

CBSE Class 12 Physics Question Paper 2017 with Solution PDF-Set 55/1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are **26** questions in all.
- (ii) This question paper has **five** sections : Section A, Section B, Section C, Section D and Section E.
- (iii) Section A contains **five** questions of **one** mark each, Section B contains **five** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains one value based question of **four** marks and Section E contains **three** questions of **five** marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in **one** question of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron} = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Nichrome and copper wires of same length and same radius are connected in series. Current I is passed through them. Which wire gets heated up more? Justify your answer. [1 mark]

Solution:

Concept:

- Joule heating is $H = I^2 R t$, where I is current, R resistance and t time.
- A uniform wire has resistance $R = \rho L/A$, with resistivity ρ , length L and cross-sectional area $A = \pi r^2$.

Step 1: Compare the wire resistances The wires have equal length and radius, so their geometrical factor L/A is identical. Nichrome has a higher resistivity than copper. Consequently, $R_{\text{nichrome}} > R_{\text{copper}}$.

Step 2: Apply the series-current condition Both wires carry the same current because they are connected in series. Over the same time interval, their heat production is proportional to resistance:

$$\frac{H_{\text{nichrome}}}{H_{\text{copper}}} = \frac{R_{\text{nichrome}}}{R_{\text{copper}}} > 1.$$

Thus copper's greater conductivity does not imply greater heating at the stipulated equal current.

Step 3: Conclusion The nichrome wire produces more heat, since its higher resistance gives greater Joule heating $I^2 R t$.

Quick Tip to Solve: For wires in series, compare resistance using $I^2 R$: the higher-resistance wire dissipates more power.

2. Do electromagnetic waves carry energy and momentum? [1 mark]

Solution:

Concept:

- Electromagnetic fields store energy; a propagating wave transports this energy through space.
- Radiation also carries momentum: for energy U travelling in one direction in vacuum, $p = U/c$, where c is the speed of light.

Step 1: Identify the transported energy The oscillating electric and magnetic fields contain energy. As the wave propagates, it can deliver that energy to matter, for example by heating an absorbing surface.

Step 2: Identify the transported momentum When radiation is absorbed or reflected, it exerts a force on the surface. This radiation pressure is evidence of momentum transfer. For complete absorption of normally incident energy U , the transferred momentum is U/c .

Step 3: Conclusion Yes. Electromagnetic waves carry both energy and momentum.

Quick Tip to Solve: Remember radiation pressure: a wave that pushes an absorbing surface must transport momentum as well as energy.

3. How does the angle of minimum deviation of a glass prism vary, if the incident violet light is replaced by red light? Give reason. [1 mark]

Solution:

Concept:

- For a prism of angle A , $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$, where δ_m is minimum deviation.
- Under normal dispersion, the refractive index of glass is smaller for red light than for violet light.

Step 1: Compare the refractive indices Red light has a longer wavelength than violet light. For the same glass prism, normal dispersion gives $\mu_r < \mu_v$. Therefore red light undergoes less refraction at the prism faces.

Step 2: Relate refractive index to deviation For fixed prism angle A ,

$$\delta_m = 2 \sin^{-1} \left(\mu \sin \frac{A}{2} \right) - A.$$

In the physical range, the inverse sine increases with its argument. A smaller refractive index therefore gives a smaller minimum deviation.

Step 3: Conclusion The angle of minimum deviation decreases when violet light is replaced by red light because $\mu_r < \mu_v$.

Quick Tip to Solve: For a glass prism, violet deviates most and red least; connect this order to their refractive indices.

4. Name the phenomenon which shows the quantum nature of electromagnetic radiation. [1 mark]

Solution:

Concept:

- In the photon model, radiation exchanges energy in discrete packets of energy $E = h\nu$.
- The photoelectric effect is electron emission from a material when radiation of sufficiently high frequency falls on it.

Step 1: Identify the phenomenon In the photoelectric effect, incident light ejects electrons from a metal surface. Emission occurs only when the frequency exceeds the threshold frequency characteristic of that surface.

Step 2: Explain the quantum evidence An electron absorbs one photon of energy $h\nu$. Energy conservation gives

$$K_{\max} = h\nu - \phi,$$

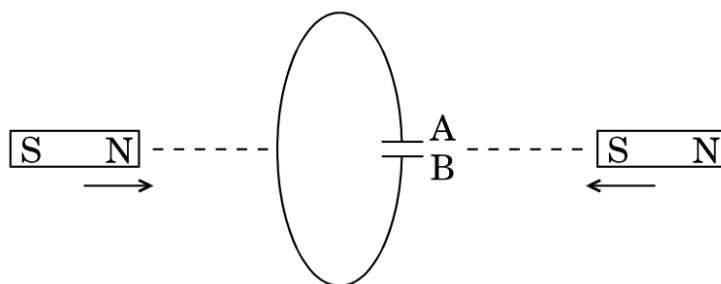
where ϕ is the work function. The dependence on photon frequency and the threshold $h\nu \geq \phi$ demonstrate the discrete energy transfer of radiation.

Step 3: Conclusion The photoelectric effect shows the quantum nature of electromagnetic radiation.

Quick Tip to Solve: Use the photoelectric threshold: raising intensity cannot cause emission if each photon has less energy than the work function.

5. Predict the polarity of the capacitor in the situation described below:

[1 mark]



Solution:

Concept:

- Lenz's law: the induced current opposes the change in magnetic flux that produces it.
- The end of a coil is north when its current is anticlockwise as seen from that end; conventional current charges the plate it enters positively.

Step 1: Determine the increasing magnetic flux The north pole approaching from the left produces a field through the loop towards the right. The south pole approaching from the right also produces a field towards the right. Hence their approaches increase rightward magnetic flux; their effects reinforce rather than cancel.

Step 2: Apply Lenz's law to the capacitor plates The induced field must point leftward. Thus the loop must act as a north pole on its left side and a south pole on its right side, repelling both approaching poles. The induced circulation is anticlockwise when viewed from the left. In the illustrated loop this drives conventional current upward along the right-hand near limb: positive charge accumulates on the lower plate B , while the upper plate A loses positive charge. The gap prevents a sustained conduction current, but the transient induced emf charges the capacitor.

Step 3: Conclusion Plate A is negative and plate B is positive.

Quick Tip to Solve: First add the flux directions of both approaching poles, then use Lenz's law and trace current up to the capacitor gap.

Section B (10 marks)

6. Draw the intensity pattern for single slit diffraction and double slit interference. Hence, state two differences between interference and diffraction patterns.

[2 marks]

Solution:

Concept:

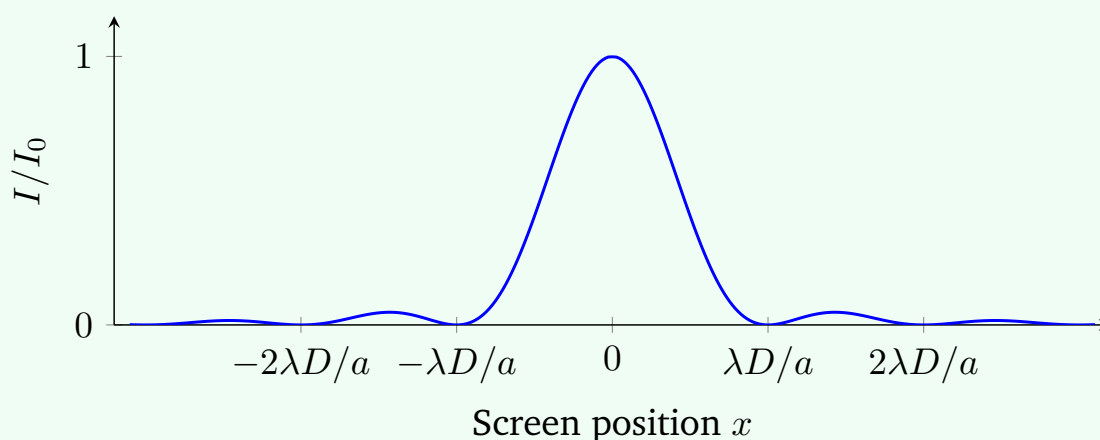
- Single-slit diffraction arises from interference of secondary waves from different parts of one slit:
 $I = I_0(\sin u/u)^2$, $u = \pi ax/(\lambda D)$.
- Two identical narrow coherent slits give $I = I_{\max} \cos^2(\pi x/\beta)$, with fringe width $\beta = \lambda D/d$. Here a is slit width, d slit separation and D screen distance.

Step 1: Draw and interpret the diffraction pattern The central maximum lies at $x = 0$. The first

Quick Tip to Solve: Locate the first diffraction minima before sketching: the central lobe spans both sides of the centre and is twice as wide as a secondary lobe.

minima occur at $x = \pm\lambda D/a$, so its width is $2\lambda D/a$. Subsequent minima occur at integer multiples of $\lambda D/a$. The secondary maxima on either side are much weaker and decrease in intensity away from the centre.

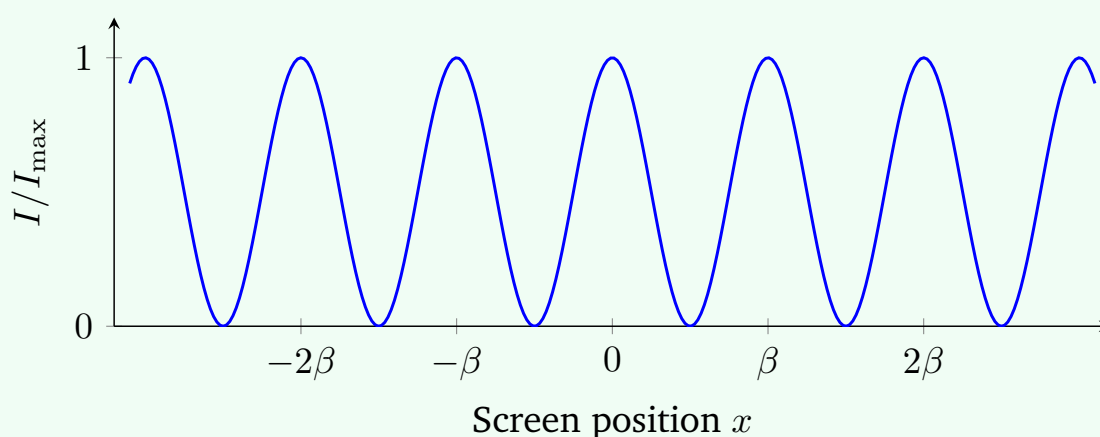
Single-slit diffraction



Single-slit diffraction intensity pattern

Step 2: Draw and compare the interference pattern For identical ideal narrow slits, bright fringes occur at $x = n\beta$ and dark fringes at $x = (n + 1/2)\beta$. The repeated peaks have equal height and spacing. Two differences follow: (i) interference bright fringes have equal width, whereas the diffraction central maximum has twice the width of each secondary lobe; (ii) ideal interference maxima have equal intensity, whereas diffraction secondary maxima are weaker than the central maximum and decrease outward. A real double-slit pattern can be modulated by a single-slit diffraction envelope; the equal-height comparison refers to the ideal interference pattern.

Double-slit interference (ideal narrow slits)



Ideal double-slit interference intensity pattern

Step 3: Conclusion The sketches show a broad, intense diffraction central maximum with weak side lobes, and equally spaced, equally intense ideal interference maxima.

Quick Tip to Solve: Locate the first diffraction minima before sketching: the central lobe spans both sides of the centre and is twice as wide as a secondary lobe.

6. Unpolarised light is passed through a polaroid P_1 . When this polarised beam passes through another polaroid P_2 and if the pass axis of P_2 makes angle θ with the pass axis of P_1 , then write the expression for the polarised beam passing through P_2 . Draw a plot showing the variation of intensity when θ varies from 0 to 2π . [2 marks]

Solution:

Concept:

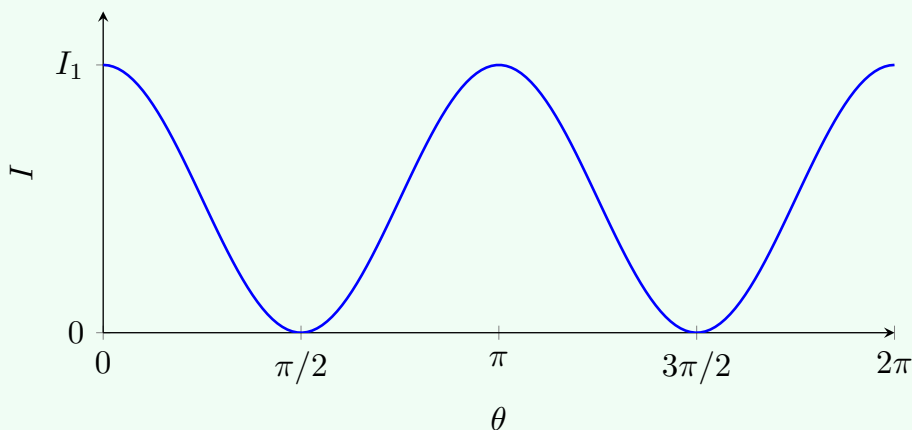
- Malus' law: an analyser transmits intensity $I = I_1 \cos^2 \theta$, where I_1 is the intensity incident on the analyser and θ is the angle between the transmission axes.
- An ideal first polaroid transmits half the intensity I_0 of an unpolarised incident beam, so $I_1 = I_0/2$.

