the vectors $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$, where $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k}$.

10(a). Find the equation of a line passing through the point (1, 2, -4) and perpendicular to two lines $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

OR

10(b). Find the equation of the plane passing through the points (-1, 2, 0), (2, 2, -1) and parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.

11(a). Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spades. Hence find the mean of the distribution.

OR

11(b). For 6 trials of an experiment, let X be a binomial variate which satisfies the relation $9P(X = 4) = P(X = 2)$. Find the probability of success.

12. Find the adjoint of the matrix $A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ and hence show that $A \cdot (\text{adj } A) = |A| I_3$.

13. Show that the function $f(x) = |x - 1| + |x + 1|$, for all $x \in \mathbb{R}$, is not differentiable at the points $x = -1$ and $x = 1$.

14. If $y = e^{m \sin^{-1} x}$, then show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2 y = 0$.

15. If $f(x) = \sqrt{x^2+1}$, $g(x) = \dfrac{x+1}{x^2+1}$ and $h(x) = 2x - 3$, then find $f'\left[h'\{g'(x)\}\right]$.

16(a). Evaluate: $\int (3-2x)\sqrt{2+x-x^2}\,dx$.

OR

16(b). Evaluate: $\int \dfrac{x^2+x+1}{(x^2+1)(x+2)}\,dx$.

17. To promote the making of toilets for women, an organisation tried to generate awareness through (i) house calls, (ii) letters, and (iii) announcements, the cost for each mode per attempt being ₹50, ₹20 and ₹40 respectively. The number of attempts made in three villages X, Y and Z is given below:

Village \\ Mode	House calls (₹50)	Letters (₹20)	Announcements (₹40)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organisation for the three villages separately, using matrices. Write one value generated by the organisation in the society.

18(a). Solve for x: $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\dfrac{8}{31}$.

OR

18(b). Prove the following: $\cot^{-1}\left(\dfrac{xy+1}{x-y}\right) + \cot^{-1}\left(\dfrac{yz+1}{y-z}\right) + \cot^{-1}\left(\dfrac{zx+1}{z-x}\right) = 0$ $(0 < xy, yx, zx < 1)$.

19. Using properties of determinants, prove the following: $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$.

Section C

20. A company manufactures three kinds of calculators: A, B and C in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind A, 4000 of kind B and 4800 of kind C. The daily output of factory I is of 50 calculators of kind A, 50 calculators of kind B, and 30 calculators of kind C. The daily output of factory II is of 40 calculators of kind A, 20 calculators of kind B and 40 calculators of kind C. The cost per day to run factory I is ₹12,000 and of factory II is ₹15,000. How many days do the two factories have to be in operation to produce the order with the minimum cost? Formulate this problem as an LPP and solve it graphically.

21. In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4 and 1 percent respectively are defective bolts. A bolts is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

22(a). Consider $f : \mathbb{R}_+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{54 + 5y} - 3}{5}\right)$.

OR

22(b). A binary operation $*$ is defined on the set $X = \mathbb{R} - \{-1\}$ by $x * y = x + y + xy, \forall\, x, y \in X$. Check whether $*$ is commutative and associative. Find its identity element and also find the inverse of each element of X.

23. Find the value of p for which the curves $x^2 = 9p(9 - y)$ and $x^2 = p(y + 1)$ cut each other at right angles.

24(a). Show that the differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is homogeneous and solve it also.

OR

24(b). Find the particular solution of the differential equation $(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$, given that $y = 0$ when $x = 1$.

25. Find the distance of the point P(3, 4, 4) from the point, where the line joining the points A(3, -4, -5) and B(2, -3, 1) intersects the plane $2x + y + z = 7$.

26. Using integration, prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ into three equal parts.

- End of Question Paper -