CBSE Class 12 Physics Question Paper 2017 with Solution PDF-Set 55/2/1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours **Maximum Marks: 70**

General Instructions

(i) **All** questions are **compulsory**. There are **26** questions in all.

(ii) This question paper has **five** sections : Section A, Section B, Section C, Section D and Section E.

(iii) Section A contains **five** questions of **one** mark each, Section B contains **five** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains one value based question of **four** marks and Section E contains **three** questions of **five** marks each.

(iv) There is no overall choice. However, an internal choice has been provided in **one** question of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$c = 3 \times 10^8$ m/s

$h = 6 \cdot 63 \times 10^{-34}$ Js

$e = 1 \cdot 6 \times 10^{-19}$ C

$\mu_o = 4\pi \times 10^{-7}$ T m A^{-1}

$\varepsilon_o = 8 \cdot 854 \times 10^{-12}$ C^2 N^{-1} m^{-2}

$\frac{1}{4\pi\varepsilon_o} = 9 \times 10^9$ N m^2 C^{-2}

Mass of electron $= 9 \cdot 1 \times 10^{-31}$ kg

Mass of neutron $= 1 \cdot 675 \times 10^{-27}$ kg

Mass of proton $= 1 \cdot 673 \times 10^{-27}$ kg

Avogadro's number $= 6 \cdot 023 \times 10^{23}$ per gram mole

Boltzmann constant $= 1 \cdot 38 \times 10^{-23}$ JK^{-1}

Section A (5 marks)

1. A point charge Q is placed at point 'O' as shown in the figure. Is the potential at point A, i.e. V_A, greater, smaller or equal to potential, V_B, at point B, when Q is (i) positive, and (ii) negative charge? [1 mark]

O• A• B•

Solution:

Concept:

- The potential of a point charge, with zero potential at infinity, is $V(r) = kQ/r$, where $k = 1/(4\pi\varepsilon_0) > 0$ and r is distance from the charge.
- Potential is a signed scalar: comparing negative potentials requires retaining the sign of Q.

Step 1: Compare the distances The printed figure places the points in the order O, A, B. Thus $r_A = OA < OB = r_B$, so $1/r_A > 1/r_B$. The difference is

$$V_A - V_B = kQ\left(\frac{1}{r_A} - \frac{1}{r_B}\right).$$

The bracket is positive.

Step 2: Apply the sign of the source charge For a positive charge, multiplication by $kQ > 0$ gives a positive potential difference. For a negative charge, multiplication by $kQ < 0$ reverses the inequality: the nearer point has a more negative potential. The absolute potential magnitude is larger near the charge in both cases, but magnitude alone does not determine the requested comparison.

Step 3: Conclusion (i) For $Q > 0$, $V_A > V_B$. (ii) For $Q < 0$, $V_A < V_B$.

Quick Tip to Solve: Write the potential difference first; its sign is the sign of Q because A is nearer O.

2. Write the expression for speed of electromagnetic waves in a medium of electrical permittivity ε and magnetic permeability μ. [1 mark]

Solution:

Concept:

- In a homogeneous linear medium, electromagnetic propagation is governed by its absolute permittivity ε and permeability μ.
- The wave equation has coefficient $\mu\varepsilon$ multiplying the second time derivative.

Step 1: Identify the wave speed For an electric-field component, Maxwell's equations give $\nabla^2 E = \mu\varepsilon\,\partial^2 E/\partial t^2$. Comparing with the standard wave equation $\nabla^2 E = (1/v^2)\partial^2 E/\partial t^2$ identifies $1/v^2 = \mu\varepsilon$.

Step 2: Express the result using the given quantities Taking the positive root gives the propagation speed. The symbols here denote absolute material properties, so no additional vacuum factors are inserted. As a check, setting $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$ gives the vacuum light speed c.

Quick Tip to Solve: Use absolute permittivity and permeability directly; relative values require the factor c.

Step 3: Conclusion

$$v = \frac{1}{\sqrt{\mu\varepsilon}}.$$

Quick Tip to Solve: Use absolute permittivity and permeability directly; relative values require the factor c.

3. Does the magnifying power of a microscope depend on the colour of the light used? Justify your answer. [1 mark]

Solution:

Concept:

- Microscope magnification depends on the focal lengths of its objective and eyepiece; for normal adjustment, $M \simeq (L/f_o)(D/f_e)$, where L is tube length and D the least distance of distinct vision.
- For a lens in air, $1/f = (n-1)(1/R_1 - 1/R_2)$; the refractive index n varies with wavelength.

Step 1: Connect colour with focal length Colour specifies the wavelength of the illuminating light. Because glass is dispersive, changing the wavelength changes its refractive index. With lens curvatures fixed, the lens-maker relation therefore changes the focal lengths of the objective and eyepiece.

Step 2: Connect focal length with magnification The magnifying-power expression contains both focal lengths in the denominator. Thus changing colour changes the magnifying power for an ordinary microscope made with refracting lenses. Under normal dispersion, violet light has a larger refractive index than red light, giving shorter focal lengths and correspondingly greater magnification for fixed geometry.

Step 3: Conclusion Yes. The refractive index, and hence the focal lengths of the lenses and the magnifying power, depend on the colour of light.

Quick Tip to Solve: Link colour to refractive index, then focal length, then magnification; do not confuse this with resolving power.

4. Draw logic symbol of an OR gate and write its truth table. [1 mark]

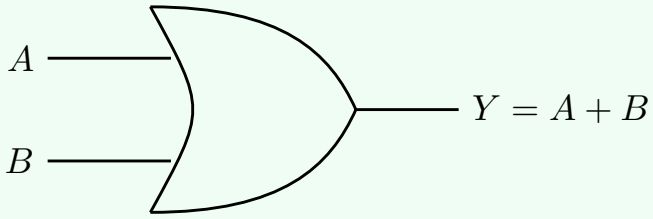
Solution:

Concept:

- For a two-input OR gate, the Boolean relation is $Y = A + B$, where $+$ means logical OR rather than ordinary addition.
- Logic 1 denotes HIGH and logic 0 denotes LOW. The output is HIGH whenever at least one input is HIGH.

Step 1: Draw the standard logic symbol The labelled symbol shows two input lines entering the curved side of the OR gate and one output line leaving its pointed side. There is no output inversion bubble, so this is an OR gate. The labels A, B, and Y agree with the Boolean expression.

Quick Tip to Solve: For OR, fill only the 00 row with 0; every other input row has output 1.



Two-input OR gate

Step 2: Enumerate all input combinations Two binary inputs have four combinations. With both inputs LOW, the output is LOW. With either input HIGH, or with both HIGH, the output is HIGH. The complete truth table is

A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

In the last row, logical $1 + 1$ equals 1, not the arithmetic value 2.

Step 3: Conclusion The OR symbol is shown above and its truth table gives $Y = 0$ only for $A = B = 0$; otherwise $Y = 1$.

Quick Tip to Solve: For OR, fill only the 00 row with 0; every other input row has output 1.

5. A photosensitive surface emits photoelectrons when red light falls on it. Will the surface emit photoelectrons when blue light is incident on it? Give reason. [1 mark]

Solution:

Concept:

- A photon has energy $E = h\nu = hc/\lambda$, where h is Planck's constant and ν its frequency.
- Photoelectric emission requires photon energy at least equal to the surface work function ϕ: $h\nu \geq \phi$.

Step 1: Use the observation with red light The stated emission under red illumination means the energy of red photons is sufficient to overcome the work function of this surface. Hence $h\nu_{\text{red}} \geq \phi$. This fixes the relevant threshold condition without needing a numerical value of ϕ.

Step 2: Compare the blue photon energy Blue light has a shorter wavelength and therefore a higher frequency than red light: $\nu_{\text{blue}} > \nu_{\text{red}}$. Consequently $h\nu_{\text{blue}} > h\nu_{\text{red}} \geq \phi$. Blue photons can also release electrons. Einstein's equation $K_{\max} = h\nu - \phi$ further shows that the maximum kinetic energy is greater under blue illumination.

Step 3: Conclusion Yes, the surface emits photoelectrons under blue light because blue photons have more energy than red photons, which already exceed the emission threshold.

Quick Tip to Solve: If a lower-frequency colour causes emission, any higher-frequency colour also crosses the same threshold.

Section B (10 marks)

6. Find the intensity at a point on a screen in Young's double slit experiment where the interfering waves of equal intensity have a path difference of (i) $\lambda/4$, and (ii) $\lambda/3$. [2 marks]

Solution:

Concept:

- A path difference Δ produces phase difference $\phi = 2\pi\Delta/\lambda$, where λ is the wavelength.
- For coherent waves, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$. If each wave has intensity I_0, then $I = 2I_0(1+\cos\phi)$ and $I_{\max} = 4I_0$.

Step 1: Path difference $\lambda/4$ Take the individual intensities as $I_1 = I_2 = I_0$. The phase difference is $\phi = (2\pi/\lambda)(\lambda/4) = \pi/2$. Substitution gives

$$I = 2I_0[1 + \cos(\pi/2)] = 2I_0.$$

Since the cosine is zero, the interference term vanishes and the resultant intensity equals the sum of the two individual intensities.

Step 2: Path difference $\lambda/3$ Here $\phi = (2\pi/\lambda)(\lambda/3) = 2\pi/3$. Using $\cos(2\pi/3) = -1/2$,

$$I = 2I_0\left(1 - \frac{1}{2}\right) = I_0.$$

The interference term is negative, so the intensity is less than the sum of the individual intensities. It is not zero: complete destructive interference requires an odd multiple of $\lambda/2$.

Step 3: Conclusion The required intensities are (i) $\boxed{2I_0 = I_{\max}/2}$ and (ii) $\boxed{I_0 = I_{\max}/4}$, where I_0 is the intensity of each wave.

Quick Tip to Solve: Convert path difference to phase first; distinguish each wave intensity I_0 from the maximum intensity $4I_0$.

7. Write two points of difference between intrinsic and extrinsic semiconductors. [2 marks]

Solution:

Concept:

- An intrinsic semiconductor is pure; an extrinsic semiconductor is formed by controlled doping of a pure semiconductor.
- Electrons and holes are mobile charge carriers. Let n_e and n_h denote their respective number densities.

Step 1: Difference in composition An intrinsic semiconductor, such as pure silicon or germanium, has no intentionally added impurity. An extrinsic semiconductor contains a small, controlled amount of suitable impurity. A pentavalent donor such as phosphorus produces n-type material, while a trivalent acceptor such as boron produces p-type material.

Quick Tip to Solve: Remember: intrinsic means pure and equal carrier densities; extrinsic means doped with either electrons or holes as majority carriers.

Step 2: Difference in carrier populations In an intrinsic semiconductor, thermally generated electrons and holes occur in pairs, so their number densities are equal: $n_e = n_h$. In an extrinsic semiconductor, the densities are unequal. Electrons are majority carriers in n-type material, $n_e > n_h$; holes are majority carriers in p-type material, $n_h > n_e$. Both types still possess minority carriers, and the crystal remains electrically neutral.

Step 3: Conclusion The two differences are purity versus intentional doping, and equal electron-hole densities versus unequal densities with a majority carrier type.

Quick Tip to Solve: Remember: intrinsic means pure and equal carrier densities; extrinsic means doped with either electrons or holes as majority carriers.

8. Distinguish between broadcast mode and point-to-point mode of communication and give one example for each. [2 marks]

Solution:

Concept:

- A communication mode specifies how a transmitter and its intended receivers are connected.
- Broadcast distributes a message to many receivers; point-to-point communication links one transmitter to one intended receiver.

Step 1: Broadcast mode and example In broadcast mode, a single transmitting station sends the same information to a large number of receivers simultaneously. The signal covers a service region, and receivers within that region may tune to it. It is therefore a one-to-many mode of communication. Radio broadcasting is an example: one radio station transmits a programme that many listeners receive on their radio sets.

Step 2: Point-to-point mode and example In point-to-point mode, communication takes place between one transmitter and one intended receiver through a link connecting the two locations. The message is meant for that particular receiving endpoint rather than an audience of many receivers. A telephone conversation between two subscribers is an example. Although both subscribers can transmit and receive during the conversation, the connection is between those two endpoints.

