

CBSE Class 12 Physics Question Paper 2017 with Solution PDF-Set 55/2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are **26** questions in all.
- (ii) This question paper has **five** sections : Section A, Section B, Section C, Section D and Section E.
- (iii) Section A contains **five** questions of **one** mark each, Section B contains **five** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains one value based question of **four** marks and Section E contains **three** questions of **five** marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in **one** question of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron} = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

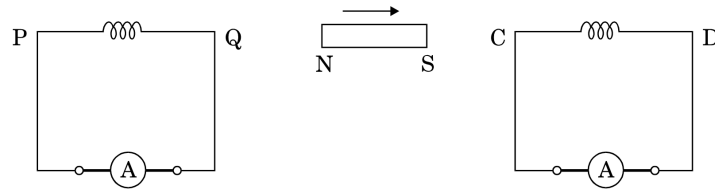
$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. A bar magnet is moved in the direction indicated by the arrow between two coils PQ and CD. Predict the direction of the induced current in each coil. [1 mark]



Solution:

Concept:

- Lenz's law: the induced current produces a magnetic field that opposes the change of magnetic flux through the coil.
- A coil face is a south pole when the current is clockwise as viewed from that face; an anticlockwise current makes that face a north pole.

Step 1: Determine the polarity of coil PQ The north pole of the magnet moves away from coil PQ. The coil opposes this recession by attracting the receding north pole. Therefore its face Q, nearest the magnet, becomes a south pole. The induced current is clockwise when coil PQ is viewed from the magnet, looking towards Q.

Step 2: Determine the polarity of coil CD The south pole of the magnet approaches coil CD. This coil opposes the approach by repelling the approaching south pole. Therefore its face C, nearest the magnet, also becomes a south pole. The induced current is clockwise when coil CD is viewed from the magnet, looking towards C. These two viewing directions are opposite in space, so specifying the viewing face is essential.

Step 3: Conclusion Both coils carry clockwise induced current when viewed from the magnet between them: towards the Q face for PQ and towards the C face for CD. Both faces adjacent to the magnet act as south poles.

Quick Tip to Solve: First make the near face attract a receding pole or repel an approaching pole; then use clockwise current for a south face.

2. Write the relation for the speed of electromagnetic waves in terms of the amplitudes of electric and magnetic fields. [1 mark]

Solution:

Concept:

- In a plane electromagnetic wave, the electric field and magnetic field oscillate in phase and are perpendicular to each other and to the propagation direction.
- Their amplitudes obey $E_0 = vB_0$, where E_0 is the electric-field amplitude, B_0 is the magnetic-field amplitude, and v is the wave speed.

Quick Tip to Solve: Put the electric amplitude in the numerator: E_0/B_0 has the dimensions of speed, while its reciprocal does not.

Step 1: Identify the two field amplitudes Let the maximum magnitude of the electric field be E_0 , measured in V m^{-1} , and the maximum magnitude of the magnetic field be B_0 , measured in T. The required speed depends on the ratio of these amplitudes.

Step 2: Express the speed using the amplitudes Rearranging the electromagnetic-wave relation $E_0 = vB_0$ gives

$$v = \frac{E_0}{B_0}.$$

For propagation in vacuum, this speed is denoted by c and equals approximately $3.0 \times 10^8 \text{ m s}^{-1}$.

Step 3: Conclusion The required relation is $c = E_0/B_0$ in vacuum, or $v = E_0/B_0$ for the wave speed in a medium.

Quick Tip to Solve: Put the electric amplitude in the numerator: E_0/B_0 has the dimensions of speed, while its reciprocal does not.

3. Nichrome and copper wires of same length and same radius are connected in series. Current I is passed through them. Which wire gets heated up more? Justify your answer. [1 mark]

Solution:

Concept:

- Joule's law gives the heat generated in a resistor as $H = I^2 R t$, where I is current, R is resistance and t is time.
- A wire of length l , radius r and resistivity ρ has resistance $R = \rho l / (\pi r^2)$.

Step 1: Compare the resistances Both wires have the same length and cross-sectional area. Their resistances therefore depend on their resistivities. Nichrome has a higher resistivity than copper, so $R_{\text{NiCr}} > R_{\text{Cu}}$.

Step 2: Apply Joule's law to the series circuit The same current flows through both wires in series. For the same time interval, $I^2 t$ is common, so the heat generated is proportional to resistance:

$$\frac{H_{\text{NiCr}}}{H_{\text{Cu}}} = \frac{R_{\text{NiCr}}}{R_{\text{Cu}}} > 1.$$

Step 3: Conclusion The nichrome wire gets heated up more because its greater resistivity gives it greater resistance and hence greater Joule heating at the same current.

Quick Tip to Solve: For wires in series, compare $I^2 R$: the common current makes the higher-resistance wire generate more heat.

4. How does the angle of minimum deviation of a glass prism vary, if the incident violet light is replaced by red light? Give reason. [1 mark]

Solution:

Concept:

- Normal dispersion in glass means that its refractive index is lower for red light than for violet light: $\mu_r < \mu_v$.
- For a prism of angle A , the minimum deviation δ_m satisfies $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$.

Step 1: Compare the refractive indices Red light has a longer wavelength than violet light. In the visible range for glass, the refractive index decreases with increasing wavelength. Replacing violet light by red light therefore reduces the refractive index of the prism for the incident radiation.

Step 2: Relate refractive index to minimum deviation For the same prism, A remains fixed. Rearranging the minimum-deviation relation gives

$$\delta_m = 2 \sin^{-1} \left[\mu \sin \left(\frac{A}{2} \right) \right] - A.$$

In the physical range of transmission through the prism, this expression increases with μ . A smaller refractive index thus corresponds to a smaller minimum deviation.

Step 3: Conclusion The angle of minimum deviation decreases when violet light is replaced by red light, because glass has a smaller refractive index for red light.

Quick Tip to Solve: Remember that red light is deviated least and violet light most by a glass prism; the same order holds for their minimum deviations.

5. Name the phenomenon which shows the quantum nature of electromagnetic radiation. [1 mark]

Solution:

Concept:

- Electromagnetic radiation exchanges energy in discrete packets called photons, each having energy $E = h\nu$, where h is Planck's constant and ν is frequency.
- In the photoelectric effect, an electron absorbs a photon and can escape from a metal if the photon energy is sufficient to overcome the work function ϕ .

Step 1: Identify the relevant observation Emission of electrons from an illuminated metal occurs only above a threshold frequency, irrespective of how intense the light is below that threshold. This observation requires energy transfer in individual quanta rather than a continuous accumulation of wave energy.

Step 2: Connect the observation with photon energy Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where $K_{\max}$ is the maximum kinetic energy of emitted electrons. It explains why the electron energy depends on frequency: one photon transfers its energy to one electron.

Step 3: Conclusion The photoelectric effect demonstrates the quantum nature of electromagnetic radiation.

Quick Tip to Solve: Associate a threshold frequency and the relation $K_{\max} = h\nu - \phi$ with the photoelectric effect and photon energy.

Section B (10 marks)

6. Find the condition under which the charged particles moving with different speeds in the presence of electric and magnetic field vectors can be used to select charged particles of a particular speed.

[2 marks]

Solution:

Concept:

- A charged particle in electric and magnetic fields experiences the Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$, where q is charge and $\vec{v}$ is velocity.
- A velocity selector uses crossed electric and magnetic fields to make the forces cancel only for particles of a chosen speed. For motion perpendicular to $\vec{B}$, the magnetic-force magnitude is $|q|vB$.

Step 1: Arrange the crossed fields Make $\vec{E}$ and $\vec{B}$ perpendicular to each other, and send the collimated particle beam perpendicular to both fields. Choose their directions so that the electric force $q\vec{E}$ opposes the magnetic force $q\vec{v} \times \vec{B}$. The condition for an undeflected particle is

$$\vec{E} + \vec{v} \times \vec{B} = 0.$$

Step 2: Balance the force magnitudes With these perpendicular directions, zero net transverse force requires

$$|q|E = |q|vB, \quad v = \frac{E}{B}.$$

Particles with a greater speed experience a larger magnetic force, whereas those with a smaller speed experience a larger electric force. They are deflected away from the straight path and can be stopped by an exit slit. The selected speed is independent of the mass and the magnitude of the charge.

Step 3: Conclusion Use mutually perpendicular electric and magnetic fields, both perpendicular to the particle beam, with opposing forces. Only particles having speed $v = E/B$ pass undeflected through the velocity selector.

Quick Tip to Solve: First make the electric and magnetic forces oppose; then cancel $|q|$ in $|q|E = |q|vB$ to obtain the selected speed E/B .

7. Identify the electromagnetic waves whose wavelengths lie in the range

(a) $10^{-11} \text{ m} < \lambda < 10^{-14} \text{ m}$

(b) $10^{-4} \text{ m} < \lambda < 10^{-6} \text{ m}$

Write one use of each.

[2 marks]

Solution:

Concept:

- Electromagnetic waves are classified by their wavelength λ ; their frequency is $\nu = c/\lambda$, where c is the speed of light in vacuum.
- Gamma rays occupy the very short wavelength region, while infrared radiation has wavelengths

Quick Tip to Solve: First order the wavelength endpoints numerically: very short wavelengths indicate gamma rays, whereas micrometre wavelengths indicate infrared radiation.

longer than visible red light and is associated with thermal emission.

Step 1: Interpret the printed wavelength endpoints The inequalities in both parts are printed in reverse numerical order: $10^{-11} > 10^{-14}$ and $10^{-4} > 10^{-6}$. Thus, taken literally, neither chained inequality admits any wavelength. For the intended identification, treat the listed values as the two endpoints of each wavelength band; the question text itself is retained as printed.

Step 2: Identify each band and give its use (a) Wavelengths between 10^{-14} m and 10^{-11} m belong to the gamma-ray region of the conventional electromagnetic spectrum. Gamma rays are used in radiotherapy to destroy cancerous cells.

(b) Wavelengths between 10^{-6} m and 10^{-4} m belong to the infrared region. Infrared radiation is used in thermal imaging to detect the heat emitted by objects, including objects that are not visible in darkness.

Step 3: Conclusion Interpreting the printed numbers as band endpoints: (a) gamma rays, used in cancer radiotherapy; (b) infrared rays, used in thermal imaging. The inequality directions in the printed question are reversed.

Quick Tip to Solve: First order the wavelength endpoints numerically: very short wavelengths indicate gamma rays, whereas micrometre wavelengths indicate infrared radiation.

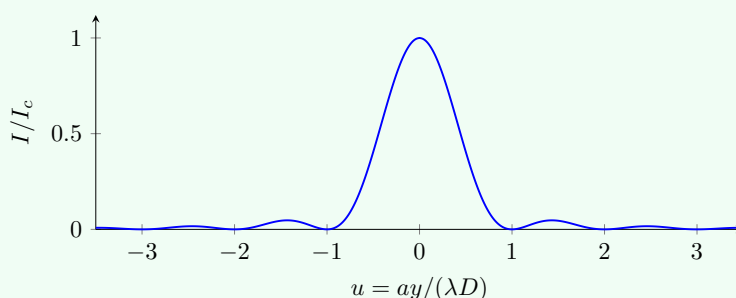
8. Draw the intensity pattern for single slit diffraction and double slit interference. Hence, state two differences between interference and diffraction patterns. [2 marks]

Solution:

Concept:

- Single-slit diffraction has intensity $I = I_c(\sin \beta / \beta)^2$, where $\beta = \pi a \sin \theta / \lambda$, a is slit width and I_c is central intensity.
- Identical narrow coherent slits give $I = I_m \cos^2(\pi dy / \lambda D)$, neglecting the diffraction envelope. Here d is slit separation, D is screen distance and y is screen displacement.

Step 1: Sketch the single-slit diffraction pattern The central maximum extends between $y = \pm \lambda D / a$. Its width is twice that of a secondary maximum, measured between bounding minima. Secondary peaks become progressively weaker. The graph uses $u = ay / (\lambda D)$.



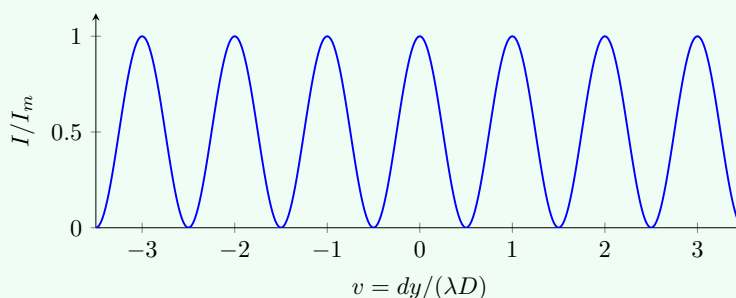
Single-slit diffraction: normalized intensity versus scaled screen position.

Step 2: Sketch and compare the interference pattern Ideal equal-slit interference has uniformly

Quick Tip to Solve: In a diffraction sketch make the central peak twice as wide and the side peaks much lower; in ideal interference repeat equal peaks at equal spacing.

spaced fringes. The graph uses $v = dy/(\lambda D)$.

Two differences are: (1) interference bright fringes have equal widths, whereas the diffraction central maximum is twice the width of a secondary maximum; (2) interference maxima have equal intensity in the stated ideal case, whereas diffraction secondary maxima are weaker than the central maximum and decrease in intensity away from it.



Ideal double-slit interference for identical narrow slits, without a diffraction envelope.

Step 3: Conclusion Diffraction has a broad, bright central maximum and weak secondary maxima. Ideal equal-slit interference has uniformly spaced bright fringes of equal width and equal peak intensity.

