CBSE Class 12 Mathematics 2015 - Set 65/2 - Compartment

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.

(ii) Please check that this Question Paper contains 26 Questions.

(iii) Questions 1 to 6 in Section-A are Very Short Answer Type Questions carrying one mark each.

(iv) Questions 7 to 19 in Section-B are Long Answer I Type Questions carrying 4 marks each.

(v) Questions 20 to 26 in Section-C are Long Answer II Type Questions carrying 6 marks each

(vi) Please write down the serial number of the Question before attempting it.

Section A

1. Write the vector equation of the line passing through $(1, 2, 3)$ and perpendicular to the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.

2. In the interval $\pi/2 < x < \pi$, find the value of x for which the matrix $\begin{pmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{pmatrix}$ is singular.

3. Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}$.

4. Write the integrating factor of the differential equation $\sqrt{x}\,\frac{dy}{dx} + y = e^{-2\sqrt{x}}$.

5. Write the direction ratio's of the vector $3\vec{a} + 2\vec{b}$ where $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$.

6. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$.

Section B

7. Evaluate $\int_0^{\pi/4} \log(1+\tan x)\,dx$.

8(a). Find $\int \frac{x}{(x^2+1)(x-1)}\,dx$.

OR

8(b). Find $\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}}\,dx$.

9(a). Four cards are drawn successively with replacement from a well shuffled deck of 52 cards. What is the probability that (i) all the four cards are spades? (ii) only 2 cards are spades?

OR

9(b). A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes. Hence find the mean of the distribution.

10. Prove that $[\vec{a}, \vec{b}+\vec{c}, \vec{d}] = [\vec{a}, \vec{b}, \vec{d}] + [\vec{a}, \vec{c}, \vec{d}]$.

11. Find the shortest distance between the following lines: $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$.

12(a). Prove that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \sin^{-1}\left(\frac{31}{25\sqrt{2}}\right)$.

OR

12(b). Solve for x: $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x$, $x > 0$.

13. For what value of λ the function defined by $f(x) = \begin{cases} \lambda(x^2+2), & \text{if } x \le 0 \\ 4x+6, & \text{if } x > 0 \end{cases}$ is continuous at $x = 0$? Hence check the differentiability of $f(x)$ at $x = 0$.

14. If $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$, then prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

15. If $y = Ae^{mx} + Be^{nx}$, then show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$.

16. Find $\int \frac{x+3}{\sqrt{5-4x-2x^2}}\,dx$.

17. A trust fund has ₹ 35,000 to be invested in two different types of bonds. The first bond pays 8% interest per annum which will be given to orphanage and second bond pays 10% interest per annum which will be given to an N.G.O. (Cancer Aid Society). Using matrix multiplication, determine how to divide ₹ 35,000 among two types of bonds if the trust fund obtains an annual total interest of ₹ 3,200. What are the values reflected in this question?

18(a). Express the matrix $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

OR

18(b). If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

19. Using properties of determinants, solve for x: $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$.

Section C

20(a). Find the solution of the differential equation $\left(x \sin^2\left(\frac{y}{x}\right) - y\right) dx + x\, dy = 0$ given $y = \frac{\pi}{4}$ when $x = 1$.

OR

20(b). Find the solution of the differential equation $\frac{dy}{dx} - 3y \cot x = \sin 2x$ given $y = 2$ when $x = \frac{\pi}{2}$.

21. Find the vector and cartesian equations of the plane passing through the line of intersection of the planes $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$ and $\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$ such that the intercepts made by the plane on x-axis and z-axis are equal.

22. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$, what is the probability that the student knows the answer given that he answered it correctly?

23. A manufacturer produces nuts and bolts. It takes 2 hours work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 2 hours on machine B to produce a package of bolts. He earns a profit of ₹ 24 per package on nuts and ₹ 18 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 10 hours a day. Make an L.P.P. from above and solve it graphically.

24. The sum of surface areas of a sphere and a cuboid with sides $\frac{x}{3}$, x and $2x$, is constant. Show that the sum of their volumes is minimum if x is equal to three times the radius of sphere.

25. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$ is an equivalence relation. Write all the equivalence classes of R.

26(a). Find the area of the region $\{(x, y) : y^2 \leq 4x,\ 4x^2 + 4y^2 \leq 9\}$, using integration.

OR

26(b). Using integration, find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

- End of Question Paper -