Step 1: Obtain the transmitted intensity After P_1 , the light is linearly polarised along its pass axis. The component of its electric-field amplitude along the pass axis of P_2 is $E_1 \cos \theta$. Since intensity is proportional to the square of amplitude,

$$I(\theta) = I_1 \cos^2 \theta = \frac{I_0}{2} \cos^2 \theta.$$

Here I_1 refers to the beam leaving P_1 , whereas I_0 refers to the original unpolarised beam.

Step 2: Plot the variation through a full turn The intensity is maximum, I_1 , at $\theta = 0, \pi, 2\pi$, when the pass axes are parallel or antiparallel. It vanishes at $\theta = \pi/2, 3\pi/2$, when the axes are crossed. Between these angles it varies smoothly as $\cos^2 \theta$; thus the graph repeats after π , not after an arbitrary linear decrease.



Intensity transmitted by P_2 as its pass axis rotates.

Step 3: Conclusion The transmitted intensity is $I = I_1 \cos^2 \theta = (I_0/2) \cos^2 \theta$, with the two complete intensity lobes shown in the graph.

Quick Tip to Solve: First define whether your incident intensity is before or after the first polaroid; this avoids an incorrect extra factor of two.

7. Identify the electromagnetic waves whose wavelengths vary as

(a) $10^{-12} \text{ m} < \lambda < 10^{-8} \text{ m}$

(b) $10^{-3} \text{ m} < \lambda < 10^{-1} \text{ m}$

Write one use for each.

[2 marks]

Solution:

Concept:

- Electromagnetic radiation is classified by wavelength λ , with frequency $\nu = c/\lambda$, where c is the speed of light in vacuum.
- The conventional school-level spectrum places X-rays in approximately 10^{-12} to 10^{-8} m and microwaves in approximately 10^{-3} to 10^{-1} m.

Step 1: Identify range (a) and its use The range $10^{-12} \text{ m} < \lambda < 10^{-8} \text{ m}$ corresponds to X-rays in the conventional electromagnetic-spectrum classification. One use is medical radiography for detecting bone fractures: X-rays pass more readily through soft tissue than through bone, producing a contrast image. The very short wavelengths distinguish this range from visible light.

Step 2: Identify range (b) and its use The range $10^{-3} \text{ m} < \lambda < 10^{-1} \text{ m}$ corresponds to microwaves. One use is radar, in which a directed microwave pulse is reflected by an object and its returning signal is detected to determine position or distance. These millimetre-to-centimetre wavelengths are longer than infrared wavelengths.

Step 3: Conclusion (a) X-rays: medical imaging of bone fractures. (b) Microwaves: radar detection and ranging.

Quick Tip to Solve: Convert powers of ten into familiar scales: millimetres to centimetres identify microwaves, while extremely short wavelengths identify X-rays.

8. Find the condition under which the charged particles moving with different speeds in the presence of electric and magnetic field vectors can be used to select charged particles of a particular speed.

[2 marks]

Solution:

Concept:

- The Lorentz force on charge q moving with velocity $\vec{v}$ is $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$.
- For a velocity selector, uniform electric and magnetic fields are mutually perpendicular, and the incident velocity is perpendicular to both; their forces must oppose each other.

Step 1: Arrange crossed fields to oppose the forces Direct the charged-particle beam perpendicular to both $\vec{E}$ and $\vec{B}$. Choose the field directions such that $\vec{v} \times \vec{B}$ points opposite to $\vec{E}$. The electric force has magnitude $|q|E$, while the magnetic force has magnitude $|q|vB$. A suitable exit slit passes only particles that travel straight through the fields.

Step 2: Balance forces to select one speed For an undeflected particle the resultant transverse force must vanish:

$$q(\vec{E} + \vec{v} \times \vec{B}) = 0, \quad |q|E = |q|vB.$$

Quick Tip to Solve: Use the cross product to check force directions first; equating magnitudes alone is insufficient if the two forces point the same way.

Hence the selected speed is $v = E/B$. Faster or slower particles experience unequal opposing forces and are deflected away from the exit slit. Reversing the charge sign reverses both forces together, so the balancing speed is independent of charge magnitude and particle mass.

Step 3: Conclusion The selector requires $\vec{E} \perp \vec{B}$, $\vec{v}$ perpendicular to both, and $\vec{E} = -\vec{v} \times \vec{B}$. Only particles with $v = E/B$ pass undeflected.

Quick Tip to Solve: Use the cross product to check force directions first; equating magnitudes alone is insufficient if the two forces point the same way.

9. A 12.5 eV electron beam is used to excite a gaseous hydrogen atom at room temperature. Determine the wavelengths and the corresponding series of the lines emitted. [2 marks]

Solution:

Concept:

- Hydrogen energy levels are $E_n = -13.6/n^2$ eV, where n is the principal quantum number; room-temperature atoms are essentially in the ground state.
- For emission from n_i to $n_f < n_i$, $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, with $R_H \simeq 1.097 \times 10^7 \text{ m}^{-1}$.

Step 1: Determine the highest accessible level The incident electron can excite an atom only if its kinetic energy is at least the required energy difference. From the ground state,

$$\Delta E_{1 \rightarrow 3} = 13.6(1 - 1/9) = 12.09 \text{ eV}, \quad \Delta E_{1 \rightarrow 4} = 13.6(1 - 1/16) = 12.75 \text{ eV}.$$

Since $12.09 < 12.5 < 12.75$, the highest populated level is $n = 3$. The electron need not transfer its entire kinetic energy in a collision; its remaining energy stays with it.

Step 2: Find all possible emission lines Atoms excited to $n = 3$ can return directly to $n = 1$, or cascade through $n = 2$. The possible transitions and wavelengths are

$$\begin{aligned} 3 \rightarrow 1: \quad \lambda &= \frac{1}{R_H(1 - 1/9)} \simeq 102.6 \text{ nm} && \text{(Lyman series),} \\ 2 \rightarrow 1: \quad \lambda &= \frac{1}{R_H(1 - 1/4)} \simeq 121.5 \text{ nm} && \text{(Lyman series),} \\ 3 \rightarrow 2: \quad \lambda &= \frac{1}{R_H(1/4 - 1/9)} \simeq 656.3 \text{ nm} && \text{(Balmer series).} \end{aligned}$$

Direct excitation to $n = 2$ also yields the same $2 \rightarrow 1$ line; no transition from $n = 4$ is possible with this beam energy.

Step 3: Conclusion Three lines can be emitted: approximately 102.6 nm and 121.5 nm in the Lyman series, and 656.3 nm in the Balmer series.

Quick Tip to Solve: Compare the beam energy with excitation thresholds first, then list all downward transitions among the accessible levels, including cascades.

10. Write two properties of a material suitable for making (a) a permanent magnet, and (b) an electromagnet. [2 marks]

Solution:

Concept:

- Retentivity is the ability to retain magnetisation after the external magnetising field is removed.
- Coercivity measures the reverse field needed to remove residual magnetisation. High coercivity resists demagnetisation; low coercivity allows easy demagnetisation.

Step 1: Choose properties for a permanent magnet (a) A permanent magnet must keep a strong magnetic field without a continuous magnetising current. Its material therefore needs high retentivity, so that substantial magnetisation remains after the applied field is removed. It also needs high coercivity, so that stray reverse fields do not readily demagnetise it. Hard magnetic materials such as suitable steel are used for this purpose.

Step 2: Choose properties for an electromagnet (b) An electromagnet should develop a strong magnetisation while current flows through its coil, but lose its magnetisation when the current is switched off. Its core therefore needs high magnetic permeability, meaning that it is readily magnetised by the coil's field, and low retentivity, meaning that very little residual magnetisation remains after removal of the field. Soft iron is a suitable example; its low coercivity also makes repeated magnetisation and demagnetisation easy.

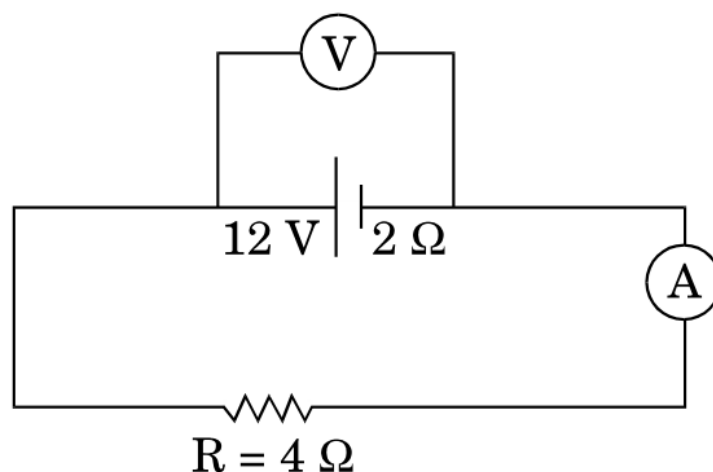
Step 3: Conclusion A permanent-magnet material requires high retentivity and high coercivity. An electromagnet-core material requires high permeability and low retentivity.

Quick Tip to Solve: Ask what happens when the external field is removed: a permanent magnet should retain magnetisation, whereas an electromagnet should release it readily.

Section C (36 marks)

11. (a) The potential difference applied across a given resistor is altered so that the heat produced per second increases by a factor of 9. By what factor does the applied potential difference change?

(b) In the figure shown, an ammeter A and a resistor of $4\ \Omega$ are connected to the terminals of the source. The emf of the source is 12 V having an internal resistance of $2\ \Omega$. Calculate the voltmeter and ammeter readings. [3 marks]



Solution:

Concept:

- Joule heating per second is electrical power: $P = V^2/R$, where V is the potential difference across a resistor of fixed resistance R .
- A cell supplying current I through external resistance R has $I = \mathcal{E}/(R + r)$ and terminal voltage $V_t = \mathcal{E} - Ir = IR$. Here $\mathcal{E}$ is its emf and r its internal resistance.

Step 1: Relate the heating rate to the applied voltage (a) Let the initial and final voltage magnitudes be V_1 and V_2 . For the same resistor, R remains fixed, so the ratio of heat produced per second is the ratio of the squared voltages:

$$\frac{P_2}{P_1} = \frac{V_2^2/R}{V_1^2/R} = \left(\frac{V_2}{V_1}\right)^2 = 9.$$

Taking the positive square root for the voltage magnitudes gives $V_2/V_1 = 3$. Thus the heating rate scales quadratically with applied voltage, rather than linearly.

Step 2: Calculate the current through the ammeter (b) Treat the meters as ideal: the ammeter has negligible resistance and the voltmeter draws negligible current. The internal resistance $r = 2\ \Omega$ is in series with the external resistance $R = 4\ \Omega$. Hence the resistance in the current path is $6\ \Omega$, giving

$$I = \frac{\mathcal{E}}{R + r} = \frac{12}{4 + 2} = 2\text{ A}.$$

This is the current measured by the series ammeter.

Step 3: Calculate the terminal voltage The voltmeter is connected across the source terminals, so it measures the terminal potential difference while the source supplies current. The internal potential

Quick Tip to Solve: Take the square root of the heating-rate factor at fixed resistance. For a loaded cell, include internal resistance in the current calculation and subtract Ir from the emf for the voltmeter reading.

drop is $Ir = (2)(2) = 4 \text{ V}$. Therefore

$$V_t = \mathcal{E} - Ir = 12 - 4 = 8 \text{ V}.$$

As a check, the potential difference across the external resistor is $IR = (2)(4) = 8 \text{ V}$, equal to the terminal voltage.

Step 4: Conclusion (a) The applied potential-difference magnitude increases by a factor of 3. (b) The voltmeter reads 8 V, and the ammeter reads 2 A.

Quick Tip to Solve: Take the square root of the heating-rate factor at fixed resistance. For a loaded cell, include internal resistance in the current calculation and subtract Ir from the emf for the voltmeter reading.

12. (a) How is amplitude modulation achieved?

(b) The frequencies of two side bands in an AM wave are 640 kHz and 660 kHz respectively. Find the frequencies of carrier and modulating signal. What is the bandwidth required for amplitude modulation? [3 marks]

Solution:

Concept:

- In amplitude modulation (AM), the amplitude of a high-frequency carrier varies in accordance with the instantaneous value of a low-frequency information signal. The carrier frequency remains unchanged.
- For a single modulating frequency f_m and carrier frequency f_c , the sideband frequencies are $f_L = f_c - f_m$ and $f_U = f_c + f_m$. The required bandwidth is $B = f_U - f_L = 2f_m$.