Step 3: Conclusion Broadcast is one-to-many communication, for example radio broadcasting. Point-to-point is communication between two specific endpoints, for example a telephone conversation.

Quick Tip to Solve: Classify the mode by the intended receiver count: many listeners means broadcast; one linked endpoint means point-to-point.

9. A light bulb and a solenoid are connected in series across an ac source of voltage. Explain, how the glow of the light bulb will be affected when an iron rod is inserted in the solenoid. [2 marks]

Solution:

Concept:

- A solenoid opposes alternating current through inductive reactance $X_L = \omega L$, where L is its inductance and ω is the source angular frequency.
- For a series resistive-inductive circuit, $Z = \sqrt{R^2 + (\omega L)^2}$ and $I_{\text{rms}} = V_{\text{rms}}/Z$. The bulb power is $P = I_{\text{rms}}^2 R_b$, where R_b is its resistance.

Step 1: Effect of inserting the iron rod Iron has a much larger magnetic permeability than air. Inserting the rod increases the magnetic flux linked with the solenoid for a given current, so its self-inductance L increases. As the supply frequency is unchanged, the inductive reactance $X_L = \omega L$ also increases. The larger self-induced opposing emf makes the solenoid a greater opposition to alternating current.

Step 2: Effect on current and heating The bulb and solenoid carry the same current because they are connected in series. With the source voltage unchanged, increasing L increases the circuit impedance $Z = \sqrt{R^2 + (\omega L)^2}$. Therefore $I_{\text{rms}} = V_{\text{rms}}/Z$ decreases. The smaller current reduces Joule heating in the bulb filament, as shown by $P = I_{\text{rms}}^2 R_b$ for a given filament resistance. Consequently the filament reaches a lower temperature and emits less light.

Step 3: Conclusion The light bulb becomes dimmer: the iron rod increases inductance and inductive reactance, reducing the series current and the bulb heating.

Quick Tip to Solve: Follow the causal chain: iron core increases L, hence X_L and impedance increase, so current and bulb brightness decrease.

10. Use the mirror equation to show that an object placed between f and $2f$ of a concave mirror forms an image beyond $2f$. [2 marks]

Solution:

Concept:

- The mirror equation is $1/f = 1/v + 1/u$, where f, u and v are signed distances from the pole.
- With the Cartesian sign convention, a concave mirror and an object in front of it have negative focal and object distances.

Step 1: Express the object position with signed distances Let the magnitude of the focal length be $F > 0$, and the object distance magnitude be x. The stated position means $F < x < 2F$. Therefore $f = -F$ and $u = -x$; the point $2f$ is the centre of curvature at distance $2F$ in front of the mirror.

Step 2: Use the mirror equation and compare the image distance Substitution gives

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{F} + \frac{1}{x} = \frac{F - x}{Fx}, \qquad v = -\frac{Fx}{x - F}.$$

Since $x > F$, $v < 0$, so the image is real and lies in front of the mirror. Also $x < 2F$ implies

Quick Tip to Solve: Use positive magnitudes only after inserting the negative signs for both f and u; divide inequalities by $x - F > 0$.

$x > 2(x - F)$. Dividing this inequality by the positive quantity $x - F$ and multiplying by F gives

$$|v| = \frac{Fx}{x - F} > 2F.$$

Thus the image distance is strictly greater than the radius of curvature.

Step 3: Conclusion The image forms beyond $2f$ in front of the concave mirror: $v < -2F$, or equivalently $|v| > 2|f|$.

Quick Tip to Solve: Use positive magnitudes only after inserting the negative signs for both f and u; divide inequalities by $x - F > 0$.

OR

10. (a) State the condition under which a large magnification can be achieved in an astronomical telescope.

(b) Give two reasons to explain why a reflecting telescope is preferred over a refracting telescope.

[2 marks]

Solution:

Concept:

- For an astronomical telescope in normal adjustment, the angular magnification magnitude is $|M| = f_o/f_e$, where f_o and f_e are the positive focal lengths of the objective and eyepiece.
- A reflecting telescope uses a concave mirror as its objective; reflection does not disperse light into different colours.

Step 1: (a) State the condition for large magnification Choose an objective of large focal length f_o and an eyepiece of small focal length f_e, so that $f_o/f_e \gg 1$. In normal adjustment, the distant object forms its image in the objective focal plane, which coincides with the eyepiece focal plane; the final image is at infinity. The angular magnification is $M = -f_o/f_e$; the minus sign describes inversion, whereas its magnitude gives the enlargement.

Step 2: (b) Explain the advantages of a reflecting telescope First, the mirror objective is free from chromatic aberration: all colours obey the same law of reflection, so their focal positions coincide. A refracting lens has a wavelength-dependent refractive index and can produce coloured fringes. Second, a large-aperture mirror can be supported from behind and made more readily than a large lens, which must be supported at its rim. The larger aperture collects more light from faint celestial objects and improves resolving power. These are two distinct reasons for preferring a reflecting telescope.

Step 3: Conclusion Large magnification requires $f_o \gg f_e$. A reflecting telescope is preferred because it avoids chromatic aberration and permits a large supported aperture with better light collection and resolution.

Quick Tip to Solve: Use the focal-length ratio for magnification; use objective aperture when explaining brightness and resolution.

Section C (36 marks)

11. (a) Define the term 'modulation index,' used in communication system. Why is its value kept less than or equal to one?

(b) A message signal of frequency 10 kHz and peak voltage of 10 V is used to modulate a carrier frequency 1 MHz and peak voltage 10 V. Determine the (i) modulation index, and (ii) side bands produced. [3 marks]

Solution:

Concept:

- In amplitude modulation, the modulation index is $\mu = A_m/A_c$, the ratio of the peak message voltage A_m to the peak unmodulated carrier voltage A_c.
- For a single sinusoidal message of frequency f_m, the sideband frequencies are $f_c - f_m$ and $f_c + f_m$, where f_c is the carrier frequency.

Step 1: (a) Define the index and explain its permitted value The modulation index measures the depth of amplitude modulation: the fractional change in the carrier amplitude caused by the message. For sinusoidal modulation, the instantaneous carrier envelope is $A_c[1 + \mu\cos(2\pi f_m t)]$. Its minimum is $A_c(1-\mu)$. If $\mu > 1$, this expression becomes negative during part of the cycle, causing overmodulation and distortion in ordinary envelope detection. Hence $0 \le \mu \le 1$ is maintained for faithful reproduction; at $\mu = 1$, the envelope just reaches zero.

Step 2: (b)(i) Calculate the modulation index The given peak message and carrier voltages are $A_m = 10\,\text{V}$ and $A_c = 10\,\text{V}$. Thus

$$\mu = \frac{A_m}{A_c} = \frac{10}{10} = 1.$$

The signal has 100 percent modulation and is at the maximum modulation depth without overmodulation.

Step 3: (b)(ii) Calculate the two sideband frequencies Convert the carrier frequency to the same unit as the message frequency: $f_c = 1\,\text{MHz} = 1000\,\text{kHz}$, while $f_m = 10\,\text{kHz}$. Therefore

$$f_{\text{LSB}} = 1000 - 10 = 990\,\text{kHz} = 0.990\,\text{MHz},$$
$$f_{\text{USB}} = 1000 + 10 = 1010\,\text{kHz} = 1.010\,\text{MHz}.$$

The sidebands lie symmetrically on either side of the carrier, each separated from it by the message frequency.

Step 4: Conclusion The modulation index is $\mu = 1$; the lower and upper sidebands are $990\,\text{kHz}$ and $1010\,\text{kHz}$, respectively.

Quick Tip to Solve: Convert MHz and kHz to one common unit before adding or subtracting the message frequency.

12. Using Bohr's postulates, derive the expression for the orbital period of the electron moving in the n^{th} orbit of hydrogen atom. [3 marks]

Solution:

Concept:

- For hydrogen, the Coulomb force provides the centripetal force: $m_e v_n^2/r_n = e^2/(4\pi\varepsilon_0 r_n^2)$, with electron mass m_e and charge magnitude e.
- Bohr quantisation requires $m_e v_n r_n = n\hbar = nh/(2\pi)$, where $n = 1, 2, \ldots$, h is Planckś constant and $\hbar = h/(2\pi)$.
- The orbital period of uniform circular motion is $T_n = 2\pi r_n/v_n$.

Step 1: Find the radius allowed by Bohr quantisation Using the stationary circular orbit model and treating the proton as fixed, the attractive electrostatic force balances the centripetal force. From angular momentum quantisation, $v_n = n\hbar/(m_e r_n)$. Insert this into the force equation:

$$\frac{n^2\hbar^2}{m_e r_n^3} = \frac{e^2}{4\pi\varepsilon_0 r_n^2}.$$

Solving gives

$$r_n = \frac{4\pi\varepsilon_0 n^2\hbar^2}{m_e e^2} = \frac{\varepsilon_0 n^2 h^2}{\pi m_e e^2}.$$

Thus the allowed radius grows as n^2.

Step 2: Find the speed in the nth orbit Substituting this radius in the quantisation condition gives

$$v_n = \frac{n\hbar}{m_e r_n} = \frac{e^2}{4\pi\varepsilon_0 n\hbar} = \frac{e^2}{2\varepsilon_0 nh}.$$

Hence the electron speed decreases inversely with n. The charge e here is a positive magnitude; the electrostatic attraction is already accounted for in the radial force equation.

Step 3: Divide the circumference by the speed The time for one complete revolution is the orbit circumference divided by its speed. Therefore

$$\begin{aligned} T_n &= \frac{2\pi r_n}{v_n} \\ &= 2\pi\left(\frac{\varepsilon_0 n^2 h^2}{\pi m_e e^2}\right)\left(\frac{2\varepsilon_0 nh}{e^2}\right) \\ &= \frac{4\varepsilon_0^2 h^3}{m_e e^4}\, n^3. \end{aligned}$$

The dependence n^3 follows because the path length increases as n^2 while the speed falls as $1/n$.

Step 4: Conclusion The orbital period in the n^{th} hydrogen orbit is $T_n = \dfrac{4\varepsilon_0^2 h^3 n^3}{m_e e^4}$, so $T_n = n^3 T_1$.

Quick Tip to Solve: Derive $r_n \propto n^2$ and $v_n \propto n^{-1}$ first; then $T_n \propto n^3$ is an immediate check.

13. A charge Q is distributed uniformly over a metallic sphere of radius R. Obtain the expressions for the electric field (E) and electric potential (V) at a point $0 < x < R$.

Show on a plot the variation of E and V with x for $0 < x < 2R$. [3 marks]

Solution:

Concept:

- In electrostatic equilibrium, excess charge on a metallic sphere lies on its outer surface; the electric field inside the conductor is zero.
- Gauss's law is $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\varepsilon_0$. The potential is defined with $V(\infty) = 0$, and $E_r = -dV/dx$.

Step 1: Field inside and outside the sphere Let x be the radial distance from the centre and put $k = 1/(4\pi\varepsilon_0)$. For $0 < x < R$, a concentric Gaussian sphere encloses no excess charge, so spherical symmetry and Gauss's law give $E(4\pi x^2) = 0$, hence $E = 0$. For $x > R$, the enclosed charge is Q, giving

$$E_r(x) = \frac{kQ}{x^2}.$$

For positive Q, this field is radially outward. Immediately inside the surface it is zero, whereas immediately outside it is kQ/R^2.