Quick Tip to Solve: In a diffraction sketch make the central peak twice as wide and the side peaks much lower; in ideal interference repeat equal peaks at equal spacing.

OR

8. Unpolarised light is passed through a polaroid P_1 . When this polarised beam passes through another polaroid P_2 and if the pass axis of P_2 makes angle θ with the pass axis of P_1 , then write the expression for the polarised beam passing through P_2 . Draw a plot showing the variation of intensity when θ varies from 0 to 2π . [2 marks]

Solution:

Concept:

- An ideal polaroid transmits the electric-field component parallel to its pass axis. Unpolarised light of intensity I_u emerges from the first polaroid with intensity $I_1 = I_u/2$.
- Malus's law states $I_2 = I_1 \cos^2 \theta$, where θ is the angle between the transmission axes and I_2 is the intensity transmitted by the second polaroid.

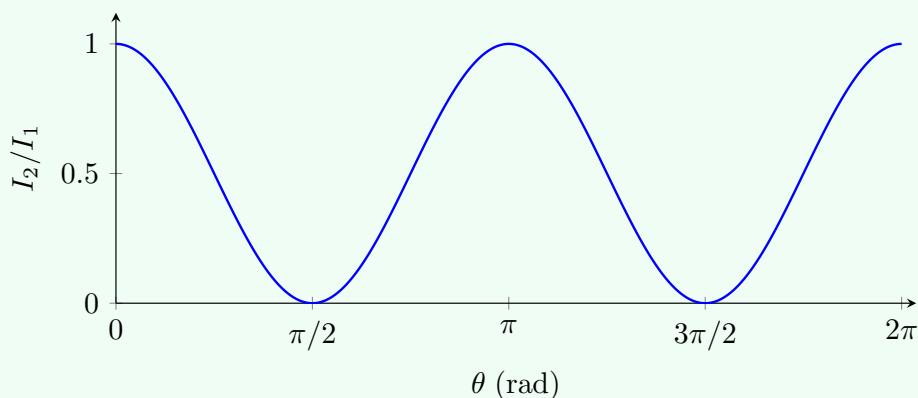
Step 1: Apply Malus's law to the second polaroid After P_1 , the beam is linearly polarised parallel to its pass axis. If its electric-field amplitude is E_1 , the component along the pass axis of P_2 is $E_1 \cos \theta$. Since intensity is proportional to the square of the amplitude,

$$I_2 = I_1 \cos^2 \theta = \frac{I_u}{2} \cos^2 \theta.$$

Here I_1 refers to the beam incident on P_2 , while I_u refers to the original unpolarised beam incident on P_1 .

Quick Tip to Solve: Define the input intensity first: use the factor one-half only for the initially unpolarised beam, then apply the cosine-squared factor at the analyser.

Step 2: Locate the maxima and minima of the graph Parallel or antiparallel axes give maxima $I_2 = I_1$ at $\theta = 0, \pi, 2\pi$. Crossed axes give zero intensity at $\theta = \pi/2, 3\pi/2$. Between these angles the curve varies smoothly as $\cos^2 \theta$, is never negative, and repeats after an angular interval π . The graph uses the normalised vertical scale I_2/I_1 .



Malus's law over $0 \leq \theta \leq 2\pi$; the maximum intensity is $I_1 = I_u/2$.

Step 3: Conclusion The intensity emerging from P_2 is $I_2 = I_1 \cos^2 \theta = (I_u/2) \cos^2 \theta$. Its plot has maxima at $0, \pi, 2\pi$ and zeros at $\pi/2, 3\pi/2$.

Quick Tip to Solve: Define the input intensity first: use the factor one-half only for the initially unpolarised beam, then apply the cosine-squared factor at the analyser.

9. The short wavelength limit for the Lyman series of the hydrogen spectrum is $913.4 \text{ \AA}$. Calculate the short wavelength limit for Balmer series of the hydrogen spectrum. [2 marks]

Solution:

Concept:

- The Rydberg formula for hydrogen is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where $n_i > n_f$ are the initial and final principal quantum numbers and R_H is the Rydberg constant.
- The shortest wavelength of a series corresponds to its largest photon energy, obtained when $n_i \rightarrow \infty$. The Lyman and Balmer series end at $n_f = 1$ and $n_f = 2$, respectively.

Step 1: Write the Lyman limit At the series limit the term $1/n_i^2$ vanishes. For transitions to the ground state,

$$\frac{1}{\lambda_L} = R_H, \quad \lambda_L = \frac{1}{R_H} = 913.4 \text{ \AA}.$$

We use the given limit directly, so a separately rounded numerical value of R_H is unnecessary.

Step 2: Relate the Balmer limit to the Lyman limit In the Balmer series the final level is $n_f = 2$. At its short wavelength limit,

$$\frac{1}{\lambda_B} = \frac{R_H}{4}, \quad \lambda_B = 4\lambda_L = 4(913.4) \text{ \AA}.$$

The factor is four because the series-limit wavelength is proportional to the square of the final quantum number.

Quick Tip to Solve: At a hydrogen series limit use $\lambda_{\min} \propto n_f^2$; changing the final level from 1 to 2 multiplies the limit by four.

Step 3: Conclusion The short wavelength limit of the Balmer series is $3653.6 \text{ \AA}$, or 365.36 nm.

Quick Tip to Solve: At a hydrogen series limit use $\lambda_{\min} \propto n_f^2$; changing the final level from 1 to 2 multiplies the limit by four.

10. Write two properties of a material suitable for making (a) a permanent magnet, and (b) an electromagnet. [2 marks]

Solution:

Concept:

- Retentivity measures how well a material retains magnetisation after the magnetising field is removed. Coercivity is the reverse magnetising field needed to remove its residual magnetisation.
- Magnetic permeability indicates how readily a material develops magnetic induction in an applied field. The desired magnetic properties depend on whether magnetism must persist or be controlled by current.

Step 1: Choose properties for a permanent magnet (a) A suitable material must have **high retentivity**, so it retains strong magnetisation after the external magnetising field is removed. It must also have **high coercivity**, so ordinary opposing magnetic fields do not demagnetise it easily. Thus its magnetic state remains stable in use; hard magnetic materials are suitable.

Step 2: Choose properties for an electromagnet (b) The core should have **high permeability**, so a modest current in the coil produces a strong magnetic field. It should also have **low retentivity**, so little residual magnetism remains when the current is switched off. These properties allow the magnet to act strongly when energised and release its load when de-energised. Soft iron is a typical suitable core material.

Step 3: Conclusion A permanent magnet requires **high retentivity and high coercivity**; an electromagnet core requires **high permeability and low retentivity**.

Quick Tip to Solve: Ask whether the magnet must stay on after the applied field is removed: permanent magnets retain magnetism, while electromagnet cores should release it.

Section C (36 marks)

11. (a) Monochromatic light of wavelength 589 nm is incident from air on a water surface. If μ for water is 1.33, find the wavelength, frequency and speed of the refracted light.

(b) A double convex lens is made of a glass of refractive index 1.55, with both faces of the same radius of curvature. Find the radius of curvature required, if the focal length is 20 cm. [3 marks]

Solution:

Concept:

- Light retains its frequency ν on refraction because the source fixes the oscillation rate. For refractive index μ , the speed is $v = c/\mu$ and the wavelength is $\lambda = \lambda_0/\mu$, taking the speed in air as c .
- For a thin lens in air, the lens maker formula is $1/f = (\mu_g - 1)(1/R_1 - 1/R_2)$, where μ_g is the refractive index of glass and R_1, R_2 are signed radii of curvature.

Step 1: Find the wavelength and speed in water (a) The incident wavelength is $\lambda_0 = 589$ nm and water has refractive index $\mu = 1.33$. Both wavelength and speed decrease on entering water:

$$\lambda_w = \frac{589}{1.33} \text{ nm} \simeq 442.9 \text{ nm},$$

$$v_w = \frac{3.0 \times 10^8}{1.33} \text{ m s}^{-1} \simeq 2.26 \times 10^8 \text{ m s}^{-1}.$$

Step 2: Calculate the unchanged frequency Convert the incident wavelength to metres before using the wave relation. Since frequency is continuous across the stationary air-water boundary,

$$\nu_w = \nu_0 = \frac{c}{\lambda_0} = \frac{3.0 \times 10^8}{589 \times 10^{-9}} \text{ Hz} \simeq 5.09 \times 10^{14} \text{ Hz}.$$

The smaller speed is therefore accounted for by the shorter wavelength, not by a change in frequency.

Step 3: Apply the lens maker formula with signed radii (b) Let the common radius magnitude be R . With light travelling left to right, a double convex lens has $R_1 = +R$, $R_2 = -R$, and $f = +20$ cm. Hence

$$\frac{1}{f} = (1.55 - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{1.10}{R},$$

$$R = 1.10f = 1.10(20) \text{ cm}.$$

Equal curvature magnitudes do not mean equal signed radii; the two centres of curvature lie on opposite sides of their surfaces.

Step 4: Conclusion (a) In water, $\lambda_w \simeq 443$ nm, $\nu_w \simeq 5.09 \times 10^{14}$ Hz, and $v_w \simeq 2.26 \times 10^8$ m s⁻¹. (b) Each lens face requires radius magnitude 22 cm.

Quick Tip to Solve: On refraction keep frequency fixed and divide speed and wavelength by the index. For a symmetric biconvex lens use $R_1 = +R$, $R_2 = -R$.

12. (a) Draw a ray diagram showing the formation of image by a reflecting telescope.

(b) Write two advantages of a reflecting telescope over a refracting telescope.

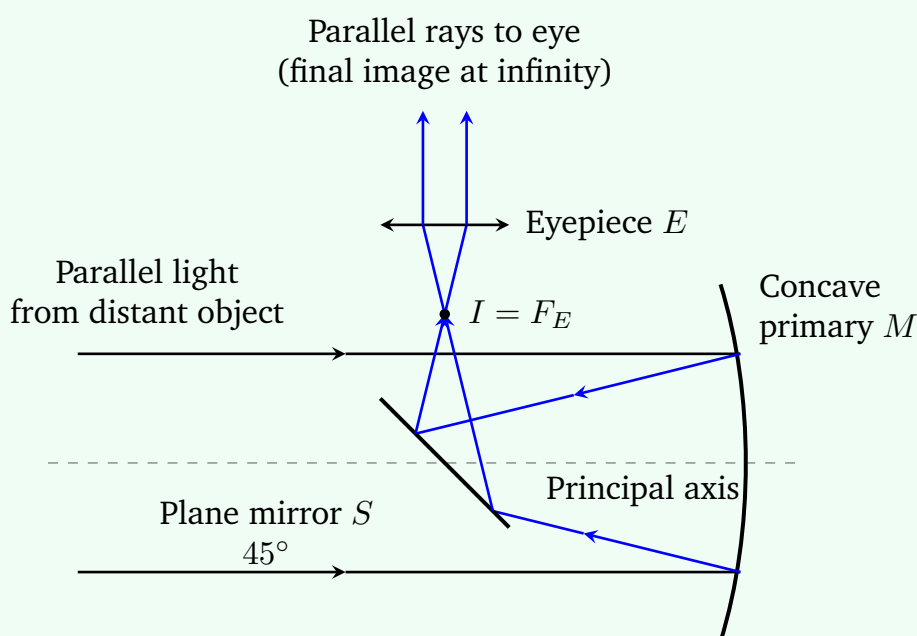
[3 marks]

Solution:

Concept:

- A reflecting telescope uses a large concave primary mirror to collect nearly parallel light from a distant object and form a real image near its focus.
- A convex eyepiece acts as a magnifier. In normal adjustment the intermediate image lies in its focal plane, so the final image is virtual and at infinity.

Step 1: (a) Trace the rays through a reflecting telescope The diagram shows the Newtonian arrangement. Parallel rays from a distant object fall on the concave primary mirror M . After reflection they converge towards its focus. A small plane mirror S , inclined at 45° to the principal axis, intercepts this converging beam and turns it towards the side of the tube. The rays meet at I , producing the real intermediate image. The convex eyepiece E is placed so that I lies in its first focal plane. Rays emerging from the eyepiece are then parallel, allowing the eye to view a magnified virtual image at infinity. The final image of an extended distant object is inverted.



Newtonian reflecting telescope in normal adjustment; the two rays represent one distant axial point.

Step 2: (b) Explain two advantages over a refracting telescope **Freedom from chromatic aberration:** reflection obeys the same law for every wavelength. Thus the primary mirror brings different colours to the same focus, unlike a refracting objective whose refractive index varies with wavelength.

Large aperture is practical: a mirror can be supported from behind, making a large objective easier to construct without sagging than a large lens supported at its rim. Its large aperture collects more light and improves resolving power, so faint objects and close details can be observed more effectively.

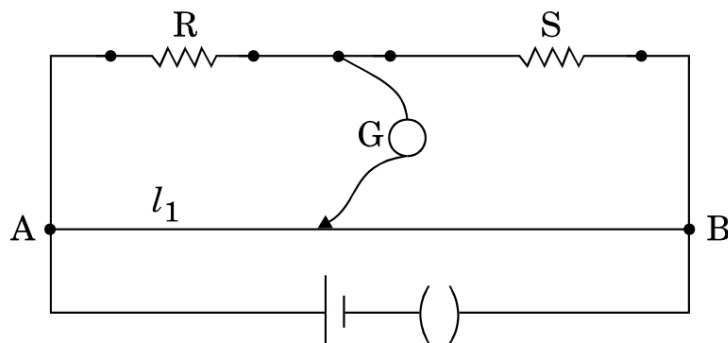
Step 3: Conclusion The primary mirror forms a real image, the small plane mirror redirects the beam, and the eyepiece magnifies the image. The two advantages are absence of chromatic aberration at the objective and the practicality of a large light-gathering, high-resolution aperture.