Step 1: Explain how amplitude modulation is produced (a) The low-frequency message signal and a high-frequency carrier are applied to a nonlinear device, such as a transistor modulator. Its nonlinear response produces sum and difference frequency components. The desired carrier and sidebands are selected by the output circuit. The resulting carrier's amplitude follows the message signal, while its carrier frequency is unchanged. For a sinusoidal message the AM signal can be written as

$$v(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t),$$

where A_c is the unmodulated carrier amplitude and μ is the modulation index. The bracket gives the varying amplitude; expanding the product gives components at f_c and $f_c \pm f_m$.

Step 2: Find the carrier and message frequencies (b) The lower and upper sidebands are 640 kHz and 660 kHz. They lie symmetrically about the carrier. Adding their equations eliminates f_m , while subtracting eliminates f_c :

$$f_c = \frac{f_U + f_L}{2} = \frac{660 + 640}{2} = 650 \text{ kHz},$$
$$f_m = \frac{f_U - f_L}{2} = \frac{660 - 640}{2} = 10 \text{ kHz}.$$

The message frequency is the distance from the carrier to either sideband, rather than the full separation of the sidebands.

Quick Tip to Solve: For two AM sideband frequencies, their average gives the carrier, half their difference gives the message frequency, and their full difference gives the bandwidth.

Step 3: Determine the required bandwidth The transmission channel must include both sidebands, extending from the lower sideband to the upper sideband. Thus

$$B = f_U - f_L = 660 - 640 = 20 \text{ kHz} = 2f_m.$$

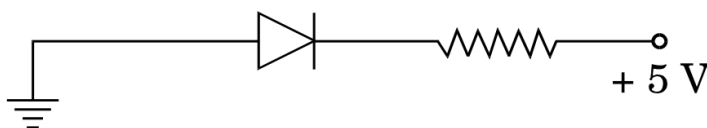
Step 4: Conclusion AM varies the carrier amplitude according to the message signal. The carrier frequency is 650 kHz, the modulating frequency is 10 kHz, and the required bandwidth is 20 kHz.

Quick Tip to Solve: For two AM sideband frequencies, their average gives the carrier, half their difference gives the message frequency, and their full difference gives the bandwidth.

13. (a) In the following diagram, is the junction diode forward biased or reverse biased?

(b) Draw the circuit diagram of a full wave rectifier and state how it works.

[3 marks]



Solution:

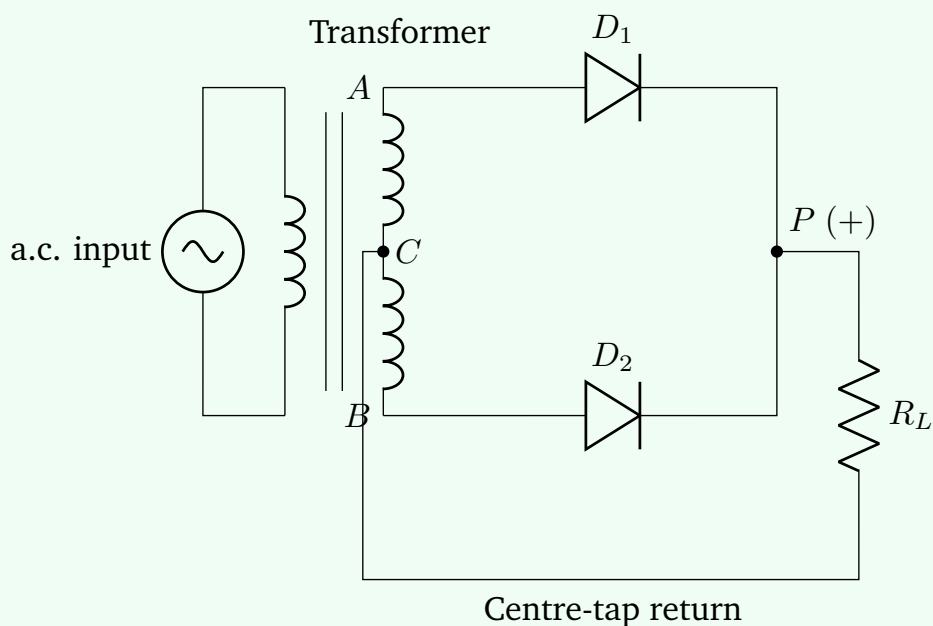
Concept:

- A p-n junction is forward biased when its p-side (anode) is at a higher potential than its n-side (cathode); the opposite polarity produces reverse bias.
- A full-wave rectifier uses both half-cycles of an alternating voltage to drive current through a load in the same direction. A centre-tapped transformer supplies two secondary voltages of opposite polarity relative to the centre tap.

Step 1: (a) Identify the polarity across the junction The vertical bar of the diode symbol denotes its cathode (n-side). In the given circuit the anode is connected to ground, so its potential is 0 V, whereas the cathode is connected through a resistor to the +5 V terminal. Thus the n-side is positive relative to the p-side. This increases the potential barrier and suppresses majority-carrier conduction; only a very small reverse leakage current can flow under normal conditions.

Step 2: (b) Connect a centre-tapped full-wave rectifier Connect the ends A and B of the centre-tapped transformer secondary to the anodes of diodes D_1 and D_2 . Join their cathodes at the positive output terminal P . Connect the load resistance R_L between P and the centre tap C . The primary is connected to the a.c. input; the transformer isolates the input and provides the required alternating secondary voltage.

Quick Tip to Solve: Locate the diode's cathode bar first; for the rectifier, trace each half-cycle through the load and check that both paths have the same load-current direction.



Centre-tapped full-wave rectifier: cathodes meet at the positive output P.

Step 3: Explain the two conducting paths During the half-cycle when A is positive relative to C, D_1 is forward biased and D_2 is reverse biased. The current path is $A \rightarrow D_1 \rightarrow P \rightarrow R_L \rightarrow C$. During the next half-cycle B becomes positive relative to C; D_2 conducts and D_1 blocks. The path is then $B \rightarrow D_2 \rightarrow P \rightarrow R_L \rightarrow C$. In both cases load current flows from P towards C. The output is therefore a unidirectional, pulsating voltage, with ripple frequency twice the a.c. input frequency; a filter would be required for a smoother d.c. output.

Step 4: Conclusion (a) The given junction diode is reverse biased. (b) The centre-tapped two-diode circuit shown rectifies both half-cycles, producing pulsating d.c. through R_L in the same direction.

Quick Tip to Solve: Locate the diode's cathode bar first; for the rectifier, trace each half-cycle through the load and check that both paths have the same load-current direction.

14. Using photon picture of light, show how Einstein's photoelectric equation can be established. Write two features of photoelectric effect which cannot be explained by wave theory. [3 marks]

Solution:

Concept:

- In the photon picture, light of frequency ν consists of photons, each carrying energy $h\nu$, where h is Planck's constant.
- The work function ϕ_0 is the minimum energy required to remove an electron from the metal surface. Its threshold frequency is $\nu_0 = \phi_0/h$.

Step 1: Apply energy conservation to photon absorption An electron absorbs the energy of one photon in a single interaction. A part of this energy is spent overcoming the binding of the electron in the metal. Any remaining energy appears as the emitted electron's kinetic energy. An electron that

Quick Tip to Solve: Separate photon energy from photon number: frequency sets the maximum electron energy, while intensity primarily sets the emitted-electron rate above threshold.

escapes without losing additional energy inside the metal has the greatest kinetic energy. Therefore energy conservation gives

$$h\nu = \phi_0 + K_{\max}.$$

Electrons starting deeper inside the metal or suffering collisions can emerge with smaller kinetic energies, so the equation refers to the maximum value, not to every photoelectron.

Step 2: Express Einstein's photoelectric equation With $K_{\max} = \frac{1}{2}mv_{\max}^2$ and $\phi_0 = h\nu_0$, the equation becomes

$$K_{\max} = h\nu - \phi_0 = h(\nu - \nu_0) = \frac{1}{2}mv_{\max}^2.$$

Here m is the electron mass and $v_{\max}$ its maximum speed. If a stopping potential of magnitude V_s just prevents even the fastest electrons from reaching the collector, then $eV_s = K_{\max}$, where e is the positive magnitude of electronic charge. This also gives $eV_s = h\nu - \phi_0$.

Step 3: State two observations that classical wave theory cannot explain First, each metal has a definite threshold frequency: below ν_0 , electrons are not emitted however intense the incident light is. In classical wave theory, increasing intensity increases the energy delivered to electrons, so it cannot account for this absolute frequency threshold. In the photon model a photon below threshold has insufficient energy to overcome ϕ_0 . Second, at a fixed frequency above threshold, the maximum kinetic energy is independent of light intensity and increases with frequency. Classical wave theory associates greater intensity with greater energy transfer and would predict a larger electron energy. In the photon model intensity changes the number of incident photons, and hence the photoelectric current, whereas the energy per photon remains $h\nu$.

Step 4: Conclusion Einstein's equation is $K_{\max} = h\nu - \phi_0 = h(\nu - \nu_0)$. The threshold frequency and the frequency dependence, together with intensity independence, of maximum kinetic energy are explained by photons but not by classical wave theory.

Quick Tip to Solve: Separate photon energy from photon number: frequency sets the maximum electron energy, while intensity primarily sets the emitted-electron rate above threshold.

15. (a) Monochromatic light of wavelength 589 nm is incident from air on a water surface. If μ for water is 1.33, find the wavelength, frequency and speed of the refracted light.

(b) A double convex lens is made of a glass of refractive index 1.55, with both faces of the same radius of curvature. Find the radius of curvature required, if the focal length is 20 cm. [3 marks]

Solution:

Concept:

- The frequency of light remains unchanged on refraction at a stationary interface. In a medium of refractive index μ , speed $v = c/\mu$ and wavelength $\lambda = v/\nu = \lambda_{\text{air}}/\mu$, taking the refractive index of air as unity.
- For a thin lens in air, the lensmaker's formula is $1/f = (\mu_g - 1)(1/R_1 - 1/R_2)$. Radii are signed according to the Cartesian convention, with light travelling from left to right.

Step 1: (a) Find the unchanged frequency The incident wavelength is $\lambda_{\text{air}} = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$.

Quick Tip to Solve: Keep frequency fixed at a refracting boundary; for an equiconvex lens in air use opposite signed radii, giving the shortcut $R = 2(\mu - 1)f$.

Taking $c = 3.00 \times 10^8 \text{ m s}^{-1}$, the frequency of the incident light is

$$\nu = \frac{c}{\lambda_{\text{air}}} = \frac{3.00 \times 10^8}{589 \times 10^{-9}} \simeq 5.09 \times 10^{14} \text{ Hz.}$$

This frequency is set by the source and is continuous across the boundary, so the refracted light has the same frequency. Refraction changes the propagation speed and wavelength, not the number of oscillations per second.

Step 2: Calculate speed and wavelength in water For water, $\mu = 1.33$. Therefore

$$v = \frac{c}{\mu} = \frac{3.00 \times 10^8}{1.33} \simeq 2.26 \times 10^8 \text{ m s}^{-1}.$$

The wavelength follows from $v = \nu\lambda$, or directly from the unchanged frequency:

$$\lambda_{\text{water}} = \frac{\lambda_{\text{air}}}{\mu} = \frac{589}{1.33} \text{ nm} \simeq 443 \text{ nm} = 4.43 \times 10^{-7} \text{ m.}$$

Both the speed and wavelength decrease by the same factor 1.33, consistent with an unchanged frequency.

Step 3: (b) Apply the signed radii in the lensmaker's formula Let R be the positive magnitude of each radius. For a double convex lens with light travelling from left to right, the centre of curvature of the first face lies to its right and that of the second face lies to its left. Thus $R_1 = +R$, $R_2 = -R$. The focal length of the converging lens is $f = +20 \text{ cm}$ and the glass refractive index is $\mu_g = 1.55$. Hence

$$\frac{1}{f} = (1.55 - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{1.10}{R},$$
$$R = 1.10f = 1.10 \times 20 \text{ cm} = 22 \text{ cm.}$$

The two surfaces have equal radius magnitudes, although their signed radii have opposite signs.

Step 4: Conclusion (a) In water, $\lambda \simeq 443 \text{ nm}$, $\nu \simeq 5.09 \times 10^{14} \text{ Hz}$, and $v \simeq 2.26 \times 10^8 \text{ m s}^{-1}$. (b) Each face needs radius of curvature 22 cm in magnitude, with $R_1 = +22 \text{ cm}$ and $R_2 = -22 \text{ cm}$.

Quick Tip to Solve: Keep frequency fixed at a refracting boundary; for an equiconvex lens in air use opposite signed radii, giving the shortcut $R = 2(\mu - 1)f$.

16. Define mutual inductance between a pair of coils. Derive an expression for the mutual inductance of two long coaxial solenoids of same length wound one over the other. [3 marks]

Solution:

Concept:

- Mutual inductance measures the flux linkage of one coil produced per unit current in the other: $N_2\Phi_{21} = MI_1$. Here Φ_{21} is flux through one turn of coil 2 due to current I_1 .
- For a long solenoid, the magnetic field inside is $B = \mu_0 nI$, where $n = N/\ell$ is turns per unit length. End effects and leakage flux are neglected.

Quick Tip to Solve: Calculate field using the turns of the current-carrying coil, then multiply flux per turn by the turns of the receiving coil.

