CBSE Class 12 Mathematics 2016 - 065-Set3 - South

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.
(ii) Questions 1 to 6 carry 1 mark each.
(iii) Questions 7 to 19 carry 4 marks each.
(iv) Questions 20 to 26 carry 6 marks each.
(v) Write the serial number of the question before attempting it.

Section A

1. If A is a square matrix such that $|A| = 5$, write the value of $|AA^T|$.

2. Find the distance between the planes $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) - 4 = 0$ and $\vec{r} \cdot (6\hat{i} - 9\hat{j} + 18\hat{k}) + 30 = 0$.

3. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\vec{a} - \sqrt{2}\vec{b}$ to be a unit vector?

4. If $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -4 \\ 3 & -2 \end{pmatrix}$, find $|AB|$.

5. If $A = \begin{pmatrix} 0 & 3 \\ 2 & -5 \end{pmatrix}$ and $kA = \begin{pmatrix} 0 & 4a \\ -8 & 5b \end{pmatrix}$, find the values of k and a.

6. If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \frac{1}{2}$, $|\vec{b}| = \frac{4}{\sqrt{3}}$ and $|\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}}$, then find $|\vec{a} \cdot \vec{b}|$.

Section B

7. Find k, if $f(x) = \begin{cases} k\sin^{-1}(x+1), & x \le 0 \\ \dfrac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is continuous at $x = 0$.

8. Differentiate $(\sin 2x)^x + \sin^{-1}\sqrt{3x}$ with respect to x.

OR

Differentiate $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ with respect to $\cos^{-1}x^2$.

9. Given that vectors $\vec{a}, \vec{b}, \vec{c}$ form a triangle such that $\vec{a} = \vec{b} + \vec{c}$. Find p, q, r, s such that area of triangle is $5\sqrt{6}$ where $\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$, $\vec{b} = s\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$.

10. There are two bags A and B. Bag A contains 3 white and 4 red balls whereas bag B contains 4 white and 3 red balls. Three balls are drawn at random (without replacement) from one of the bags and are found to be two white and one red. Find the probability that these were drawn from bag B.

11. Ishan wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by $5300\,\text{m}^2$. Using matrices, find the dimensions of the plot. Also give reason why he wants to donate the plot for a school.

12. Solve the differential equation: $2y\,e^{x/y}\,dx + \left(y - 2x\,e^{x/y}\right)\,dy = 0$.

13. Solve the differential equation: $(x+1)\dfrac{dy}{dx} - y = e^{3x}(x+1)^3$.

14. Find: $\displaystyle\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$.

15. Prove that $2\sin^{-1}\left(\dfrac{3}{5}\right) - \tan^{-1}\left(\dfrac{17}{31}\right) = \dfrac{\pi}{4}$.

OR

Solve the equation for x: $\cos\left(\tan^{-1}x\right) = \sin\left(\cot^{-1}\dfrac{3}{4}\right)$.

16. Find: $\displaystyle\int \frac{1-\sin x}{\sin x\,(1+\sin x)}\,dx$.

17. Evaluate: $\displaystyle\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\,dx$.

OR

Evaluate: $\displaystyle\int_0^1 \cot^{-1}(1 - x + x^2)\,dx$.

18. Find equation of normal to the curve $ay^2 = x^3$ at the point whose x-coordinate is am^2.

19. Find the equation of plane passing through the points $A(3,2,1)$, $B(4,2,-2)$ and $C(6,5,-1)$ and hence find the value of λ for which $A(3,2,1)$, $B(4,2,-2)$, $C(6,5,-1)$ and $D(\lambda,5,5)$ are coplanar.

OR

Find the coordinates of the point where the line $\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$ meets the plane which is perpendicular to the vector $\vec{n} = \hat{i} + \hat{j} + 3\hat{k}$ and at a distance of $\dfrac{4}{\sqrt{11}}$ from origin.

Section C

20. Three numbers are selected at random (without replacement) from first six positive integers. If X denotes the smallest of the three numbers obtained, find the probability distribution of X. Also find the mean and variance of the distribution.

21. A diet is to contain at least 80 units of Vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available costing Rs.5 per unit and Rs.6 per unit respectively. One unit of food F_1 contains 4 units of vitamin A and 3 units of minerals whereas one unit of food F_2 contains 3 units of vitamin A and 6 units of minerals. Formulate this as a linear programming problem. Find the minimum cost of diet that consists of mixture of these two foods and also meets minimum nutritional requirement.

22. Using properties of determinants, prove that: $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$.

OR

Using elementary row operations, find the inverse of the following matrix: $A = \begin{pmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{pmatrix}$.

23. Find the equation of the plane containing two parallel lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$. Also, find if the plane thus obtained contains the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$ or not.

24. Using integration, find the area of the region $\{(x, y) : y^2 \leq 6ax \text{ and } x^2 + y^2 \leq 16a^2\}$.

25. Let $f : \mathbb{N} \to \mathbb{N}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : \mathbb{N} \to S$ is invertible (where S is range of f). Find the inverse of f and hence find $f^{-1}(31)$ and $f^{-1}(87)$.

26. Determine the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is strictly increasing or strictly decreasing.

OR

Find the maximum and minimum values of $f(x) = \sec x + \log \cos^2 x,\ 0 < x < 2\pi$.

- End of Question Paper -