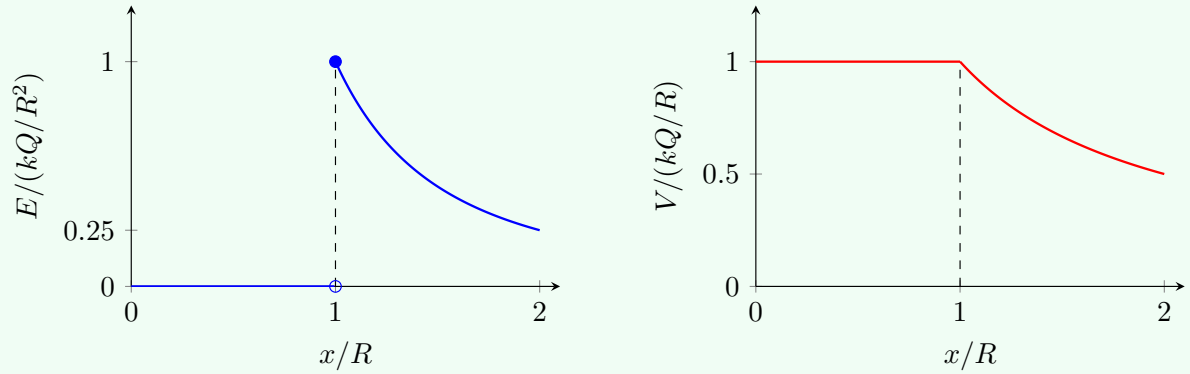
Step 2: Potential and the requested variations Outside the sphere, integration from infinity gives

$$V(x) = -\int_{\infty}^{x} \frac{kQ}{r^2}\, dr = \frac{kQ}{x}.$$

Inside, the zero electric field implies no potential difference between any two points. Therefore

$$V(x) = V(R) = \frac{kQ}{R}, \qquad 0 < x < R.$$

The potential is continuous at the surface, while its slope changes from zero to $-kQ/R^2$. From R to $2R$, the field falls as $1/x^2$, reaching $kQ/(4R^2)$; the potential falls as $1/x$, reaching $kQ/(2R)$. The normalized plots below display these distinct variations for positive Q; a negative charge reverses the signs.

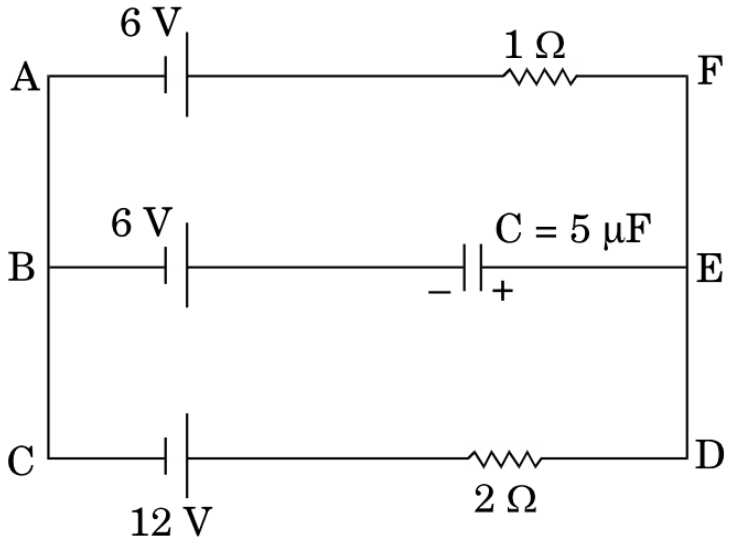


Normalized field and potential versus radial distance, for $Q > 0$; $k = 1/(4\pi\varepsilon_0)$.

Step 3: Conclusion For $0 < x < R$, $\boxed{E = 0, \quad V = Q/(4\pi\varepsilon_0 R)}$. Outside, $E_r = kQ/x^2$ and $V = kQ/x$, with a field jump and continuous potential at $x = R$.

Quick Tip to Solve: First identify a metallic sphere: its interior field is zero, but its interior potential equals the nonzero surface potential.

14. In the given circuit, with steady current, calculate the potential difference across the capacitor and the charge stored in it. [3 marks]



Solution:

Concept:

- A capacitor in a steady DC circuit carries no current. Its branch is open for finding the outer-loop current, but its potential difference need not be zero.
- Kirchhoff's loop rule gives the current from the algebraic sum of emfs divided by resistance. The capacitor charge magnitude is $q = C|V_C|$.

Step 1: Find the steady current in the outer loop The left vertical wire makes A, B, C one node, and the right vertical wire makes F, E, D another. Every cell has its positive terminal on the right, as shown by its longer plate. The capacitor blocks the middle-branch current after charging, so the $1\,\Omega$ and $2\,\Omega$ resistors lie in the remaining closed loop. The $12\,\text{V}$ and top $6\,\text{V}$ cells oppose each other around this loop. Taking the direction from C to D along the bottom branch,

$$12 - 2I - I - 6 = 0, \qquad I = \frac{12-6}{2+1} = 2\,\text{A}.$$

Thus current flows rightwards in the bottom branch and leftwards in the top branch.

Step 2: Find the node voltage and capacitor voltage Choose the left node potential as zero. Going from C to D, the cell raises the potential by $12\,\text{V}$, and the resistor lowers it by $I(2\,\Omega) = 4\,\text{V}$. Hence $V_E = V_D = 8\,\text{V}$. This also follows from the top branch: traversing against its current gives $6 + 2 = 8\,\text{V}$. In the middle branch, the $6\,\text{V}$ cell raises the potential of the capacitor's left plate to $6\,\text{V}$. Its right plate is connected directly to E, at $8\,\text{V}$, so

$$V_{\text{right}} - V_{\text{left}} = 8 - 6 = 2\,\text{V}.$$

Step 3: Calculate the stored charge With $C = 5\,\mu\text{F}$,

$$q = CV_C = (5 \times 10^{-6})(2) = 1.0 \times 10^{-5}\,\text{C} = 10\,\mu\text{C}.$$

Quick Tip to Solve: Open the capacitor branch only to calculate steady current; then compare the two actual plate potentials, including the middle cell.

The right plate has charge $+10\,\mu$C, and the left plate has charge $-10\,\mu$C, consistent with the polarity shown. The charge stored means the magnitude on either plate; the net charge of the two plates together is zero.

Step 4: Conclusion The capacitor potential difference is $\boxed{2\,\text{V}}$, with its right plate positive. The stored charge is $\boxed{10\,\mu\text{C}}$.

Quick Tip to Solve: Open the capacitor branch only to calculate steady current; then compare the two actual plate potentials, including the middle cell.

15. A long charged cylinder of linear charge density $+\lambda_1$ is surrounded by a hollow coaxial conducting cylinder of linear charge density $-\lambda_2$. Use Gauss's law to obtain expressions for the electric field at a point (i) in the space between the cylinders, and (ii) outside the larger cylinder. [3 marks]

Solution:

Concept:

- For long coaxial cylinders, cylindrical symmetry makes the electric field radial, with a constant magnitude at a fixed radial distance r, neglecting end effects.
- For a coaxial Gaussian cylinder of length L, only the curved surface contributes flux: $\Phi = E_r(2\pi rL)$. Gauss's law equates this to $Q_{\text{enc}}/\varepsilon_0$.

Step 1: Field in the space between the cylinders Let the inner cylinder radius be a, and the inner and outer radii of the hollow conducting cylinder be b and c, respectively. For $a < r < b$, choose a coaxial Gaussian cylinder of radius r and length L. Its end-cap normals lie along the axis, perpendicular to the radial electric field, so their flux is zero. The enclosed charge is only $+\lambda_1 L$. Therefore

$$E_r(2\pi rL) = \frac{\lambda_1 L}{\varepsilon_0}, \qquad \vec{E} = \frac{\lambda_1}{2\pi\varepsilon_0 r}\,\hat{r}.$$

This field is outward from the common axis and falls as $1/r$. Charges on the surrounding cylinder are outside this Gaussian surface.

Step 2: Field outside the larger cylinder For $r > c$, a coaxial Gaussian cylinder encloses both cylinders. The hollow conductor's specified total charge per unit length is $-\lambda_2$; thus

$$Q_{\text{enc}} = (\lambda_1 - \lambda_2)L.$$

Applying the same flux calculation gives

$$\vec{E} = \frac{\lambda_1 - \lambda_2}{2\pi\varepsilon_0 r}\,\hat{r}.$$

Its direction is outward if $\lambda_1 > \lambda_2$, inward if $\lambda_1 < \lambda_2$, and it vanishes if the two positive magnitudes are equal. The signed expression handles all three cases.

Step 3: Check consistency with the conducting shell Within the conducting material, $b < r < c$, electrostatic equilibrium requires $E = 0$. A Gaussian cylinder there therefore encloses zero net charge:

Quick Tip to Solve: Count only the charge enclosed by your Gaussian cylinder; outside the shell use its total charge, not just its inner-surface induced charge.

the shell's inner surface carries $-\lambda_1$ per unit length. Since the entire shell carries $-\lambda_2$, its outer surface carries $\lambda_1 - \lambda_2$. This redistribution agrees with the external result and does not change the prescribed total charge of the hollow conductor.

Step 4: Conclusion Between the cylinders, $\boxed{\vec{E} = \lambda_1 \hat{r}/(2\pi\varepsilon_0 r)}$. Outside the larger cylinder, $\boxed{\vec{E} = (\lambda_1 - \lambda_2)\hat{r}/(2\pi\varepsilon_0 r)}$, where r is distance from the common axis.

Quick Tip to Solve: Count only the charge enclosed by your Gaussian cylinder; outside the shell use its total charge, not just its inner-surface induced charge.

16. Using Biot-Savart law, deduce the expression for the magnetic field at a point (x) on the axis of a circular current carrying loop of radius R. How is the direction of the magnetic field determined at this point? [3 marks]

Solution:

Concept:

- Biot-Savart law: $d\mathbf{B} = \dfrac{\mu_0 I}{4\pi}\dfrac{d\boldsymbol{\ell} \times \hat{\mathbf{s}}}{s^2}$, where $\mathbf{s}$ joins the current element to the observation point.
- Rotational symmetry cancels the components perpendicular to the loop axis; the axial components add. The right-hand thumb rule gives the direction.

Step 1: Find the field of an element Let P be on the axis, distance x from the centre of the circular loop of radius R, carrying current I. Every element lies at distance $s = \sqrt{R^2 + x^2}$ from P. The tangent $d\boldsymbol{\ell}$ is perpendicular to $\mathbf{s}$, so the sine factor in Biot-Savart law is unity:

$$dB = \frac{\mu_0 I\, d\ell}{4\pi(R^2 + x^2)}.$$

Step 2: Add the axial components Let α be the angle between $d\mathbf{B}$ and the loop axis. Geometry gives $\cos\alpha = R/s$. For each element, the diametrically opposite element produces an equal transverse component in the opposite direction. Their axial components point in the same direction. Thus

$$\begin{aligned} B &= \oint dB\cos\alpha \\ &= \frac{\mu_0 I R}{4\pi(R^2 + x^2)^{3/2}} \oint d\ell \\ &= \frac{\mu_0 I R(2\pi R)}{4\pi(R^2 + x^2)^{3/2}}. \end{aligned}$$

The integral of the element length is the circumference $2\pi R$. At $x = 0$, this gives $\mu_0 I/(2R)$, the standard field at the centre of a single circular loop.

Step 3: Determine the field direction Curl the fingers of the right hand in the current direction around the loop. The extended thumb gives the axial magnetic-field direction at P. Viewed from the observation side, anticlockwise current produces a field towards the observer, whereas clockwise current produces a field away from the observer. Reversing the current reverses the field.

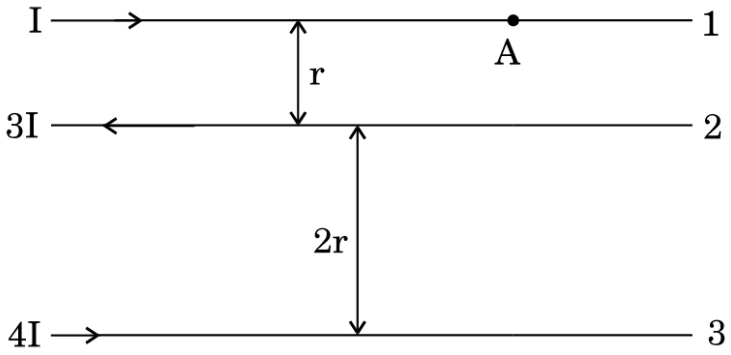
Quick Tip to Solve: Cancel the transverse components by symmetry before integration, and check the final expression by setting $x = 0$.

Step 4: Conclusion $\boxed{B(x) = \dfrac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}}$, directed along the axis according to the right-hand thumb rule.

