CBSE Class 12 Physics Question Paper 2017 with Solution PDF-Set 55/2/2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours **Maximum Marks: 70**

General Instructions

(i) **All** questions are **compulsory**. There are **26** questions in all.

(ii) This question paper has **five** sections : Section A, Section B, Section C, Section D and Section E.

(iii) Section A contains **five** questions of **one** mark each, Section B contains **five** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains one value based question of **four** marks and Section E contains **three** questions of **five** marks each.

(iv) There is no overall choice. However, an internal choice has been provided in **one** question of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$c = 3 \times 10^8$ m/s

$h = 6.63 \times 10^{-34}$ Js

$e = 1.6 \times 10^{-19}$ C

$\mu_o = 4\pi \times 10^{-7}$ T m A^{-1}

$\varepsilon_o = 8.854 \times 10^{-12}$ C^2 N^{-1} m^{-2}

$\frac{1}{4\pi\varepsilon_o} = 9 \times 10^9$ N m^2 C^{-2}

Mass of electron $= 9.1 \times 10^{-31}$ kg

Mass of neutron $= 1.675 \times 10^{-27}$ kg

Mass of proton $= 1.673 \times 10^{-27}$ kg

Avogadro's number $= 6.023 \times 10^{23}$ per gram mole

Boltzmann constant $= 1.38 \times 10^{-23}$ JK^{-1}

Section A (5 marks)

1. A photosensitive surface emits photoelectrons when red light falls on it. Will the surface emit photoelectrons when blue light is incident on it? Give reason. [1 mark]

Solution:

Concept:

- A photon carries energy $E = h\nu$, where h is Planck's constant and ν is the frequency.
- Photoemission requires $h\nu \geq \phi$, where ϕ is the work function of the given surface.

Step 1: Apply the threshold condition Red light already causes emission, so its photon energy is sufficient to overcome the work function: $h\nu_{\text{red}} \geq \phi$. The work function remains the same for this surface.

Step 2: Compare blue and red photons Blue light has a shorter wavelength and hence a higher frequency than red light. Therefore $h\nu_{\text{blue}} > h\nu_{\text{red}} \geq \phi$. Blue photons can also liberate electrons; the excess photon energy becomes their kinetic energy.

Step 3: Conclusion Yes. The surface emits photoelectrons under blue light because blue photons have more energy than the red photons that already cause emission.

Quick Tip to Solve: Compare photon frequencies with the same surface threshold, not the intensities of the two lights.

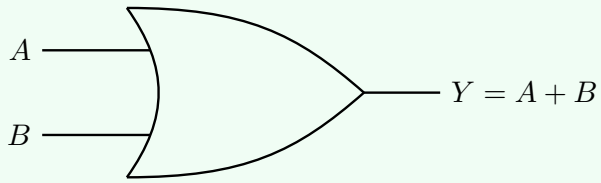
2. Draw logic symbol of an OR gate and write its truth table. [1 mark]

Solution:

Concept:

- An OR gate gives a high output if either input or both inputs are high.
- Its Boolean relation is $Y = A + B$, where $+$ denotes logical OR, not ordinary numerical addition.

Step 1: Draw the two-input OR symbol The two input terminals are labelled A and B, and the output is Y. The curved input boundary and pointed output side identify the standard OR-gate symbol. There is no inversion bubble.



Standard two-input OR gate

Quick Tip to Solve: For an OR gate, fill only the 00 row with 0; every other output entry is 1.

Step 2: List all input combinations The four possible pairs of binary inputs give the truth table

A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

In particular, when both inputs are 1 the output remains the binary value 1.

Step 3: Conclusion The OR output is 0 only for $(A, B) = (0, 0)$; for all other input combinations it is 1.

Quick Tip to Solve: For an OR gate, fill only the 00 row with 0; every other output entry is 1.

3. A point charge Q is placed at point 'O' as shown in the figure. Is the potential at point A, i.e. V_A, greater, smaller or equal to potential, V_B, at point B, when Q is (i) positive, and (ii) negative charge? [1 mark]

O• A• B•

Solution:

Concept:

- The potential at distance r from a point charge is $V = kQ/r$, with $k = 1/(4\pi\varepsilon_0) > 0$.
- Potential is a signed scalar measured with zero potential at infinity.

Step 1: Read the distances in the printed figure The figure places A between O and B. Thus $r_A = OA < r_B = OB$, and $1/r_A > 1/r_B$. Compare signed potentials, rather than just their magnitudes.

Step 2: Compare for each sign of the charge For $Q > 0$, multiplying by $kQ > 0$ preserves the inequality, so $V_A > V_B$. For $Q < 0$, multiplying by the negative number kQ reverses it, so $V_A < V_B$: the nearer point has a more negative potential.

Step 3: Conclusion (i) For positive Q, $V_A > V_B$. (ii) For negative Q, $V_A < V_B$.

Quick Tip to Solve: Near a negative charge, potential has a larger magnitude but a smaller signed value.

4. Write the expression for speed of electromagnetic waves in a medium of electrical permittivity ε and magnetic permeability μ. [1 mark]

Solution:

Concept:

- The propagation speed depends on the absolute permittivity ε and absolute permeability μ of the medium.

Quick Tip to Solve: Use absolute permittivity and permeability; relative values would require the factor c.

- For an ordinary homogeneous medium, Maxwell's wave equation relates wave speed to these electromagnetic properties.

Step 1: Identify the wave-equation coefficient The electromagnetic wave equation contains the coefficient $\mu\varepsilon$ multiplying the second time derivative. Comparing it with the standard wave equation gives $1/v^2 = \mu\varepsilon$.

Step 2: Take the positive speed Speed is the positive magnitude of the propagation velocity, so $v = (\mu\varepsilon)^{-1/2}$. Here the quantities are the medium's absolute properties, not their relative values. For vacuum this reduces to $c = 1/\sqrt{\mu_0\varepsilon_0}$.

Step 3: Conclusion The speed is

$$\boxed{v = \frac{1}{\sqrt{\mu\varepsilon}}}.$$

Quick Tip to Solve: Use absolute permittivity and permeability; relative values would require the factor c.

5. Does the magnifying power of a microscope depend on the colour of the light used? Justify your answer. [1 mark]

Solution:

Concept:

- For a compound microscope in normal adjustment, $M \simeq (L/f_o)(D/f_e)$, where L is the optical tube length and D the least distance of distinct vision.
- The objective and eyepiece focal lengths f_o, f_e depend on the refractive index of their lens material.

Step 1: Connect colour to focal length The refractive index of ordinary lens glass varies with wavelength. By the lens-maker relation, $1/f = (n-1)(1/R_1 - 1/R_2)$ for a thin lens in air. Thus changing the colour changes the lens focal lengths even though its curvatures stay fixed.

Step 2: Relate focal lengths to magnification The magnifying power varies inversely with the objective and eyepiece focal lengths. Under ordinary dispersion, blue light has a higher refractive index than red light and therefore shorter focal lengths. For the same microscope geometry and viewing condition, blue light consequently gives greater magnifying power than red light.

Step 3: Conclusion Yes. Magnifying power depends on colour because refractive index, and hence the focal lengths of the microscope lenses, depend on wavelength.

Quick Tip to Solve: Link colour to refractive index, then to focal length, before using the magnification formula.

Section B (10 marks)

6. Two metallic wires P and Q of the same material and same length but different cross-sectional areas A_1 and A_2 are joined together and then connected to a source of emf. Find the ratio of the drift velocities of free electrons in the wires P and Q, if the wires are connected (i) in series, and (ii) in parallel (i.e. side by side). [2 marks]

Solution:

Concept:

- The drift-speed magnitude is $v_d = I/(neA)$, where n is the free-electron density, e the electron-charge magnitude and A the cross-sectional area.
- For the same material, n and resistivity ρ are equal; each wire has resistance $R = \rho l/A$.

Step 1: Series connection The same current I flows through both wires. With P of area A_1 and Q of area A_2,

$$\frac{v_P}{v_Q} = \frac{I/(neA_1)}{I/(neA_2)} = \frac{A_2}{A_1}.$$

The wire of smaller area requires a larger drift speed to carry the same current.

Step 2: Parallel connection The potential difference V is the same across both equal-length wires. Their currents are $I_P = VA_1/(\rho l)$ and $I_Q = VA_2/(\rho l)$, so $I_P/I_Q = A_1/A_2$. Substituting in the drift-speed ratio,

$$\frac{v_P}{v_Q} = \frac{I_P}{I_Q}\frac{A_2}{A_1} = 1.$$

Equivalently, each branch has the same electric field $E = V/l$ and the same material properties, giving equal drift-speed magnitudes.

Step 3: Conclusion (i) In series, $\boxed{v_P : v_Q = A_2 : A_1}$. (ii) In parallel, $\boxed{v_P : v_Q = 1 : 1}$.

Quick Tip to Solve: In series use equal currents; in parallel first use equal voltages and R proportional to 1/A.

7. Distinguish between broadcast mode and point-to-point mode of communication and give one example for each. [2 marks]

Solution:

Concept:

- A communication system transfers information from a transmitter to one or more receivers through a channel.
- The distinction between these modes is the number of intended receiving users and the nature of the communication link.

Step 1: Broadcast mode In broadcast communication, one transmitter sends the same information to a large number of receivers spread over an area. The transmission is intended for the public or a group of listeners, rather than a single individually selected receiver. For example, a radio station transmits a programme that many radio receivers can receive simultaneously.