Quick Tip to Solve: In a Newtonian sketch, place the small plane mirror before the primary focus and put the redirected image in the eyepiece focal plane.

13. (a) Write the principle of working of a metre bridge.

(b) In a metre bridge, the balance point is found at a distance l_1 with resistances R and S as shown in the figure.

An unknown resistance X is now connected in parallel to the resistance S and the balance point is found at a distance l_2 . Obtain a formula for X in terms of l_1 , l_2 and S . [3 marks]



Solution:

Concept:

- At Wheatstone-bridge balance, galvanometer current is zero and the ratio of the gap resistances equals the ratio of the wire resistances.
- For a uniform wire, resistance is proportional to length. Resistances S and X in parallel have equivalent resistance $S' = SX/(S + X)$.

Step 1: (a) State the metre-bridge principle The gap junction and jockey have equal potential at balance. For a uniform bridge wire of length $L = 100$ cm, a balance length l measured from A gives $R/S = l/(L - l)$.

Step 2: (b) Write both balance conditions Initially, the gap resistances are R, S . Connecting X across S changes the right gap to $S' = SX/(S + X)$; the left gap R is unchanged. Thus

$$R = \frac{Sl_1}{L - l_1}, \quad R = \frac{SX}{S + X} \frac{l_2}{L - l_2}.$$

Step 3: Eliminate R and isolate X Equate the expressions for R , cancel S , and cross-multiply:

$$\begin{aligned} l_1(S + X)(L - l_2) &= Xl_2(L - l_1), \\ Sl_1(L - l_2) &= X\{l_2(L - l_1) - l_1(L - l_2)\}, \\ Sl_1(L - l_2) &= XL(l_2 - l_1). \end{aligned}$$

The parallel combination reduces the right-gap resistance, shifting the balance towards B. Thus $l_2 > l_1$

Quick Tip to Solve: Measure both lengths from A. Adding a parallel resistor to S shifts the balance towards B; check that $l_2 > l_1$.

ensures a positive resistance.

Step 4: Conclusion For balance lengths in centimetres, the unknown resistance is

$$X = \frac{Sl_1(L - l_2)}{L(l_2 - l_1)} = \frac{Sl_1(100 - l_2)}{100(l_2 - l_1)}.$$

Quick Tip to Solve: Measure both lengths from A. Adding a parallel resistor to S shifts the balance towards B; check that $l_2 > l_1$.

14. Define mutual inductance between a pair of coils. Derive an expression for the mutual inductance of two long coaxial solenoids of same length wound one over the other. [3 marks]

Solution:

Concept:

- Mutual inductance measures the magnetic coupling between two coils. For a linear medium, the flux linkage of coil 2 due to current I_1 in coil 1 is $N_2\Phi_{21} = MI_1$, where N_2 is its number of turns.
- The field inside a long air-core solenoid is $B = \mu_0 nI$, where $n = N/\ell$ is the number of turns per unit length. End effects and the external field are neglected.

Step 1: Define mutual inductance The mutual inductance of a pair of coils is the magnetic flux linkage with one coil per unit current in the other coil:

$$M = \frac{N_2\Phi_{21}}{I_1}.$$

Equivalently, from Faraday's law, the induced emf in coil 2 is

$$\varepsilon_2 = -M \frac{dI_1}{dt}.$$

Thus its magnitude equals the emf induced in one coil when the current in the other changes at unit rate. The minus sign expresses Lenz's law. Its SI unit is the henry, H.

Step 2: Calculate the common magnetic flux Let the two coaxial solenoids have common length ℓ , turn numbers N_1, N_2 , and effective common cross-sectional area A . For closely wound coils one over the other, their areas are approximately equal; if their radii differ, A is the smaller solenoid's area. Take solenoid 1 to be the inner one. A current I_1 produces

$$B_1 = \mu_0 \frac{N_1}{\ell} I_1.$$

Since the field is approximately uniform inside and negligible outside it, the flux linked with each turn of solenoid 2 is

$$\Phi_{21} = B_1 A = \mu_0 \frac{N_1 A}{\ell} I_1.$$

Step 3: Obtain the flux linkage and divide by current Multiplying the flux per turn by all N_2 turns

Quick Tip to Solve: First find the field using the source coil's turns, then multiply the common flux by the receiving coil's turns; this produces the product $N_1 N_2$.

gives

$$N_2 \Phi_{21} = \frac{\mu_0 N_1 N_2 A}{\ell} I_1.$$

Comparing this relation with $N_2 \Phi_{21} = M I_1$ identifies the mutual inductance. Writing $n_1 = N_1/\ell$ and $n_2 = N_2/\ell$ gives the equivalent turn-density form. Interchanging the coils gives the same value of mutual inductance.

Step 4: Conclusion For two long coaxial air-core solenoids,

$$M = \frac{\mu_0 N_1 N_2 A}{\ell} = \mu_0 n_1 n_2 A \ell.$$

With a uniform linear core of relative permeability μ_r , replace μ_0 by $\mu_0 \mu_r$.

Quick Tip to Solve: First find the field using the source coil's turns, then multiply the common flux by the receiving coil's turns; this produces the product $N_1 N_2$.

OR

14. Define self-inductance of a coil. Obtain the expression for the energy stored in an inductor L connected across a source of emf. [3 marks]

Solution:

Concept:

- Self-inductance L relates a coil's own flux linkage $N\Phi$ to its current i : $N\Phi = Li$, for a linear magnetic medium and fixed geometry.
- The self-induced emf $\varepsilon_L = -L di/dt$ opposes a change of current. Work supplied against this back emf is stored as energy in the inductor's magnetic field.

Step 1: Define self-inductance The self-inductance of a coil is its magnetic flux linkage per unit current through it:

$$L = \frac{N\Phi}{i}.$$

Equivalently, it is the magnitude of the induced emf when the current changes at unit rate. A coil has inductance 1 H when a current change of 1 A s^{-1} induces an emf of magnitude 1 V. The negative sign in $\varepsilon_L = -L di/dt$ shows opposition to the change, rather than a negative inductance.

Step 2: Find the work needed to increase the current Connect an initially unenergised inductor of constant inductance L across a source of emf. As the current increases from zero, the source must do work against the induced emf. The voltage required across an ideal inductor is

$$V_L = L \frac{di}{dt}.$$

During a small time dt , charge $dq = i dt$ passes through it. The work transferred to the magnetic field

Quick Tip to Solve: Use the opposing voltage $L di/dt$, multiply by $i dt$, then integrate $Li di$; this avoids missing the factor $1/2$.

is therefore

$$dW = V_L dq = \left(L \frac{di}{dt} \right) i dt = Li di.$$

This work is positive during an increasing current and builds up the magnetic field.

Step 3: Integrate from zero to the final current If the current reaches I , add the incremental work from $i = 0$ to $i = I$:

$$U = \int_0^I Li di = L \left[\frac{i^2}{2} \right]_0^I.$$

Here L is taken outside the integral because it is constant. If the coil also has resistance R , the source supplies an additional amount of heat: its instantaneous power is $\mathcal{E}i = Li di/dt + i^2 R$. Only the first term contributes to magnetic energy. Thus stored energy depends on the current reached, not on assigning all source work to the field in a resistive circuit.

Step 4: Conclusion The energy stored in an inductor carrying current I is

$$U = \frac{1}{2} LI^2 \text{ joules}.$$

This energy resides in its magnetic field and can be returned to the circuit as the current decreases.

Quick Tip to Solve: Use the opposing voltage $L di/dt$, multiply by $i dt$, then integrate $Li di$; this avoids missing the factor $1/2$.

15. Explain giving reasons for the following:

- (a) Photoelectric current in a photocell increases with the increase in the intensity of the incident radiation.
- (b) The stopping potential (V_0) varies linearly with the frequency (ν) of the incident radiation for a given photosensitive surface with the slope remaining the same for different surfaces.
- (c) Maximum kinetic energy of the photoelectrons is independent of the intensity of incident radiation.

[3 marks]

Solution:

Concept:

- One photon of frequency ν transfers energy $h\nu$ to one electron. The work function ϕ is the minimum energy needed to escape the surface.
- Einstein's equation is $K_{\max} = h\nu - \phi = eV_0$, where e is the electron charge magnitude and V_0 is the stopping-potential magnitude.

Step 1: (a) Relate intensity to electron emission At fixed frequency above threshold, a larger intensity supplies more photons per second to the same illuminated area. More photons can eject more electrons per second. Thus the photocurrent increases under the same collection conditions. When all emitted electrons are collected, the saturation current is

$$I_s = e\dot{N}_e \propto \mathcal{I},$$

Quick Tip to Solve: Use intensity to track photon number and current; use frequency to track photon energy, maximum kinetic energy and stopping potential.

where $\dot{N}_e$ is the electron emission rate and $\mathcal{I}$ is radiation intensity. The increase concerns electron number, not the energy received by each electron.

Step 2: (b) Obtain the slope of stopping potential At the stopping potential, even the fastest electrons are prevented from reaching the collector, so $eV_0 = K_{\max}$. Hence

$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e}.$$

For a given surface, ϕ is constant: this is a straight line in ν , with slope h/e and intercept $-\phi/e$. Since h and e are universal constants, the slope is identical for different surfaces. Different work functions change the intercept and threshold frequency $\nu_0 = \phi/h$. The physical emission relation applies above threshold.

Step 3: (c) Distinguish photon energy from photon number Each electron absorbs energy $h\nu$ from one photon, of which at least ϕ is spent in escaping. Its maximum remaining energy is

$$K_{\max} = h\nu - \phi.$$

At fixed frequency, greater intensity changes neither $h\nu$ nor ϕ . It therefore increases the number of photoelectrons without changing their maximum kinetic energy, in the usual photoelectric regime.

Step 4: Conclusion (a) Greater photon flux increases photocurrent. (b) Stopping potential varies linearly with frequency, with universal slope h/e . (c) Maximum kinetic energy depends on frequency and work function, and is independent of intensity at fixed frequency.

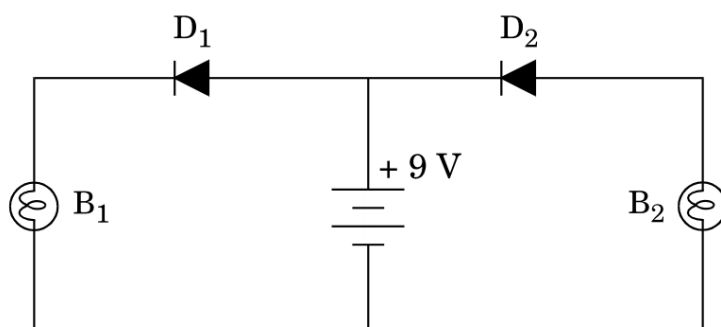
Quick Tip to Solve: Use intensity to track photon number and current; use frequency to track photon energy, maximum kinetic energy and stopping potential.

16. (a) In the following diagram, which bulb out of B_1 and B_2 will glow and why?

(b) Draw a diagram of an illuminated p-n junction solar cell.

(c) Explain briefly the three processes due to which generation of emf takes place in a solar cell.

[3 marks]



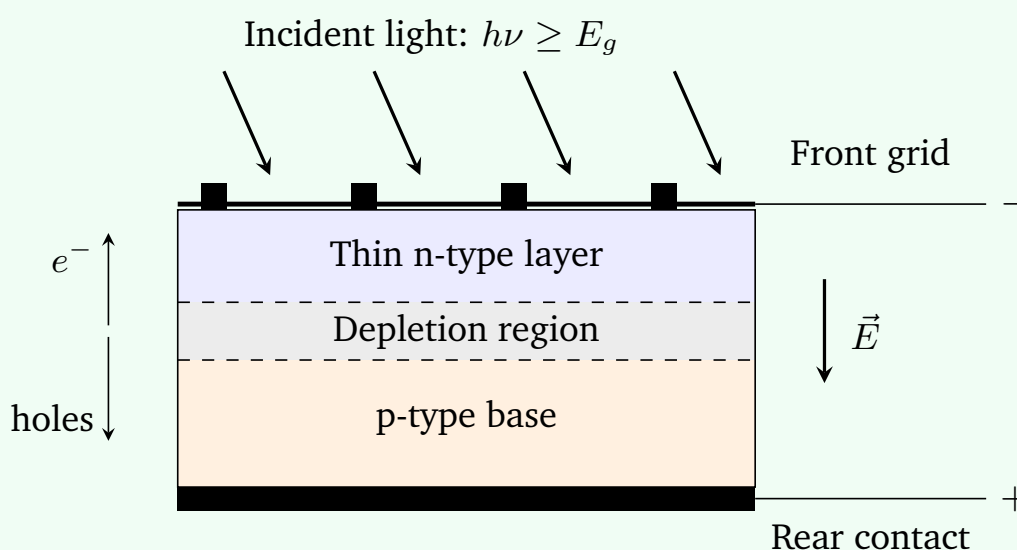
Solution:

Concept:

- A p-n junction diode conducts appreciable current in forward bias: its p-side (anode) is at a higher potential than its n-side (cathode). Reverse bias gives negligible current.
- A solar cell converts incident light into electrical energy. Photons of energy $h\nu \geq E_g$, where E_g is the semiconductor band gap, create electron-hole pairs near the junction.
- The built-in electric field in the depletion region separates the photogenerated carriers. Their collection produces a potential difference between the two contacts without an external bias.