Quick Tip to Solve: Cancel the transverse components by symmetry before integration, and check the final expression by setting $x = 0$.

OR

16. The figure shows three infinitely long straight parallel current carrying conductors. Find the
(i) magnitude and direction of the net magnetic field at point A lying on conductor 1,
(ii) magnetic force on conductor 2. [3 marks]



Solution:

Concept:

- An infinitely long straight wire carrying current J produces $B = \mu_0 J/(2\pi d)$ at perpendicular distance d; its direction follows the right-hand grip rule.
- Parallel wires exert force per unit length $F/L = \mu_0 J_1 J_2/(2\pi d)$. Currents in the same direction attract and opposite currents repel. Fields and forces must be added as vectors.

Step 1: (i) Find the fields at A The printed figure has currents I to the right in conductor 1, $3I$ to the left in conductor 2 and $4I$ to the right in conductor 3. The separations are r between 1 and 2, and $2r$ between 2 and 3. Thus A is r above conductor 2 and $3r$ above conductor 3. The field at A means the external field produced by the other conductors; the ideal line conductor's own field at its axis is not included. By the right-hand rule, conductor 2 gives a field into the plane of the paper, and conductor 3 gives a field out of the plane:

$$B_2 = \frac{3\mu_0 I}{2\pi r}, \qquad B_3 = \frac{4\mu_0 I}{6\pi r}.$$

Step 2: (i) Subtract the opposite fields Since $B_2 > B_3$, the resultant follows conductor 2's field:

$$B_A = B_2 - B_3 = \frac{\mu_0 I}{2\pi r}\left(3 - \frac{4}{3}\right) = \frac{5\mu_0 I}{6\pi r}.$$

The direction is into the plane of the paper, perpendicular to the plane containing the three conductors.

Quick Tip to Solve: At A, use the total distance $3r$ to wire 3. For the force, compare the two repulsions before choosing its direction.

Step 3: (ii) Find the force on conductor 2 Conductor 2 carries current opposite to both other conductors, so both interactions are repulsive. Conductor 1 pushes it downwards, towards conductor 3; conductor 3 pushes it upwards, towards conductor 1. Their force magnitudes per unit length are

$$\frac{F_{21}}{L} = \frac{\mu_0(3I)I}{2\pi r} = \frac{3\mu_0 I^2}{2\pi r}, \qquad \frac{F_{23}}{L} = \frac{\mu_0(3I)(4I)}{2\pi(2r)} = \frac{6\mu_0 I^2}{2\pi r}.$$

Hence the net force per unit length is the upward contribution minus the downward contribution. Because the conductors are infinitely long and no length is specified, force per unit length is the physically meaningful quantity; for a finite segment of length L, multiply by L.

Step 4: Conclusion (i) $\boxed{B_A = 5\mu_0 I/(6\pi r)}$, into the paper. (ii) $\boxed{F_2/L = 3\mu_0 I^2/(2\pi r)}$, upwards towards conductor 1; for length L, $F_2 = 3\mu_0 I^2 L/(2\pi r)$.

Quick Tip to Solve: At A, use the total distance $3r$ to wire 3. For the force, compare the two repulsions before choosing its direction.

17. (a) State the law of radioactive decay. Write the SI unit of 'activity'.

(b) There are $4\sqrt{2} \times 10^6$ radioactive nuclei in a given radioactive sample. If the half life of the sample is 20 s, how many nuclei will decay in 10 s? [3 marks]

Solution:

Concept:

- Radioactive decay obeys $-dN/dt = \lambda N$, where N is the number of undecayed nuclei and λ is the decay constant. Hence $N = N_0 e^{-\lambda t}$.
- Activity is the number of decays per unit time, $A = \lambda N$. The half-life $T_{1/2}$ satisfies $\lambda = \ln 2/T_{1/2}$, so $N = N_0\, 2^{-t/T_{1/2}}$.

Step 1: (a) State the law and the unit At any instant, the rate of disintegration of a radioactive sample is directly proportional to the number of radioactive nuclei present at that instant:

$$\frac{dN}{dt} = -\lambda N.$$

The negative sign indicates that the population of undecayed nuclei decreases with time, while the activity $A = -dN/dt$ is positive. Integration gives the exponential decay law $N(t) = N_0 e^{-\lambda t}$. The SI unit of activity is the becquerel (Bq), with $1\,\text{Bq} = 1\,\text{decay s}^{-1}$.

Step 2: (b) Calculate the nuclei remaining after 10 s Here $N_0 = 4\sqrt{2} \times 10^6$, $T_{1/2} = 20\,\text{s}$ and $t = 10\,\text{s}$. The elapsed time is half of one half-life, not a full half-life. Therefore

$$\begin{aligned} N(10) &= N_0\, 2^{-10/20} \\ &= (4\sqrt{2} \times 10^6)\frac{1}{\sqrt{2}} \\ &= 4 \times 10^6. \end{aligned}$$

Quick Tip to Solve: Compute the exponential surviving fraction first, then subtract from the initial number whenever the question asks how many nuclei decay.

The surviving fraction is $1/\sqrt{2}$, rather than $1/2$: the decay law is exponential, so the number cannot be found by linearly interpolating between the initial population and its half-life value.

Step 3: Subtract to obtain the nuclei that decay The question asks for the population lost, so subtract the surviving population from the initial population:

$$\begin{aligned} N_{\text{decayed}} &= N_0 - N(10) \\ &= (4\sqrt{2} - 4) \times 10^6 \\ &= 4(\sqrt{2} - 1) \times 10^6 \approx 1.657 \times 10^6. \end{aligned}$$

This is less than half the initial population, as expected because only half of a half-life has elapsed.

Step 4: Conclusion (a) $-dN/dt = \lambda N$; the SI activity unit is Bq. (b) $\boxed{4(\sqrt{2} - 1) \times 10^6 \approx 1.66 \times 10^6}$ nuclei decay in 10 s.

Quick Tip to Solve: Compute the exponential surviving fraction first, then subtract from the initial number whenever the question asks how many nuclei decay.

18. (a) How are electromagnetic waves produced? Explain.

(b) A plane electromagnetic wave is travelling through a medium along the +ve z-direction. Depict the electromagnetic wave showing the directions of the oscillating electric and magnetic fields. [3 marks]

Solution:

Concept:

- An accelerating charge produces electromagnetic radiation. Oscillation of a charge continuously changes its velocity and generates time-dependent electric and magnetic fields.
- A plane electromagnetic wave is transverse: $\vec{E} \perp \vec{B}$, and both fields are perpendicular to propagation. Energy flows along $\vec{E} \times \vec{B}$.

Step 1: (a) Explain the production of electromagnetic waves A charge oscillating to and fro, for example in an alternating-current antenna, is accelerated. Its changing electric field produces a changing magnetic field through Maxwell's displacement-current term. The changing magnetic field produces a changing electric field by Faraday's law. These coupled field disturbances propagate away from the accelerating charge as an electromagnetic wave. Once generated, the wave propagates independently of the source and requires no material medium. A stationary charge produces a static electric field, whereas acceleration is the essential condition for electromagnetic radiation.

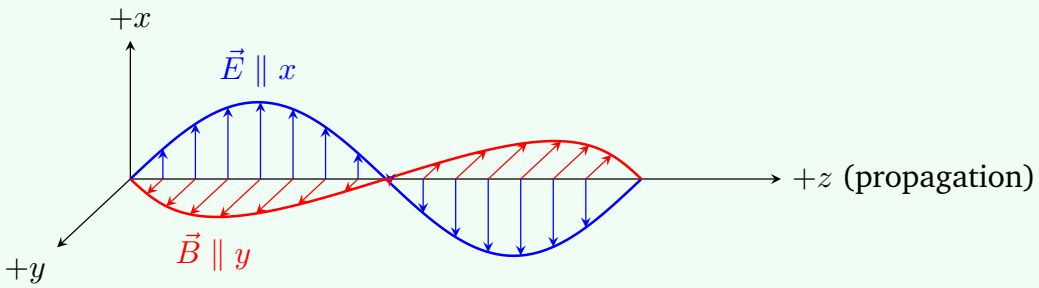
Step 2: (b) Represent the transverse fields along positive z Choose the electric field along the x-axis and the magnetic field along the y-axis. A suitable wave in the medium is

$$\begin{aligned} \vec{E}(z,t) &= E_0 \sin(kz - \omega t)\hat{x}, \\ \vec{B}(z,t) &= B_0 \sin(kz - \omega t)\hat{y}. \end{aligned}$$

Here k is the wave number, ω is the angular frequency and E_0, B_0 are the field amplitudes. The fields oscillate in phase, reaching zero and their extrema together. Since $\hat{x} \times \hat{y} = \hat{z}$, the propagation is along

Quick Tip to Solve: Apply the right-hand rule to electric field cross magnetic field to verify the direction of wave propagation.

$+z$. Both directions reverse together on the negative half-cycle, preserving the direction of energy flow. The sketch uses separate arbitrary scales for the two field amplitudes.



In-phase transverse fields with propagation along $+z$.

Step 3: Conclusion Accelerated charges produce electromagnetic waves. For propagation along $+z$, the fields can oscillate along x and y, respectively, with $\vec{E} \times \vec{B}$ along $+z$.

Quick Tip to Solve: Apply the right-hand rule to electric field cross magnetic field to verify the direction of wave propagation.

19. A source of ac voltage $v = v_0 \sin \omega t$ is connected across a pure inductor of inductance L. Derive the expressions for the instantaneous current in the circuit. Show that average power dissipated in the circuit is zero. [3 marks]

Solution:

Concept:

- For an ideal inductor of inductance L, the applied voltage balances the self-induced emf: $v = L\, di/dt$.
- Inductive reactance is $X_L = \omega L$. The current amplitude is $i_0 = v_0/X_L$, and current lags voltage by $\pi/2$.
- Instantaneous power is $p = vi$. Average power is the time average over one complete period $T = 2\pi/\omega$.

Step 1: Derive the instantaneous current The induced emf is $-L\, di/dt$; applying the loop equation to the pure inductor gives

$$L\frac{di}{dt} = v_0 \sin \omega t.$$

On integrating,

$$i = -\frac{v_0}{\omega L} \cos \omega t + C.$$

For the usual sinusoidal ac current with no dc component, $C = 0$. Therefore, defining $i_0 = v_0/(\omega L)$,

$$i(t) = -i_0 \cos \omega t = i_0 \sin(\omega t - \pi/2).$$

The voltage has phase ωt, whereas the current has phase $\omega t - \pi/2$. Thus the current lags the applied voltage by a quarter cycle. The constant C would describe a separately specified initial dc current in

Quick Tip to Solve: Start from voltage equals L times the current derivative; integrating sine gives negative cosine and the required quarter-cycle lag.

an ideal lossless inductor, rather than the sinusoidal ac component sought here.

Step 2: Calculate instantaneous and average power Multiplying the applied voltage by the sinusoidal current,

$$p(t) = (v_0 \sin \omega t)(-i_0 \cos \omega t)$$
$$= -\frac{v_0 i_0}{2} \sin 2\omega t.$$

Hence the average over a complete period is

$$P_{\text{av}} = \frac{1}{T}\int_0^T p(t)\,dt = -\frac{v_0 i_0}{2T}\int_0^T \sin 2\omega t\,dt = 0.$$

Positive and negative power intervals cancel: the inductor stores energy in its magnetic field, $U = Li^2/2$, and subsequently returns it to the source. With zero resistance there is no irreversible heating. Equivalently, $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos(\pi/2) = 0$.

Step 3: Conclusion The ac current is $i(t) = \dfrac{v_0}{\omega L}\sin(\omega t - \pi/2)$, lagging voltage by $90°$. The average power dissipated in a pure inductor is zero.

Quick Tip to Solve: Start from voltage equals L times the current derivative; integrating sine gives negative cosine and the required quarter-cycle lag.

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20. (a) Draw a plot showing the variation of photoelectric current with collector potential for different frequencies but same intensity of incident radiation.

(b) Use Einstein's photoelectric equation to explain the observations from this graph.

