

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 100

General Instructions

- (i) All questions are compulsory.
- (ii) Questions 1 to 4 carry 1 mark each.
- (iii) Questions 5 to 12 carry 2 marks each.
- (iv) Questions 13 to 23 carry 4 marks each.
- (v) Questions 24 to 29 carry 6 marks each.

Section A

1. If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$.

2. Determine the value of the constant k so that the function $f(x) = \begin{cases} \frac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$ is continuous at $x = 0$.

3. Evaluate: $\int_2^3 3^x dx$.

4. If a line makes angles 90° and 60° respectively with the positive directions of x and y axes, find the angle which it makes with the positive direction of z -axis.

Section B

5. Show that all the diagonal elements of a skew symmetric matrix are zero.

6. Find $\frac{dy}{dx}$ at $x = 1, y = \frac{\pi}{4}$ if $\sin^2 y + \cos xy = K$.

7. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

8. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on $\mathbb{R}$.

9. Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.

10. Prove that if E and F are independent events, then the events E and F' are also independent.

11. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is Rs. 100 and that on a bracelet is Rs. 300. Formulate an L.P.P. for finding how many of each should be produced daily to maximize the profit? It is being given that at least one of each must be produced.

12. Find: $\int \frac{dx}{x^2 + 4x + 8}$.

13. Prove that $\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right] = \frac{2b}{a}$.

14. Using properties of determinants, prove that

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y).$$

OR

Let $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}$; find a matrix D such that $CD - AB = O$.

15. Differentiate the function $(\sin x)^x + \sin^{-1}\sqrt{x}$ with respect to x .

OR

If $x^m y^n = (x+y)^{m+n}$, prove that $\frac{d^2 y}{dx^2} = 0$.

16. Find: $\int \frac{2x}{(x^2+1)(x^2+2)^2} dx$.

17. Evaluate: $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$.

OR

Evaluate: $\int_0^{3/2} |x \sin \pi x| dx$.

18. Prove that $x^2 - y^2 = c(x^2 + y^2)^2$ is the general solution of the differential equation $(x^3 - 3xy^2) dx = (y^3 - 3x^2y) dy$, where c is a parameter.

19. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, then

(a) Let $c_1 = 1$ and $c_2 = 2$, find c_3 which makes $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar.

(b) If $c_2 = -1$ and $c_3 = 1$, show that no value of c_1 can make $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar.

20. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a} + \vec{b} + \vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$.

21. The random variable X can take only the values 0, 1, 2, 3. Given that $P(X=0) = P(X=1) = p$ and $P(X=2) = P(X=3)$ such that $\sum p_i x_i^2 = 2 \sum p_i x_i$, find the value of p .

22. Often it is taken that a truthful person commands more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. Do you also agree that the value of truthfulness leads to more respect in the society?

23. Solve the following L.P.P. graphically: Minimise $Z = 5x + 10y$ subject to constraints $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$ and $x, y \geq 0$.

Section D

24. Use product $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$ to solve the system of equations $x + 3z = 9$, $-x + 2y - 2z = 4$, $2x - 3y + 4z = -3$.

25. Consider $f : \mathbb{R}_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$. Hence find (i) $f^{-1}(10)$ (ii) y if $f^{-1}(y) = \frac{4}{3}$, where $\mathbb{R}_+$ is the set of all non-negative real numbers.

OR

Discuss the commutativity and associativity of binary operation $*$ defined on $A = \mathbb{Q} - \{1\}$ by the rule $a * b = a - b + ab$ for all $a, b \in A$. Also find the identity element of $*$ in A and hence find the invertible elements of A .

26. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

27. Using integration, find the area of region bounded by the triangle whose vertices are $(-2, 1)$, $(0, 4)$ and $(2, 3)$.

OR

Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration.

28. Solve the differential equation $x \frac{dy}{dx} + y = x \cos x + \sin x$, given that $y = 1$ when $x = \frac{\pi}{2}$.

29. Find the equation of the plane through the line of intersection of $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} - \hat{j}) + 4 = 0$ and perpendicular to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$. Hence find whether the plane thus obtained contains the line $x - 1 = 2y - 4 = 3z - 12$.

OR

Find the vector and Cartesian equations of a line passing through $(1, 2, -4)$ and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

- End of Question Paper -