Step 1: Define mutual inductance The mutual inductance of a pair of coils is the flux linkage with either coil due to unit current in the other. Equivalently, it is the magnitude of emf induced in one coil when current in the other changes at unit rate:

$$\varepsilon_2 = -M \frac{dI_1}{dt}.$$

The minus sign expresses opposition to the change of linked flux. Its SI unit is henry (H).

Step 2: Find the flux linked with the second solenoid Let the coaxial solenoids have common length ℓ , numbers of turns N_1, N_2 , and common linked cross-sectional area A . For solenoids wound closely one over the other, their areas are effectively equal; if unequal, A is the smaller area. A current I_1 in the first produces

$$B_1 = \mu_0 \frac{N_1}{\ell} I_1.$$

This field is uniform within the common area and directed along the axis. The flux through each turn of the second is $\Phi_{21} = B_1 A$, so its total flux linkage is

$$N_2 \Phi_{21} = \frac{\mu_0 N_1 N_2 A}{\ell} I_1.$$

Step 3: Identify the coefficient of current Compare the calculated flux linkage with $N_2 \Phi_{21} = M I_1$. The coefficient is independent of current for an air core. Interchanging the two coils gives the same coefficient, so $M_{12} = M_{21}$. For a uniform linear magnetic core of permeability μ , replace μ_0 by μ .

Step 4: Conclusion Thus the mutual inductance of the two long air-core coaxial solenoids is

$$M = \frac{\mu_0 N_1 N_2 A}{\ell} = \mu_0 n_1 n_2 A \ell.$$

Quick Tip to Solve: Calculate field using the turns of the current-carrying coil, then multiply flux per turn by the turns of the receiving coil.

OR

16. Define self-inductance of a coil. Obtain the expression for the energy stored in an inductor L connected across a source of emf. [3 marks]

Solution:

Concept:

- Self-inductance is flux linkage per unit current: $N\Phi = LI$, for a linear coil with constant inductance L . Its induced emf is $\varepsilon_L = -L dI/dt$.
- Energy stored in the magnetic field is the work done against the induced back emf while the current grows. Instantaneous electrical power is $P = VI$.

Step 1: Define self-inductance The self-inductance of a coil is the magnetic flux linkage with the coil

Quick Tip to Solve: Use the induced opposing voltage with the instantaneous current: integrate $LI dI$ from zero, rather than multiplying the final power by time.

per unit current flowing through it. Equivalently, it equals the magnitude of emf induced in the same coil when its current changes at unit rate. Thus

$$L = \frac{N\Phi}{I}, \quad \varepsilon_L = -L \frac{dI}{dt}.$$

The negative sign is required by Lenz's law: the induced emf opposes the change of current. The SI unit of L is the henry (H).

Step 2: Calculate the work during current growth Connect the inductor across a source of emf and let the current build from zero. As its magnetic field increases, the back emf opposes the source. The applied voltage required to overcome this inductive opposition is $V_L = L dI/dt$. During a small time dt , charge $dq = I dt$ passes through the coil. The work supplied to its magnetic field is therefore

$$dU = V_L dq = \left(L \frac{dI}{dt} \right) I dt = LI dI.$$

This expression concerns the inductive part of the source work; any resistance also dissipates heat and is not included in the stored energy.

Step 3: Integrate to the final current Assume L remains constant and let the final current be I_f . Starting with no current and no magnetic field, integrate the incremental work:

$$U = \int_0^{I_f} LI dI = L \left[\frac{I^2}{2} \right]_0^{I_f}.$$

The energy depends on the final current, regardless of how quickly that current is established. It is stored in the magnetic field and can be returned to the circuit as the current decreases.

Step 4: Conclusion The magnetic energy stored in an inductor of inductance L carrying current I_f is

$$U = \frac{1}{2} LI_f^2.$$

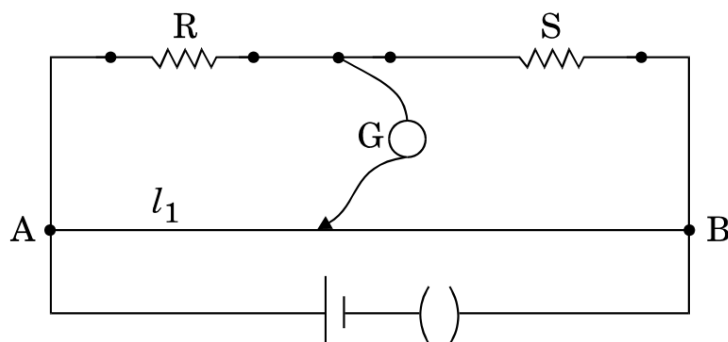
With L in henry and current in ampere, U is in joule.

Quick Tip to Solve: Use the induced opposing voltage with the instantaneous current: integrate $LI dI$ from zero, rather than multiplying the final power by time.

17. (a) Write the principle of working of a metre bridge.

(b) In a metre bridge, the balance point is found at a distance l_1 with resistances R and S as shown in the figure.

An unknown resistance X is now connected in parallel to the resistance S and the balance point is found at a distance l_2 . Obtain a formula for X in terms of l_1 , l_2 and S . [3 marks]



Solution:

Concept:

- A metre bridge operates on the Wheatstone bridge principle: at balance, the galvanometer current is zero and the resistance ratios of the two arms are equal.
- For a uniform bridge wire, resistance is proportional to length. Let $a = 100$ cm be its total length; all lengths in the ratios must use the same unit.
- The parallel combination of S and X has resistance $S_p = SX/(S + X)$.

Step 1: (a) State the balance principle At the null point, the galvanometer joins two points of equal potential and carries no current. Hence the ratio of the resistances in the two gaps equals the ratio of the resistances of the corresponding wire segments. Since the wire has uniform cross-sectional area and resistivity, its resistance ratio is its length ratio. With R in the left gap and S in the right gap, the length is measured from the left end A, as shown in the question.

Step 2: (b) Write the two balance equations Initially the balance length from A is l_1 , giving

$$\frac{R}{S} = \frac{l_1}{a - l_1}.$$

After X is connected in parallel with S , the left-gap resistance stays R , while the right-gap resistance becomes $S_p = SX/(S + X)$. The new balance condition is

$$\frac{R}{S_p} = \frac{l_2}{a - l_2}.$$

The same length unit must be used for a, l_1, l_2 .

Step 3: Eliminate the unchanged resistance R Divide the second balance equation by the first to obtain

$$\frac{S}{S_p} = \frac{l_2(a - l_1)}{l_1(a - l_2)}.$$

Quick Tip to Solve: Divide the two balance equations to eliminate R immediately; a parallel resistance on the right must move the balance point towards B.

Since $S/S_p = (S + X)/X = 1 + S/X$,

$$\begin{aligned}\frac{S}{X} &= \frac{l_2(a - l_1) - l_1(a - l_2)}{l_1(a - l_2)} \\ &= \frac{a(l_2 - l_1)}{l_1(a - l_2)}.\end{aligned}$$

For positive finite resistances, adding a parallel branch lowers the right-gap resistance, so $R/S_p > R/S$ and $l_2 > l_1$. This also checks the sign of the denominator.

Step 4: Conclusion The metre bridge uses the Wheatstone bridge null-balance principle. The required unknown resistance is

$$X = \frac{S l_1 (a - l_2)}{a (l_2 - l_1)}, \quad a = 100 \text{ cm}.$$

Thus, when l_1, l_2 are expressed in centimetres, $X = S l_1 (100 - l_2) / [100 (l_2 - l_1)]$.

Quick Tip to Solve: Divide the two balance equations to eliminate R immediately; a parallel resistance on the right must move the balance point towards B.

18. Draw a block diagram of a generalized communication system. Write the functions of each of the following:

- (a) Transmitter
- (b) Channel
- (c) Receiver

[3 marks]

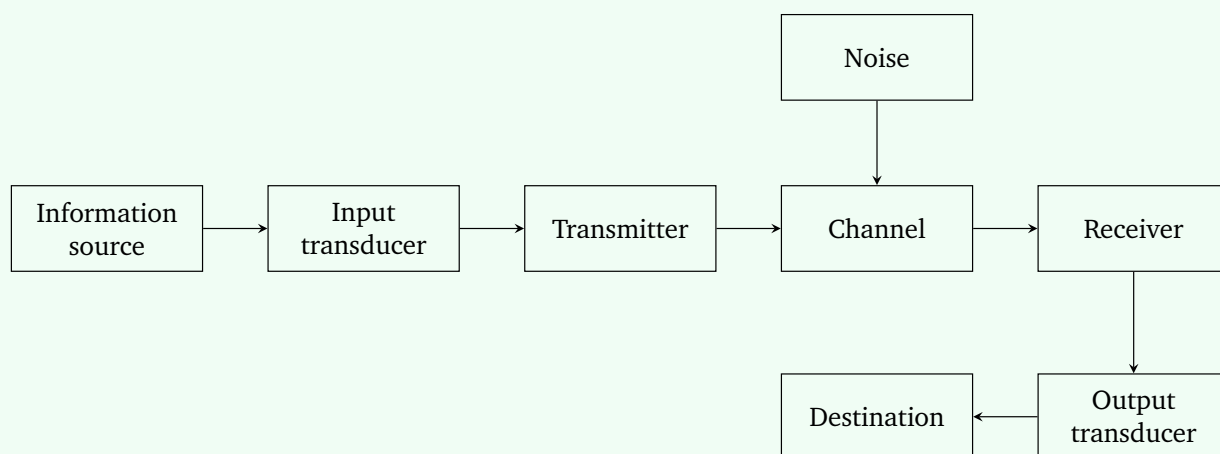
Solution:

Concept:

- Communication transfers information from a source to a destination through a transmitter, a channel and a receiver.
- Input and output transducers convert messages to and from electrical signals. Noise is an unwanted disturbance superposed on the signal during transmission.

Step 1: Draw the generalized communication system The arrows show the direction of information transfer. The input transducer converts the message into an electrical signal, which enters the transmitter. The channel carries the transmitted signal to the receiver. After recovery, the message reaches the destination through the output transducer. Noise entering the channel may disturb the received signal.

Quick Tip to Solve: Associate transmitter with preparation, channel with propagation, and receiver with recovery of the message.



Generalized communication system with noise entering the channel.

Step 2: (a) Transmitter The transmitter processes the message signal into a form suitable for transmission through the selected channel. It amplifies the signal and, when required, modulates a suitable carrier with the message. In radio communication, for example, an audio signal modulates a high-frequency carrier so that the resulting signal can be radiated efficiently by an antenna.

Step 3: (b) Channel The channel is the physical medium connecting the transmitter to the receiver. It transports the signal between them. A conducting wire, an optical fibre or free space can serve as a channel. While travelling through it, the signal may be attenuated and may acquire unwanted noise.

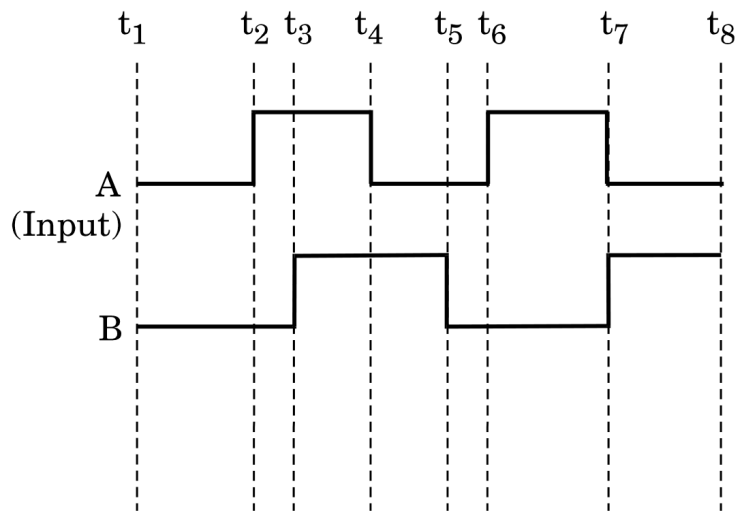
Step 4: (c) Receiver The receiver extracts the original message from the incoming signal. It selects the desired signal, amplifies it as necessary and demodulates it if a modulated carrier has been used. The recovered electrical message goes to the output transducer, which converts it into the required form for the destination.

Step 5: Conclusion The transmitter prepares and sends the signal, the channel carries it, and the receiver recovers the original message. The diagram includes the transducers, destination and channel noise.

Quick Tip to Solve: Associate transmitter with preparation, channel with propagation, and receiver with recovery of the message.

19. (a) Write the functions of the three segments of a transistor.

(b) The figure shows the input waveforms A and B for 'AND' gate. Draw the output waveform and write the truth table for this logic gate. [3 marks]



Solution:

Concept:

- A bipolar transistor has emitter, base and collector regions. Carrier injection from the emitter and collection by the collector are controlled by the thin base region.
- An AND gate obeys $Y = A \cdot B$: its output is high only when both inputs are simultaneously high. A high level represents logic 1, and a low level represents logic 0.