(c) What change will you observe if intensity of incident radiation is changed but the frequency remains the same? [3 marks]

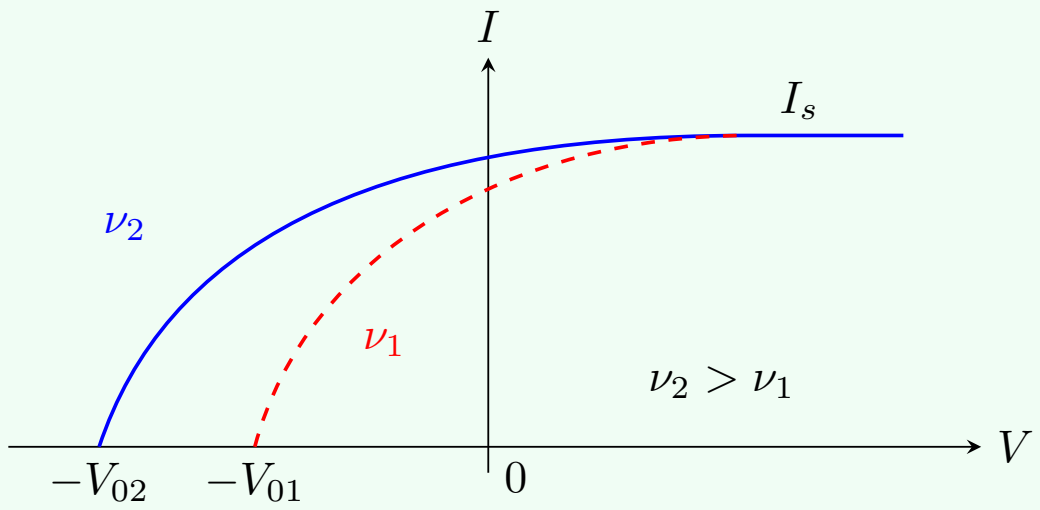
Solution:

Concept:

- Einsteiń equation is $h\nu = \phi + K_{\max}$, where ϕ is the metal work function and ν is the incident frequency.
- The stopping potential magnitude V_0 satisfies $eV_0 = K_{\max}$. In the usual CBSE comparison, saturation current depends on intensity, whereas stopping potential depends on frequency.

Step 1: (a) Compare current-potential curves Consider two frequencies $\nu_2 > \nu_1 > \nu_0$, where $\nu_0 = \phi/h$ is the threshold frequency. Plot collector potential horizontally and photoelectric current vertically. With the same incident intensity, the curves have the same saturation current in the standard school-level model. The higher-frequency curve meets the negative-potential axis farther to the left, since $V_{02} > V_{01}$. At sufficiently positive collector potential all emitted electrons are collected, giving the common plateau.

Quick Tip to Solve: Read frequency from the negative voltage intercept and intensity from the saturation-current plateau.



Equal intensity: common saturation current, different stopping potentials.

Step 2: (b) Explain the observations Each electron absorbs energy $h\nu$ from a photon. The energy ϕ liberates it; the remainder supplies its maximum kinetic energy. Thus

$$eV_0 = h\nu - \phi.$$

Increasing frequency increases $K_{\max}$ and therefore the magnitude of the retarding potential required to stop even the fastest electrons. This shifts the intercept to a more negative collector potential. Raising positive collector potential increases collection of emitted electrons until saturation; it does not supply the energy responsible for photoemission.

Step 3: (c) Change intensity at fixed frequency Increasing intensity increases the rate of photoelectron emission and hence raises the saturation current; reducing intensity lowers it. Since photon energy $h\nu$ and the work function remain fixed, $K_{\max}$ and V_0 remain unchanged. The new curves therefore share the same negative-potential intercept but have different saturation plateaus.

Step 4: Conclusion For equal intensity, higher frequency gives a larger stopping-potential magnitude with the same saturation current. At fixed frequency, changing intensity changes saturation current while leaving stopping potential unchanged.

Quick Tip to Solve: Read frequency from the negative voltage intercept and intensity from the saturation-current plateau.

21. (a) State the condition under which a charged particle moving with velocity $\vec{v}$ goes undeflected in a magnetic field $\vec{B}$.

(b) An electron, after being accelerated through a potential difference of 10^4 V, enters a uniform magnetic field of $0 \cdot 04$ T, perpendicular to its direction of motion. Calculate the radius of curvature of its trajectory. [3 marks]

Solution:

Concept:

- Magnetic force is $\vec{F} = q(\vec{v} \times \vec{B})$, with magnitude $|q|vB \sin\theta$. A magnetic field changes direction of velocity but does no work.
- Acceleration through potential difference V gives an electron kinetic energy $eV = mv^2/2$. For perpendicular entry, $evB = mv^2/r$.

Step 1: (a) Condition for no deflection For a moving charged particle in a nonzero magnetic field, no magnetic deflection requires $\vec{v} \times \vec{B} = 0$. Thus its velocity must be parallel or antiparallel to the field: $\theta = 0$ or $180°$. In either case $\sin\theta = 0$ and the Lorentz magnetic force vanishes. A perpendicular velocity instead gives the maximum force and a curved path.

Step 2: (b) Find the speed acquired by the electron Taking the electron initially at rest, its electric potential energy loss becomes kinetic energy. Using $e = 1.60 \times 10^{-19}$ C and $m_e = 9.11 \times 10^{-31}$ kg,

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2(1.60 \times 10^{-19})(10^4)}{9.11 \times 10^{-31}}} = 5.93 \times 10^7 \text{ m s}^{-1}.$$

The usual nonrelativistic treatment applies to this board-level calculation.

Step 3: Balance magnetic and centripetal forces Because the velocity is perpendicular to the field, the force magnitude is evB. It remains perpendicular to velocity and provides the centripetal force without changing speed. Hence

$$r = \frac{m_e v}{eB} = \frac{(9.11 \times 10^{-31})(5.93 \times 10^7)}{(1.60 \times 10^{-19})(0.04)} = 8.44 \times 10^{-3} \text{ m}.$$

The sign of the electron charge determines the direction of bending, while its magnitude is used for the radius.

Step 4: Conclusion The particle is undeflected when $\vec{v} \parallel \vec{B}$ or $\vec{v}$ is antiparallel to $\vec{B}$. The electron trajectory has radius $r \approx 8.4$ mm.

Quick Tip to Solve: Combine energy and circular motion directly as $r = \sqrt{2mV/e}/B$, using the charge magnitude.

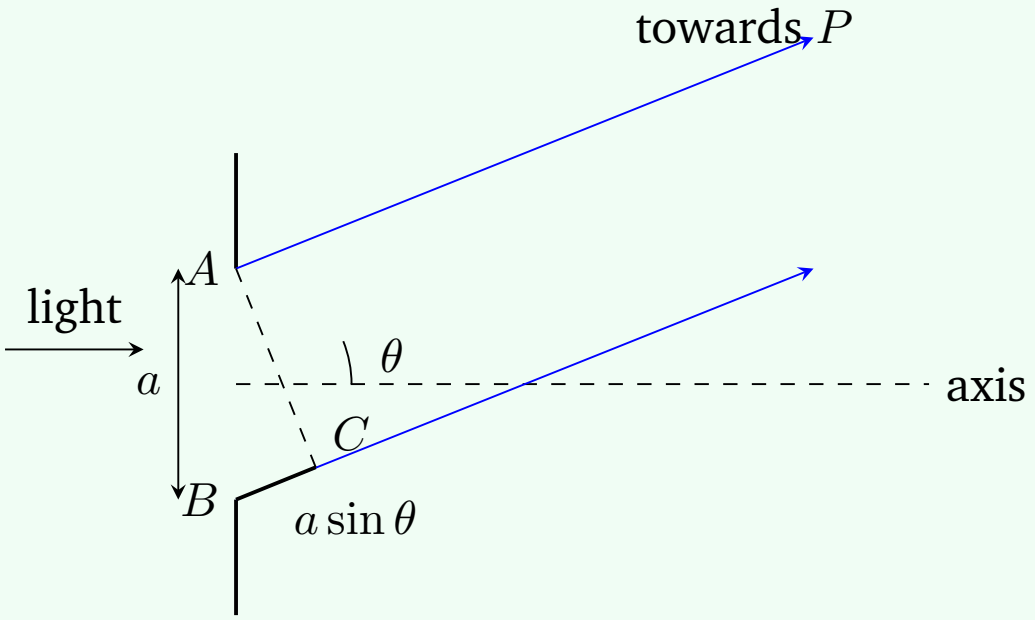
22. A monochromatic light of wavelength λ is incident normally on a narrow slit of width 'a' to produce a diffraction pattern on the screen placed at a distance D from the slit. With the help of a relevant diagram, deduce the conditions for obtaining maxima and minima on the screen. Use these conditions to show that angular width of central maximum is twice the angular width of secondary maximum. [3 marks]

Solution:

Concept:

- In Fraunhofer single-slit diffraction, the path difference between extreme rays is $a\sin\theta$, where a is slit width and θ is measured from the central axis.
- For small angles, $\sin\theta \simeq \theta \simeq y/D$. Diffraction intensity is $I = I_0(\sin\beta/\beta)^2$, where $\beta = \pi a\sin\theta/\lambda$.

Step 1: Use the diffraction geometry Let the slit endpoints be A and B, separated by a. Rays from different points of the slit reaching a distant point P are effectively parallel at angle θ to the axis. Their extreme path difference is $a\sin\theta$. The diagram shows this geometry; the incident wavefront is normal to the slit and all points initially have the same phase.



Single-slit geometry and extreme-ray path difference.

Step 2: Deduce the minima If $a\sin\theta = \lambda$, divide the slit into two equal halves. Each ray from one half pairs with a ray from the other half separated by $a/2$; their path difference is $\lambda/2$, so they cancel. More generally, divide into $2n$ equal portions to obtain pairwise cancellation when

$$a\sin\theta_n = n\lambda, \qquad n = \pm1, \pm2, \ldots.$$

Thus $y_n \simeq n\lambda D/a$. The case $n = 0$ is a maximum, since all contributions then arrive in phase.

Step 3: Deduce the maxima and compare angular widths The central maximum is at $\theta = 0$. Secondary maxima lie approximately midway between successive minima:

$$a\sin\theta \simeq (n + \tfrac{1}{2})\lambda, \qquad n = 1, 2, \ldots,$$

Quick Tip to Solve: Measure each bright fringe between its bounding minima; the central fringe spans both first minima.

on either side. This is the usual approximate maxima condition; the exact condition obtained by differentiating intensity is $\tan\beta = \beta$, excluding $\beta = 0$. The central maximum extends between the first minima $-\lambda/a$ and $+\lambda/a$, so its small-angle width is $2\lambda/a$. Each secondary maximum lies between adjacent minima, separated by λ/a. Therefore the central maximum has twice the angular width of a secondary maximum in the small-angle approximation.

Step 4: Conclusion Minima: $a\sin\theta = n\lambda$, $n \neq 0$. Secondary maxima approximately satisfy $a\sin\theta = (n + \frac{1}{2})\lambda$. The angular widths are $2\lambda/a$ and λ/a, giving the ratio $2 : 1$.

Quick Tip to Solve: Measure each bright fringe between its bounding minima; the central fringe spans both first minima.

Section D (4 marks)

23. Sunil and his parents were travelling to their village in their car. On the way his mother noticed some grey coloured panels installed on the roof of a low building. She enquired from Sunil what those panels were and Sunil told his mother that those were solar panels.

(a) What were the values displayed by Sunil and his mother? State one value for each.

(b) In what way would the use of solar panels prove to be very useful?

(c) Name the semiconductor device used in solar panels. Briefly explain with the help of a diagram, how this device works. [4 marks]

Solution:

Concept:

- A solar cell is a semiconductor p-n junction that converts incident light directly into electrical energy by the photovoltaic effect.
- Photons with energy $h\nu \geq E_g$, where E_g is the semiconductor band gap, can create electron-hole pairs. The junction's built-in electric field separates these charges.