Quick Tip to Solve: Identify the intended audience: a public radio programme is broadcast, while a call between two subscribers is point-to-point.

Step 2: Point-to-point mode In point-to-point communication, the communication link connects one transmitting user with one intended receiving user. Information is directed between these two endpoints; the link may permit communication in both directions. For example, a telephone conversation establishes communication between two particular subscribers. Other subscribers are not the intended recipients of that conversation.

Step 3: Conclusion Broadcast: one transmitter to many receivers; example, radio broadcasting. Point-to-point: communication between two specific endpoints; example, a telephone conversation.

Quick Tip to Solve: Identify the intended audience: a public radio programme is broadcast, while a call between two subscribers is point-to-point.

8. Use the mirror equation to show that an object placed between f and $2f$ of a concave mirror forms an image beyond $2f$. [2 marks]

Solution:

Concept:

- The mirror equation is $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$, where f, u and v are signed distances from the pole.
- Under the Cartesian sign convention, a concave mirror has $f < 0$, and an object in front of it has $u < 0$.

Step 1: Express the specified object position using positive magnitudes Let $F > 0$ be the magnitude of the focal length and $U > 0$ the magnitude of the object distance. Thus $f = -F$ and $u = -U$. The statement that the object is between the focus and the centre of curvature means

$$F < U < 2F.$$

The endpoints are excluded, so $U - F$ is strictly positive.

Step 2: Apply the mirror equation and compare the image distance with twice the focal length Substituting the signed distances gives

$$\frac{1}{v} = -\frac{1}{F} + \frac{1}{U} = \frac{F - U}{FU},$$
$$v = -\frac{FU}{U - F}.$$

Therefore $v < 0$, so the image lies in front of the mirror. Its distance magnitude is $V = -v = FU/(U - F)$. Since $U - F > 0$,

$$V - 2F = \frac{FU - 2F(U - F)}{U - F}$$
$$= \frac{F(2F - U)}{U - F} > 0.$$

Both numerator and denominator are positive because $F < U < 2F$.

Step 3: Conclusion Hence $V > 2F$, or equivalently $v < -2F$. The real image is formed beyond the

Quick Tip to Solve: Use positive distance magnitudes after setting f and u negative; the inequality follows from F(2F-U)/(U-F)>0.

centre of curvature, at a distance greater than $2F$ in front of the concave mirror.

Quick Tip to Solve: Use positive distance magnitudes after setting f and u negative; the inequality follows from F(2F-U)/(U-F)>0.

OR

8. (a) State the condition under which a large magnification can be achieved in an astronomical telescope.

(b) Give two reasons to explain why a reflecting telescope is preferred over a refracting telescope.

[2 marks]

Solution:

Concept:

- For an astronomical telescope in normal adjustment, the magnitude of angular magnification is $|M| = f_o/f_e$, where f_o and f_e are the positive focal-length magnitudes of the objective and eyepiece.
- A reflector uses a concave mirror as its objective. Reflection avoids the wavelength-dependent refraction that causes chromatic aberration in a lens.

Step 1: (a) Condition for large magnification Choose an objective with a large focal length f_o and an eyepiece with a small focal length f_e, so that

$$f_o \gg f_e, \qquad |M| = \frac{f_o}{f_e} \gg 1.$$

Thus the required condition is a large objective-to-eyepiece focal-length ratio. The inverted image changes the sign of the signed magnification, but not its magnitude.

Step 2: (b) Two advantages of a reflecting telescope First, a mirror objective does not produce chromatic aberration: light of different colours obeys the same law of reflection, so the mirror does not separate their focal positions as a refracting lens does. Second, a large mirror can be supported from its back and is easier to construct than a large lens, which must be supported around its edge. A larger aperture collects more light from faint astronomical objects and improves resolving power. These are distinct optical and construction advantages.

Step 3: Conclusion (a) Large magnification requires f_o/f_e to be large. (b) A reflecting telescope avoids chromatic aberration and allows a large, conveniently supported objective aperture.

Quick Tip to Solve: For magnification compare focal lengths; for the reflector advantages mention absence of chromatic aberration and back support of a large mirror.

9. Find the intensity at a point on a screen in Young's double slit experiment where the interfering waves of equal intensity have a path difference of (i) $\lambda/4$, and (ii) $\lambda/3$. [2 marks]

Solution:

Concept:

- For two coherent waves, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$, where ϕ is their phase difference.
- A path difference Δ corresponds to $\phi = 2\pi\Delta/\lambda$. Let each wave separately have intensity I_0; then $I = 2I_0(1+\cos\phi)$.

Step 1: (i) Path difference equal to one quarter wavelength For $\Delta = \lambda/4$, the phase difference is

$$\phi = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}.$$

Hence the interference term vanishes because $\cos(\pi/2) = 0$:

$$I_{(i)} = 2I_0\,(1+0) = 2I_0.$$

The intensities add at this position, even though the waves are coherent, because they are in phase quadrature.

Step 2: (ii) Path difference equal to one third wavelength For $\Delta = \lambda/3$,

$$\phi = \frac{2\pi}{3}, \qquad \cos\phi = -\frac{1}{2}.$$

Substitution gives

$$I_{(ii)} = 2I_0\left(1 - \frac{1}{2}\right) = I_0.$$

The negative interference term reduces the resultant intensity, but does not make it zero. For reference, the central maximum has $I_{\max} = 4I_0$; therefore these intensities are $I_{\max}/2$ and $I_{\max}/4$, respectively.

Step 3: Conclusion Taking I_0 as the intensity due to either slit alone: (i) $I = 2I_0$ for $\lambda/4$; (ii) $I = I_0$ for $\lambda/3$.

Quick Tip to Solve: Always define I_0 as one-slit intensity; convert path difference to phase before evaluating the cosine interference term.

10. Write two points of difference between n-type and p-type semiconductors. [2 marks]

Solution:

Concept:

- An extrinsic semiconductor is formed by doping pure silicon or germanium with a small amount of a suitable impurity.
- The majority carrier is the carrier present in greater concentration; donor and acceptor impurities determine whether electrons or holes are the majority carriers.

Step 1: First difference: dopant used An n-type semiconductor is obtained by doping silicon

Quick Tip to Solve: Remember donor supplies an electron (n-type); acceptor leaves a hole (p-type). Pair the dopant and majority carrier consistently.

or germanium with a pentavalent donor impurity, such as phosphorus or arsenic. Four valence electrons participate in bonding and the extra electron is readily available for conduction. A p-type semiconductor is obtained using a trivalent acceptor impurity, such as boron or aluminium. The missing electron in bonding creates a hole, and the impurity can accept an electron.

Step 2: Second difference: majority and minority charge carriers In an n-type semiconductor, electrons are the majority carriers and holes are the minority carriers, so the electron concentration n exceeds the hole concentration p. In a p-type semiconductor, holes are the majority carriers and electrons are the minority carriers, so $p > n$. Thus electrical conduction is mainly by electrons in n-type material and mainly by holes in p-type material. Both kinds remain electrically neutral overall; the letters describe the dominant mobile carriers, not a net charge on the crystal.

Step 3: Conclusion n-type: pentavalent donor doping, with electrons as majority carriers. p-type: trivalent acceptor doping, with holes as majority carriers.

Quick Tip to Solve: Remember donor supplies an electron (n-type); acceptor leaves a hole (p-type). Pair the dopant and majority carrier consistently.

Section C (36 marks)

11. A charge Q is distributed uniformly over a metallic sphere of radius R. Obtain the expressions for the electric field (E) and electric potential (V) at a point $0 < x < R$.
Show on a plot the variation of E and V with x for $0 < x < 2R$. [3 marks]

Solution:

Concept:

- A metallic sphere is a conductor. In electrostatic equilibrium its excess charge resides on its surface and the electric field inside the metal is zero.
- Gauss's law is $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\varepsilon_0$. Spherical symmetry gives $E(4\pi x^2) = Q/\varepsilon_0$ outside, where x is distance from the centre.
- Potential is measured relative to infinity: $V(x) = -\int_{\infty}^{x} \vec{E} \cdot d\vec{l}$. Write $k = 1/(4\pi\varepsilon_0)$.

Step 1: Obtain the interior field Choose a concentric Gaussian sphere of radius $x < R$. All excess charge is on the metallic surface at R, so this Gaussian surface encloses no charge. By symmetry the radial field has the same value everywhere on it. Hence

$$E(x)\,4\pi x^2 = 0 \quad \Rightarrow \quad E(x) = 0, \qquad 0 < x < R.$$

A nonzero field inside the metal would move its free charges, contradicting electrostatic equilibrium.