Step 1: (a) Check the bias of each diode The battery's positive terminal is connected to the central upper junction and its negative terminal to the lower common wire. For D_1 , the central connection is the anode and the left connection, identified by the bar, is the cathode. Thus D_1 is forward biased and current passes through D_1 and B_1 to the negative terminal. For D_2 , the bar or cathode is connected to the positive terminal; its anode connects through B_2 to the negative terminal. Hence D_2 is reverse biased and the current through B_2 is negligible.

Step 2: (b) Draw the illuminated junction The diagram shows a thin n-type layer over a p-type base. The front metal grid admits light while collecting electrons; the rear metal contact collects holes. The electric field in the depletion region points from the n-side to the p-side. Under illumination the n-contact becomes negative and the p-contact becomes positive.



Illuminated p-n junction solar cell (schematic; no external bias).

Step 3: (c) Explain generation, separation and collection **Generation:** Absorption of photons with $h\nu \geq E_g$ generates electron-hole pairs in and near the depletion region.

Separation: The junction field drives electrons toward the n-side, opposite to the field, and holes

Quick Tip to Solve: Identify a diode's cathode from its bar before tracing current. For a solar cell remember: light creates, the junction field separates, and the contacts collect.

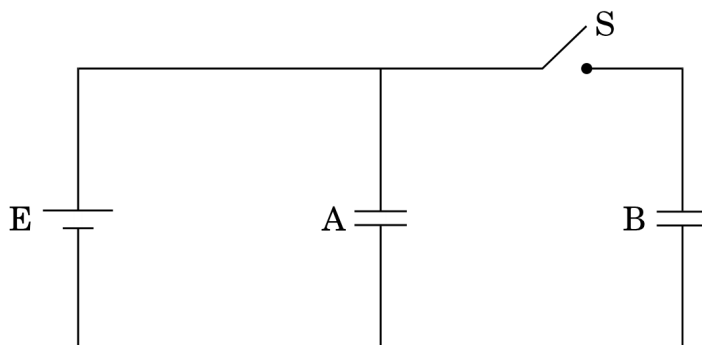
toward the p-side, along the field. This separates the charges before they recombine.

Collection: Electrons reaching the n-side and holes reaching the p-side are collected by their respective metal contacts. The accumulation of opposite charges establishes an emf across the terminals. When an external load is connected, conventional current flows through it from the p-contact to the n-contact.

Step 4: Conclusion Only B_1 glows: D_1 is forward biased, whereas D_2 is reverse biased. The illuminated solar cell produces emf by generation of electron-hole pairs, their separation by the junction field, and their collection at the contacts.

Quick Tip to Solve: Identify a diode's cathode from its bar before tracing current. For a solar cell remember: light creates, the junction field separates, and the contacts collect.

17. Two identical parallel plate capacitors A and B are connected to a battery of V volts with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K . Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. [3 marks]



Solution:

Concept:

- Complete insertion of a dielectric of relative permittivity K changes an air-filled capacitance C to KC .
- The stored energy is $U = \frac{1}{2}CV^2 = Q^2/(2C)$. A battery-connected capacitor retains its voltage, whereas an isolated capacitor retains its charge.

Step 1: Find the initial energy Let the initial capacitance of each identical capacitor be C . With switch S closed, both A and B are connected in parallel across the battery, so each has potential difference V and charge $Q = CV$. The total initial energy is

$$U_i = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2.$$

Step 2: Identify the quantities fixed after opening S From the printed circuit, opening S disconnects the upper plate of B from the battery, but A remains connected across the battery. Thus the potential difference across A stays V . No charge can enter or leave the isolated upper plate of B, so its charge

Quick Tip to Solve: Trace the open switch first: A stays at fixed voltage, while B stays at fixed charge. Their energies change by factors K and $1/K$, respectively.

magnitude remains CV . Merely opening the switch does not change either capacitor's energy before the dielectric is inserted.

Step 3: Calculate the final energies After the dielectric fills both gaps, each capacitance is KC . The battery supplies extra charge to A to maintain its voltage. For B, the fixed charge gives the reduced voltage V/K . Therefore

$$U_{A,f} = \frac{1}{2}KCV^2,$$

$$U_{B,f} = \frac{(CV)^2}{2KC} = \frac{CV^2}{2K},$$

$$U_f = \frac{CV^2}{2} \left(K + \frac{1}{K} \right).$$

Use voltage conservation for A and charge conservation for B; the two capacitors do not obey the same constraint after the switch is opened.

Step 4: Conclusion The required ratio of total energy before to that after dielectric insertion is

$$U_i : U_f = 2K : (K^2 + 1).$$

Quick Tip to Solve: Trace the open switch first: A stays at fixed voltage, while B stays at fixed charge. Their energies change by factors K and $1/K$, respectively.

18(a). How is amplitude modulation achieved ?

[3 marks total]

Solution:

Concept:

- Amplitude modulation (AM) varies the amplitude of a high-frequency carrier in accordance with the instantaneous value of a low-frequency message signal, while keeping the carrier frequency and phase fixed.
- If the carrier amplitude is A_c and the modulating amplitude is A_m , the modulation index is $\mu = A_m/A_c$; ordinary undistorted AM requires $\mu \leq 1$.

Step 1: Combine the carrier and message in a modulator Let the carrier be $c(t) = A_c \cos \omega_c t$ and the message be $m(t) = A_m \cos \omega_m t$, where $\omega_c \gg \omega_m$. Apply their sum to a nonlinear device, such as a transistor operated in its nonlinear region. Its output contains a product term proportional to $m(t)c(t)$, which generates components at the sum and difference frequencies.

Step 2: Select the required components A suitable band-pass filter selects the carrier and its two sidebands, rejecting unwanted low-frequency and harmonic components. With suitable gain, the resulting wave is

$$s(t) = A_c[1 + \mu \cos \omega_m t] \cos \omega_c t.$$

Its slowly varying amplitude, $A_c[1 + \mu \cos \omega_m t]$, follows the message. The spectral components occur at f_c , $f_c - f_m$ and $f_c + f_m$, where f_c and f_m are the carrier and message frequencies.

Quick Tip to Solve: For AM, identify the changing quantity as the carrier amplitude; the message appears in its envelope, with sidebands on either side of the carrier.

Step 3: Conclusion AM is achieved by varying the carrier amplitude in proportion to the instantaneous message signal, using a nonlinear modulator followed by a band-pass filter. The carrier frequency and phase remain unchanged.

Quick Tip to Solve: For AM, identify the changing quantity as the carrier amplitude; the message appears in its envelope, with sidebands on either side of the carrier.

18(b). The frequencies of two side bands in an AM wave are 640 kHz and 660 kHz respectively. Find the frequencies of carrier and modulating signal. What is the bandwidth required for amplitude modulation ?

Solution:

Concept:

- For a single-frequency message in AM, the upper and lower sideband frequencies are $f_U = f_c + f_m$ and $f_L = f_c - f_m$, where f_c is the carrier frequency and f_m is the modulating frequency.
- The bandwidth spans both sidebands: $B = f_U - f_L = 2f_m$. The carrier is at the midpoint of the sideband frequencies.

Step 1: Obtain the carrier frequency The lower frequency is $f_L = 640$ kHz, and the higher is $f_U = 660$ kHz. Adding the two sideband relations cancels the modulating frequency:

$$2f_c = f_U + f_L = 660 + 640 = 1300 \text{ kHz.}$$

Hence the carrier frequency is $f_c = 650$ kHz. It lies midway between the two sidebands, as required for amplitude modulation.

Step 2: Obtain the message frequency and bandwidth Subtracting the lower-sideband relation from the upper-sideband relation eliminates the carrier frequency:

$$2f_m = f_U - f_L = 660 - 640 = 20 \text{ kHz.}$$

Thus $f_m = 10$ kHz. Each sideband is 10 kHz away from the carrier, and transmission must include the entire interval from the lower to the upper sideband. Therefore the required bandwidth is

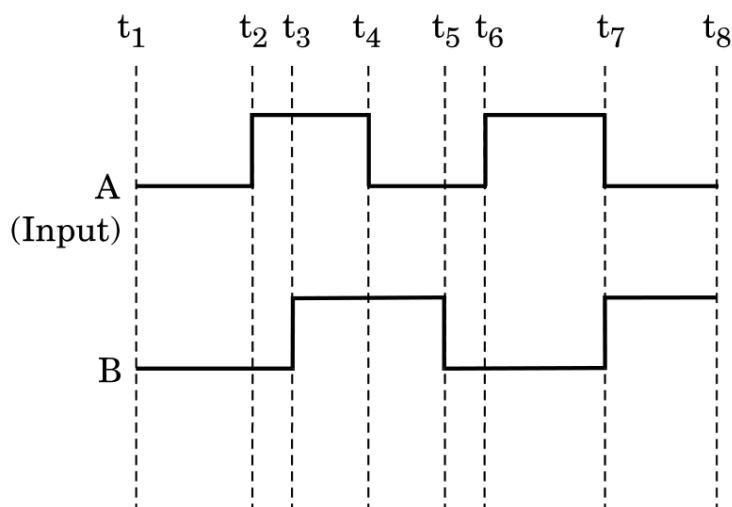
$$B = 2f_m = 2 \times 10 = 20 \text{ kHz.}$$

Step 3: Conclusion The carrier frequency is 650 kHz, the modulating frequency is 10 kHz, and the AM bandwidth is 20 kHz.

Quick Tip to Solve: Take the average of the two sideband frequencies for the carrier, half their difference for the message, and their full difference for bandwidth.

19. (a) Draw the circuit diagram for studying the characteristics of a transistor in common emitter configuration. Explain briefly and show how input and output characteristics are drawn.

(b) The figure shows input waveforms A and B to a logic gate. Draw the output waveform for an OR gate. Write the truth table for this logic gate and draw its logic symbol. [3 marks]



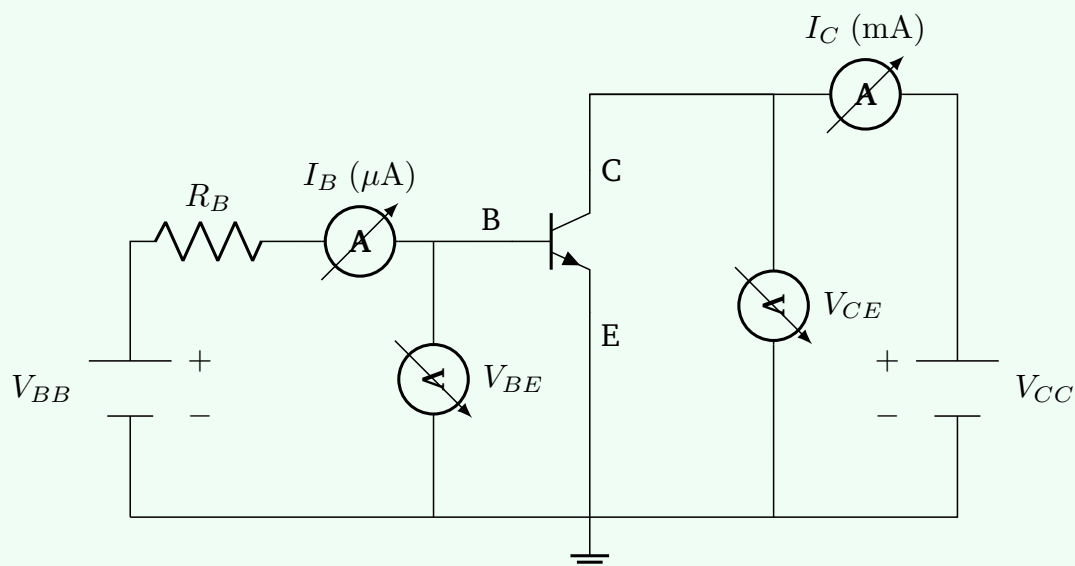
Solution:

Concept:

- In the common-emitter (CE) arrangement, the emitter is common to the input base-emitter circuit and the output collector-emitter circuit. In active operation, the base-emitter junction is forward biased and the collector-base junction is reverse biased.
- The input characteristic is the plot of base current I_B against base-emitter voltage V_{BE} at constant V_{CE} . The output characteristic is the plot of collector current I_C against collector-emitter voltage V_{CE} at constant I_B .
- An OR gate gives $Y = A + B$, where $+$ denotes logical OR: the output is high if either or both inputs are high, and is low only when both inputs are low.

Step 1: (a) Connect the common-emitter test circuit Use an n-p-n transistor with a common return through the emitter. The adjustable supply V_{BB} forward biases the base-emitter junction, while V_{CC} supplies the collector circuit. Connect a microammeter in series with the base and a milliammeter in series with the collector. Connect voltmeters across base-emitter and collector-emitter to measure V_{BE} and V_{CE} . The resistor R_B limits the base current. The positive terminals of both supplies face the base and collector respectively, as shown.

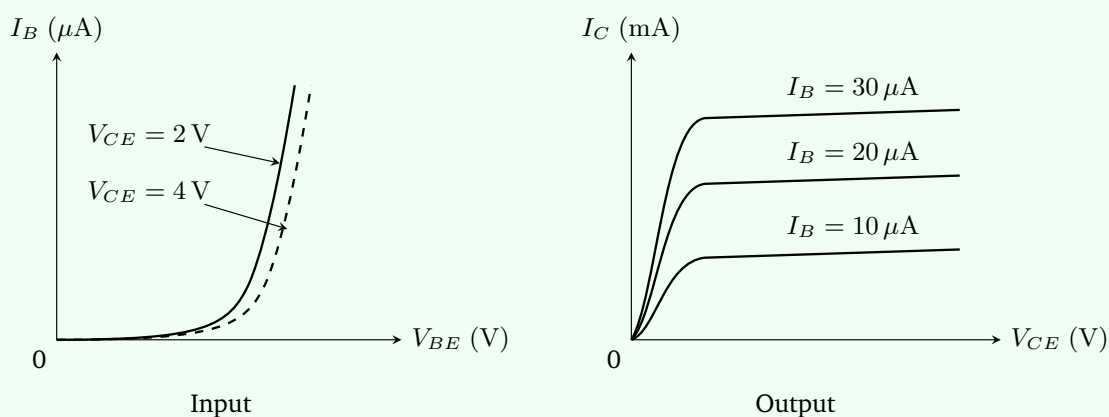
Quick Tip to Solve: Write the quantity held constant beside each transistor curve. For an OR waveform, mark only intervals in which both inputs are zero; the output is high everywhere else.