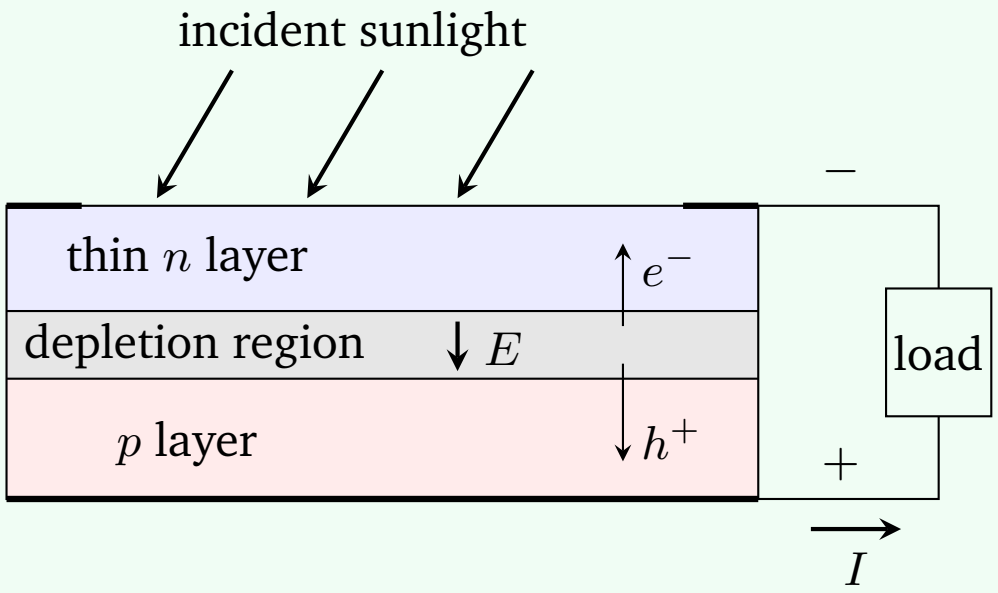
Step 1: (a) Identify the values displayed Sunil displays scientific awareness: he recognises the panels as solar panels and explains them to his mother. His mother displays curiosity and a willingness to learn: she notices an unfamiliar installation and asks what it is. One value for each is therefore scientific awareness for Sunil and curiosity for his mother.

Step 2: (b) Explain the usefulness of solar panels Solar panels supply electrical energy using sunlight, a renewable source. Their operation does not involve burning fuel, so they help reduce air pollution and dependence on fossil fuels. They are particularly useful in villages and remote places where grid electricity may be unavailable or unreliable. Roof-mounted panels can use otherwise unused space to supply electricity for lighting and other needs.

Step 3: (c) Name the device and explain its working The semiconductor device is a solar cell. It consists of a p-n junction with a thin illuminated upper layer so that light can reach the junction region. When suitable photons are absorbed near the depletion region, electron-hole pairs are generated. The built-in electric field, directed from the n side towards the p side, drives electrons towards the n side and holes towards the p side before they recombine. The p contact becomes positive and the n

Quick Tip to Solve: For solar-cell working, connect the three stages explicitly: light creates carriers, the junction field separates them, and an external load receives current.

contact negative. This separation establishes a photovoltage. Connecting an external load permits conventional current to flow from the p contact through the load to the n contact, delivering electrical power. No external bias is needed: the energy delivered to the load comes from the incident light.



Illuminated solar cell: the junction field separates carriers and the external load receives current.

Step 4: Conclusion Sunil shows scientific awareness and his mother curiosity. Solar panels provide renewable, low-pollution electricity, including in remote locations. Their solar cells generate electricity by light-induced carrier generation and charge separation at a p-n junction.

Quick Tip to Solve: For solar-cell working, connect the three stages explicitly: light creates carriers, the junction field separates them, and an external load receives current.

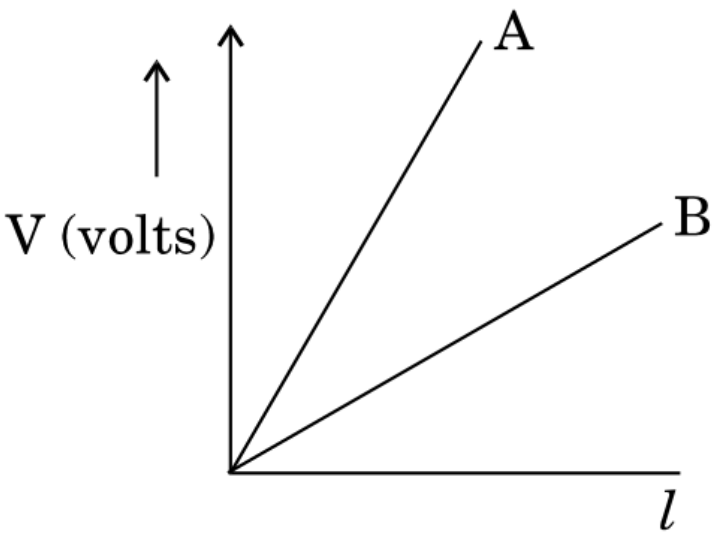
Section E (15 marks)

24. (a) (i) State the principle on which a potentiometer works. How can a given potentiometer be made more sensitive?

(ii) In the graph shown below for two potentiometers, state with reason which of the two potentiometers, A or B, is more sensitive.

(b) Two metallic wires, P_1 and P_2 of the same material and same length but different cross-sectional areas, A_1 and A_2 are joined together and connected to a source of emf. Find the ratio of the drift velocities of free electrons in the two wires when they are connected (i) in series, and (ii) in parallel.

[5 marks]



Solution:

Concept:

- A uniform potentiometer wire carrying steady current has potential drop proportional to its length: $V = kl$, where k is the potential gradient. At balance, the unknown emf is $\mathcal{E} = kl$.
- Drift speed obeys $I = neAv_d$, where n is the number density of free electrons, e their charge magnitude and A the wire's cross-sectional area. Wires of the same material have the same n and resistivity ρ at the same temperature.

Step 1: (a)(i) State the principle and improve sensitivity For a uniform wire, resistance per unit length is constant. With a steady current through it, Ohm's law gives

$$V = IR = I\frac{\rho l}{A} = kl, \qquad k = \frac{I\rho}{A}.$$

Thus the potential difference across any length of the potentiometer wire is directly proportional to that length, provided its cross-section and temperature remain uniform and the current is constant. At the null point, the galvanometer draws no current from the test cell, so the measured balancing potential equals its emf. A potentiometer is more sensitive when it detects a smaller potential difference for a given measurable change in balancing length. Since $\Delta\mathcal{E} = k\Delta l$, this requires a smaller potential gradient k. For the given wire, increase the series resistance in the driving circuit using the

Quick Tip to Solve: Read the slope of a V-versus-length graph as the potential gradient; for drift speed, first identify whether current or voltage is common.

rheostat, thereby reducing the steady current and the voltage drop across the wire. The total wire drop must still exceed the emf to be measured so that a balance point exists.

Step 2: (a)(ii) Compare the two graph slopes The graph has V on the vertical axis and l on the horizontal axis. Therefore its slope is the potential gradient, $k = V/l$. Line B is less steep than line A, giving $k_B < k_A$. For the same small emf, its balancing length $l = \mathcal{E}/k$ is larger on B; equivalently, a given length resolution corresponds to a smaller voltage on B. Hence potentiometer B is more sensitive.

Step 3: (b)(i) Obtain the drift-speed ratio in series In a series connection, charge conservation gives the same current I through both wires. Since their material is the same, their carrier number densities are equal. Consequently,

$$v_{d1} = \frac{I}{neA_1}, \qquad v_{d2} = \frac{I}{neA_2}, \qquad \frac{v_{d1}}{v_{d2}} = \frac{A_2}{A_1}.$$

The narrower wire has a greater drift speed because the same charge per second must cross a smaller area. Equal length is not needed for this series ratio.

Step 4: (b)(ii) Obtain the drift-speed ratio in parallel Let L be the common length and V the common terminal potential difference across the parallel branches. Their resistances and currents are

$$R_1 = \frac{\rho L}{A_1}, \quad R_2 = \frac{\rho L}{A_2}, \qquad \frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{A_1}{A_2}.$$

Substituting these branch currents into the drift-speed relation yields

$$\frac{v_{d1}}{v_{d2}} = \frac{I_1}{I_2}\frac{A_2}{A_1} = 1.$$

Alternatively, each wire has electric field V/L, so identical material gives identical drift speed. The greater current in the thicker wire is exactly balanced by its greater area. These are ratios of drift-speed magnitudes; electrons drift opposite to conventional current.

Step 5: Conclusion The potentiometer principle is $V \propto l$ for a uniform wire carrying steady current. Reducing its potential gradient improves sensitivity, and B is more sensitive because its graph slope is smaller. The drift-speed ratios are $v_{d1} : v_{d2} = A_2 : A_1$ in series and $1 : 1$ in parallel.

Quick Tip to Solve: Read the slope of a V-versus-length graph as the potential gradient; for drift speed, first identify whether current or voltage is common.

OR

24. (a) Define the capacitance of a capacitor. Obtain the expression for the capacitance of a parallel plate capacitor in vacuum in terms of plate area A and separation d between the plates.

(b) A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor but has a thickness $\frac{3d}{4}$. Find the ratio of the capacitance with dielectric inside it to its capacitance without the dielectric. [5 marks]

Solution:

Concept:

- Capacitance is the charge stored on either plate per unit potential difference: $C = Q/V$. Here Q is the magnitude of the charge on a plate and V is the potential difference between the plates. Its SI unit is the farad, $1\,\text{F} = 1\,\text{C/V}$.
- For large parallel plates, neglecting edge effects, the vacuum field between plates is $E_0 = \sigma/\varepsilon_0$, where $\sigma = Q/A$ and ε_0 is the permittivity of vacuum.
- A dielectric of relative permittivity K reduces the field within it to E_0/K at a given free charge. Potential differences across successive layers add.

Step 1: (a) Define capacitance and find the plate field A capacitor has two conductors carrying equal and opposite charges $+Q$ and $-Q$. Its capacitance measures how much charge it stores for a given potential difference: $C = Q/V$. For fixed geometry and medium, it is independent of Q and V. Consider two parallel conducting plates of area A, separated by a small distance d, in vacuum. Each plate produces a field $\sigma/(2\varepsilon_0)$. Between the plates these fields are in the same direction and add, while outside they cancel in the ideal large-plate approximation. Thus

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}.$$

Step 2: (a) Obtain the vacuum capacitance The field is uniform between the plates, so the magnitude of their potential difference is

$$V_0 = E_0 d = \frac{Qd}{\varepsilon_0 A}.$$

Using the definition of capacitance gives

$$C_0 = \frac{Q}{V_0} = \frac{\varepsilon_0 A}{d}.$$

This derivation assumes the plate dimensions are much larger than their separation, so fringing fields can be neglected. Increasing area increases the capacity to store charge; increasing separation raises the potential difference for the same charge and therefore reduces capacitance.

Step 3: (b) Add the potential drops across the two media The printed slab thickness is $t = 3d/4$, leaving a total vacuum thickness $d - t = d/4$. Since the slab covers the entire plate area, the vacuum and dielectric regions lie successively along the electric field. For a free plate charge Q, the fields are

Quick Tip to Solve: For a full-area slab of thickness t, replace the plate separation by the effective separation $d - t + t/K$; insert the printed thickness before simplifying.

$Q/(\varepsilon_0 A)$ in vacuum and $Q/(K\varepsilon_0 A)$ in the dielectric. Hence

$$V = \frac{Q}{\varepsilon_0 A}(d-t) + \frac{Q}{K\varepsilon_0 A}t$$
$$= \frac{Qd}{4\varepsilon_0 A}\left(1 + \frac{3}{K}\right).$$

If vacuum gaps occur on both sides of the slab, their potential drops still add to the same vacuum contribution because their combined thickness is $d/4$.