Step 2: Obtain the interior potential For $x > R$, Gauss's law gives $E = kQ/x^2$, radially outward for positive Q. Integrating from infinity to the surface gives

$$V(R) = -\int_{\infty}^{R} \frac{kQ}{r^2}\,dr = \frac{kQ}{R}.$$

The field between the surface and an interior point is zero, so there is no potential difference within

Quick Tip to Solve: Draw zero interior field and constant interior potential; join only the potential continuously at the metallic surface.

the conductor:

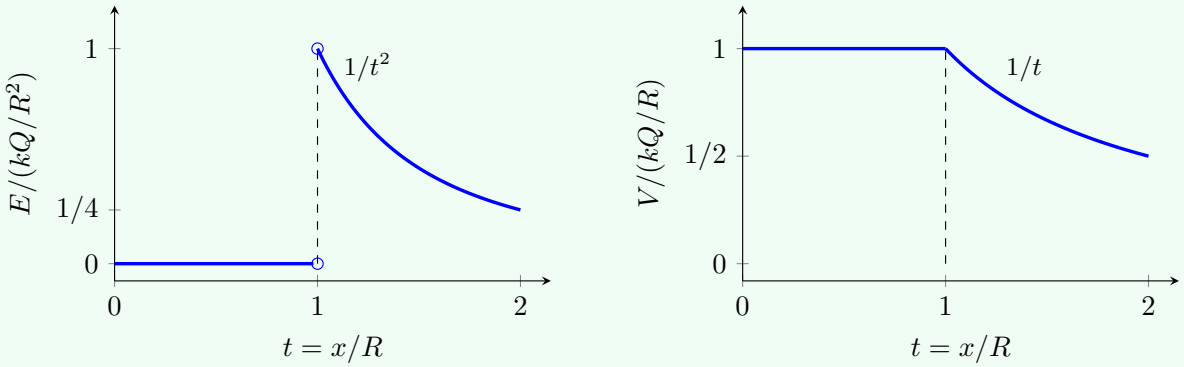
$$V(x) - V(R) = -\int_R^x 0\, dr = 0 \quad \Rightarrow \quad V(x) = \frac{kQ}{R}, \qquad 0 < x < R.$$

Zero interior field does not mean zero interior potential; the conductor is an equipotential at its surface value.

Step 3: Plot the field and potential The expressions for the full requested interval are

$$E(x) = \begin{cases} 0, & 0 < x < R, \\ kQ/x^2, & R < x < 2R, \end{cases} \qquad V(x) = \begin{cases} kQ/R, & 0 < x \le R, \\ kQ/x, & R < x < 2R. \end{cases}$$

The plots below assume $Q > 0$. The field jumps from zero just inside to kQ/R^2 just outside, then decreases as $1/x^2$. The potential stays constant inside and joins continuously to its exterior $1/x$ curve. At $2R$, the field and potential are $kQ/(4R^2)$ and $kQ/(2R)$. For negative Q, the signed radial field and potential reverse sign.



Field and potential for $Q > 0$, with $t = x/R$ and $k = 1/(4\pi\varepsilon_0)$.

Step 4: Conclusion For $0 < x < R$, $\boxed{E = 0}$ and $\boxed{V = Q/(4\pi\varepsilon_0 R)}$, taking $V(\infty) = 0$. The field is discontinuous at the charged surface; the potential is continuous there.

Quick Tip to Solve: Draw zero interior field and constant interior potential; join only the potential continuously at the metallic surface.

12. Define electric flux. Write its SI unit.

Using Gauss's law, deduce an expression for electric field intensity due to an infinitely long straight uniformly charged wire. [3 marks]

Solution:

Concept:

- Electric flux through a surface measures the electric field passing through it: $\Phi_E = \int_S \vec{E} \cdot d\vec{A}$, where $d\vec{A}$ is the oriented area element.
- Gauss's law for a closed surface is $\oint_S \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\varepsilon_0$.

Quick Tip to Solve: Use a coaxial cylinder: only its curved area $2\pi r L$ contributes flux, so L cancels immediately against the enclosed charge λL.

- For an infinitely long uniformly charged wire, cylindrical symmetry makes the field radial, with magnitude depending only on perpendicular distance r from the wire. Its linear charge density is $\lambda = Q/L$.

Step 1: Define flux and state its unit For a uniform electric field passing through a plane area A, electric flux is the scalar product of field and area vector:

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta,$$

where θ is the angle between the field and the normal to the surface. For a nonuniform field or curved surface, sum the contributions of the small surface elements using the integral in the concept. Its SI unit is

$$[\Phi_E] = (\text{N C}^{-1})(\text{m}^2) = \text{N m}^2\ \text{C}^{-1}.$$

Step 2: Choose a cylindrical Gaussian surface Take a closed cylinder of radius r and length L, coaxial with the wire. The enclosed charge is λL. On the curved surface, the electric field is radial and normal to the surface, and its magnitude $E(r)$ is constant. Thus its flux is $E(r)(2\pi rL)$. On each flat end cap, the area vector is parallel to the wire while the field lies perpendicular to the wire, so $\vec{E} \cdot d\vec{A} = 0$. Both end caps contribute zero flux.

Step 3: Apply Gauss's law Equating the total outward flux to the enclosed charge divided by ε_0,

$$E_r(2\pi rL) = \frac{\lambda L}{\varepsilon_0} \quad \Rightarrow \quad E_r = \frac{\lambda}{2\pi\varepsilon_0 r}.$$

The length L cancels, as it must for an infinitely long wire. The field decreases inversely with the perpendicular distance, rather than as its square. A positive wire produces an outward field and a negative wire an inward field.

Step 4: Conclusion Electric flux is $\Phi_E = \int_S \vec{E} \cdot d\vec{A}$, with SI unit N m^2 C^{-1}. For an infinitely long wire,

$$\boxed{\vec{E}(r) = \frac{\lambda}{2\pi\varepsilon_0 r}\,\hat{r}},$$

where $\hat{r}$ points outward from the wire; the magnitude is $|\lambda|/(2\pi\varepsilon_0 r)$.

Quick Tip to Solve: Use a coaxial cylinder: only its curved area $2\pi rL$ contributes flux, so L cancels immediately against the enclosed charge λL.

13. A proton and an α-particle move perpendicular to a magnetic field. Find the ratio of radii of the circular paths described by them when both (i) have equal momenta, and (ii) were accelerated through the same potential difference. [3 marks]

Solution:

Concept:

- For perpendicular motion in a uniform magnetic field, the magnetic force supplies the centripetal force: $qvB = mv^2/r$. Thus $r = p/(qB)$, where $p = mv$ is momentum and q is the positive charge magnitude.
- A proton has charge e and mass m_p. An α-particle has charge $2e$ and mass approximately $4m_p$.
- Acceleration from rest through a potential difference V gives kinetic energy $qV = p^2/(2m)$, so $p = \sqrt{2mqV}$ in the nonrelativistic regime.

Step 1: Establish the orbit radius relation Since each velocity is perpendicular to the same magnetic field, the magnetic force has magnitude qvB. It stays perpendicular to the velocity, changing its direction without changing its speed. Equating this force to the centripetal force,

$$qvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB} = \frac{p}{qB}.$$

Therefore the radius depends on momentum divided by charge; the mass does not enter independently when the momentum is specified.

Step 2: Compare radii for equal momenta For case (i), set $p_p = p_\alpha = p$. The proton and alpha-particle radii are

$$r_p = \frac{p}{eB}, \qquad r_\alpha = \frac{p}{2eB}.$$

Hence

$$\frac{r_p}{r_\alpha} = \frac{2e}{e} = 2.$$

The alpha-particle's double charge gives it half the radius for the same momentum.

Step 3: Compare radii for the same accelerating potential For case (ii), take the usual acceleration from rest through the same potential difference V. The kinetic energies are eV and $2eV$, respectively, so the momenta are no longer equal. Substitution into the orbit relation gives

$$r = \frac{\sqrt{2mqV}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$$

Cancelling the common V and B,

$$\frac{r_p}{r_\alpha} = \sqrt{\frac{m_p/e}{4m_p/(2e)}} = \frac{1}{\sqrt{2}}.$$

The larger mass-to-charge ratio of the alpha-particle makes its orbit larger in this case. Equal potential difference means energy proportional to charge, not equal energy or equal momentum.

Quick Tip to Solve: Keep the ratio order proton to alpha fixed: use $r \propto 1/q$ for equal momentum, but $r \propto \sqrt{m/q}$ for equal accelerating potential.

Step 4: Conclusion The requested proton-to-alpha radius ratios are

$$\boxed{r_p : r_\alpha = 2 : 1 \quad \text{(equal momenta)}}, \qquad \boxed{r_p : r_\alpha = 1 : \sqrt{2} \quad \text{(same potential difference)}}.$$

Quick Tip to Solve: Keep the ratio order proton to alpha fixed: use $r \propto 1/q$ for equal momentum, but $r \propto \sqrt{m/q}$ for equal accelerating potential.