Step 1: (a) Functions of the transistor regions The emitter supplies a large number of majority charge carriers for transistor action. It is heavily doped to make this injection efficient. The base is very thin and lightly doped; it controls the carrier flow from emitter to collector while permitting most injected carriers to cross it with little recombination. The collector collects these carriers after they cross the base. It is moderately doped and generally has a larger physical size to dissipate the heat generated during operation. In normal active operation the emitter-base junction is forward biased and the collector-base junction is reverse biased.

Step 2: (b) Write the AND gate truth table Applying $Y = A \cdot B$ to all four input pairs gives

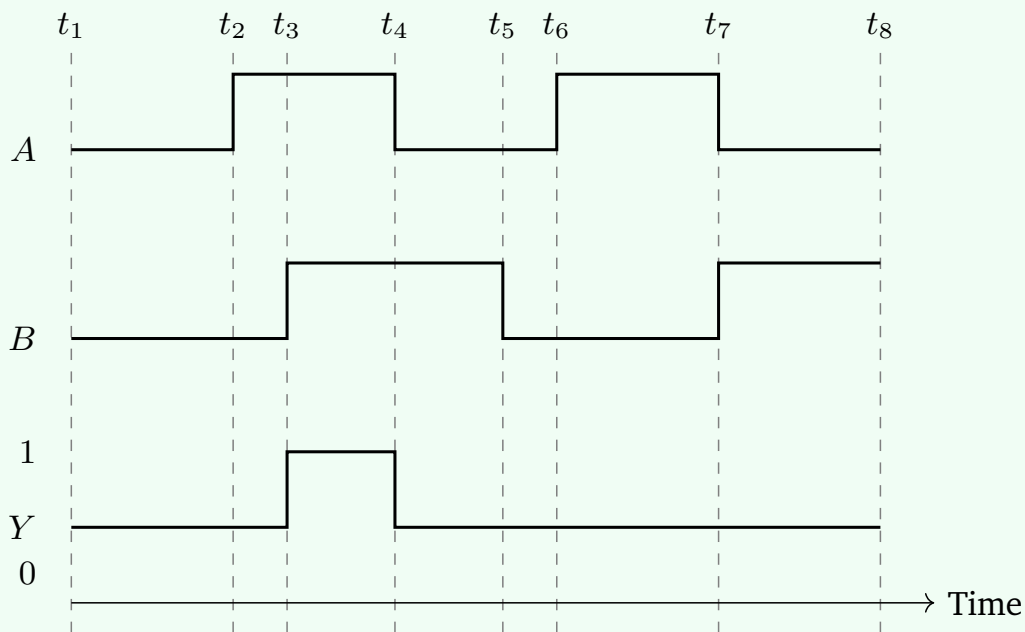
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

A single high input is insufficient: either low input forces the output to zero.

Step 3: Read the overlap and draw the output waveform In the printed waveforms, A is high from t_2 to t_4 and from t_6 to t_7 . Input B is high from t_3 to t_5 and from t_7 to t_8 . Their only finite common high interval is $t_3 < t < t_4$. Hence the output rises at t_3 , falls at t_4 , and stays low over every other interval shown. At t_7 , one input falls as the other rises, so ideal simultaneous switching creates no

Quick Tip to Solve: For an AND waveform, shade only the time intervals where both input traces are high; those intervals alone become output pulses.

finite output pulse.



AND output is high only from t_3 to t_4 ; input traces are repeated for comparison.

Step 4: Conclusion The emitter injects carriers, the base controls their passage, and the collector collects them. For the AND gate, $Y = 1$ only between t_3 and t_4 ; the truth table and output waveform are shown.

Quick Tip to Solve: For an AND waveform, shade only the time intervals where both input traces are high; those intervals alone become output pulses.

20. (a) Draw a ray diagram depicting the formation of the image by an astronomical telescope in normal adjustment.

(b) You are given the following three lenses. Which two lenses will you use as an eyepiece and as an objective to construct an astronomical telescope? Give reason. [3 marks]

Lenses	Power (D)	Aperture (cm)
L_1	3	8
L_2	6	1
L_3	10	1

Solution:

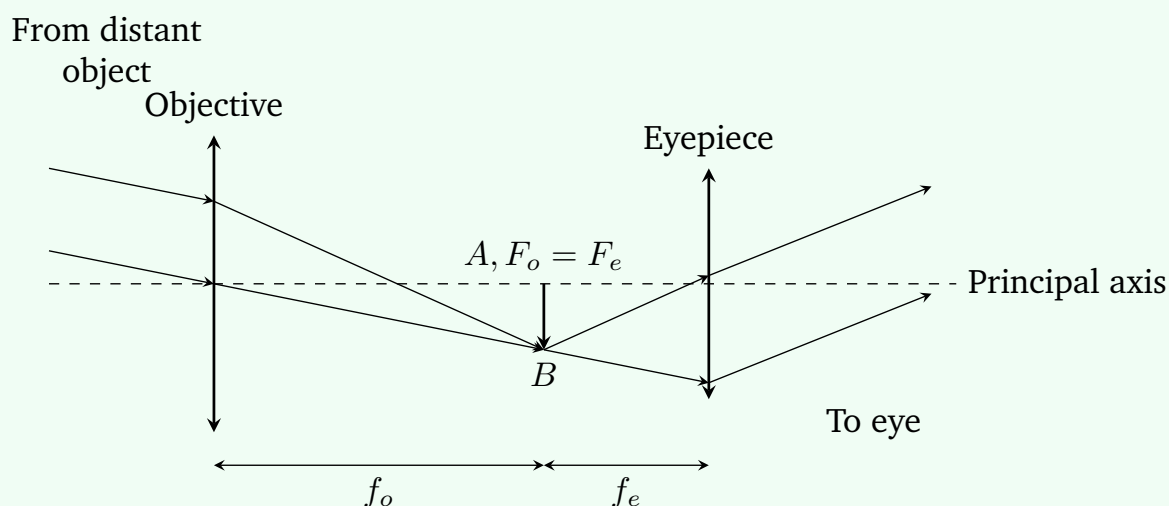
Concept:

- In normal adjustment, the real image formed by the objective lies in the focal plane of the eyepiece. The final image is at infinity, so the eye views it without accommodation.

Quick Tip to Solve: Choose the lowest positive power with the widest aperture for the objective, and the highest positive power for the eyepiece.

- For positive lens power P in dioptres, $f = 1/P$ in metres. The telescope has length $f_o + f_e$ and angular magnification $M = -f_o/f_e$, where the minus sign indicates inversion.

Step 1: (a) Formation of the image in normal adjustment Parallel rays from a distant object enter the large objective at a small angle to the principal axis. The objective converges them to a real, inverted intermediate image AB in its focal plane. This plane is also the first focal plane of the eyepiece: $F_o = F_e$. Rays from B emerge parallel after refraction through the eyepiece, producing an inverted, magnified final image at infinity. The ray diagram below shows one bundle from an off-axis point; the lower intermediate point makes the inversion explicit.



Astronomical telescope in normal adjustment; the final rays are parallel.

Step 2: (b) Compare the three focal lengths The printed lens table gives powers 3, 6 and 10 D. Therefore

$$f_1 = \frac{1}{3} \text{ m} = 33.3 \text{ cm}, \quad f_2 = \frac{1}{6} \text{ m} = 16.7 \text{ cm}, \quad f_3 = \frac{1}{10} \text{ m} = 10 \text{ cm}.$$

An objective should have a long focal length for a larger image and a large aperture to collect more light from distant faint objects. Lens L_1 has both the longest focal length and the largest aperture, 8 cm. The eyepiece should have a short focal length to increase angular magnification; between the two lenses with aperture 1 cm, L_3 has the shorter focal length. For this choice, $|M| = 33.3/10 \simeq 3.33$, and the normal-adjustment separation is about 43.3 cm.

Step 3: Conclusion Use L_1 as the objective and L_3 as the eyepiece. Their common focal plane gives the final image at infinity in normal adjustment.

Quick Tip to Solve: Choose the lowest positive power with the widest aperture for the objective, and the highest positive power for the eyepiece.

21. (a) State Biot - Savart law and express this law in the vector form.

(b) Two identical circular coils, P and Q each of radius R , carrying currents 1 A and $\sqrt{3}\text{ A}$ respectively, are placed concentrically and perpendicular to each other lying in the XY and YZ planes. Find the magnitude and direction of the net magnetic field at the centre of the coils. [3 marks]

Solution:

Concept:

- A current element $I d\vec{l}$ produces a magnetic field perpendicular to both the element and the displacement vector from it to the observation point.
- At the centre of a circular single-turn coil of radius R , $B = \mu_0 I / (2R)$. Its direction is normal to the coil plane, determined by the right-hand rule.

Step 1: (a) State Biot-Savart law The magnetic field contribution dB due to a short current element is directly proportional to the current I , element length dl , and $\sin \theta$, and inversely proportional to the square of the separation r . Here θ is the angle between $d\vec{l}$ and $\vec{r}$, which points from the element to the field point. In vacuum its scalar and vector forms are

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin \theta}{r^2}, \quad d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \hat{r})}{r^2}.$$

Equivalently the vector denominator is r^3 when the numerator contains $\vec{r}$ instead of $\hat{r}$.

Step 2: (b) Combine the fields of the perpendicular coils Coil P lies in the XY plane, so its field is along the Z axis; coil Q lies in the YZ plane, so its field is along the X axis. Their magnitudes are

$$B_P = \frac{\mu_0}{2R}, \quad B_Q = \frac{\sqrt{3}\mu_0}{2R}.$$

Since they are perpendicular, their magnitudes must be combined by Pythagoras:

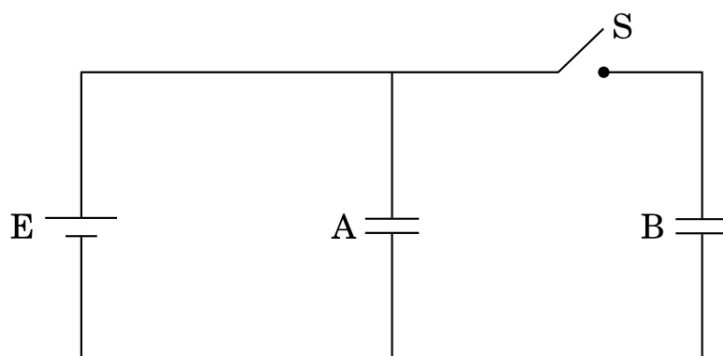
$$B_{\text{net}} = \sqrt{B_P^2 + B_Q^2} = \frac{\mu_0}{2R} \sqrt{1 + 3} = \frac{\mu_0}{R} \text{ T}$$

when R is expressed in metres. Let θ be the angle of the resultant from the field of Q toward that of P . Then $\tan \theta = B_P / B_Q = 1/\sqrt{3}$, giving $\theta = 30^\circ$. If both fields point along positive axes, $\vec{B} = \mu_0(\sqrt{3}\hat{i} + \hat{k})/(2R)$. The printed question does not specify current circulation senses, so the signs of the axis components must follow the actual right-hand-rule directions; a unique signed direction cannot be inferred.

Step 3: Conclusion The resultant has magnitude $\mu_0/R \text{ T}$, lies in the XZ plane, and makes 30° with the field of coil Q (60° with that of P). Axis signs depend on current senses.

Quick Tip to Solve: First take the normal to each coil plane; perpendicular coil fields add by Pythagoras, with their ratio giving the resultant angle.

22. Two identical parallel plate capacitors A and B are connected to a battery of V volts with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K . Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. [3 marks]



Solution:

Concept:

- For a capacitor, $U = \frac{1}{2}CV^2 = Q^2/(2C)$. Filling its entire gap with dielectric constant K changes its capacitance from C to KC .
- A capacitor connected to an ideal battery retains its voltage; an isolated capacitor retains its free charge. The open switch determines which condition applies.

Step 1: Find the initial energy Let the capacitance of each identical capacitor before inserting the dielectric be C . With switch S closed, both capacitors are in parallel with the battery, so each has voltage V , charge CV , and energy $\frac{1}{2}CV^2$. The initial total energy is consequently

$$U_{\text{before}} = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2.$$

Opening the switch alone does not change these initial charges. In the printed circuit, the switch is on the upper lead to B , whereas the upper lead to A remains joined directly to the battery.

Step 2: Use constant voltage for capacitor A After opening S , capacitor A still has both terminals connected across the battery. When dielectric is inserted, its capacitance becomes KC while its voltage stays V . The battery supplies additional charge, giving

$$Q_A = KCV, \quad U_A = \frac{1}{2}KCV^2.$$

Its energy increases because the voltage source maintains the potential difference.

Step 3: Use constant charge for capacitor B and add The upper plate of B is isolated by the open switch, so its free charge remains $Q_B = CV$. Although its lower plate is still attached to the common lower wire, there is no conducting path that can transfer charge to or from its isolated upper plate. Its new voltage is $V_B = Q_B/(KC) = V/K$, and

$$U_B = \frac{(CV)^2}{2KC} = \frac{CV^2}{2K}.$$

Quick Tip to Solve: Trace both leads after opening the switch: A remains at fixed voltage, whereas B has fixed charge; use a different energy formula for each.