Step 4: (b) Calculate the capacitance ratio Dividing the charge by the total potential difference yields

$$C = \frac{Q}{V} = \frac{4K\varepsilon_0 A}{d(K+3)}.$$

Therefore the dimensionless ratio is

$$\frac{C}{C_0} = \frac{4K\varepsilon_0 A}{d(K+3)}\frac{d}{\varepsilon_0 A} = \frac{4K}{K+3}.$$

As a check, $K = 1$ restores the vacuum value. For very large K, the dielectric potential drop becomes negligible and the remaining vacuum gap limits the capacitance to $4C_0$. No battery connection assumption is needed to obtain capacitance, because it depends only on the geometry and the medium.

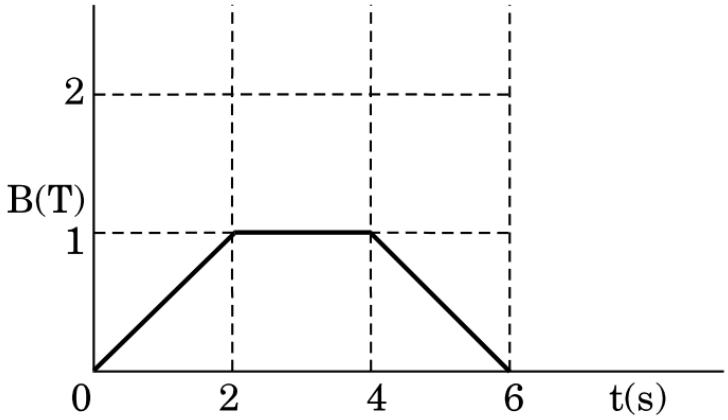
Step 5: Conclusion (a) $C = Q/V$, and for parallel plates in vacuum $\boxed{C_0 = \varepsilon_0 A/d}$. (b) For slab thickness $3d/4$, $\boxed{C/C_0 = 4K/(K+3)}$.

Quick Tip to Solve: For a full-area slab of thickness t, replace the plate separation by the effective separation $d - t + t/K$; insert the printed thickness before simplifying.

25. (a) State Faraday's law of electromagnetic induction.

(b) The magnetic field through a circular loop of wire 12 cm in radius and $8 \cdot 5\ \Omega$ resistance, changes with time as shown in the figure. The magnetic field is perpendicular to the plane of the loop. Calculate the induced current in the loop and plot it as a function of time.

(c) Show that Lenz's law is a consequence of conservation of energy. [5 marks]



Solution:

Concept:

- Faraday's law relates induced emf to the rate of change of magnetic flux: $\varepsilon = -N\, d\Phi_B/dt$, where N is the number of turns and Φ_B is the flux through each turn.
- For a single loop in a uniform perpendicular magnetic field, $\Phi_B = AB$, so $I = \varepsilon/R = -(A/R)\, dB/dt$. The sign refers to a chosen positive direction around the loop.
- Lenz's law specifies that the induced current opposes the change of flux that produces it. Mechanical work supplied against this opposition accounts for the electrical energy produced.

Step 1: (a) State Faraday's law Whenever the magnetic flux linked with a circuit changes, an emf is induced in the circuit. Its magnitude equals the time rate of change of flux linkage:

$$\varepsilon = -\frac{d(N\Phi_B)}{dt} = -N\frac{d\Phi_B}{dt}.$$

For a closed circuit this emf drives an induced current. The negative sign expresses Lenz's law and determines the direction, while the magnitude depends on how rapidly the flux changes.

Step 2: (b) Calculate the area and read the field slopes The radius is $r = 12\,\text{cm} = 0.12\,\text{m}$, and the resistance is $R = 8.5\,\Omega$. Therefore,

$$A = \pi r^2 = \pi(0.12)^2 = 0.04524\,\text{m}^2.$$

The printed graph shows a linear rise from 0 to $1\,\text{T}$ during 0 to $2\,\text{s}$, a constant field during 2 to $4\,\text{s}$, and a linear fall to zero during 4 to $6\,\text{s}$. Thus the successive values of dB/dt are $+0.5$, 0, and $-0.5\,\text{T s}^{-1}$.

Step 3: (b) Obtain the current and its time graph Choose the field direction shown by the positive scalar B as the positive normal; the right-hand rule then defines positive circulation. Using $I =$

Quick Tip to Solve: Read the slope of the field graph, not its height: a constant nonzero field gives zero induced current, and reversing the slope reverses the current.

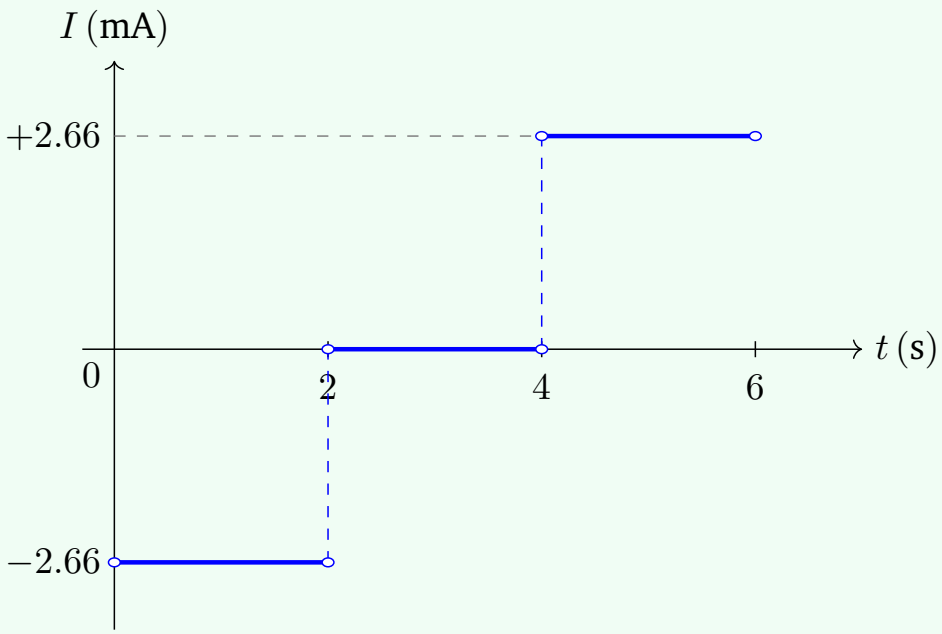
$-(A/R)dB/dt$, the nonzero current magnitude is

$$I_0 = \frac{0.04524 \times 0.5}{8.5} = 2.66 \times 10^{-3}\,\text{A} = 2.66\,\text{mA}.$$

Hence the signed current is

$$I(t) = \begin{cases} -2.66\,\text{mA}, & 0 < t < 2\,\text{s}, \\ 0, & 2 < t < 4\,\text{s}, \\ +2.66\,\text{mA}, & 4 < t < 6\,\text{s}. \end{cases}$$

During the rise the induced field opposes the applied field; during the fall it supports the applied field. Accordingly the two nonzero currents flow in opposite directions. Since the field's inward or outward orientation is not specified, absolute clockwise or anticlockwise directions cannot be assigned without an additional convention. The graph shows the interval values, with ideal jumps at the corners.



Signed induced current versus time; positive circulation follows the chosen positive field normal.

Step 4: (c) Relate Lenz's law to conservation of energy Consider pushing the north pole of a magnet towards a conducting loop. Its increasing flux induces a current whose near face behaves as a north pole, repelling the approaching magnet. An external agent must therefore do work against the magnetic force. This mechanical work supplies the electrical energy that is dissipated as heat in the loop, at a rate I^2R. If instead the induced current assisted the approach, the magnet would accelerate towards the loop while the current continued to produce heat. Both mechanical energy and electrical energy would increase without a corresponding energy supply. This would violate conservation of energy. Withdrawal is likewise opposed. Lenz's law therefore follows from conservation of energy.

Step 5: Conclusion Faraday's law is $\varepsilon = -N\,d\Phi_B/dt$. With the stated circulation convention, the current is $-2.66\,\text{mA}$ from 0 to $2\,\text{s}$, zero from 2 to $4\,\text{s}$, and $+2.66\,\text{mA}$ from 4 to $6\,\text{s}$. Lenz's opposition ensures that external work supplies the induced electrical energy.

Quick Tip to Solve: Read the slope of the field graph, not its height: a constant nonzero field gives zero induced current, and reversing the slope reverses the current.

OR

25. (a) Describe, with the help of a suitable diagram, the working principle of a step-up transformer. Obtain the relation between input and output voltages in terms of the number of turns of primary and secondary windings and the currents in the input and output circuits.

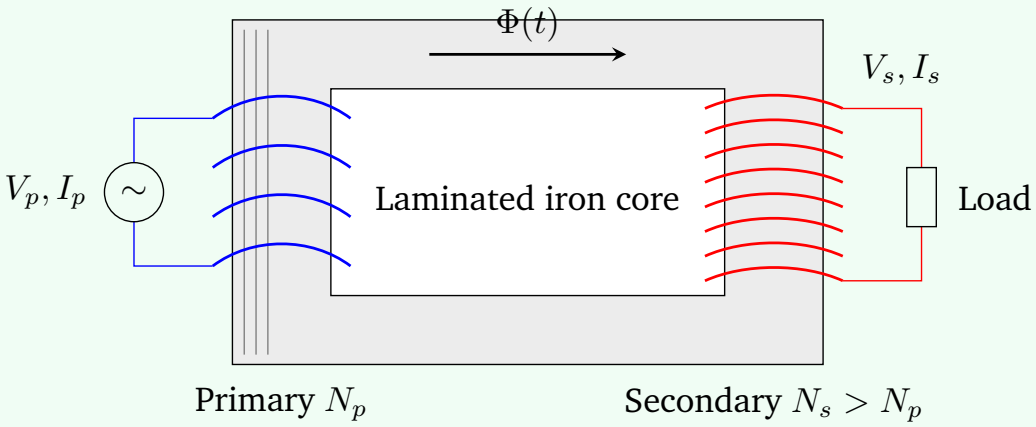
(b) Given the input current 15 A and the input voltage of 100 V for a step-up transformer having 90% efficiency, find the output power and the voltage in the secondary if the output current is 3 A. [5 marks]

Solution:

Concept:

- A transformer operates by mutual induction: alternating current in the primary produces a changing magnetic flux in a common iron core, inducing an emf in the secondary.
- Faraday's law gives $e = -N\, d\Phi/dt$, where N is the winding's number of turns and Φ is the flux through each turn.
- In an ideal transformer, input power equals output power: $V_p I_p = V_s I_s$. For a real transformer, efficiency is $\eta = P_{\text{out}}/P_{\text{in}}$.

Step 1: (a) Construction and working of a step-up transformer Two insulated windings, the primary with N_p turns and the secondary with N_s turns, are wound on a common laminated soft-iron core. The primary is connected to an alternating-voltage source and the secondary to a load. In a step-up transformer, $N_s > N_p$. Alternating primary current produces alternating magnetic flux in the core. This flux links both windings and induces an alternating emf in the secondary by mutual induction. The laminated core reduces eddy-current losses and provides a path of low magnetic reluctance. A steady DC supply cannot provide the continuously changing flux required for transformer action.



Step-up transformer: both windings link the alternating flux in a common laminated iron core; the secondary has more turns.

Step 2: (a) Derive the voltage and current ratios Neglecting leakage flux, the same instantaneous flux Φ links each turn of both windings. Faraday's law gives

$$e_p = -N_p \frac{d\Phi}{dt}, \qquad e_s = -N_s \frac{d\Phi}{dt}.$$

Quick Tip to Solve: For a transformer with stated efficiency, first multiply input power by efficiency, then divide by output current to obtain the secondary voltage.

Thus their emf magnitudes, and hence their rms voltages for negligible winding voltage drops, obey

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

For an ideal transformer, no energy is lost, so $V_p I_p = V_s I_s$. Therefore

$$\boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}}.$$

Increasing the voltage therefore decreases the current in an ideal step-up transformer. The inverse current relation assumes a lossless transformer and a resistive load, or equal power factors. For the real transformer in part (b), use its stated efficiency instead of imposing ideal power conservation.

Step 3: (b) Calculate the secondary power Using the input voltage and current given in the question,

$$P_{\text{in}} = V_p I_p = (100)(15) = 1500\,\text{W}.$$

The efficiency is $\eta = 90/100 = 0.90$, hence

$$P_{\text{out}} = \eta P_{\text{in}} = 0.90(1500) = 1350\,\text{W}.$$

The difference $1500 - 1350 = 150\,\text{W}$ represents the power lost in the transformer.