14. A monochromatic light of wavelength λ is incident normally on a narrow slit of width 'a' to produce a diffraction pattern on the screen placed at a distance D from the slit. With the help of a relevant diagram, deduce the conditions for obtaining maxima and minima on the screen. Use these conditions to show that angular width of central maximum is twice the angular width of secondary maximum. [3 marks]

Solution:

Concept:

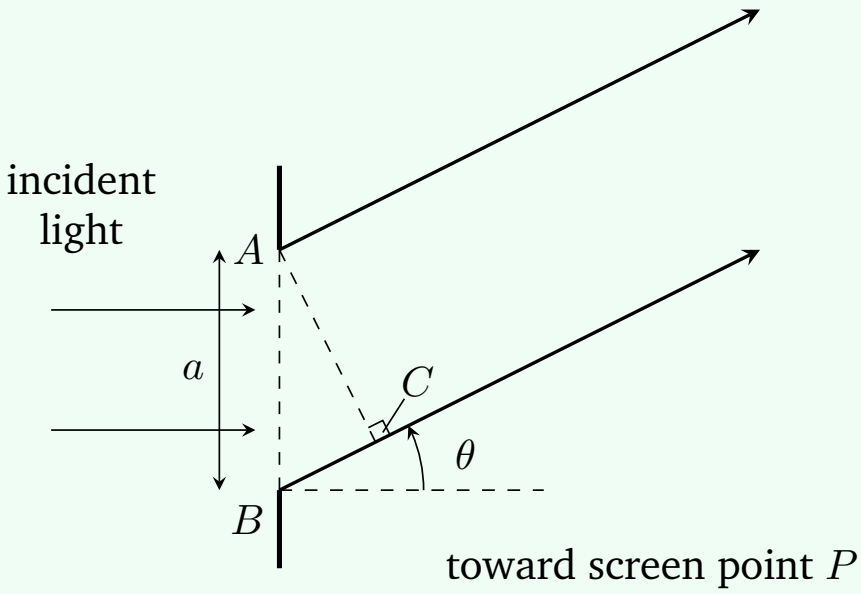
- Huygens principle: points across the slit act as coherent secondary sources. Extreme rays diffracted at angle θ have path difference $a \sin\theta$, where a is slit width.
- For Fraunhofer diffraction, $I = I_0(\sin\beta/\beta)^2$, with $\beta = \pi a \sin\theta/\lambda$. Here λ is wavelength and I_0 is central intensity.
- For small angles, $\sin\theta \simeq \theta \simeq y/D$, where y is displacement from the centre of a distant screen at distance D.

Step 1: Relate the ray geometry to path difference Let A and B be the upper and lower edges of the slit. Draw AC perpendicular to the diffracted rays, with C on the ray from B. The extra distance travelled by the lower ray is

$$BC = AB\sin\theta = a\sin\theta.$$

The sketch shows the effectively parallel rays directed toward a point above the screen centre. In the far-field approximation, their small convergence angle is neglected.

Quick Tip to Solve: Measure fringe width between its neighbouring minima: the central fringe spans both sides of zero, while a secondary fringe spans one interval.



Slit geometry: AC is perpendicular to the rays and $BC = a\sin\theta$.

Step 2: Deduce the condition for dark fringes If $a\sin\theta = \lambda$, divide the slit into two equal halves. Each source in one half pairs with a source a distance $a/2$ away, giving path difference $\lambda/2$ and phase difference π. Every pair cancels. For $a\sin\theta = m\lambda$, divide the slit into $2m$ equal strips and pair successive strips similarly. Thus minima occur at

$$a\sin\theta = \pm m\lambda, \qquad m = 1, 2, 3, \ldots.$$

Their small-angle screen positions are $y_m = \pm m\lambda D/a$. Zero angle is not a minimum: all secondary waves there are in phase.

Step 3: Deduce the conditions for bright fringes At $\theta = 0$, all waves add in phase to give the central maximum. Secondary maxima lie between adjacent minima. Differentiating the intensity gives their exact condition $\tan\beta = \beta$, excluding zero, which is handled by its limiting value. In the school approximation the side peaks are near

$$a\sin\theta \simeq \pm(m + \tfrac{1}{2})\lambda, \qquad m = 1, 2, 3, \ldots.$$

This half-integral condition is approximate; the minimum condition is exact within the Fraunhofer model.

Step 4: Compare widths between bounding minima The central maximum extends between the first minima on opposite sides of zero. For $\lambda/a \ll 1$, its angular width is

$$\Delta\theta_c \simeq \lambda/a - (-\lambda/a) = 2\lambda/a.$$

A secondary maximum occupies the interval between successive minima on the same side, giving

$$\Delta\theta_s \simeq (m+1)\lambda/a - m\lambda/a = \lambda/a.$$

Quick Tip to Solve: Measure fringe width between its neighbouring minima: the central fringe spans both sides of zero, while a secondary fringe spans one interval.

The corresponding linear widths are $2D\lambda/a$ and $D\lambda/a$. The factor of two uses the small-angle approximation; exact angular widths involve inverse sines.

Step 5: Conclusion Minima satisfy $a\sin\theta = \pm m\lambda$; the central maximum is at zero angle, and side maxima approximately satisfy $a\sin\theta = \pm(m+\frac{1}{2})\lambda$. The central angular width $2\lambda/a$ is twice the secondary width λ/a for small angles.

Quick Tip to Solve: Measure fringe width between its neighbouring minima: the central fringe spans both sides of zero, while a secondary fringe spans one interval.

15. Using Bohr's postulates, derive the expression for the orbital period of the electron moving in the n^{th} orbit of hydrogen atom. [3 marks]

Solution:

Concept:

- In Bohr's model the electron moves in a stationary circular orbit without radiating energy. Electrostatic attraction supplies the centripetal force: $m_e v_n^2/r_n = e^2/(4\pi\varepsilon_0 r_n^2)$.
- Allowed angular momenta obey $m_e v_n r_n = n\hbar = nh/(2\pi)$, where $n = 1, 2, \ldots$, m_e is electron mass, e is the magnitude of its charge, and h is Planck's constant.
- Orbital period is distance travelled in one revolution divided by speed: $T_n = 2\pi r_n/v_n$. The nucleus is treated as fixed in the usual hydrogen Bohr model.

Step 1: Obtain the speed from the quantisation condition Use $m_e v_n r_n = nh/(2\pi)$ to express the speed in terms of radius:

$$v_n = \frac{nh}{2\pi m_e r_n}.$$

The Coulomb force equation also gives

$$v_n^2 = \frac{e^2}{4\pi\varepsilon_0 m_e r_n}.$$

These two equations must hold together; arbitrary radii are not permitted by the angular-momentum postulate.

Step 2: Derive the allowed orbital radius and speed Square the quantised-speed expression and substitute it into the force equation:

$$\frac{n^2h^2}{4\pi^2 m_e^2 r_n^2} = \frac{e^2}{4\pi\varepsilon_0 m_e r_n}.$$

Cancel common factors and solve for the radius:

$$r_n = \frac{\varepsilon_0 h^2}{\pi m_e e^2} n^2.$$

Quick Tip to Solve: First recall $r_n \propto n^2$ and $v_n \propto 1/n$; then $T_n = 2\pi r_n/v_n$ must scale as n^3, which checks your final expression.

Substituting this radius back into the angular-momentum relation gives

$$v_n = \frac{e^2}{2\varepsilon_0 h n}.$$

Thus the radius grows as n^2, whereas the speed decreases as $1/n$.

Step 3: Calculate the time for one revolution A circular orbit has circumference $2\pi r_n$. Divide by its constant orbital speed:

$$\begin{aligned} T_n &= \frac{2\pi r_n}{v_n} \\ &= 2\pi \left(\frac{\varepsilon_0 h^2 n^2}{\pi m_e e^2}\right)\left(\frac{2\varepsilon_0 h n}{e^2}\right) \\ &= \frac{4\varepsilon_0^2 h^3}{m_e e^4} n^3. \end{aligned}$$

The factor n^3 follows from combining the larger circumference with the slower speed. This period describes motion within one stationary orbit; it is not the inverse frequency of radiation emitted in a transition between orbits.

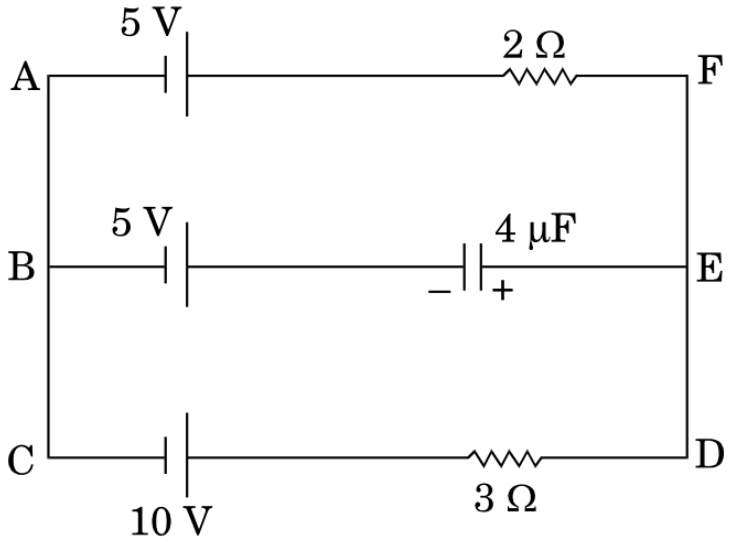
Step 4: Conclusion The electron's orbital period in the nth hydrogen orbit is

$$\boxed{T_n = \frac{4\varepsilon_0^2 h^3 n^3}{m_e e^4} = T_1 n^3.}$$

Here $T_1 = 4\varepsilon_0^2 h^3/(m_e e^4)$ has units of seconds.

Quick Tip to Solve: First recall $r_n \propto n^2$ and $v_n \propto 1/n$; then $T_n = 2\pi r_n / v_n$ must scale as n^3, which checks your final expression.

16. In the given circuit, with steady current, calculate the potential drop across the capacitor in terms of V. [3 marks]



Solution:

Concept:

- A capacitor behaves as an open circuit in the steady state of a dc circuit: its charging current has fallen to zero, although its voltage and charge can remain nonzero.
- Kirchhoff's loop rule states that the algebraic sum of potential changes around a closed loop is zero. Crossing a cell from its short plate to its long plate raises the potential by its emf.
- The wires connect A, B, C to one common potential V_L, and F, E, D to another common potential V_R. Across a resistor, potential falls by IR in the direction of current.