Hence the total final stored energy is

$$U_{\text{after}} = \frac{CV^2}{2} \left(K + \frac{1}{K} \right).$$

The energy of the capacitors alone need not be conserved, since the battery and the mechanical process of dielectric insertion also exchange energy.

Step 4: Conclusion

$$\frac{U_{\text{before}}}{U_{\text{after}}} = \frac{2}{K + 1/K} = \frac{2K}{K^2 + 1}.$$

Thus the required before-to-after ratio is $2K : (K^2 + 1)$.

Quick Tip to Solve: Trace both leads after opening the switch: A remains at fixed voltage, whereas B has fixed charge; use a different energy formula for each.

Section D (4 marks)

23. Asha's mother read an article in the newspaper about a disaster that took place at Chernobyl. She could not understand much from the article and asked a few questions from Asha regarding the article. Asha tried to answer her mother's questions based on what she learnt in Class XII Physics.

(a) What was the installation at Chernobyl where the disaster took place? What, according to you, was the cause of this disaster?

(b) Explain the process of release of energy in the installation at Chernobyl.

(c) What, according to you, were the values displayed by Asha and her mother?

[4 marks]

Solution:

Concept:

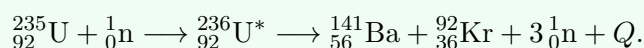
- Nuclear fission is the splitting of a heavy nucleus into lighter nuclei after absorbing a neutron; energy and further neutrons are released.
- The energy released is $Q = \Delta mc^2$, where Δm is the decrease in total rest mass and c is the speed of light. The products have a greater binding energy per nucleon.
- A reactor uses a controlled chain reaction: neutrons from one fission cause further fissions, while neutron-absorbing control rods regulate the reaction rate.

Step 1: (a) Identify the installation and the cause of the disaster The installation was a nuclear reactor in a nuclear power station at Chernobyl. The accident involved loss of safe control of the reactor during a test: unsafe operating conditions and deficiencies in reactor design and safety procedures allowed a rapid rise in reactor power. The resulting overheating and steam explosions damaged the reactor, and a fire dispersed radioactive material into the surroundings. In physical terms, failure to keep the fission reaction and heat removal under safe control caused the disaster; it was not a normal, controlled release of electrical energy.

Step 2: (b) Explain how fission releases energy When a uranium-235 nucleus absorbs a slow neutron, it forms an excited uranium-236 nucleus which can split into two lighter nuclei. One possible fission

Quick Tip to Solve: Keep the three requested answers separate: identify the reactor accident, explain fission and its mass defect, then connect each value to the behaviour of Asha or her mother.

channel is



Both nucleon number and charge are conserved: $235 + 1 = 141 + 92 + 3$ and $92 = 56 + 36$. The total rest mass of the final particles is smaller than that of the initial particles. This mass difference appears as energy, $Q = \Delta mc^2$, approximately 200 MeV per fission. Much of it is carried as kinetic energy of the fission fragments and is converted into heat as they slow down in the reactor material.

Step 3: (b) Relate the chain reaction to power generation The released neutrons can induce fission in other uranium nuclei, establishing a chain reaction. In normal operation, a moderator slows neutrons and control rods absorb surplus neutrons so that the reaction proceeds at a controlled rate. A coolant removes the heat; this heat produces steam which drives a turbine connected to an electrical generator. Loss of safe reaction control and cooling can produce dangerous overheating and release radioactive products, as occurred in the disaster.

Step 4: (c) Identify the values displayed Asha's mother displayed curiosity, awareness of current events and a willingness to learn by asking questions about what she did not understand. Asha displayed scientific understanding, helpfulness, respect for her mother and willingness to share knowledge. By explaining a public event using the physics she had learnt, she also showed a scientific attitude and concern for informed understanding of nuclear safety.

Step 5: Conclusion Chernobyl was a nuclear power reactor installation whose unsafe operation led to loss of reactor control and a catastrophic release of radioactive material. Its normal energy source was a controlled nuclear-fission chain reaction, with mass converted into energy through $Q = \Delta mc^2$. Asha and her mother showed helpfulness, scientific curiosity, respect and willingness to learn.

Quick Tip to Solve: Keep the three requested answers separate: identify the reactor accident, explain fission and its mass defect, then connect each value to the behaviour of Asha or her mother.

Section E (15 marks)

24. (a) Derive an expression for the electric field $\vec{E}$ due to a dipole of length ' $2a$ ' at a point distant r from the centre of the dipole on the axial line.

(b) Draw a graph of E versus r for $r \gg a$.

(c) If this dipole were kept in a uniform external electric field $\vec{E}_0$, diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases.

[5 marks]

Solution:

Concept:

- An electric dipole has charges $-q$ and $+q$ separated by $2a$. Its dipole moment is $\vec{p} = q(2a)\hat{p}$, directed from negative to positive charge.
- Coulomb's law gives $E = kq/d^2$, where $k = 1/(4\pi\epsilon_0)$. Fields add vectorially; on the axial line beyond either charge their contributions oppose each other.
- In a uniform external field, the net force is zero but the torque is $\vec{\tau} = \vec{p} \times \vec{E}_0$, of magnitude $pE_0 \sin \theta$.

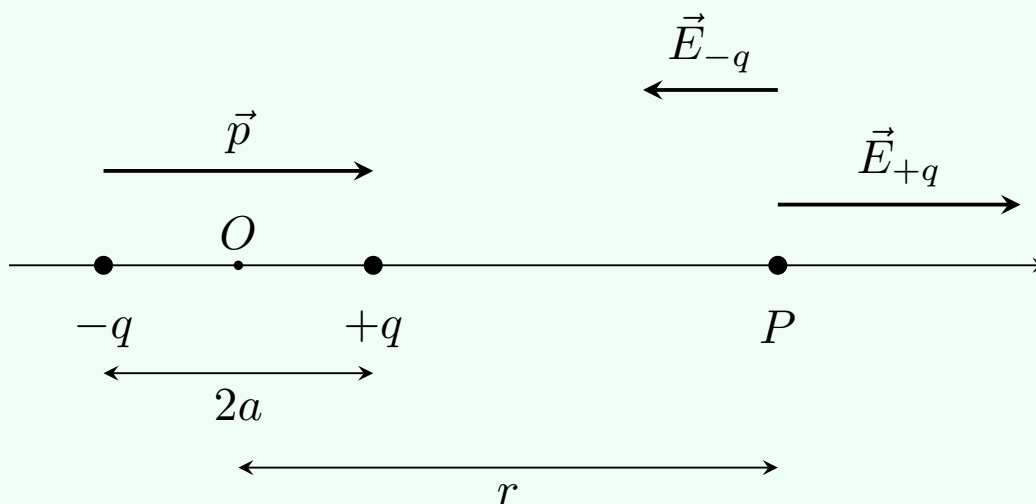
Quick Tip to Solve: Subtract the two axial Coulomb fields before approximating; use minimum or maximum potential energy to distinguish the two zero-torque positions.

The potential energy is $U = -pE_0 \cos \theta$.

Step 1: (a) Calculate the two axial fields Place $-q$ at $x = -a$, $+q$ at $x = a$, and the observation point P at $x = r > a$. The distances of P from $+q$ and $-q$ are $r - a$ and $r + a$, respectively. The positive charge produces a field along $\hat{p}$, whereas the negative charge produces a field opposite to $\hat{p}$. Thus superposition gives

$$\vec{E} = kq \left[\frac{1}{(r - a)^2} - \frac{1}{(r + a)^2} \right] \hat{p}.$$

The nearer positive charge has the stronger contribution, so the resultant points in the direction of the dipole moment.



Axial point beyond the positive charge; the two charge fields oppose.

Step 2: Simplify and obtain the distant-field expression Combining the fractions explicitly,

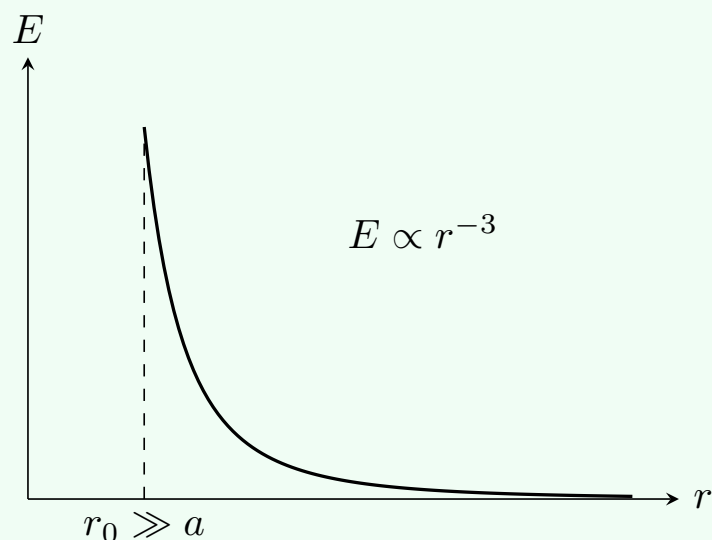
$$\begin{aligned} \vec{E} &= kq \frac{(r + a)^2 - (r - a)^2}{(r^2 - a^2)^2} \hat{p} \\ &= k \frac{4qar}{(r^2 - a^2)^2} \hat{p} = \frac{1}{4\pi\epsilon_0} \frac{2r\vec{p}}{(r^2 - a^2)^2}. \end{aligned}$$

On the other outer half of the axis the resultant also points along $\vec{p}$. The displayed formula applies to an external axial point, with r its distance from the centre. For $r \gg a$, neglect a^2 relative to r^2 to obtain

$$\vec{E} \simeq \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}.$$

Step 3: (b) Plot the field in the far-field region The magnitude satisfies $E = Cr^{-3}$, with $C = 2p/(4\pi\epsilon_0) > 0$. It decreases rapidly as r increases and approaches zero asymptotically. For example, doubling the distance reduces the field to one eighth. The graph below begins at a distance $r_0 \gg a$: extrapolating the approximation to the dipole centre would be invalid.

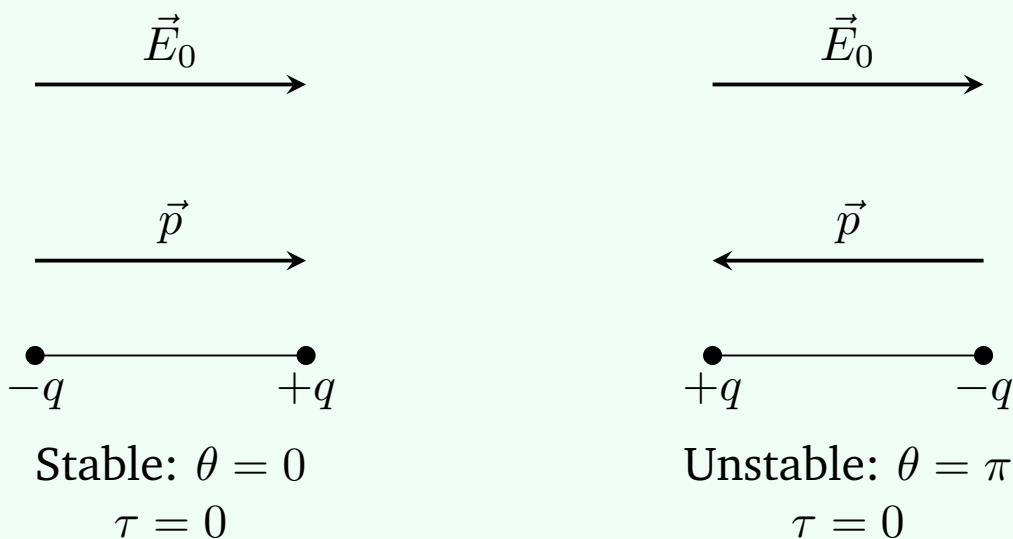
Quick Tip to Solve: Subtract the two axial Coulomb fields before approximating; use minimum or maximum potential energy to distinguish the two zero-torque positions.



Qualitative axial-field curve for distances beginning at $r_0 \gg a$.

Step 4: (c) Identify and draw both equilibria At equilibrium, $\sin \theta = 0$, so $\theta = 0$ or π . For the parallel orientation, $\theta = 0$, the energy $U = -pE_0$ is minimum and a small angular displacement produces a restoring torque: this is stable equilibrium. Its torque is $\tau = pE_0 \sin 0 = 0$.

For the antiparallel orientation, $\theta = \pi$, the energy $U = +pE_0$ is maximum. A small displacement produces a torque that takes the dipole further away from this orientation: this is unstable equilibrium. Its torque is $\tau = pE_0 \sin \pi = 0$. Zero torque at the exact orientation therefore does not by itself establish stability.



Dipole orientations in a uniform field: stable parallel and unstable antiparallel.

Step 5: Conclusion The external axial field is $\vec{E} = 2r\vec{p}/[4\pi\epsilon_0(r^2 - a^2)^2]$, reducing to $2\vec{p}/(4\pi\epsilon_0r^3)$ for $r \gg a$. The far-field graph decreases as r^{-3} . Parallel and antiparallel dipoles are in stable and unstable equilibrium, respectively, with zero torque in both exact positions.