Common-emitter circuit with adjustable DC supplies and a shared emitter return.

Step 2: (a) Record and plot the characteristics For the input characteristic, first set a fixed V_{CE} . Increase V_{BB} gradually and record successive pairs (V_{BE}, I_B) , maintaining V_{CE} at its chosen value. Plot I_B vertically against V_{BE} horizontally; repeat for another fixed V_{CE} . The curves resemble a forward-biased diode characteristic: the base current rises rapidly after the junction begins conducting appreciably.

For the output characteristic, set a chosen I_B using V_{BB} . Vary V_{CC} , record (V_{CE}, I_C) , and readjust the input if necessary to keep I_B constant. Repeat for successively larger base currents. At very small V_{CE} , the transistor is in saturation; beyond the knee, I_C is nearly independent of V_{CE} for each fixed I_B . In this active region, a larger I_B gives a larger I_C . The sketches show the operating range below breakdown.



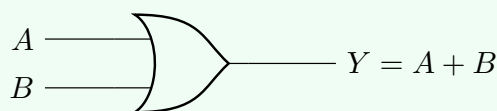
Schematic CE input and output characteristics; curve parameters are held constant.

Step 3: (b) Apply the OR rule and write its truth table Compare the two inputs in each interval between successive dashed lines. Their pairs (A, B) , from t_1 to t_8 , are $(0, 0)$, $(1, 0)$, $(1, 1)$, $(0, 1)$, $(0, 0)$, $(1, 0)$, $(0, 1)$. Thus the output levels in these seven intervals are 0, 1, 1, 1, 0, 1, 1. At t_7 , input A falls as B rises, so

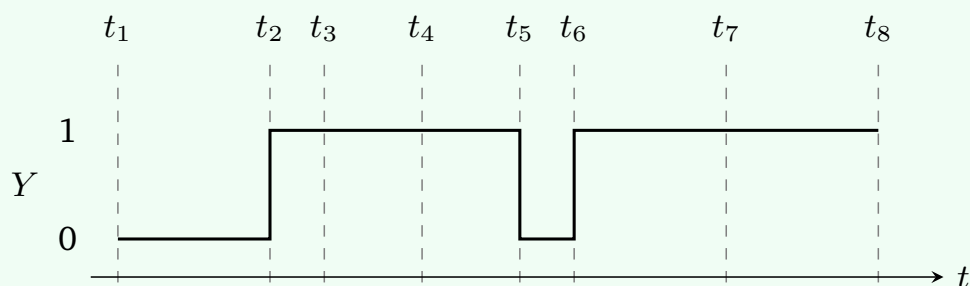
Quick Tip to Solve: Write the quantity held constant beside each transistor curve. For an OR waveform, mark only intervals in which both inputs are zero; the output is high everywhere else.

the ideal OR output stays high. The truth table is

A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1



OR gate logic symbol.



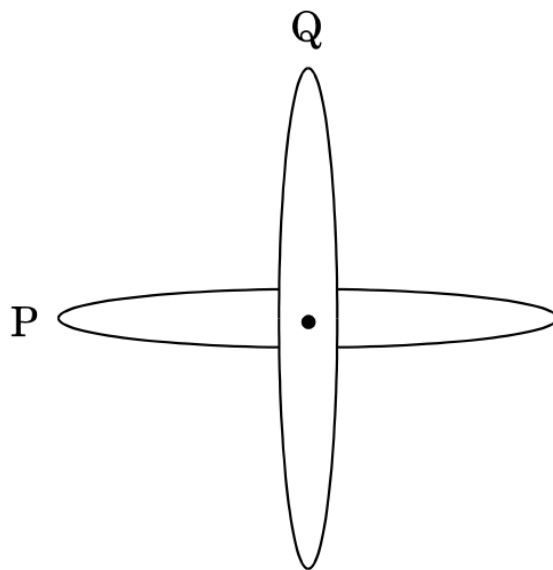
Ideal OR output for the given input waveforms (shown only over the printed time interval).

Step 4: Conclusion The CE circuit gives I_B -versus- V_{BE} input curves at fixed V_{CE} , and I_C -versus- V_{CE} output curves at fixed I_B . For the OR gate, $Y = 0$ during t_1 to t_2 and t_5 to t_6 ; $Y = 1$ during t_2 to t_5 and t_6 to t_8 .

Quick Tip to Solve: Write the quantity held constant beside each transistor curve. For an OR waveform, mark only intervals in which both inputs are zero; the output is high everywhere else.

20. Two identical loops P and Q each of radius 5 cm are lying in perpendicular planes such that they have a common centre as shown in the figure. Find the magnitude and direction of the net magnetic field at the common centre of the two coils, if they carry currents equal to 3 A and 4 A respectively.

[3 marks]



Solution:

Concept:

- The magnetic field at the centre of a circular loop is $B = \mu_0 I / (2R)$, where I is its current, R its radius and $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$.
- Each loop's field is perpendicular to its plane, with its sense given by the right-hand thumb rule. Fields from loops in perpendicular planes are therefore perpendicular vectors.
- Superposition gives $\vec{B} = \vec{B}_P + \vec{B}_Q$. For perpendicular fields, $B = \sqrt{B_P^2 + B_Q^2}$ and $\tan \theta = B_Q / B_P$, with θ measured from $\vec{B}_P$ towards $\vec{B}_Q$.

Step 1: Calculate the two perpendicular field components Convert the common radius to SI units: $R = 5 \text{ cm} = 0.05 \text{ m}$. The currents in P and Q are 3 A and 4 A, respectively. Thus

$$B_P = \frac{4\pi \times 10^{-7} \times 3}{2 \times 0.05} = 12\pi \times 10^{-6} \text{ T},$$

$$B_Q = \frac{4\pi \times 10^{-7} \times 4}{2 \times 0.05} = 16\pi \times 10^{-6} \text{ T}.$$

Their directions are along the respective loop axes. Since these axes are perpendicular, adding the scalar magnitudes would give an incorrect resultant.

Step 2: Combine the fields and determine the inclination Apply the Pythagorean rule to the perpendicular components:

$$B = \sqrt{(12\pi)^2 + (16\pi)^2} \times 10^{-6} = 20\pi \times 10^{-6} \text{ T}.$$

The resultant lies in the plane containing the two field vectors. Its angle with the field due to P

Quick Tip to Solve: Equal radii make the field ratio equal to the current ratio. Recognise the perpendicular 3 : 4 : 5 triangle to obtain both the resultant magnitude and its inclination quickly.

satisfies

$$\tan \theta = \frac{B_Q}{B_P} = \frac{4}{3}, \quad \theta = 53.1^\circ.$$

It consequently makes 36.9° with the field due to Q . The printed figure gives no current arrows, so a unique absolute sense such as upward or leftward cannot be inferred. For any specified current senses, use the right-hand rule for each field and take the resultant between those directed field vectors.

Step 3: Conclusion The net field is $B = 20\pi \times 10^{-6} \text{ T} \approx 6.28 \times 10^{-5} \text{ T}$. It is inclined at 53.1° to $\vec{B}_P$ towards $\vec{B}_Q$, or 36.9° to $\vec{B}_Q$; its absolute sense depends on the unspecified current directions.

Quick Tip to Solve: Equal radii make the field ratio equal to the current ratio. Recognise the perpendicular 3 : 4 : 5 triangle to obtain both the resultant magnitude and its inclination quickly.

21. Draw a block diagram of a generalized communication system. Write the functions of each of the following:

(a) Transmitter

(b) Channel

(c) Receiver

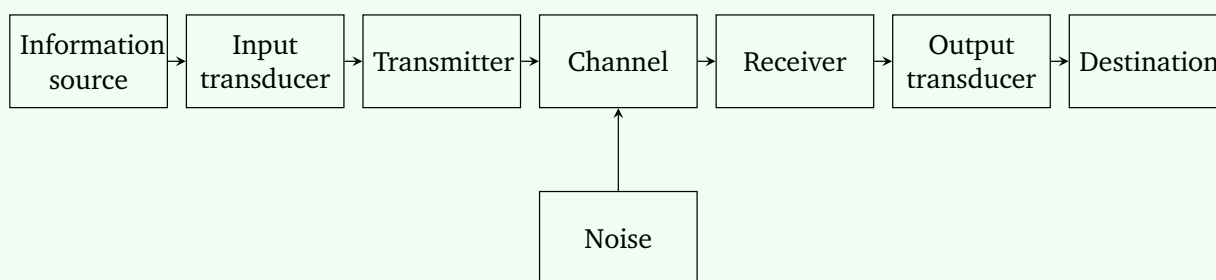
[3 marks]

Solution:

Concept:

- Communication transfers information from a source to a destination. The essential system consists of a transmitter, a communication channel and a receiver.
- An input transducer converts the original message, such as sound, into an electrical signal; an output transducer converts the recovered electrical signal into a usable form.
- Noise is an unwanted disturbance added to the signal during transmission. The receiver must recover the desired information from the received signal.

Step 1: Draw the generalized system The arrows in the block diagram show the direction in which information travels. A microphone can serve as the input transducer and a loudspeaker as the output transducer. The noise input indicates that unwanted signals may enter the communication channel.



Generalized communication system with an unwanted noise input.

Step 2: State the function of the transmitter The transmitter processes the electrical message into a form suitable for transmission through the chosen channel. In a radio system it modulates a high-frequency carrier with the message and amplifies the resulting signal to provide the power needed for transmission.

Quick Tip to Solve: Draw arrows from source to destination and add noise at the channel; describe the three core functions as prepare, carry and recover.

Step 3: State the function of the channel The channel is the physical medium that carries the transmitted signal from the transmitter to the receiver. Examples include a pair of wires, an optical fibre and free space. The signal may become attenuated or acquire noise while passing through the channel.

Step 4: State the function of the receiver The receiver accepts the signal arriving through the channel, selects the desired signal and amplifies it. It then demodulates or decodes the signal to recover the original message, which the output transducer presents to the destination.

Step 5: Conclusion The transmitter prepares and sends the message signal, the channel carries it, and the receiver recovers the information for the destination.

Quick Tip to Solve: Draw arrows from source to destination and add noise at the channel; describe the three core functions as prepare, carry and recover.

22(a). The potential difference applied across a given resistor is altered so that the heat produced per second increases by a factor of 9. By what factor does the applied potential difference change?

[3 marks total]

Solution:

Concept:

- Joule's law gives the heat developed in a resistor in time t as $H = I^2 R t$, where I is the current and R is the resistance.
- Heat produced per second is electrical power: $P = H/t = VI = V^2/R$. For a given resistor with constant resistance, power is proportional to the square of the applied potential difference.

Step 1: Compare the initial and final heating rates Let the initial and final potential differences be V_1 and V_2 , and the corresponding rates of heat production be P_1 and P_2 . Since the same resistance R is used,

$$P_1 = \frac{V_1^2}{R}, \quad P_2 = \frac{V_2^2}{R}.$$

Dividing cancels the resistance and avoids needing its numerical value:

$$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1} \right)^2.$$

Step 2: Take the square root of the power ratio The heating rate increases by a factor of nine, so $P_2/P_1 = 9$. Hence the ratio of the magnitudes of the applied voltages is

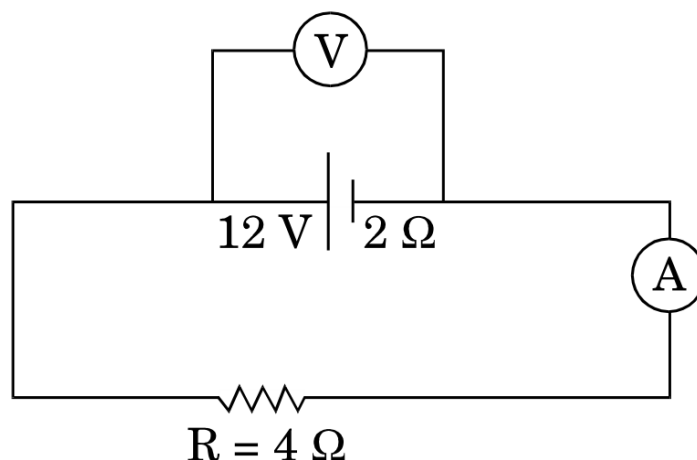
$$\frac{|V_2|}{|V_1|} = \sqrt{9} = 3.$$

This concerns the size of the potential difference; reversing its polarity would not change the heat produced in the resistor.

Step 3: Conclusion The applied potential difference must increase in magnitude by a factor of 3, assuming the resistor's resistance remains constant.

Quick Tip to Solve: For the same resistance use $V \propto \sqrt{P}$; take the square root of the heating-rate factor to find the voltage factor.

22(b). In the figure shown, an ammeter A and a resistor of $4\ \Omega$ are connected to the terminals of the source. The emf of the source is 12 V having an internal resistance of $2\ \Omega$. Calculate the voltmeter and ammeter readings.



Solution:

Concept:

- A source of emf $\mathcal{E}$ and internal resistance r , connected to an external resistance R , supplies current $I = \mathcal{E}/(R + r)$.
- While a cell supplies current, its terminal potential difference is $V = \mathcal{E} - Ir = IR$. An ideal voltmeter measures this terminal voltage and draws negligible current; an ideal ammeter has negligible resistance.