Step 1: Remove the capacitor branch from the steady-current loop No current flows through the middle branch containing the $4\,\mu$F capacitor. The two resistors therefore lie in the single outer current loop. All three cells have their positive, long plates on the right. Hence the outer $10\,\text{V}$ and $5\,\text{V}$ cells oppose one another in that loop. Choose current I from C to D along the bottom branch, then from F to A along the upper branch. Kirchhoff's rule gives

$$10 - 3I - 2I - 5 = 0, \qquad I = \frac{10-5}{3+2} = 1\,\text{A}.$$

The positive result confirms the chosen direction.

Step 2: Find the voltage between the common rails Traverse the lower branch from C to D. The cell raises the potential by $10\,\text{V}$, then the $3\,\Omega$ resistor reduces it by $3I$:

$$V_R - V_L = 10 - 3(1) = 7\,\text{V}.$$

Check using the upper branch from A to F: the cell raises potential by $5\,\text{V}$, and traversal against the current through $2\,\Omega$ adds another $2\,\text{V}$. Thus the same rail voltage is $5 + 2 = 7\,\text{V}$.

Step 3: Find the potential difference across the capacitor Let X be the capacitor's left plate, immediately to the right of the middle $5\,\text{V}$ cell. That cell fixes

$$V_X = V_L + 5\,\text{V}.$$

The right capacitor plate is directly connected to E, so its potential is $V_R = V_L + 7\,\text{V}$. Therefore

$$V_E - V_X = (V_L + 7) - (V_L + 5) = 2\,\text{V}.$$

The right plate is positive, as indicated in the printed figure. Capacitance is not needed to obtain this voltage; it determines the stored charge once the circuit has reached steady state.

Step 4: Conclusion The potential difference across the capacitor is $\boxed{2\,\text{V}}$, with the right plate at higher potential. A traversal from the positive right plate to the negative left plate gives a potential drop of $2\,\text{V}$.

Quick Tip to Solve: Open the capacitor branch for dc steady state, calculate the common rail voltage, then subtract the middle cell's emf to obtain the capacitor voltage.

17. (a) How are electromagnetic waves produced? Explain.

(b) A plane electromagnetic wave is travelling through a medium along the +ve z-direction. Depict the electromagnetic wave showing the directions of the oscillating electric and magnetic fields. [3 marks]

Solution:

Concept:

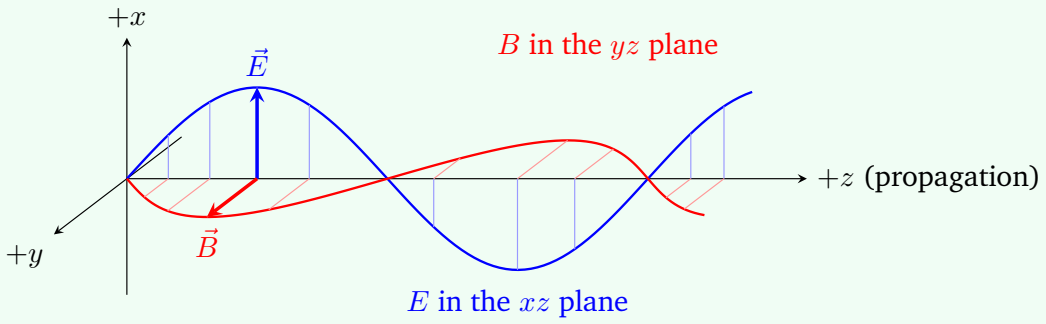
- An accelerating charge emits electromagnetic radiation. An oscillating charge is an example of an accelerating charge whose fields vary periodically.
- A time-varying electric field produces a magnetic field (Maxwell's displacement-current law), and a time-varying magnetic field produces an electric field (Faraday's law).
- In a plane electromagnetic wave, $\vec{E} \perp \vec{B}$, and both fields are perpendicular to the propagation direction, which is along $\vec{E} \times \vec{B}$.

Step 1: Explain the production of the wave (a) Consider a charge oscillating in an antenna. Its velocity changes continuously, so it is accelerated. The electric field associated with the charge consequently changes with time. This changing electric field produces a changing magnetic field; the changing magnetic field in turn produces a changing electric field. These coupled disturbances propagate away from the source as an electromagnetic wave. Once emitted, the wave does not require a material medium for its propagation. A stationary charge gives only an electrostatic field and does not produce such radiation.

Step 2: Choose the field directions and depict the wave (b) For propagation along $+z$, choose the electric field along the x-axis and the magnetic field along the y-axis. One consistent representation is

$$\vec{E} = E_0 \sin(kz - \omega t)\,\hat{x}, \qquad \vec{B} = B_0 \sin(kz - \omega t)\,\hat{y}.$$

Here k is the wave number and ω the angular frequency. The fields have the same frequency and are in phase: their maxima, minima and zero values occur together. Since $\hat{x} \times \hat{y} = \hat{z}$, the wave travels along the required $+z$ direction. When both fields reverse in the next half-cycle, their cross product still points along $+z$. In an ordinary homogeneous medium the amplitudes obey $E_0/B_0 = v$, where v is the wave speed in that medium.



The electric and magnetic fields oscillate in perpendicular planes; arrowed field samples show the directions at a positive crest.

Step 3: Conclusion Electromagnetic waves originate from accelerated charges and propagate as mutually coupled electric and magnetic disturbances. For a wave moving along $+z$, $\vec{E}$ may oscillate along x and $\vec{B}$ along y, with $\vec{E} \times \vec{B}$ along $+z$.

Quick Tip to Solve: Choose the two transverse axes so that the right-hand rule for $\vec{E} \times \vec{B}$ gives the stated propagation direction; draw both sinusoids in phase.

18. (a) Write the process of β^--decay. How can radioactive nuclei emit β-particles even though they do not contain them? Why do all electrons emitted during β-decay not have the same energy?

(b) A heavy nucleus splits into two lighter nuclei. Which one of the two – parent nucleus or the daughter nuclei has more binding energy per nucleon? [3 marks]

Solution:

Concept:

- In β^--decay, the weak interaction converts a neutron into a proton, an electron and an electron antineutrino: $n \to p + e^- + \bar{\nu}_e$.
- Conservation of charge and nucleon number gives ${}^{A}_{Z}X \to {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$: the mass number A remains fixed while atomic number Z increases by one.
- Binding energy per nucleon is B/A. A larger value means a more tightly bound nucleus; the binding-energy curve rises towards intermediate-mass nuclei from the heavy-nucleus region.

Step 1: Write the beta-minus decay process (a) Within a radioactive nucleus, a neutron undergoes the transformation

$$n \longrightarrow p + e^- + \bar{\nu}_e.$$

Thus the complete nuclear process is

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\nu}_e.$$

The emitted electron is the beta particle. It is created during the decay by the weak interaction and was not present as an electron confined inside the nucleus beforehand. The proton remains in the daughter nucleus. Hence the number of nucleons is unchanged, and the nuclear charge increases by one elementary charge, balancing the negative charge carried away by the electron.

Step 2: Explain the spread of electron energies The energy released in a decay to a specified daughter state is shared among the electron, the antineutrino and the recoil of the daughter nucleus. Different decays can divide this available energy differently while conserving total energy and momentum. Therefore the electron kinetic energy is not fixed: the emitted beta electrons form a continuous energy spectrum up to an endpoint. The antineutrino carries the varying remainder, with a small contribution from nuclear recoil. This differs from a decay with only two final particles, whose energies would be fixed by the conservation laws for a specified transition.

Step 3: Compare binding energy per nucleon after fission (b) When a heavy nucleus undergoes energy-releasing fission into two lighter nuclei, the products lie closer to the intermediate-mass region of the binding-energy curve. They are more tightly bound per nucleon than the original heavy nucleus. Thus the daughter nuclei have higher binding energy per nucleon. Their greater total binding energy corresponds to a reduction in the total rest mass; this mass difference is released as kinetic energy and radiation according to $Q = \Delta m\, c^2$.

Step 4: Conclusion Beta-minus emission creates an electron through $n \to p + e^- + \bar{\nu}_e$; varying energy sharing with the antineutrino produces the continuous electron spectrum. In heavy-nucleus fission, the lighter daughter nuclei have greater binding energy per nucleon than the parent.

Quick Tip to Solve: For beta-minus decay keep A unchanged and increase Z by one; for fission compare binding energy per nucleon on the heavy side of the binding-energy curve.

19. What is sky wave propagation? Which frequency range is suitable for sky wave propagation and why? Over which range of frequencies can communication through free space using radio waves take place? [3 marks]

Solution:

Concept:

- Radio waves can reach a receiver by ground-wave, sky-wave or space-wave propagation, depending on their frequency and the path between the stations.
- The ionosphere contains free electrons and ions. Its refractive behaviour can bend suitable radio waves back towards the Earth's surface.
- Frequency and wavelength obey $c = f\lambda$ in free space, where c is the speed of electromagnetic waves, f the frequency and λ the wavelength.