Quick Tip to Solve: Subtract the two axial Coulomb fields before approximating; use minimum or maximum potential energy to distinguish the two zero-torque positions.

24. (a) Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge density σ .

(b) An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Obtain the expression for the amount of work done in bringing a point charge q from infinity to a point, distant r , in front of the charged plane sheet.

[5 marks]

Solution:

Concept:

- Gauss's theorem is $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\epsilon_0$, where ϵ_0 is the permittivity of free space.
- An infinite uniformly charged plane has translational symmetry parallel to its surface and reflection symmetry across it. Its electric field is normal to the plane, with the same magnitude on both sides.
- The work done by an external agent in moving a charge slowly is $W_{\text{ext}} = q\Delta V$. The work done by the electric field is its negative. A reference at infinity is usable only if the potential difference from infinity converges.

Step 1: (a) Choose a Gaussian pillbox Take a short cylindrical Gaussian surface whose flat faces each have area A , with one face on each side of the thin sheet. The cylinder's axis is perpendicular to the sheet. Symmetry rules out a tangential component of $\mathbf{E}$. For a positively charged sheet the field points away from the sheet on both sides; hence it is parallel to each flat face's outward normal. The field is perpendicular to the outward normal on the curved surface, so the curved surface contributes zero flux.

Step 2: (a) Apply Gauss's theorem The enclosed charge is $Q_{\text{enc}} = \sigma A$. For positive σ , the flux through the two flat faces is $EA + EA$, giving

$$2EA = \frac{\sigma A}{\epsilon_0}, \quad E = \frac{\sigma}{2\epsilon_0}.$$

Thus the field magnitude is $|\sigma|/(2\epsilon_0)$, independent of the distance from the sheet. It points away from a positive sheet and towards a negative sheet. The factor of two arises because both flat faces contribute flux.

Step 3: (b) First calculate work from a finite reference point Put the sheet at $x = 0$, and let the point in front of it be $x = r > 0$. On this side, $\mathbf{E} = E\hat{\mathbf{x}}$, where $E = \sigma/(2\epsilon_0)$ and $\sigma > 0$. Start at $x = R > r$. The potential difference is

$$V(r) - V(R) = - \int_R^r E dx = \frac{\sigma}{2\epsilon_0}(R - r).$$

Consequently, for a slow displacement the external work is

$$W_{\text{ext}}(R \rightarrow r) = \frac{q\sigma}{2\epsilon_0}(R - r),$$

while the electric field does work $W_{\text{field}} = q\sigma(r - R)/(2\epsilon_0)$. For a positive charge, bringing it closer opposes the repulsive force and requires positive external work.

Quick Tip to Solve: Before using infinity as the potential reference, test the integral of the field: a constant field integrated over an infinite distance diverges.

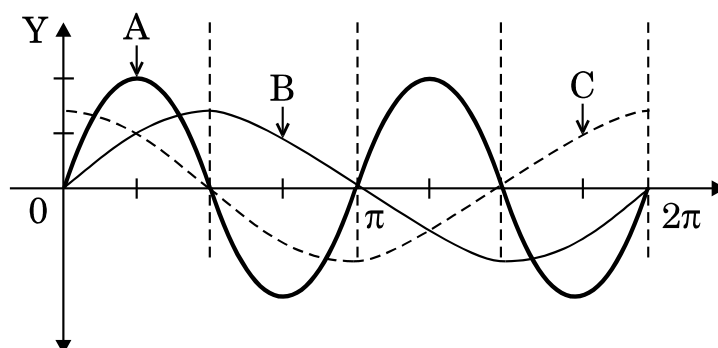
Step 4: (b) Examine the printed reference at infinity The printed question specifies an infinitely large sheet and a starting point at infinity. Taking $R \rightarrow \infty$ in the finite-reference expression shows that the work does not converge for nonzero q : it tends to $+\infty$ for $q > 0$ and $-\infty$ for $q < 0$ for the external agent. The field is constant even arbitrarily far away, so one cannot consistently set $V(\infty) = 0$ and obtain a finite potential at r . If instead the sheet itself is chosen as the zero-potential reference, $V(r) - V(0) = -\sigma r/(2\epsilon_0)$, giving $-q\sigma r/(2\epsilon_0)$ for external work from the sheet to r ; that expression does not represent work from infinity.

Step 5: Conclusion (a) The field magnitude is $E = |\sigma|/(2\epsilon_0)$, perpendicular to the plane. (b) As printed, there is no finite work from infinity for a nonzero charge: $W_{\text{ext}} = \lim_{R \rightarrow \infty} q\sigma(R - r)/(2\epsilon_0)$, which diverges. Between finite points the expression is $q\sigma(R - r)/(2\epsilon_0)$; a finite answer proportional to r requires a finite reference such as the sheet.

Quick Tip to Solve: Before using infinity as the potential reference, test the integral of the field: a constant field integrated over an infinite distance diverges.

25. A device X is connected to an ac source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph:

- Identify the device ' X '.
- Which of the curves A , B and C represent the voltage, current and the power consumed in the circuit? Justify your answer.
- How does its impedance vary with frequency of the ac source? Show graphically.
- Obtain an expression for the current in the circuit and its phase relation with ac voltage. [5 marks]



Solution:

Concept:

- For a capacitor of capacitance C_X , the charge is $q = C_X V$ and the current is $i = dq/dt$. The symbol C_X distinguishes capacitance from curve C in the question.

Quick Tip to Solve: First locate the sine curve that starts at zero; a current already at its positive peak leads that voltage by a quarter cycle and identifies a capacitor.

- Capacitive reactance is $X_C = 1/(\omega C_X) = 1/(2\pi f C_X)$, where f is the source frequency and $\omega = 2\pi f$. Current leads voltage by $\pi/2$.
- Instantaneous power is $p = Vi$. For an ideal capacitor its cycle average is zero: energy is alternately stored in its electric field and returned to the source.

Step 1: (a) Identify the device from the phase relation The curve that begins at zero and initially rises must be the given voltage $V = V_0 \sin \omega t$. This is curve B. Curve C has its positive maximum at $t = 0$, becomes zero when B reaches its positive maximum, and is negative when B passes through zero at $\omega t = \pi$. Thus C leads B by a quarter cycle. A capacitor has exactly this current-voltage relation, so the device X is an ideal capacitor. An inductor would have current lagging the voltage, whereas a resistor would have voltage and current in phase.

Step 2: (b) Assign the voltage, current and power curves Curve B represents voltage, curve C represents current, and curve A represents instantaneous power. In particular,

$$V = V_0 \sin \omega t, \quad i = I_0 \cos \omega t.$$

Their product is

$$p = V_0 I_0 \sin \omega t \cos \omega t = \frac{V_0 I_0}{2} \sin 2\omega t.$$

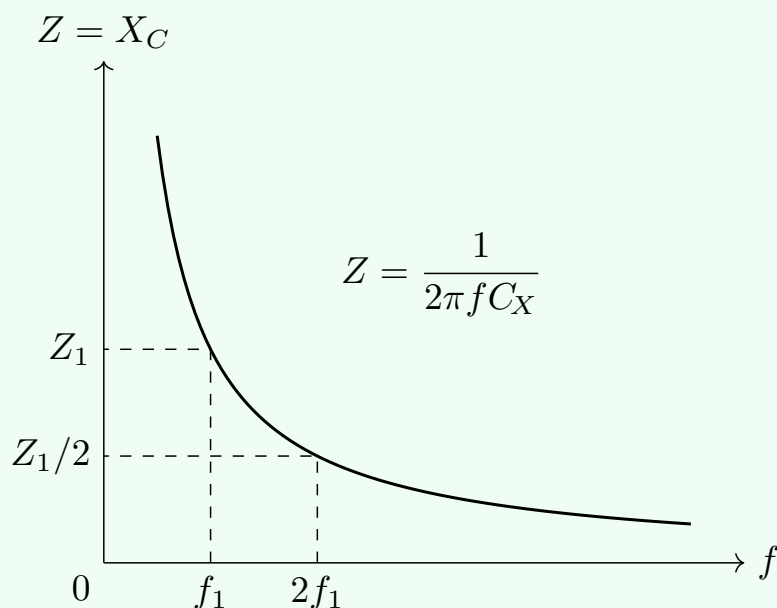
Therefore power has twice the source frequency, as curve A shows: it completes two oscillations while B and C complete one. During the first quarter cycle both voltage and current are positive and A is positive; during the next quarter voltage is positive but current is negative, so A is negative. The same pattern repeats in the second half-cycle. The negative portions indicate energy flowing back to the source, rather than negative dissipation. Consequently the net energy consumed over a complete cycle and the average power are zero.

Step 3: (c) Show how impedance depends on frequency For this pure capacitive circuit the impedance magnitude is

$$Z = X_C = \frac{1}{2\pi f C_X}.$$

For a fixed capacitance, Z is inversely proportional to f . Doubling the frequency halves the impedance. Its graph is a decreasing rectangular hyperbola in the positive quadrant. As $f \rightarrow 0^+$, $Z \rightarrow \infty$; as $f \rightarrow \infty$, $Z \rightarrow 0$. Thus the capacitor blocks steady direct current after charging, but offers less opposition at higher alternating-current frequencies.

Quick Tip to Solve: First locate the sine curve that starts at zero; a current already at its positive peak leads that voltage by a quarter cycle and identifies a capacitor.



Impedance magnitude of a capacitor decreases inversely with source frequency.

Step 4: (d) Derive the instantaneous current Using the charge-voltage relation and differentiating with respect to time gives

$$\begin{aligned}
 q(t) &= C_X V_0 \sin \omega t, \\
 i(t) &= \frac{dq}{dt} = \omega C_X V_0 \cos \omega t \\
 &= I_0 \sin \left(\omega t + \frac{\pi}{2} \right), \\
 I_0 &= \omega C_X V_0 = \frac{V_0}{X_C}.
 \end{aligned}$$

The extra phase $+\pi/2$ in the current expression means that its maximum occurs a quarter period before the voltage maximum. This agrees with the positions of the peaks and zero crossings of the printed curves.

Step 5: Conclusion X is an ideal capacitor: B is voltage, C is current and A is instantaneous power. Its impedance magnitude is $Z = 1/(2\pi f C_X)$, and $i = \omega C_X V_0 \sin(\omega t + \pi/2)$; the current leads the voltage by 90° , while the average power is zero.

Quick Tip to Solve: First locate the sine curve that starts at zero; a current already at its positive peak leads that voltage by a quarter cycle and identifies a capacitor.

25. (a) Draw a labelled diagram of an ac generator. Obtain the expression for the emf induced in the rotating coil of N turns each of cross-sectional area A , in the presence of a magnetic field $\vec{B}$.

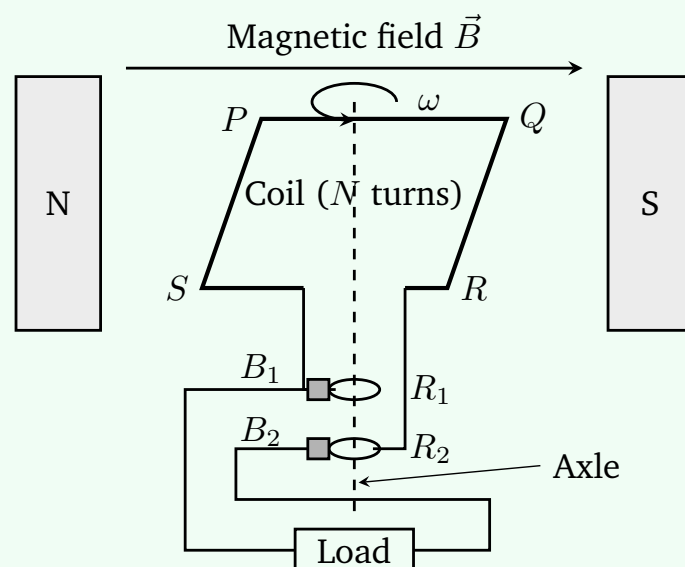
(b) A horizontal conducting rod 10 m long extending from east to west is falling with a speed 5.0 m s^{-1} at right angles to the horizontal component of the Earth's magnetic field, $0.3 \times 10^{-4} \text{ Wb m}^{-2}$. Find the instantaneous value of the emf induced in the rod. [5 marks]

Solution:

Concept:

- An ac generator converts mechanical energy into electrical energy by rotating a coil in a magnetic field. Faraday's law gives the induced emf as $\varepsilon = -d\Phi_N/dt$, where Φ_N is the total flux linkage of the coil.
- For a coil of N turns and area A , in a uniform field of magnitude B , the flux linkage is $\Phi_N = NBA \cos \theta$, where θ is the angle between the field and the normal to the coil.
- A straight rod of length l moving with speed v perpendicular to both its length and a magnetic field has motional emf of magnitude $\varepsilon = Blv$.

