

CBSE Class 12 Mathematics 2017 - 65-3

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 100

General Instructions

- (i) All questions are compulsory.
- (ii) Questions 1 to 4 carry 1 mark each.
- (iii) Questions 5 to 12 carry 2 marks each.
- (iv) Questions 13 to 23 carry 4 marks each.
- (v) Questions 24 to 29 carry 6 marks each.
- (vi) Answer Section A questions in one word, one sentence or as required by the question.
- (vii) There is no overall choice. Internal choice is provided in three questions of 4 marks each and three questions of 6 marks each. Attempt only one alternative in these questions.
- (viii) Use of calculators is not permitted. Logarithmic tables may be requested if required.

Section A

1. Determine the value of k for which the following function is continuous at $x = 3$: $f(x) =$

$$\begin{cases} \frac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}.$$

2. If for any 2×2 square matrix A , $A(\text{adj } A) = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$, then write the value of $|A|$.

3. Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2.5y + 5z = 20$.

4. Find: $\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$.

Section B

5. Find: $\int \frac{dx}{5 - 8x - x^2}$.

6. Two tailors, A and B , earn Rs. 300 and Rs. 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

7. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

8. The x -coordinate of a point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its z -coordinate.

9. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on $\mathbb{R}$.

10. Find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in $[-\sqrt{3}, 0]$.

11. If A is a skew-symmetric matrix of order 3, then prove that $\det A = 0$.

12. The volume of a sphere is increasing at the rate of $8 \text{ cm}^3/\text{s}$. Find the rate at which its surface area is increasing when the radius of the sphere is 12 cm.

Section C

13. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X .

14. Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

15. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance? Is regularity required only in school? Justify your answer.

16. If $\tan^{-1}\left(\frac{x-3}{x-4}\right) + \tan^{-1}\left(\frac{x+3}{x+4}\right) = \frac{\pi}{4}$, then find the value of x .

17. Using properties of determinants, prove that

$$\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3.$$

OR

Find matrix A such that $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}.$

18. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

OR

If $e^y(x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

19. Evaluate: $\int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx$.

OR

Evaluate: $\int_1^4 \{|x-1| + |x-2| + |x-4|\} dx$.

20. Solve the following linear programming problem graphically: Maximise $Z = 7x + 10y$ subject to the constraints $4x + 6y \leq 240$, $6x + 3y \leq 240$, $x \geq 10$, $x \geq 0$, $y \geq 0$.

21. Find: $\int \frac{e^x dx}{(e^x - 1)^2(e^x + 2)}.$

22. If $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 2\hat{j} - 3\hat{k}$, then express $\vec{b}$ in the form of $\vec{b} = \vec{b}_1 + \vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$.

23. Find the general solution of the differential equation $\frac{dy}{dx} - y = \sin x$.

Section D

24. Using the method of integration, find the area of the triangle ABC , coordinates of whose vertices are $A(4, 1)$, $B(6, 6)$ and $C(8, 4)$.

OR

Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$.

25. Find the particular solution of the differential equation $(x - y) \frac{dy}{dx} = (x + 2y)$, given that $y = 0$ when $x = 1$.

26. Find the coordinates of the point where the line through the points $(3, -4, -5)$ and $(2, -3, 1)$, crosses the plane determined by the points $(1, 2, 3)$, $(4, 2, -3)$ and $(0, 4, 3)$.

OR

A variable plane which remains at a constant distance $3p$ from the origin cuts the coordinate axes at A, B, C . Show that the locus of the centroid of triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.

27. Consider $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbb{R} - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

OR

Let $A = \mathbb{Q} \times \mathbb{Q}$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Determine, whether $*$ is commutative and associative. Then, with respect to $*$ on A : (i) Find the identity element in A . (ii) Find the invertible elements of A .

28. If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$, find A^{-1} . Hence using A^{-1} solve the system of equations $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$.

29. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

- End of Question Paper -