Step 1: Define sky-wave propagation In sky-wave propagation, a radio wave transmitted upwards from an antenna is returned towards the Earth by the ionosphere and reaches a distant receiving antenna. At school level this is described as reflection by the ionosphere; physically, the wave is progressively refracted through the ionised layers. This return path allows reception beyond the direct line of sight and hence beyond the ordinary horizon of the transmitting antenna. Repeated returns between the Earth and the ionosphere can extend the communication distance further.

Step 2: State the suitable band and explain the upper limit The standard band suitable for sky-wave communication is approximately

$$3\,\text{MHz to } 30\,\text{MHz}.$$

These high-frequency radio waves can be returned to the Earth by the ionosphere rather than travelling only along the ground. At frequencies much above $30\,\text{MHz}$, the ionosphere generally does not provide sufficient bending to return the waves; they pass through into space. Consequently this higher-frequency region is ordinarily used for line-of-sight or satellite links. Lower-frequency waves suffer different absorption and ground-propagation effects, so they are not the standard high-frequency sky-wave band. The exact usable frequency also depends on ionisation, time of day and the angle of incidence.

Step 3: State the broader radio communication range Communication through free space using radio waves can take place over the much broader approximate range

$$20\,\text{kHz to } 300\,\text{GHz}.$$

This is the overall radio-frequency communication range, not the frequency band of a single propagation mechanism. Within it, different bands use ground, sky or space paths as appropriate. In particular, the higher-frequency waves that escape the ionosphere can still communicate with satellites or receivers within direct line of sight.

Step 4: Conclusion Sky waves return to the Earth through the ionosphere, enabling long-distance communication. The suitable sky-wave band is approximately 3–$30\,\text{MHz}$, while the overall free-space radio communication range is approximately $20\,\text{kHz}$–$300\,\text{GHz}$.

Quick Tip to Solve: Keep the two requested ranges separate: use 3–$30\,\text{MHz}$ for the ionospheric sky-wave band and $20\,\text{kHz}$–$300\,\text{GHz}$ for the broader radio communication range.

20. (a) Draw a graph showing variation of photocurrent with anode potential for a particular intensity of incident radiation. Mark saturation current and stopping potential.

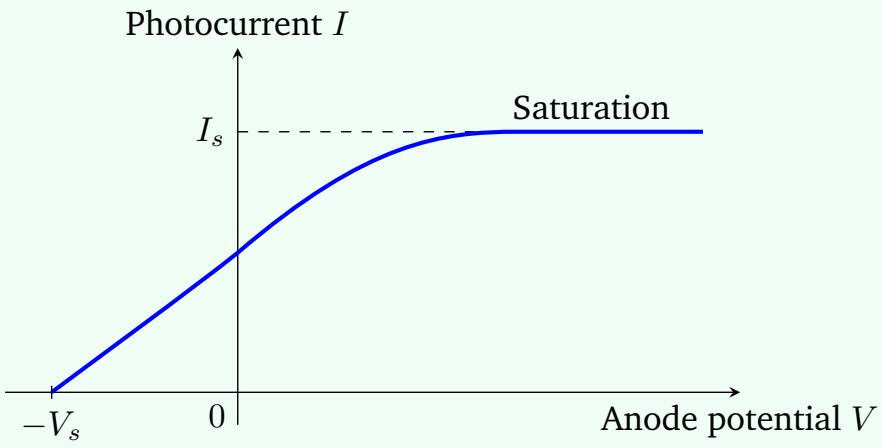
(b) By how much would the stopping potential for a given photosensitive surface go up if the frequency of the incident radiations were to be increased from 4×10^{15} Hz to 8×10^{15} Hz? [3 marks]

Solution:

Concept:

- At fixed incident intensity and frequency, increasing positive anode potential collects more emitted electrons until saturation current I_s is reached.
- The stopping potential V_s is the magnitude of negative anode potential that stops the fastest photoelectrons: $eV_s = K_{\max}$.
- Einstein's photoelectric equation is $eV_s = h\nu - \phi$, where h is Planck's constant, ν the frequency and ϕ the surface work function.

Step 1: Draw and interpret the photocurrent curve For part (a), put anode potential on the horizontal axis and photocurrent on the vertical axis. A positive anode attracts photoelectrons; the current rises as more emitted electrons reach it. At a sufficiently large positive potential all emitted electrons are collected, so the current approaches the horizontal saturation level I_s. At zero anode potential some electrons still reach the anode because of their initial kinetic energy, so current is nonzero. A negative anode potential retards them. Current falls to zero at $-V_s$, where even the fastest electrons are just prevented from reaching the anode. The sketch assumes fixed intensity and frequency above threshold.



Photocurrent at fixed intensity; V_s is the magnitude of stopping potential.

Step 2: Eliminate the unchanged work function For part (b), the same photosensitive surface has the same work function at both frequencies. Apply Einstein's equation and subtract:

$$eV_{s1} = h\nu_1 - \phi,$$
$$eV_{s2} = h\nu_2 - \phi,$$
$$e(V_{s2} - V_{s1}) = h(\nu_2 - \nu_1).$$

The increase in stopping-potential magnitude thus depends on frequency change, not on the unknown work function or intensity.

Quick Tip to Solve: For the same surface, subtract the two photoelectric equations first: the work function cancels, leaving $\Delta V_s = h\Delta\nu/e$.

Step 3: Substitute the frequency change The difference between 8×10^{15} Hz and 4×10^{15} Hz is 4×10^{15} Hz. Using $h = 6.63 \times 10^{-34}$ J s and $e = 1.6 \times 10^{-19}$ C,

$$\Delta V_s = \frac{h\Delta\nu}{e} = \frac{(6.63 \times 10^{-34})(4 \times 10^{15})}{1.6 \times 10^{-19}} = 16.575\,\text{V}.$$

Joule per coulomb gives volts. The anode potential required to stop the electrons becomes more negative by this amount.

Step 4: Conclusion The graph has saturation current I_s and a negative-potential intercept at $-V_s$. The stopping-potential magnitude increases by approximately $\boxed{16.6\,\text{V}}$.

Quick Tip to Solve: For the same surface, subtract the two photoelectric equations first: the work function cancels, leaving $\Delta V_s = h\Delta\nu/e$.

21. A source of ac voltage $v = v_0 \sin \omega t$ is connected across a pure inductor of inductance L. Derive the expressions for the instantaneous current in the circuit. Show that average power dissipated in the circuit is zero. [3 marks]

Solution:

Concept:

- A pure inductor has negligible resistance. Its induced emf opposes the change of current; the applied voltage balances this emf, so $v = L\,di/dt$.
- Inductive reactance is $X_L = \omega L$, where ω is angular frequency and L is inductance. The current amplitude is $i_0 = v_0/X_L$.
- Instantaneous power supplied is $p = vi$. Its average over a complete period $T = 2\pi/\omega$ measures the net rate of energy transfer.

Step 1: Integrate the voltage-current equation For the specified applied voltage $v = v_0 \sin \omega t$, Kirchhoff's loop equation gives

$$L\frac{di}{dt} = v_0 \sin \omega t.$$

Separating the differential and integrating,

$$di = \frac{v_0}{L} \sin \omega t\, dt,$$
$$i(t) = -\frac{v_0}{\omega L} \cos \omega t + C.$$

For the usual purely alternating steady-state current, the mean current is zero, hence $C = 0$. A nonzero constant would represent an additional initial dc current in an ideal lossless inductor; it is absent in the standard ac solution.

Step 2: Identify the amplitude and phase difference Use $-\cos \omega t = \sin(\omega t - \pi/2)$ to express the current in the same sinusoidal convention as the voltage:

$$i(t) = \frac{v_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) = i_0 \sin\left(\omega t - \frac{\pi}{2}\right).$$

Quick Tip to Solve: Start from $v = L\,di/dt$, then integrate once. Rewrite minus cosine as a delayed sine to read the current lag immediately.

Thus current lags applied voltage by $\pi/2$, or a quarter-cycle, and its amplitude is $i_0 = v_0/(\omega L)$. Increasing frequency increases the opposition to current even though resistance is zero.

Step 3: Average the instantaneous power over one cycle Multiply voltage and current, then use $2\sin\theta\cos\theta = \sin 2\theta$:

$$p(t) = -v_0 i_0 \sin\omega t \cos\omega t = -\frac{v_0 i_0}{2}\sin 2\omega t.$$

Therefore

$$P_{\text{av}} = \frac{1}{T}\int_0^T p(t)\,dt = -\frac{v_0 i_0}{2T}\int_0^T \sin 2\omega t\,dt = 0.$$

Positive and negative power contributions cancel. Energy is stored in the magnetic field, $U = Li^2/2$, when current magnitude rises, and returned to the source when it falls. There is no resistive heating in a pure inductor.

Step 4: Conclusion The instantaneous ac current is $\boxed{i = (v_0/\omega L)\sin(\omega t - \pi/2)}$, lagging voltage by $90°$. The average power dissipated over a full cycle is $\boxed{0}$.

Quick Tip to Solve: Start from $v = L\,di/dt$, then integrate once. Rewrite minus cosine as a delayed sine to read the current lag immediately.

22. Using Biot-Savart law, deduce the expression for the magnetic field at a point (x) on the axis of a circular current carrying loop of radius R. How is the direction of the magnetic field determined at this point? [3 marks]

Solution:

Concept:

- Biot-Savart law gives $d\vec{B} = (\mu_0 I/4\pi)(d\vec{\ell}\times\hat{r})/r^2$, where I is current and $\vec{r}$ points from the current element to the observation point.
- On the axis of a circular loop, all elements have the same distance $r = \sqrt{R^2+x^2}$ from the point. Rotational symmetry cancels components perpendicular to the axis.
- Axial components add, and their direction is given by the right-hand grip rule: curled fingers follow current and the thumb indicates the field along the axis.