Step 1: Calculate the current in the series circuit The ammeter is in series with the external resistor, while the voltmeter is connected across the source terminals. Therefore the ammeter measures the load current. Including the internal resistance gives

$$R_{\text{total}} = R + r = 4 + 2 = 6\ \Omega.$$

Applying the circuit equation $\mathcal{E} = I(R + r)$,

$$I = \frac{12}{6} = 2\text{ A}.$$

Step 2: Find the voltage across the source terminals The source loses a potential difference Ir across its internal resistance. The voltmeter therefore measures less than the emf:

$$V = \mathcal{E} - Ir = 12 - (2)(2) = 8\text{ V}.$$

As a check, the terminal voltage also equals the potential drop across the external resistor, since the ideal ammeter has zero voltage drop:

$$IR = (2)(4) = 8\text{ V}.$$

Quick Tip to Solve: Include internal resistance when finding current, then subtract Ir from the emf to obtain the voltmeter reading across a loaded source.

Step 3: Conclusion The voltmeter reads 8 V and the ammeter reads 2 A , taking the meters to be ideal.

Quick Tip to Solve: Include internal resistance when finding current, then subtract Ir from the emf to obtain the voltmeter reading across a loaded source.

Section D (4 marks)

23. Asha's mother read an article in the newspaper about a disaster that took place at Chernobyl. She could not understand much from the article and asked a few questions from Asha regarding the article. Asha tried to answer her mother's questions based on what she learnt in Class XII Physics.

(a) What was the installation at Chernobyl where the disaster took place? What, according to you, was the cause of this disaster?

(b) Explain the process of release of energy in the installation at Chernobyl.

(c) What, according to you, were the values displayed by Asha and her mother?

[4 marks]

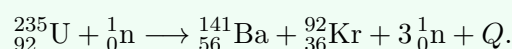
Solution:

Concept:

- Nuclear fission splits a heavy nucleus into lighter nuclei, releasing neutrons and energy; the emitted neutrons can sustain a chain reaction.
- The energy released is $Q = \Delta mc^2$, where Δm is the decrease in total rest mass and c is the speed of light. The products have greater binding energy per nucleon.

Step 1: (a) Identify the installation and the cause The installation at Chernobyl was a nuclear power station containing nuclear fission reactors. In normal operation, neutron absorption and cooling keep the fission chain reaction and temperature under control. A failure to maintain safe control of the reactor caused a rapid increase in power and overheating, leading to explosions and the release of radioactive material. Thus, the essential physics was loss of control of the chain reaction and heat production in a nuclear reactor; it was not the detonation of a nuclear bomb.

Step 2: (b) Explain the energy-producing process When a uranium-235 nucleus absorbs a neutron, it forms an excited, unstable uranium-236 nucleus, which splits into two lighter nuclei. One representative fission channel is



Both nucleon number and charge are conserved: $235 + 1 = 141 + 92 + 3$ and $92 = 56 + 36$. The combined rest mass of the products is smaller than that of the initial particles; the mass difference appears as energy, approximately 200 MeV per fission. Much of it initially appears as kinetic energy of the fragments and is converted into heat as they slow down in the fuel. The released neutrons cause further fissions, producing a chain reaction. In a power station, reactor heat produces steam that drives a turbine coupled to an electric generator. Control rods absorb excess neutrons so that the normal chain reaction remains controlled.

Step 3: (c) State the values shown by each person Asha displayed scientific understanding, willingness

Quick Tip to Solve: For a nuclear power question, connect fission to mass defect, the neutron chain reaction and useful heat; distinguish controlled reactor operation from a runaway accident.

to apply classroom learning to real events, helpfulness and respect for her mother. Her mother displayed curiosity, a desire to understand scientific events and openness to learning from her daughter. Their conversation also shows the value of seeking a reasoned explanation of a disaster and developing awareness of responsible use of nuclear energy.

Step 4: Conclusion Chernobyl was a nuclear power installation where loss of safe reactor control led to disaster. Its energy came from a fission chain reaction and the conversion of mass into energy. Asha showed scientific awareness and helpfulness; her mother showed curiosity and willingness to learn.

Quick Tip to Solve: For a nuclear power question, connect fission to mass defect, the neutron chain reaction and useful heat; distinguish controlled reactor operation from a runaway accident.

Section E (15 marks)

24. (a) Define wavefront. Use Huygens' principle to verify the laws of refraction.

(b) How is linearly polarised light obtained by the process of scattering of light? Find the Brewster angle for air-glass interface, when the refractive index of glass = 1.5. [5 marks]

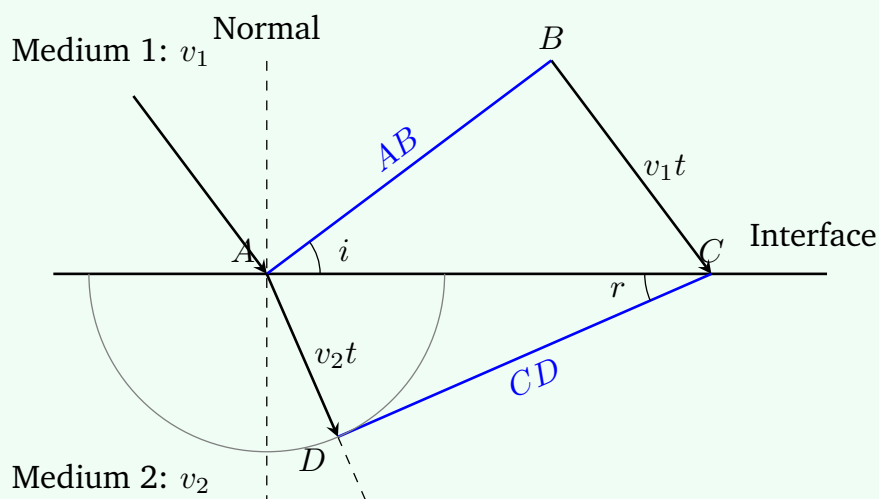
Solution:

Concept:

- A wavefront is a surface joining points of a wave that vibrate in the same phase. According to Huygens' principle, every point on a wavefront emits secondary wavelets; their forward envelope gives the new wavefront.
- Rays are normal to wavefronts. For speeds v_1, v_2 in two media, their refractive indices are $n_1 = c/v_1$ and $n_2 = c/v_2$.
- An oscillating electric dipole does not radiate along its own axis. This explains complete linear polarisation of singly scattered light observed perpendicular to the incident beam.
- Brewster's law for polarisation by reflection is $\tan i_B = n_2/n_1$, where i_B is the polarising angle measured from the normal.

Step 1: (a) Construct the refracted wavefront Consider a plane incident wavefront AB approaching a plane interface between two transparent, isotropic media. Point A reaches the interface first. During the time t taken by the disturbance at B to reach C , it travels a distance $BC = v_1 t$. In the same time the secondary wavelet from A in the second medium grows to radius $AD = v_2 t$. Draw the tangent CD from C to this wavelet. It is the refracted wavefront, and AD , perpendicular to CD , gives the refracted ray direction. The construction below illustrates $v_2 < v_1$.

Quick Tip to Solve: In the Huygens triangles, use the common hypotenuse AC to cancel both AC and time. Keep right-angle scattering distinct from Brewster-angle reflection.



Huygens' construction: AB is the incident wavefront and CD the refracted wavefront.

Step 2: (a) Verify both laws of refraction Since AB is perpendicular to the incident ray BC , the angle BAC equals the angle of incidence i . Similarly, CD is perpendicular to AD , so the angle ACD equals the angle of refraction r . The right triangles ABC and ADC therefore give

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}.$$

Dividing these relations yields

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{n_2}{n_1}, \quad n_1 \sin i = n_2 \sin r.$$

This is Snell's law. The secondary-wavelet construction is symmetric on either side of the plane containing the incident ray and the interface normal. The refracted wavefront's normal therefore also lies in that plane. Thus the incident ray, refracted ray and normal at the point of incidence are coplanar, verifying the other law.

Step 3: (b) Explain polarisation by scattering Let unpolarised light travel along the x -axis through a gas or a dilute suspension of small particles. Its electric field can be resolved into components along y and z , which induce dipole oscillations in those directions. Observe scattered light along the y -axis, at 90° to the incident beam. The dipole component oscillating along y cannot radiate towards this observer. Only the z -directed component contributes, so the observed scattered electric field has one fixed direction: the light is linearly polarised, perpendicular to the xy scattering plane. This complete polarisation describes single scattering at a right angle; scattering at other angles is generally only partially polarised.

Step 4: (b) Calculate the Brewster angle The numerical question concerns polarisation by reflection at the air-glass interface. Taking the refractive index of air as unity and that of glass as 1.5, Brewster's law gives

$$\tan i_B = \frac{1.5}{1} = 1.5, \quad i_B = \tan^{-1}(1.5) = 56.3^\circ.$$

At this incidence angle, the reflected beam is completely linearly polarised with its electric field

Quick Tip to Solve: In the Huygens triangles, use the common hypotenuse AC to cancel both AC and time. Keep right-angle scattering distinct from Brewster-angle reflection.

perpendicular to the plane of incidence; reflected and refracted rays are mutually perpendicular.

Step 5: Conclusion Huygens' construction verifies coplanarity and $n_1 \sin i = n_2 \sin r$. Single scattering through 90° gives linearly polarised light perpendicular to the scattering plane. The Brewster angle for light incident from air onto glass is 56.3° .

Quick Tip to Solve: In the Huygens triangles, use the common hypotenuse AC to cancel both AC and time. Keep right-angle scattering distinct from Brewster-angle reflection.

OR

24. (a) Draw a ray diagram to show the image formation by a combination of two thin convex lenses in contact. Obtain the expression for the power of this combination in terms of the focal lengths of the lenses.

(b) A ray of light passing from air through an equilateral glass prism undergoes minimum deviation when the angle of incidence is $\frac{3}{4}$ th of the angle of prism. Calculate the speed of light in the prism.

[5 marks]

Solution:

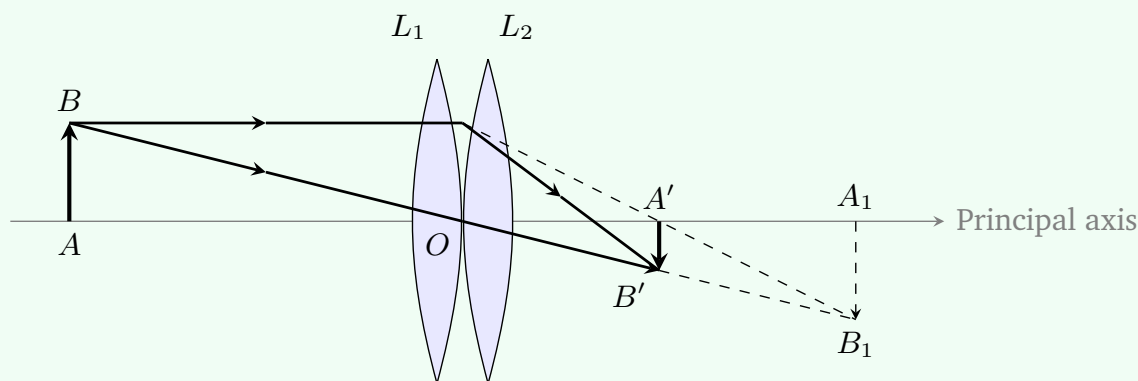
Concept:

- For a thin lens in air, the Cartesian lens formula is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, where u , v and f are signed object distance, image distance and focal length.
- Lenses in contact have negligible separation. The image formed by the first lens acts as the object for the second lens; their distances are measured from the same effective optical centre.
- Lens power is $P = 1/F$, with equivalent focal length F in metres and power in dioptres. For refraction from air into glass, $n = \sin i / \sin r$ and the speed in glass is c/n .

Step 1: (a) Construct the ray diagram for the contact combination Let AB be an object before two thin convex lenses L_1 and L_2 , of focal lengths f_1 and f_2 . Their common optical centre is O . A ray from B , parallel to the principal axis, is converged by the two lenses to the final image point B' . A second ray through O passes undeviated in the thin-lens approximation. Their intersection gives the real, inverted image $A'B'$ for the object position shown.

The dashed construction shows A_1B_1 , the image that L_1 alone would form if L_2 were absent. The converging beam approaching L_2 is directed towards B_1 , so A_1B_1 is a virtual object for L_2 . The outlines distinguish the two lenses, but their thicknesses and separation are neglected in all distance measurements.

Quick Tip to Solve: For lenses in contact, add their lens equations to cancel the intermediate image distance. At minimum deviation in an equilateral prism, immediately set both internal refraction angles to 30° .



Two thin convex lenses in contact: solid rays form $A'B'$; dashed rays show the image A_1B_1 due to L_1 alone. Lens thickness is exaggerated.

Step 2: Apply the lens formula separately to the two lenses Take incident light from left to right, so distances to the right of O are positive. Let the object distance be u , the intermediate image distance be v_1 , and the final image distance be v . For L_1 ,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}.$$

Since the lenses are in contact, the object distance for L_2 is v_1 , with its Cartesian sign retained. Thus

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

In the illustrated arrangement $v_1 > 0$: the virtual object for the second lens is on its outgoing side. Treating this distance as negative would give the wrong combined power.

Step 3: Obtain the equivalent focal length and power Add the two equations. The intermediate-image term cancels:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

A single equivalent lens producing the same final image satisfies

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}.$$

Comparing these expressions gives

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}, \quad F = \frac{f_1 f_2}{f_1 + f_2}.$$

Therefore, when the focal lengths are in metres,

$$P = \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = P_1 + P_2.$$

Both convex lenses have positive power, so the combination has greater converging power and a shorter focal length than either lens separately.