Step 4: (b) Calculate the secondary voltage With the given secondary current $I_s = 3\,\text{A}$, the output power is $P_{\text{out}} = V_s I_s$. Therefore

$$V_s = \frac{P_{\text{out}}}{I_s} = \frac{1350}{3} = 450\,\text{V}.$$

This exceeds the input voltage, as expected for a step-up transformer. Equivalently, $V_s/V_p = \eta I_p/I_s = 0.90(15/3) = 4.5$ for the supplied numerical data.

Step 5: Conclusion The step-up transformer works by mutual induction. Ideally, $V_s/V_p = N_s/N_p = I_p/I_s$, with $N_s > N_p$. For the specified transformer, the output power is $\boxed{1350\,\text{W}}$ and the secondary voltage is $\boxed{450\,\text{V}}$.

Quick Tip to Solve: For a transformer with stated efficiency, first multiply input power by efficiency, then divide by output current to obtain the secondary voltage.

26. (a) A point object is placed on the principal axis of a convex spherical surface of radius of curvature R, which separates the two media of refractive indices n_1 and n_2 $(n_2 > n_1)$. Draw the ray diagram and deduce the relation between the object distance (u), image distance (v) and the radius of curvature (R) for refraction to take place at the convex spherical surface from rarer to denser medium.

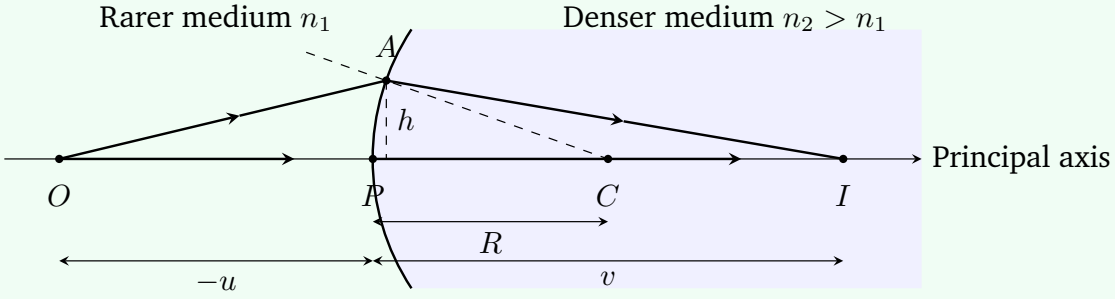
(b) A converging lens has a focal length of 20 cm in air. It is made of a material of refractive index $1 \cdot 6$. If it is immersed in a liquid of refractive index $1 \cdot 3$, find its new focal length. [5 marks]

Solution:

Concept:

- Snell's law at the surface is $n_1 \sin i = n_2 \sin r$, where i and r are measured from the normal. For paraxial rays, small angles in radians obey $\sin\theta \simeq \tan\theta \simeq \theta$.
- Use the Cartesian sign convention: distances are measured from the pole P, positive along incident light. For the arrangement shown, $u < 0$, $R > 0$, and the real image has $v > 0$.
- The thin-lens maker's formula in a medium is $1/f = (n_{\text{lens}}/n_{\text{medium}} - 1)(1/R_1 - 1/R_2)$. Immersion changes the relative refractive index but leaves the radii unchanged.

Step 1: (a) Draw the refraction geometry Let O be the axial object, P the pole, C the centre of curvature, and I the real image for the illustrated object position. An axial ray goes undeviated through P. A paraxial ray OA strikes the spherical surface at a small height h above the axis and refracts along AI. Since $n_2 > n_1$, it bends towards the normal AC. The figure is schematic; the derivation uses rays arbitrarily close to the axis.



Paraxial refraction from rarer to denser medium; AC is the normal at A.

Step 2: (a) Express incidence and refraction angles Let α, β and γ be the positive small angles made with the axis by OA, AI and AC, respectively. The incident ray rises, whereas the normal and refracted ray slope downwards; hence $i = \alpha + \gamma$ and $r = \gamma - \beta$. To first order, the sag of the surface at A is negligible, so

$$\alpha \simeq \frac{h}{-u}, \qquad \beta \simeq \frac{h}{v}, \qquad \gamma \simeq \frac{h}{R}.$$

These expressions refer to the geometry drawn, with positive lengths $-u$, v and R.

Quick Tip to Solve: When a lens is immersed, take the ratio of the two lens-maker formulae: the curvature factor cancels, but use the lens index divided by the medium index.

Step 3: (a) Apply Snell's law and rearrange Applying the small-angle form of Snell's law gives

$$n_1(\alpha+\gamma) = n_2(\gamma-\beta),$$
$$n_1\alpha + n_2\beta = (n_2-n_1)\gamma,$$
$$-\frac{n_1 h}{u} + \frac{n_2 h}{v} = \frac{(n_2-n_1)h}{R}.$$

Cancel the nonzero height h. This derivation assumes a small aperture and paraxial refraction, so the rays meet at a common first-order image. With algebraic Cartesian distances, the resulting relation also handles virtual images by assigning their signed image distance.

Step 4: (b) Compare lens power in air and in liquid Put $K = 1/R_1 - 1/R_2$. For the same lens, K is constant. Taking the refractive index of air as unity,

$$\frac{1}{f_{\text{air}}} = (1.6-1)K = \frac{3}{5}K,$$
$$\frac{1}{f_{\text{liquid}}} = \left(\frac{1.6}{1.3} - 1\right)K = \frac{3}{13}K,$$
$$\frac{f_{\text{liquid}}}{f_{\text{air}}} = \frac{3/5}{3/13} = \frac{13}{5}.$$

Thus $f_{\text{liquid}} = 20 \times 13/5 = 52\,\text{cm}$. The relative index remains greater than unity, so the lens stays converging. Its reduced refractive-index contrast decreases its power and increases its focal length.

Step 5: Conclusion (a) For paraxial refraction at the spherical surface,

$$\boxed{\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2-n_1}{R}}.$$

(b) The new focal length is $\boxed{f_{\text{liquid}} = +52\,\text{cm}}$.

Quick Tip to Solve: When a lens is immersed, take the ratio of the two lens-maker formulae: the curvature factor cancels, but use the lens index divided by the medium index.

OR

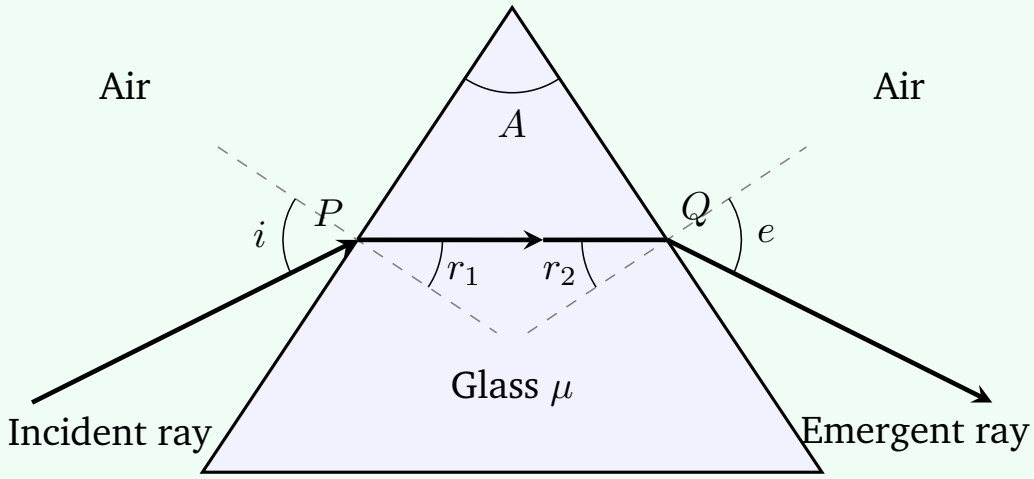
26. (a) Draw the ray diagram showing refraction of light through a glass prism and hence obtain the relation between the refractive index μ of the prism, angle of prism and angle of minimum deviation.
(b) Determine the value of the angle of incidence for a ray of light travelling from a medium of refractive index $\mu_1 = \sqrt{2}$ into the medium of refractive index $\mu_2 = 1$, so that it just grazes along the surface of separation. [5 marks]

Solution:

Concept:

- For a prism in air, Snell's law at the first face is $\sin i = \mu \sin r_1$; at the second face it is $\mu \sin r_2 = \sin e$. Here i, e are the external incidence and emergence angles, r_1, r_2 are the internal refraction angles, and all are measured from their respective normals.
- Prism geometry gives $r_1 + r_2 = A$, where A is the prism angle. The total deviation is $\delta = i + e - A$. At minimum deviation, the path is symmetric: $i = e$ and $r_1 = r_2$.
- For light passing from a denser medium to a rarer medium, grazing emergence means a refracted angle of 90°. The corresponding incidence angle is the critical angle C, with $\sin C = \mu_2/\mu_1$.

Step 1: (a) Draw the ray path through the prism In the diagram, PQ is the refracted ray inside the glass prism. The dashed lines through P and Q are normals to the two refracting faces. The incident ray bends towards the normal on entering glass; the emergent ray bends away from the normal on leaving glass. The symmetric ray shown is the minimum-deviation path. The angles i, e, r_1, and r_2 must be measured from the normals, rather than from the faces. The deviation δ is the angle between the original incident direction and the final emergent direction.



Symmetric ray path through a prism at minimum deviation; dashed lines are face normals.

Step 2: (a) Obtain the geometrical relations The normals to the two faces and the internal ray form a triangle in which the angle between the normals is $180^\circ - A$. Its angle sum gives

$$r_1 + r_2 + (180^\circ - A) = 180^\circ, \qquad r_1 + r_2 = A.$$

At the first face the ray changes direction through $i - r_1$, and at the second face it changes through

Quick Tip to Solve: At minimum deviation first set both internal angles to A/2. For grazing emergence set the refracted angle to 90 degrees in Snell's law, not the incident angle.

$e - r_2$, both towards the base of the prism. Adding these deviations yields

$$\delta = (i - r_1) + (e - r_2) = i + e - A.$$

Thus the total deviation depends on both the external ray angles and the fixed prism angle.

Step 3: (a) Apply the minimum-deviation condition and Snell's law At minimum deviation δ_m, the incident and emergent rays make equal angles with their respective normals. This symmetric condition follows from reversibility of the ray path at the turning point of the deviation curve. Consequently,

$$i = e, \qquad r_1 = r_2 = \frac{A}{2}.$$

Substituting in the deviation relation gives

$$\delta_m = 2i - A, \qquad i = \frac{A + \delta_m}{2}.$$

For air outside the prism, Snell's law at the entrance face gives the refractive index relative to air:

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin\big((A + \delta_m)/2\big)}{\sin(A/2)}.$$

The angles must use the same angular units throughout. This result applies to the wavelength for which the minimum deviation is measured.

Step 4: (b) Impose grazing emergence at the separating surface The incident medium has $\mu_1 = \sqrt{2}$, while the receiving medium has $\mu_2 = 1$, so the ray travels from a denser to a rarer medium. Just grazing the surface means the refracted ray lies along it: its angle with the normal is $r = 90^\circ$. Applying Snell's law,

$$\mu_1 \sin i = \mu_2 \sin r,$$
$$\sqrt{2} \sin i = 1 \sin 90^\circ,$$
$$\sin i = \frac{1}{\sqrt{2}}.$$

The physical incidence angle lies between 0° and 90°, selecting $i = 45^\circ$. This is the critical-angle condition: a larger incidence angle would cause total internal reflection instead of a grazing transmitted ray.

Step 5: Conclusion (a) The required prism relation is $\mu = \dfrac{\sin\big((A + \delta_m)/2\big)}{\sin(A/2)}$. (b) The required angle of incidence, measured from the normal, is $\boxed{45^\circ}$.

Quick Tip to Solve: At minimum deviation first set both internal angles to A/2. For grazing emergence set the refracted angle to 90 degrees in Snell's law, not the incident angle.