Step 1: (a) Construction and working of an ac generator A rectangular armature coil $PQRS$, containing N turns, rotates on an axle between the north and south poles of a magnet. Its two ends connect to two separate slip rings R_1 and R_2 , which rotate with the coil. Stationary carbon brushes B_1 and B_2 maintain contact with the rings and connect the coil to the external load. As the coil rotates, its flux linkage changes continuously and an emf is induced. The direction of emf reverses every half revolution, giving an alternating output. The slip rings preserve the connection of each coil end to its brush throughout the rotation.



Slip rings R_1, R_2 ; carbon brushes B_1, B_2

AC generator: rotating armature, separate slip rings, stationary brushes and external load (schematic).

Step 2: (a) Derive the instantaneous induced emf Let the coil rotate with constant angular speed ω .

Quick Tip to Solve: In the generator formula measure the angle from the coil normal; for the falling rod first convert the horizontal field to tesla and then multiply by length and speed.

Choose $t = 0$ when its normal is parallel to the magnetic field, so that $\theta = \omega t$. The flux through one turn is $BA \cos \omega t$, and hence the total flux linkage is

$$\Phi_N = NBA \cos \omega t.$$

Applying Faraday's law gives

$$\begin{aligned}\varepsilon(t) &= -\frac{d}{dt}(NBA \cos \omega t) \\ &= NBA\omega \sin \omega t = \varepsilon_0 \sin \omega t, \quad \varepsilon_0 = NBA\omega.\end{aligned}$$

Here ε_0 is the peak emf, and $\omega = 2\pi f$, with f the rotational frequency. The emf is zero when the coil normal is parallel to the field, and its magnitude is maximum when the normal is perpendicular to the field. The chosen initial orientation fixes the sine phase; reversing the rotation or terminal reference reverses the sign.

Step 3: (b) Evaluate the motional emf in the falling rod The east-west rod is perpendicular to the horizontal component of the Earth's field, and its downward velocity is perpendicular to both. Therefore the magnitude is $\varepsilon = B_H lv$. Only the horizontal component contributes along the rod: any vertical field component is parallel to the velocity and gives no magnetic force from $\vec{v} \times \vec{B}$. Substituting the printed values,

$$\begin{aligned}B_H &= 0.3 \times 10^{-4} \text{ T} = 3.0 \times 10^{-5} \text{ T}, \\ \varepsilon &= (3.0 \times 10^{-5})(10)(5.0) \text{ V} \\ &= 1.5 \times 10^{-3} \text{ V}.\end{aligned}$$

This is the instantaneous potential difference between the ends at the stated speed, arising from charge separation under the magnetic force on the mobile charges.

Step 4: Conclusion The generator output is $\varepsilon(t) = NBA\omega \sin \omega t$, with peak emf $NBA\omega$. The falling rod has instantaneous induced emf of magnitude $1.5 \times 10^{-3} \text{ V} = 1.5 \text{ mV}$.

Quick Tip to Solve: In the generator formula measure the angle from the coil normal; for the falling rod first convert the horizontal field to tesla and then multiply by length and speed.

26. (a) Define wavefront. Use Huygens' principle to verify the laws of refraction.

(b) How is linearly polarised light obtained by the process of scattering of light? Find the Brewster angle for air - glass interface, when the refractive index of glass = 1.5. [5 marks]

Solution:

Concept:

- A wavefront is the locus of all points of a wave that are in the same phase at a given instant. Rays in an isotropic medium are perpendicular to the wavefront.
- Huygens' principle states that every point on a wavefront acts as a source of secondary wavelets; their forward common envelope gives the wavefront at a later time.
- For a medium of refractive index n , the speed of light is $v = c/n$. Brewster's law is $\tan i_B = n_2/n_1$,

Quick Tip to Solve: In Huygens' refraction triangles use the same time and the same hypotenuse AC; their sine ratio immediately becomes the speed ratio.

where i_B is the polarising angle of incidence.

Step 1: (a) Construct the refracted wavefront Consider a plane incident wavefront AB approaching a plane interface between two isotropic media with light speeds v_1 and v_2 . Point A first reaches the interface. In a time t , the other end B reaches the interface at C , so $BC = v_1 t$. During the same interval the secondary wavelet from A in the second medium has radius $AD = v_2 t$. Draw the tangent CD from C to this wavelet, touching it at D . The tangent CD is the refracted wavefront, and AD is perpendicular to it. Similarly, BC is perpendicular to the incident wavefront AB .

Step 2: (a) Verify Snell's law and the plane of refraction Let i and r be the angles of the incident and refracted rays with the interface normal. Since each ray is normal to its wavefront, the angles between AB and AC , and between CD and AC , are respectively i and r . The right triangles ABC and ADC give

$$\begin{aligned}\sin i &= \frac{BC}{AC} = \frac{v_1 t}{AC}, \\ \sin r &= \frac{AD}{AC} = \frac{v_2 t}{AC}, \\ \frac{\sin i}{\sin r} &= \frac{v_1}{v_2} = \frac{n_2}{n_1}.\end{aligned}$$

Thus $n_1 \sin i = n_2 \sin r$, the law of refraction. The construction takes place in the plane containing the incident ray and the interface normal. The refracted ray, normal to CD , lies in this same plane. Hence the incident ray, refracted ray and normal at the point of incidence are coplanar, verifying the other law. If $n_2 > n_1$, then $v_2 < v_1$ and $r < i$, consistent with bending towards the normal.

Step 3: (b) Explain polarisation by scattering Let unpolarised light travel along the x -direction and be observed after scattering along the y -direction, perpendicular to the incident beam. Its electric field drives the electrons of a small scattering particle to oscillate in directions perpendicular to x , which may be resolved along y and z . An oscillating electric dipole does not radiate along its own axis. Thus the y -directed oscillation contributes no radiation towards the observer on the y -axis, whereas the z -directed oscillation does. The observed scattered light therefore has its electric field along z , perpendicular to the scattering plane xy , and is linearly polarised. Complete polarisation occurs for observation at 90° in the ideal single-scattering description; at general scattering angles the light is only partially polarised.

Step 4: (b) Calculate the Brewster angle At the Brewster angle, reflected light is completely linearly polarised and the reflected and refracted rays are perpendicular. Consequently $i_B + r = 90^\circ$. Substituting $\sin r = \cos i_B$ in Snell's law gives $\tan i_B = n_{\text{glass}}/n_{\text{air}}$. For air take $n_{\text{air}} \simeq 1$, so

$$i_B = \tan^{-1}(1.5) = 56.31^\circ.$$

This angle is measured from the normal to the air–glass surface, not from the surface itself.

Step 5: Conclusion Huygens' construction gives $n_1 \sin i = n_2 \sin r$ and coplanarity of the incident ray, refracted ray and normal. Scattered light observed at right angles to the incident beam is linearly polarised perpendicular to the scattering plane. The Brewster angle for incidence from air onto glass is $i_B \simeq 56.3^\circ$.

Quick Tip to Solve: In Huygens' refraction triangles use the same time and the same hypotenuse AC ; their sine ratio immediately becomes the speed ratio.

26. (a) Draw a ray diagram to show the image formation by a combination of two thin convex lenses in contact. Obtain the expression for the power of this combination in terms of the focal lengths of the lenses.

(b) A ray of light passing from air through an equilateral glass prism undergoes minimum deviation when the angle of incidence is $\frac{3}{4}$ th of the angle of prism. Calculate the speed of light in the prism.

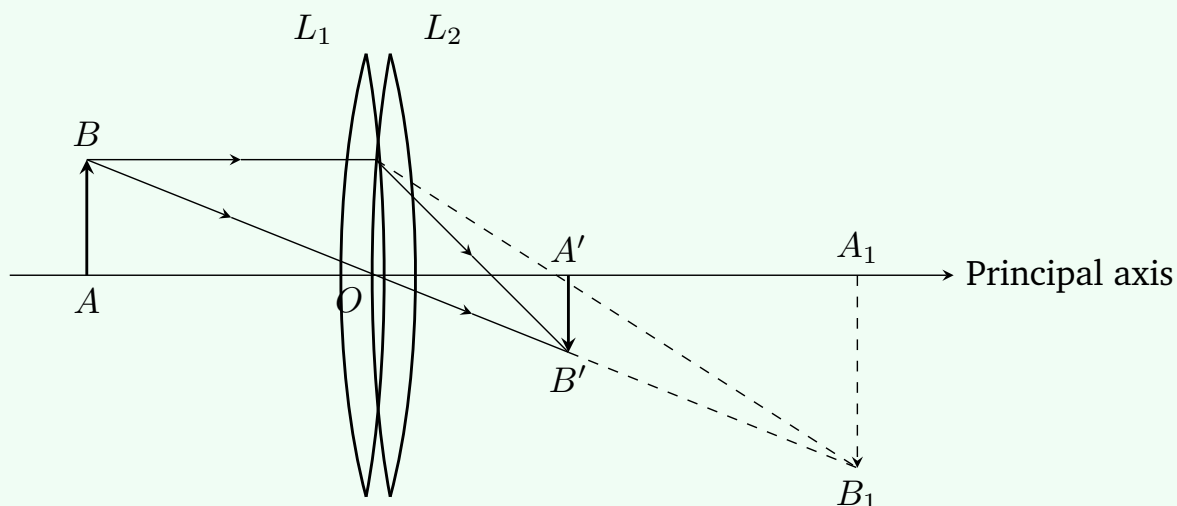
[5 marks]

Solution:

Concept:

- For a thin lens in air, the Cartesian lens formula is $1/v - 1/u = 1/f$. Distances are measured from the optical centre, with positive distances along the direction of incident light.
- For lenses in contact, the image formed by the first lens acts as the object for the second lens. Their separation is negligible, so both use the same reference plane.
- Lens power is $P = 1/f$, with f in metres and P in dioptres. At minimum deviation through a prism, $i = e$ and $r_1 = r_2 = A/2$.
- For a prism in air, $\mu = \sin i / \sin r$, and the light speed in glass is $v_{\text{glass}} = c/\mu$.

Step 1: (a) Ray diagram and intermediate image Let AB be the object, L_1 and L_2 the two thin convex lenses in contact, and O their effectively common optical centre. The diagram shows a representative case with a real, inverted final image. A ray from B parallel to the principal axis is refracted by the combination towards B' . A ray through O travels undeviated in the thin-lens approximation. Their intersection gives the final image $A'B'$. The dashed continuation shows the intermediate image A_1B_1 that the first lens alone would form; the second lens intercepts this converging bundle before it reaches that point. The small visual separation of the lens symbols is only to distinguish them: their physical separation is taken as zero.



Contact lenses: solid rays meet at the final image; dashed rays locate the image of the first lens alone. Schematic, not to scale.

Step 2: (a) Derive the equivalent focal length and power Let the object distance be u , the intermediate

Quick Tip to Solve: Add the two signed lens equations to eliminate the intermediate image distance; for minimum deviation, immediately set the internal refraction angle to half the prism angle.

image distance be v_1 , and the final image distance be v . For the first lens, of focal length f_1 ,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}.$$

For the second lens, the converging rays would meet at the intermediate image on its outgoing side. Thus that image is a virtual object for the second lens, whose signed object distance is v_1 . For focal length f_2 ,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

Adding cancels the intermediate-image term:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

An equivalent single lens of focal length F gives the same object and final image, and hence satisfies $1/v - 1/u = 1/F$. Comparison gives $1/F = 1/f_1 + 1/f_2$, or $F = f_1 f_2 / (f_1 + f_2)$. Consequently, with all focal lengths in metres, the power is $P = 1/f_1 + 1/f_2 = P_1 + P_2$. Both convex-lens powers are positive, so the combination has a shorter focal length than either lens.

Step 3: (b) Find the prism angles and refractive index An equilateral prism has $A = 60^\circ$. The given incidence angle is therefore $i = (3/4)A = 45^\circ$. At minimum deviation the ray path is symmetric, so $e = i = 45^\circ$ and $r_1 = r_2 = A/2 = 30^\circ$. Snell's law at the first air-glass face gives

$$\mu = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}.$$

As a consistency check, the minimum deviation is $\delta_{\min} = i + e - A = 30^\circ$. Substituting it into $\mu = \sin[(A + \delta_{\min})/2] / \sin(A/2)$ gives the same refractive index.

Step 4: (b) Calculate the light speed Using $c = 3.0 \times 10^8 \text{ m s}^{-1}$ for light in air to the usual approximation,

$$v_{\text{glass}} = \frac{c}{\mu} = \frac{3.0 \times 10^8}{\sqrt{2}} \text{ m s}^{-1} = 2.12 \times 10^8 \text{ m s}^{-1}.$$

This is less than the speed in air, as expected for glass with refractive index greater than unity.

Step 5: Conclusion (a) The contact combination has $P = P_1 + P_2 = 1/f_1 + 1/f_2$, in dioptres for focal lengths in metres. (b) The speed of light in the prism is $2.12 \times 10^8 \text{ m s}^{-1}$.

Quick Tip to Solve: Add the two signed lens equations to eliminate the intermediate image distance; for minimum deviation, immediately set the internal refraction angle to half the prism angle.