Quick Tip to Solve: For lenses in contact, add their lens equations to cancel the intermediate image distance. At minimum deviation in an equilateral prism, immediately set both internal refraction angles to 30° .

Step 4: (b) Find the angles at minimum deviation An equilateral prism has prism angle $A = 60^\circ$. The given incidence angle is

$$i = \frac{3}{4}A = 45^\circ.$$

At minimum deviation, the path through the prism is symmetric: $i = e$ and $r_1 = r_2$. Since $r_1 + r_2 = A$,

$$r_1 = r_2 = \frac{A}{2} = 30^\circ.$$

As a consistency check, the minimum deviation is $\delta_m = 2i - A = 30^\circ$.

Step 5: Use Snell's law to calculate the speed Taking the refractive index of air as unity, Snell's law at the first face gives

$$n = \frac{\sin i}{\sin r_1} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}.$$

Use $c = 3.0 \times 10^8 \text{ m s}^{-1}$. The speed of light in the prism is

$$v_{\text{glass}} = \frac{c}{n} = \frac{3.0 \times 10^8}{\sqrt{2}} \approx 2.12 \times 10^8 \text{ m s}^{-1}.$$

This is smaller than the speed in air, as expected for glass with refractive index greater than unity.

Step 6: Conclusion (a) The power of two thin convex lenses in contact is $P = P_1 + P_2 = \frac{1}{f_1} + \frac{1}{f_2}$, in dioptres when f_1, f_2 are in metres. (b) The speed of light in the prism is $2.12 \times 10^8 \text{ m s}^{-1}$.

Quick Tip to Solve: For lenses in contact, add their lens equations to cancel the intermediate image distance. At minimum deviation in an equilateral prism, immediately set both internal refraction angles to 30° .

25. (a) Derive an expression for the electric field $\vec{E}$ due to a dipole of length ' $2a$ ' at a point distant r from the centre of the dipole on the axial line.

(b) Draw a graph of E versus r for $r \gg a$.

(c) If this dipole were kept in a uniform external electric field $\vec{E}_0$, diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases.

[5 marks]

Solution:

Concept:

- A dipole consists of charges $-q$ and $+q$, separated by $2a$. Its dipole moment is $\vec{p} = 2aq\hat{p}$, directed from the negative to the positive charge.
- By superposition, the resultant field is the vector sum of the Coulomb fields of the two charges, with $k = 1/(4\pi\epsilon_0)$.
- In a uniform external field, a dipole experiences zero net force but a torque $\vec{\tau} = \vec{p} \times \vec{E}_0$. Its potential energy is $U = -pE_0 \cos \theta$, where θ is the angle between $\vec{p}$ and $\vec{E}_0$.

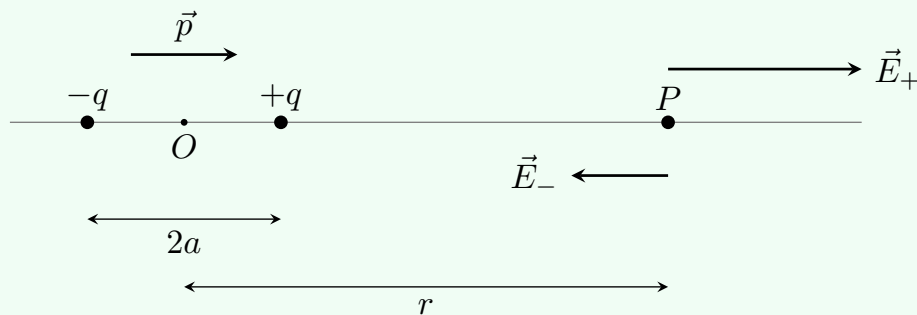
Step 1: (a) Resolve the fields at an axial point Put $-q$ at $x = -a$, $+q$ at $x = a$, and the observation

Quick Tip to Solve: On the external axial line, subtract the two fields before using $r \gg a$. For equilibrium, first set $\sin \theta = 0$, then use minimum or maximum dipole energy to decide stability.

point P at $x = r$, with $r > a$. The distances from the positive and negative charges are $r - a$ and $r + a$, respectively. At P , the positive charge's field points along $\hat{p}$, whereas the negative charge's field points opposite to $\hat{p}$:

$$\vec{E}_+ = \frac{kq}{(r-a)^2} \hat{p}, \quad \vec{E}_- = -\frac{kq}{(r+a)^2} \hat{p}.$$

The nearer positive charge produces the larger field, so the resultant points along the dipole moment.



Axial observation point and the two opposite field contributions.

Step 2: (a) Combine the two Coulomb fields Subtract the magnitudes before making any approximation:

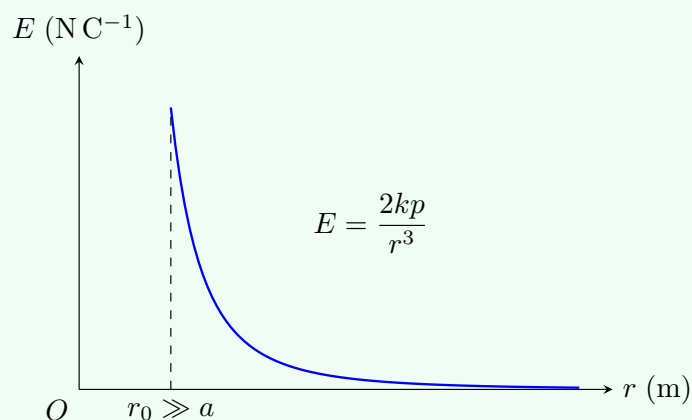
$$\begin{aligned} \vec{E} &= kq \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right] \hat{p} \\ &= kq \frac{(r+a)^2 - (r-a)^2}{(r^2 - a^2)^2} \hat{p} \\ &= \frac{4kqar}{(r^2 - a^2)^2} \hat{p} = \frac{1}{4\pi\epsilon_0} \frac{2r\vec{p}}{(r^2 - a^2)^2}. \end{aligned}$$

This is the exact field at an external axial point. At an external point on the other side of the dipole, its direction is also parallel to $\vec{p}$. For a distant point, $a^2/r^2 \ll 1$, and hence

$$\vec{E} = \frac{2k\vec{p}}{r^3} \left(1 - \frac{a^2}{r^2} \right)^{-2} \simeq \frac{2k\vec{p}}{r^3}.$$

Step 3: (b) Plot the far-field variation With p fixed, the field magnitude in the region $r \gg a$ varies as $E = 2kp/r^3$. It decreases rapidly and approaches zero asymptotically as r increases. For example, doubling a sufficiently large distance reduces the field to one-eighth. The sketch starts at a distance $r_0 \gg a$; the far-field expression is not extended into the region near the charges. The falling curve has an upward curvature since its slope becomes less negative at larger distances.

Quick Tip to Solve: On the external axial line, subtract the two fields before using $r \gg a$. For equilibrium, first set $\sin \theta = 0$, then use minimum or maximum dipole energy to decide stability.



Far-field graph, shown only for $r \geq r_0 \gg a$.

Step 4: (c) Show stable equilibrium For $\vec{p}$ parallel to $\vec{E}_0$, $\theta = 0$. The forces on the two charges are equal, opposite and collinear, so

$$\vec{\tau} = \vec{p} \times \vec{E}_0 = \vec{0}, \quad \tau = pE_0 \sin 0 = 0.$$

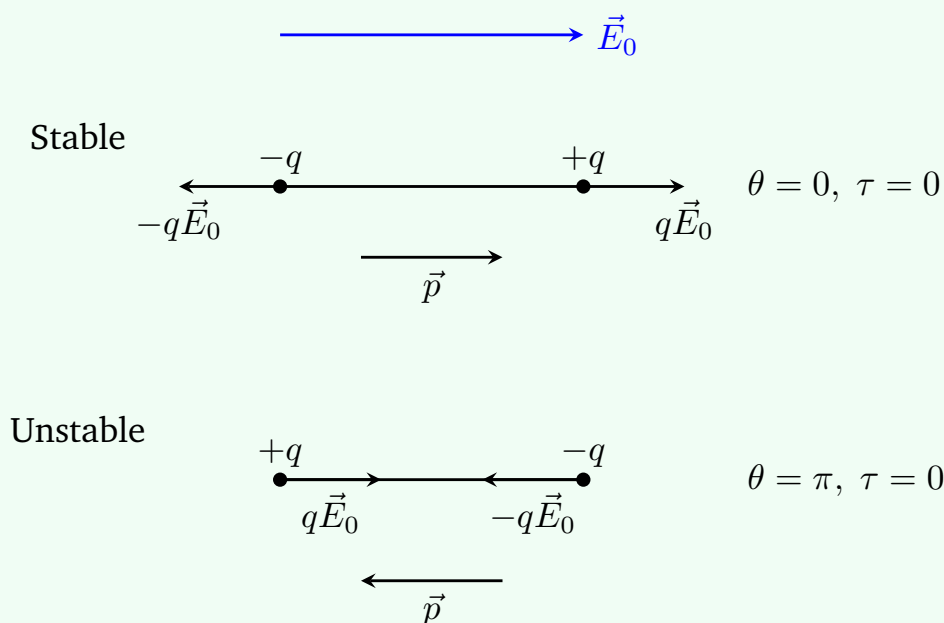
Here $U = -pE_0$, the minimum possible energy. A small angular displacement δ produces a restoring torque: with increasing angle taken positive, $\tau_\theta = -pE_0 \sin \delta \simeq -pE_0 \delta$. The dipole therefore tends to return to its original orientation, establishing stable equilibrium.

Step 5: (c) Show unstable equilibrium For $\vec{p}$ antiparallel to $\vec{E}_0$, $\theta = \pi$. At this exact orientation,

$$\vec{\tau} = \vec{p} \times \vec{E}_0 = \vec{0}, \quad \tau = pE_0 \sin \pi = 0.$$

However, $U = +pE_0$, the maximum possible energy. If $\theta = \pi + \delta$, the signed torque becomes $\tau_\theta = -pE_0 \sin(\pi + \delta) \simeq +pE_0 \delta$. It increases the departure from equilibrium, so the orientation is unstable. Thus zero torque alone does not decide stability; the response to a small rotation does.

Quick Tip to Solve: On the external axial line, subtract the two fields before using $r \gg a$. For equilibrium, first set $\sin \theta = 0$, then use minimum or maximum dipole energy to decide stability.



Stable and unstable orientations in the same uniform external field.

Step 6: Conclusion The axial field is $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2r\vec{p}}{(r^2 - a^2)^2}$, reducing to $\frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}$ for $r \gg a$. The graph falls as r^{-3} . Parallel alignment is stable and antiparallel alignment is unstable; the torque is zero at both exact equilibrium orientations.

Quick Tip to Solve: On the external axial line, subtract the two fields before using $r \gg a$. For equilibrium, first set $\sin \theta = 0$, then use minimum or maximum dipole energy to decide stability.

OR

25. (a) Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge density σ .

(b) An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Obtain the expression for the amount of work done in bringing a point charge q from infinity to a point, distant r , in front of the charged plane sheet. [5 marks]

Solution:

Concept:

- Gauss's law states $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is enclosed charge and ϵ_0 is the permittivity of free space.
- An infinite uniform sheet has a field normal to the sheet, with equal magnitudes on its two sides. The surface charge density is $\sigma = Q/A$.
- For slow transport without changing kinetic energy, external work is $W_{\text{ext}} = q\Delta V = -q \int_i^f \vec{E} \cdot d\vec{l}$. Work by the electric field has the opposite sign.

Quick Tip to Solve: Before using infinity as the zero of potential, check whether the field integral converges. For an infinite sheet, calculate work between two finite distances first.

Step 1: (a) Choose a Gaussian pillbox Consider a uniformly charged thin sheet in free space. By translational symmetry along the sheet and reflection symmetry, its electric field has no component parallel to the sheet and has the same magnitude on either side at corresponding points. Choose a small cylindrical Gaussian surface passing through the sheet, with its axis perpendicular to the sheet and each plane face of area A . For positive σ , the field points outward from the sheet on both sides. Thus it is parallel to the outward area vector of each plane face. On the curved surface it is perpendicular to the area vector, giving zero flux there.

$$\Phi = EA + EA + 0 = 2EA, \quad Q_{\text{enc}} = \sigma A.$$

Step 2: (a) Apply Gauss's law and identify the direction Substituting the flux and the enclosed charge into Gauss's law gives

$$2EA = \frac{\sigma A}{\epsilon_0} \Rightarrow E = \frac{\sigma}{2\epsilon_0} \quad (\sigma > 0).$$

The area cancels and the result contains no distance: the field of an ideal infinite sheet is uniform on each side. In general its magnitude is $|\sigma|/(2\epsilon_0)$; it is directed normally away from a positive sheet and towards a negative sheet. Both faces contribute to the flux, which explains the factor of two in the denominator.

Step 3: (b) First calculate work from a finite reference point Let x measure perpendicular distance in front of the positively charged sheet, so $\vec{E} = (\sigma/2\epsilon_0)\hat{x}$ for $x > 0$. Start with the charge at a finite distance $R > r$ and move it slowly to $x = r$. The potential difference and external work are

$$V(r) - V(R) = - \int_R^r \frac{\sigma}{2\epsilon_0} dx = \frac{\sigma}{2\epsilon_0}(R - r),$$

$$W_{\text{ext}}(R \rightarrow r) = q[V(r) - V(R)] = \frac{q\sigma}{2\epsilon_0}(R - r).$$

For a positive charge, moving towards the sheet is against electric repulsion, so an external agent does positive work. The electric field does $W_{\text{field}} = -W_{\text{ext}}$.

