

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 100

General Instructions

- (i) All questions are compulsory.
- (ii) Questions 1 to 4 carry 1 mark each.
- (iii) Questions 5 to 12 carry 2 marks each.
- (iv) Questions 13 to 23 carry 4 marks each.
- (v) Questions 24 to 29 carry 6 marks each.
- (vi) Answer Section A questions in one word, one sentence or as required by the question.
- (vii) There is no overall choice. Internal choice is provided in three questions of 4 marks each and three questions of 6 marks each. Attempt only one alternative in these questions.
- (viii) Use of calculators is not permitted. Logarithmic tables may be requested if required.

Section A

1. Write the distance of the point $(3, -5, 12)$ from x -axis.

2. Evaluate: $\int_0^{2\pi} \cos^5 x \, dx$.

3. For what value of k is the function $f(x) = \begin{cases} \frac{\sin 5x}{3x} + \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ continuous at $x = 0$?

4. If $|A| = 3$ and $A^{-1} = \begin{pmatrix} 3 & -1 \\ -\frac{5}{3} & \frac{2}{3} \end{pmatrix}$, then write the $\text{adj } A$.

5. Find: $\int \frac{dx}{\sqrt{3-2x-x^2}}.$

6. A company produces two types of goods A and B , that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can procure a maximum of 9 g of silver and 8 g of gold. If each unit of type A brings a profit of Rs. 40 and that of type B Rs. 50, formulate LPP to maximize profit.

7. If $P(A) = 0.4$, $P(B) = p$, $P(A \cup B) = 0.6$ and A and B are given to be independent events, find the value of p .

8. A line passes through the point with position vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ and is perpendicular to the plane $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 5\hat{k}) = 7$. Find the equation of the line in cartesian and vector forms.

9. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$ is decreasing for all $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right).$

10. Find $\frac{dy}{dx}$ at $t = \frac{2\pi}{3}$ when $x = 10(t - \sin t)$ and $y = 12(1 - \cos t).$

11. If A and B are square matrices of order 3 such that $|A| = -1$, $|B| = 3$, then find the value of $|2AB|.$

12. The radius r of a right circular cylinder is increasing uniformly at the rate of 0.3 cm/s and its height h is decreasing at the rate of 0.4 cm/s. When $r = 3.5$ cm and $h = 7$ cm, find the rate of change of the curved surface area of the cylinder. [Use $\pi = \frac{22}{7}$].

Section C

13. There are 4 cards numbered 1 to 4, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X .

14. If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = 4\hat{i} - 7\hat{j} + \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 6$.

15. Evaluate: $\int_{-2}^1 |x^3 - x| dx$.

OR

Find: $\int e^{2x} \sin(3x + 1) dx$.

16. In a shop X , 30 tins of pure ghee and 40 tins of adulterated ghee which look alike, are kept for sale while in shop Y , similar 50 tins of pure ghee and 60 tins of adulterated ghee are there. One tin of ghee is purchased from one of the randomly selected shops and is found to be adulterated. Find the probability that it is purchased from shop Y . What measures should be taken to stop adulteration?

17. Find: $\int \frac{e^x dx}{(2 + e^x)(4 + e^{2x})}$.

18. If $xy = e^{(x-y)}$, then show that $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$.

OR

If $\log y = \tan^{-1} x$, then show that $(1 + x^2) \frac{d^2 y}{dx^2} + (2x - 1) \frac{dy}{dx} = 0$.

19. Using properties of determinants show that

$$\begin{vmatrix} 1 & 1 & 1+x \\ 1 & 1+y & 1 \\ 1+z & 1 & 1 \end{vmatrix} = xyz + yz + zx + xy.$$

OR

Find matrix X so that $X \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$.

20. Solve the following LPP graphically:
Maximise $Z = 105x + 90y$ subject to the constraints $x + y \leq 50$, $2x + y \leq 80$, $x \geq 0$, $y \geq 0$.

21. Find the general solution of the differential equation $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$.

22. Prove that: $\tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right] = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$, $-1 \leq x \leq 1$.

23. Using vectors, find the area of triangle ABC , with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$.

Section D

24. Using the method of integration, find the area of the triangle ABC , coordinates of whose vertices are $A(1, 2)$, $B(2, 0)$ and $C(4, 3)$.

OR

Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.

25. A wire of length 34 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a rectangle whose length is twice its breadth. What should be the lengths of the two pieces, so that the combined area of the square and the rectangle is minimum?

26. Let $A = \mathbb{R} - \{3\}$, $B = \mathbb{R} - \{1\}$. Let $f : A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3} \forall x \in A$. Show that f is bijective. Also, find (i) x , if $f^{-1}(x) = 4$ (ii) $f^{-1}(7)$.

OR

Let $A = \mathbb{R} \times \mathbb{R}$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ad + bc, bd)$ for all $(a, b), (c, d) \in \mathbb{R} \times \mathbb{R}$. (i) Show that $*$ is commutative on A . (ii) Show that $*$ is associative on A . (iii) Find the identity element of $*$ in A .

27. Find the vector equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. Hence find whether the plane thus obtained contains the line $\frac{x+2}{3} = \frac{y-3}{4} = \frac{z}{5}$ or not.

OR

Find the image P' of the point P having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$. Hence find the length of PP' .

28. If $A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{pmatrix}$, find A^{-1} and hence solve the system of equations $x - 2y = 10$, $2x + y + 3z = 8$ and $-2y + z = 7$.

29. Find the particular solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$, given that $y = 0$ when $x = 1$.

- End of Question Paper -