Step 1: Set up the geometry at the axial point Let O be the loop centre, R its radius, and P the observation point a distance x from O along the axis. For an element of length $d\ell$, the displacement to P has radial and axial components. The tangent $d\vec{\ell}$ is perpendicular to both these components, so its angle with $\vec{r}$ is $90°$. Thus Biot-Savart law gives the element's field magnitude

$$dB = \frac{\mu_0 I}{4\pi}\frac{d\ell}{R^2+x^2}.$$

This field is perpendicular to $\vec{r}$. If α is the angle between $d\vec{B}$ and the axis, geometry gives $\cos\alpha = R/r$.

Step 2: Add only the surviving axial components Each element has a diametrically opposite partner. Their transverse magnetic field components are equal and opposite, while their axial components

Quick Tip to Solve: First cancel the transverse components by symmetry; then multiply the elemental field by $R/\sqrt{R^2+x^2}$ before integrating the circumference.

point in the same direction. The total field is therefore purely axial. Project each elemental field:

$$dB_{\text{axis}} = dB\frac{R}{r} = \frac{\mu_0 I R\, d\ell}{4\pi(R^2+x^2)^{3/2}}.$$

Since R and r are constant around the loop, integrating its circumference gives

$$B(x) = \frac{\mu_0 I R}{4\pi(R^2+x^2)^{3/2}} \oint d\ell,$$

$$\oint d\ell = 2\pi R,$$

$$B(x) = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}.$$

At $x = 0$, this reduces to $\mu_0 I/(2R)$, checking the result.

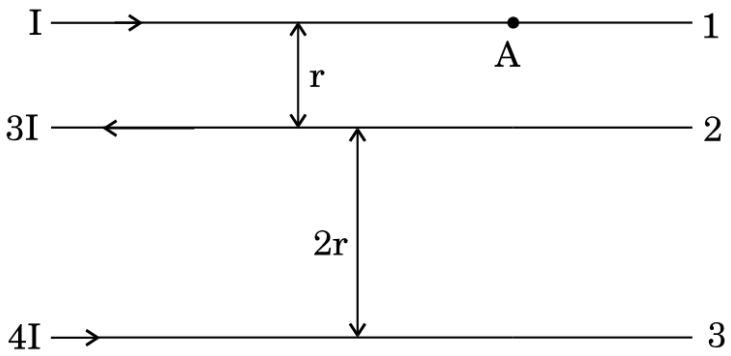
Step 3: Determine the field direction Curl the fingers of the right hand in the direction of current around the loop. The extended thumb points along its magnetic field on the axis. Viewed from P, an anticlockwise current gives a field towards the observer; a clockwise current gives a field away from the observer. Reversing current reverses the field.

Step 4: Conclusion For a single circular loop, $\boxed{B(x) = \mu_0 I R^2/[2(R^2+x^2)^{3/2}]}$. The field lies on the loop axis and its direction follows the right-hand grip rule.

Quick Tip to Solve: First cancel the transverse components by symmetry; then multiply the elemental field by $R/\sqrt{R^2+x^2}$ before integrating the circumference.

OR

22. The figure shows three infinitely long straight parallel current carrying conductors. Find the
(i) magnitude and direction of the net magnetic field at point A lying on conductor 1,
(ii) magnetic force on conductor 2. [3 marks]



Solution:

Concept:

- An infinitely long straight wire carrying current I_w produces a magnetic field $B = \mu_0 I_w/(2\pi d)$ at perpendicular distance d. The right-hand thumb rule gives its direction.
- Magnetic fields add vectorially. At a point on a conductor, the external field is found from the other conductors; its own ideal line-current field is not included.
- The force per unit length between two parallel currents is $F/L = \mu_0 I_a I_b/(2\pi d)$. Like-directed currents attract and opposite-directed currents repel.

Step 1: (i) Find the external fields at A The figure gives $I_1 = I$ to the right, $I_2 = 3I$ to the left, and $I_3 = 4I$ to the right. The separation between wires 1 and 2 is r, while that between wires 2 and 3 is $2r$. Therefore A, on wire 1, is $3r$ above wire 3. For the left-directed current in wire 2, the field above the wire is into the page. For the right-directed current in wire 3, the field above the wire is out of the page. Thus

$$B_{2A} = \frac{3\mu_0 I}{2\pi r}, \qquad B_{3A} = \frac{4\mu_0 I}{2\pi(3r)}.$$

These fields oppose each other. Subtract the smaller outward field from the larger inward field:

$$B_A = \frac{\mu_0 I}{2\pi r}\left(3 - \frac{4}{3}\right) = \frac{5\mu_0 I}{6\pi r}.$$

The net external magnetic field is into the plane of the page.

Step 2: (ii) Combine the forces on conductor 2 Wire 2 carries current opposite to both wires 1 and 3, so both interactions are repulsive. Wire 1 is above wire 2 and pushes it downward; wire 3 is below wire 2 and pushes it upward. For a length L of conductor 2, the two forces per unit length are

$$\frac{F_{21}}{L} = \frac{\mu_0(3I)(I)}{2\pi r} = \frac{3\mu_0 I^2}{2\pi r} \quad \text{downward},$$
$$\frac{F_{23}}{L} = \frac{\mu_0(3I)(4I)}{2\pi(2r)} = \frac{6\mu_0 I^2}{2\pi r} \quad \text{upward}.$$

The upward force is larger. Hence the resultant is

$$\frac{F_{2,\text{net}}}{L} = \frac{6\mu_0 I^2 - 3\mu_0 I^2}{2\pi r} = \frac{3\mu_0 I^2}{2\pi r},$$

directed upward, towards conductor 1. Since no finite wire length is specified, force per unit length is the physically meaningful result; for a segment of length L, multiply it by L.

Step 3: Conclusion (i) The net external field at A is $\boxed{5\mu_0 I/(6\pi r)}$, into the page, in tesla when SI values are used. (ii) The net force per unit length on conductor 2 is $\boxed{3\mu_0 I^2/(2\pi r)}$, upward towards conductor 1, in N m^{-1}. A length L experiences $F_2 = 3\mu_0 I^2 L/(2\pi r)$.

Quick Tip to Solve: Use $3r$, not $2r$, for the distance from wire 3 to A; draw the two repulsive force arrows on wire 2 before subtracting their magnitudes.

Section D (4 marks)

23. Sunil and his parents were travelling to their village in their car. On the way his mother noticed some grey coloured panels installed on the roof of a low building. She enquired from Sunil what those panels were and Sunil told his mother that those were solar panels.

(a) What were the values displayed by Sunil and his mother? State one value for each.

(b) In what way would the use of solar panels prove to be very useful?

(c) Name the semiconductor device used in solar panels. Briefly explain with the help of a diagram, how this device works. [4 marks]

Solution:

Concept:

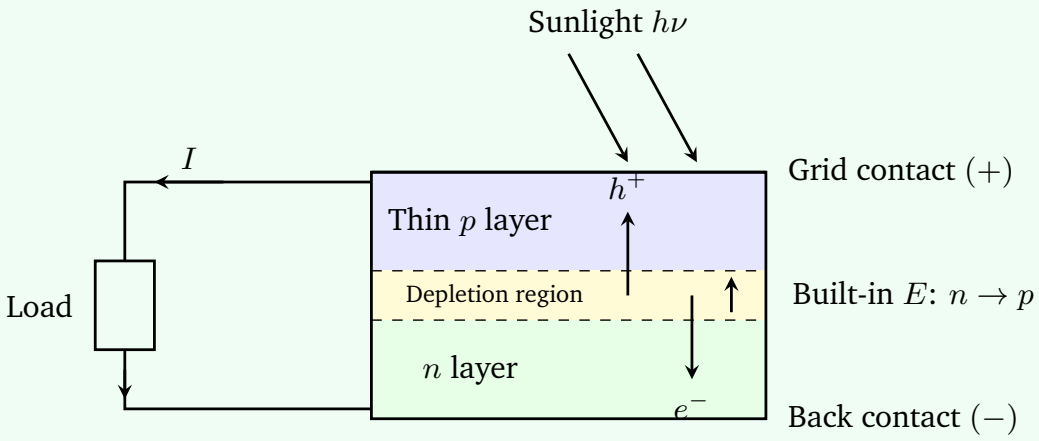
- A solar cell is a large-area p-n junction that converts light energy into electrical energy by the photovoltaic effect, without an external bias supply.
- Photons with energy $h\nu \geq E_g$, where E_g is the semiconductor band gap, create electron-hole pairs. The built-in electric field of the depletion region separates these charge carriers.
- Solar energy is renewable. Using it to generate electricity conserves fossil fuels and reduces pollution during electricity generation.

Step 1: (a) Identify a value shown by each person Sunil shows scientific awareness: he recognises the panels and explains their purpose to his mother. His mother shows curiosity, or an enquiring attitude: she notices an unfamiliar device and asks what it is. These values are supported directly by the incident; no additional behaviour needs to be assumed.