Step 4: (b) Examine the stated starting point at infinity The printed question specifies an infinitely large sheet and a starting point at infinity. Its field does not decrease with distance, so taking that limit gives

$$W_{\text{ext}}(\infty \rightarrow r) = \lim_{R \rightarrow \infty} \frac{q\sigma}{2\epsilon_0}(R - r).$$

For nonzero q , this is divergent: it is $+\infty$ for $q > 0$ and $-\infty$ for $q < 0$. The magnitude of the required energy transfer is unbounded; for $q = 0$, the work is zero. Consequently, one cannot set $V(\infty) = 0$ to obtain a finite potential for an ideal infinite sheet. A finite reference point must be specified. For example, choosing $V(0) = 0$ gives $V(r) = -\sigma r/(2\epsilon_0)$ and work $-q\sigma r/(2\epsilon_0)$ from the sheet to r ; that expression describes different endpoints from those printed.

Step 5: Conclusion (a) $E = |\sigma|/(2\epsilon_0)$, normal to the sheet, away from positive charge and towards negative charge. (b) From a finite distance R to r , $W_{\text{ext}} = q\sigma(R - r)/(2\epsilon_0)$. From infinity the work has no finite value for nonzero q ; its magnitude diverges because the field remains constant.

Quick Tip to Solve: Before using infinity as the zero of potential, check whether the field integral converges. For an infinite sheet, calculate work between two finite distances first.

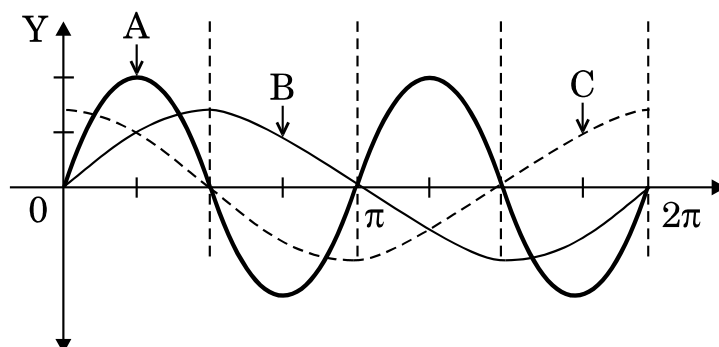
26. A device X is connected to an ac source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph:

(a) Identify the device 'X'.

(b) Which of the curves A , B and C represent the voltage, current and the power consumed in the circuit? Justify your answer.

(c) How does its impedance vary with frequency of the ac source? Show graphically.

(d) Obtain an expression for the current in the circuit and its phase relation with ac voltage. [5 marks]



Solution:

Concept:

- A capacitor of capacitance C obeys $q = CV$ and $i = dq/dt$. Its current therefore depends on the rate of change of the applied voltage.
- For angular frequency $\omega = 2\pi f$, the capacitive reactance is $X_C = 1/(\omega C)$. This is the magnitude of the impedance of an ideal capacitor.
- Instantaneous electrical power is $p = Vi$. An ideal reactive element stores and returns energy, so its average power over one complete cycle is zero.

Step 1: (a) Identify the phase pattern and the device The applied voltage $V_0 \sin \omega t$ must start at zero and increase positively. In the given graph, curve B has precisely this behaviour and completes one oscillation between 0 and 2π . The dashed curve C has a positive maximum when B is zero, and reaches zero when B reaches its positive maximum. Thus the current leads the voltage by one-quarter of a cycle, or $\pi/2$. This is the characteristic behaviour of an ideal **capacitor**; hence device X is a capacitor. For an inductor the current would lag, while for a resistor it would be in phase.

Step 2: (b) Match and justify all three curves Curve B represents the voltage and curve C represents the current:

$$V = V_0 \sin \omega t, \quad i = I_0 \cos \omega t.$$

Quick Tip to Solve: First identify the single-frequency sine curve as voltage. A current maximum at its rising zero crossing means a capacitor; the double-frequency curve is the product Vi .

Their product gives the instantaneous power:

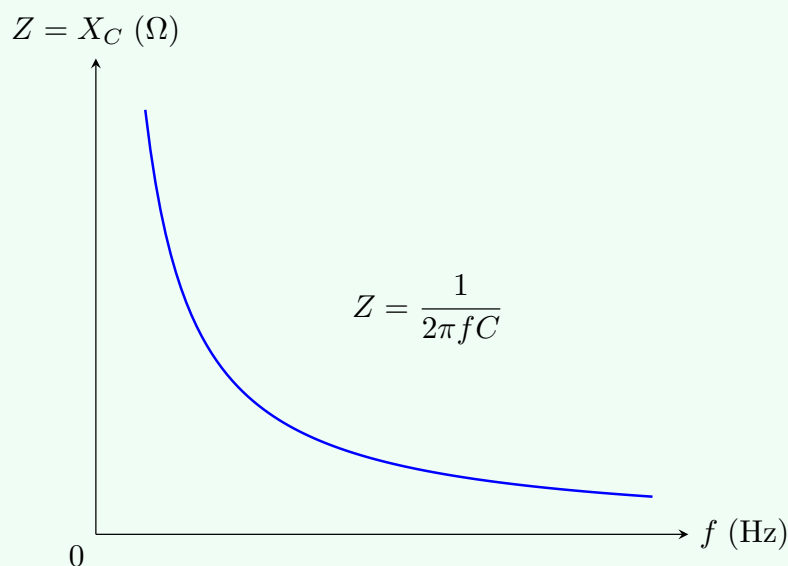
$$p = V_0 I_0 \sin \omega t \cos \omega t = \frac{V_0 I_0}{2} \sin 2\omega t.$$

Therefore curve A , which completes two oscillations in one voltage cycle, represents power. It is positive while voltage and current have the same sign and negative while their signs are opposite, exactly as shown. Its equal positive and negative areas mean zero average power: during the positive intervals the capacitor receives energy, and during the negative intervals it returns stored energy to the source. Thus the power curve is instantaneous power exchanged, not a nonzero net dissipation.

Step 3: (c) Plot the impedance against source frequency For a fixed capacitance, the impedance magnitude is

$$Z = X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

Hence Z is inversely proportional to f . Doubling the frequency halves the impedance. The graph in the first quadrant is a rectangular hyperbola: Z tends to infinity as f tends to zero from above and tends to zero as f becomes very large. The curve approaches both axes without meeting them at a finite positive frequency.



Capacitive impedance versus frequency for a fixed capacitance.

Step 4: (d) Derive the current and its phase relation The charge on the capacitor follows the applied voltage:

$$q(t) = CV(t) = CV_0 \sin \omega t.$$

Quick Tip to Solve: First identify the single-frequency sine curve as voltage. A current maximum at its rising zero crossing means a capacitor; the double-frequency curve is the product Vi .

Differentiating with respect to time gives

$$\begin{aligned}i(t) &= \frac{dq}{dt} = \omega CV_0 \cos \omega t \\&= I_0 \sin \left(\omega t + \frac{\pi}{2} \right), \\I_0 &= \omega CV_0 = \frac{V_0}{X_C}.\end{aligned}$$

The positive phase addition of $\pi/2$ shows that current leads the applied voltage by 90° . Equivalently, the voltage lags the current by one-quarter period. This derived phase relation also independently verifies the curve assignments made from the graph.

Step 5: Conclusion X is an ideal capacitor. Curves A , B , and C represent instantaneous power, voltage, and current respectively. Its impedance magnitude $Z = 1/(2\pi fC)$ decreases hyperbolically with frequency, and $i = \omega CV_0 \sin(\omega t + \pi/2)$ leads the voltage by $\pi/2$.

Quick Tip to Solve: First identify the single-frequency sine curve as voltage. A current maximum at its rising zero crossing means a capacitor; the double-frequency curve is the product Vi .

OR

26. (a) Draw a labelled diagram of an ac generator. Obtain the expression for the emf induced in the rotating coil of N turns each of cross-sectional area A , in the presence of a magnetic field $\vec{B}$.

(b) A horizontal conducting rod 10 m long extending from east to west is falling with a speed 5.0 ms^{-1} at right angles to the horizontal component of the Earth's magnetic field, $0.3 \times 10^{-4} \text{ Wb m}^{-2}$. Find the instantaneous value of the emf induced in the rod. [5 marks]

Solution:

Concept:

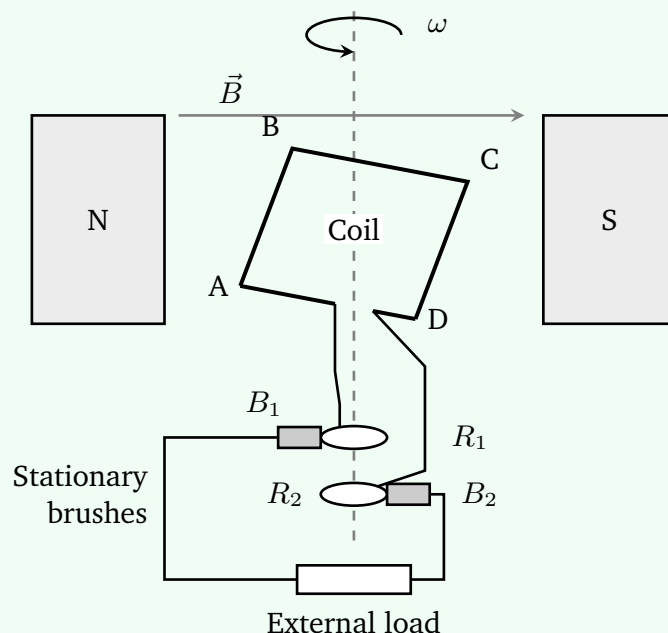
- Faraday's law gives the induced emf as $\varepsilon = -d(N\Phi)/dt$, where $N\Phi$ is the magnetic flux linkage of a coil of N turns.
- In a uniform magnetic field of magnitude B , the flux through one plane turn of area A is $\Phi = BA \cos \theta$; θ is the angle between the field and the normal to the coil.
- A straight conductor of length l , moving with velocity perpendicular to both its length and the magnetic field, develops a motional emf of magnitude Blv .

Step 1: (a) Construction and working of the ac generator A rectangular armature coil rotates between the north and south pole pieces of a magnet. Its two ends connect to separate, continuous slip rings R_1 and R_2 , which rotate with the coil. Stationary carbon brushes B_1 and B_2 touch these rings and connect the coil to an external load. The shaft is driven mechanically, with its axis perpendicular to the magnetic field. The schematic shows the two separate ring connections; the dashed shaft is a mechanical axis, not an electrical connection.

As the coil rotates, its magnetic flux changes and an emf is induced. The direction of this emf reverses after each half-turn. Continuous slip rings maintain contact with the same coil ends, so the output

Quick Tip to Solve: Measure the flux angle from the coil's normal, and use only the field component perpendicular to the velocity when finding the rod's motional emf.

across the load is alternating.



AC generator: rotating coil, separate slip rings and stationary brushes.

Step 2: (a) Express the changing flux linkage Let the coil rotate uniformly with angular speed ω . Choose $t = 0$ when its normal is parallel to $\vec{B}$, so that the angle between the normal and the field at time t is $\theta = \omega t$. Each of the N turns has area A , and the field is uniform over the coil. Thus the flux linkage is

$$N\Phi(t) = NBA \cos(\omega t).$$

Flux is maximum when the plane of the coil is perpendicular to the field. It is zero when that plane is parallel to the field. The induced emf depends on the rate of change of this flux, not on the flux alone.

Step 3: (a) Apply Faraday's law Differentiating the flux linkage gives

$$\begin{aligned} \varepsilon(t) &= -\frac{d}{dt}[NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t). \end{aligned}$$

Hence the peak emf is $\varepsilon_0 = NBA\omega$. The emf is zero at maximum or minimum flux and has maximum magnitude when the flux passes through zero. A different choice of starting orientation changes only the phase of the sinusoidal expression. Reversing the direction in which the terminals are defined changes its sign. Mechanical work supplies the electrical energy delivered to the external load.

Step 4: (b) Calculate the emf in the falling rod The rod lies east–west, its velocity is vertically downward, and the horizontal magnetic field is perpendicular to its motion and to the rod. Therefore the full length of the rod contributes to the motional emf. The relevant data are

$$l = 10 \text{ m}, \quad v = 5.0 \text{ m s}^{-1}, \quad B_H = 0.3 \times 10^{-4} \text{ T}.$$

Quick Tip to Solve: Measure the flux angle from the coil's normal, and use only the field component perpendicular to the velocity when finding the rod's motional emf.

Using the horizontal component of the field,

$$\begin{aligned} |\varepsilon| &= B_H lv \\ &= (0.3 \times 10^{-4})(10)(5.0) \text{ V} \\ &= 1.5 \times 10^{-3} \text{ V}. \end{aligned}$$

The speed given is instantaneous, so this is the emf at that instant. The vertical component of the Earth's field, if present, is parallel to the rod's velocity and produces no magnetic force from this motion. With the horizontal field directed northward, $\vec{v} \times \vec{B}_H$ points eastward, making the east end positive relative to the west end.

Step 5: Conclusion (a) The generator produces $\varepsilon(t) = NBA\omega \sin(\omega t)$, with peak emf $NBA\omega$, for the stated initial orientation. (b) The instantaneous emf across the rod is $1.5 \times 10^{-3} \text{ V} = 1.5 \text{ mV}$.

Quick Tip to Solve: Measure the flux angle from the coil's normal, and use only the field component perpendicular to the velocity when finding the rod's motional emf.