Step 2: (b) Explain the usefulness of solar panels Solar panels convert freely available sunlight into electricity. They provide a renewable source of energy and can supply power to homes and villages, including places where a grid connection is difficult. Their use reduces dependence on coal, oil and other exhaustible fuels and avoids smoke and greenhouse-gas emissions during operation. Thus they help conserve conventional energy resources while providing useful electrical power. Several cells are connected in a panel to obtain a suitable voltage and current for the required load.

Step 3: (c) Name the device and explain carrier separation The semiconductor device is a **solar cell**, commonly made from silicon. A thin p-type layer above an n-type layer forms a large-area junction. A metallic grid on the illuminated surface allows sunlight to enter and collects charge; a back contact provides the other terminal. Light absorbed in or near the depletion region generates electron-hole pairs. The built-in field points from the positively charged donor ions on the n side towards the negatively charged acceptor ions on the p side. It drives electrons to the n region and holes to the p region, preventing their immediate recombination. This separation produces a potential difference: the p terminal is positive relative to the n terminal.

Quick Tip to Solve: In a solar-cell diagram, label the illuminated junction, the carrier directions and the load; remember that the p contact is positive and supplies conventional current to the load.



Solar cell under illumination: holes move to p, electrons to n, and conventional current flows through the external load from p to n.

Step 4: Explain how the cell supplies a load Connecting an external load allows conventional current to flow from the positive p contact through the load to the negative n contact. Electron flow through the external circuit is opposite to this current. Continued illumination creates more electron-hole pairs, maintaining the output current and allowing light energy to be delivered as electrical energy to the load. The diagram shows carrier separation inside the cell and conventional current in the external circuit; the built-in field is an internal junction field, not an applied battery field.

Step 5: Conclusion Sunil displays scientific awareness and his mother displays curiosity. Solar panels provide renewable, comparatively clean electricity and conserve fossil fuels. Their working device is the solar cell: light creates electron-hole pairs, the junction field separates them, and the resulting photovoltage drives current through an external load.

Quick Tip to Solve: In a solar-cell diagram, label the illuminated junction, the carrier directions and the load; remember that the p contact is positive and supplies conventional current to the load.

Section E (15 marks)

24. (a) A point object is placed on the principal axis of a convex spherical surface of radius of curvature R, which separates the two media of refractive indices n_1 and n_2 $(n_2 > n_1)$. Draw the ray diagram and deduce the relation between the object distance (u), image distance (v) and the radius of curvature (R) for refraction to take place at the convex spherical surface from rarer to denser medium.

(b) A converging lens has a focal length of 20 cm in air. It is made of a material of refractive index $1 \cdot 6$. If it is immersed in a liquid of refractive index $1 \cdot 3$, find its new focal length. [5 marks]

Solution:

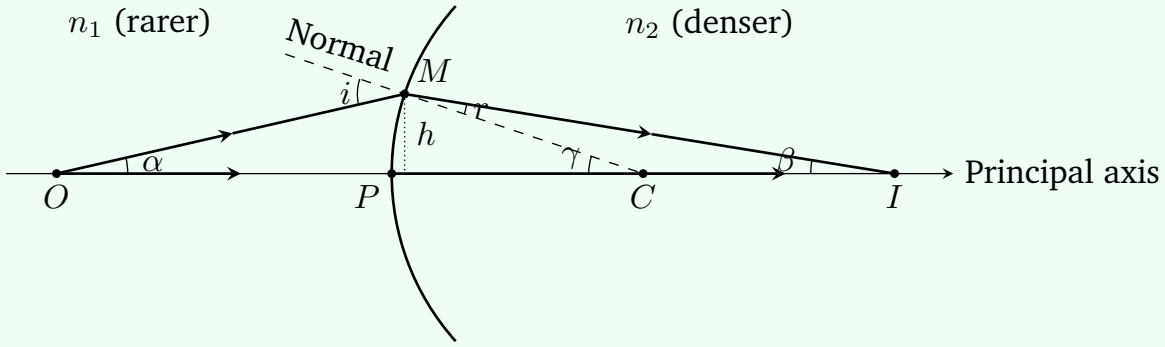
Concept:

- For refraction at a spherical surface, Snell's law is $n_1 \sin i = n_2 \sin r$. For paraxial rays, all angles are small, so $\sin\theta \simeq \tan\theta \simeq \theta$ in radians.

Quick Tip to Solve: For immersion of an unchanged lens, take the ratio of the lens-maker equations so the two radii cancel; use the relative index $n_{\text{lens}}/n_{\text{liquid}}$.

- Use the Cartesian sign convention: measure distances from the pole P, with positive direction along the incident light. For a real object on the left and a convex surface with centre on the right, $u < 0$ and $R > 0$.
- The lens-maker's relation in a surrounding medium is $1/f = (n_{\text{lens}}/n_{\text{medium}} - 1)(1/R_1 - 1/R_2)$. The lens curvatures stay fixed when it is immersed.

Step 1: (a) Draw the refracted ray and identify the normal Let O be the point object, P the pole, C the centre of curvature and I the real image in the illustrated configuration. A paraxial ray from O strikes the spherical surface at M, a small height h above the principal axis. The radius MC is the normal at M; the refracted ray bends towards this normal because $n_2 > n_1$. The axial ray passes through P without deviation. The intersection of the two refracted rays locates I. Let α, β and γ be the small positive angles made with the principal axis by OM, MI and MC, respectively. The diagram represents the real-image case; the resulting signed equation also applies when the image is virtual.



Paraxial refraction from n_1 to $n_2 > n_1$. The radius MC is the normal; the drawing is schematic.

Step 2: (a) Apply Snell's law to the small angles From the ray geometry, the angle of incidence is $i = \alpha + \gamma$, while the angle of refraction is $r = \gamma - \beta$. Applying Snell's law in the paraxial approximation gives

$$n_1(\alpha + \gamma) = n_2(\gamma - \beta),$$
$$n_1\alpha + n_2\beta = (n_2 - n_1)\gamma.$$

For a small aperture, the axial displacement of M from P is negligible to first order. Therefore, in terms of positive geometric lengths,

$$\alpha \simeq \frac{h}{OP}, \qquad \beta \simeq \frac{h}{PI}, \qquad \gamma \simeq \frac{h}{PC}.$$

Substitute these expressions and cancel the common nonzero height h:

$$\frac{n_1}{OP} + \frac{n_2}{PI} = \frac{n_2 - n_1}{PC}.$$

Step 3: (a) Introduce the signed distances With light travelling from left to right, the object is on the negative side of the pole, so $OP = -u$. For the illustrated real image, $PI = v$, and the centre of this

Quick Tip to Solve: For immersion of an unchanged lens, take the ratio of the lens-maker equations so the two radii cancel; use the relative index $n_{\text{lens}}/n_{\text{liquid}}$.

convex surface lies on the positive side, so $PC = R$. Hence the spherical refraction relation is

$$\boxed{\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}}.$$

This is a paraxial relation. In another image configuration the appropriate signed value of v is inserted in the same equation; one must not replace u by a positive object-distance magnitude in the boxed expression.

Step 4: (b) Compare the lens powers in air and in liquid Write $K = 1/R_1 - 1/R_2$, which depends only on the unchanged lens geometry. Taking the refractive index of air as unity, the original focal length $f_{\text{air}} = 20\,\text{cm}$ gives

$$\frac{1}{f_{\text{air}}} = (1.6 - 1)K = 0.6K.$$

In the liquid, the relative refractive index is $1.6/1.3 = 16/13$. Thus

$$\frac{1}{f_{\text{liquid}}} = \left(\frac{16}{13} - 1\right) K = \frac{3}{13}K.$$

Dividing the two expressions eliminates the unknown radii:

$$\frac{f_{\text{liquid}}}{f_{\text{air}}} = \frac{0.6}{3/13} = 2.6,$$
$$f_{\text{liquid}} = 2.6 \times 20\,\text{cm} = 52\,\text{cm}.$$

The lens remains converging because its refractive index still exceeds that of the liquid. Its power decreases because the refractive-index contrast is reduced, so its positive focal length increases.

Step 5: Conclusion (a) For paraxial refraction at the convex spherical surface, $n_2/v - n_1/u = (n_2 - n_1)/R$, with Cartesian signed distances. (b) The new focal length is $\boxed{+52\,\text{cm}}$; the lens remains converging.

Quick Tip to Solve: For immersion of an unchanged lens, take the ratio of the lens-maker equations so the two radii cancel; use the relative index $n_{\text{lens}}/n_{\text{liquid}}$.

OR

24. (a) Draw the ray diagram showing refraction of light through a glass prism and hence obtain the relation between the refractive index μ of the prism, angle of prism and angle of minimum deviation.
(b) Determine the value of the angle of incidence for a ray of light travelling from a medium of refractive index $\mu_1 = \sqrt{2}$ into the medium of refractive index $\mu_2 = 1$, so that it just grazes along the surface of separation. [5 marks]

Solution:

Concept:

- Snell's law at a plane boundary is $n_1 \sin i = n_2 \sin r$, where i and r are measured from the normal, and n_1, n_2 are the refractive indices of the incident and refracting media.
- For a prism in air, A is the angle between its refracting faces, r_1, r_2 are the internal refraction angles, and i, e are the external incidence and emergence angles. Geometry gives $r_1 + r_2 = A$ and deviation $\delta = i + e - A$.
- At minimum deviation, the ray passes symmetrically through the prism: $i = e$ and $r_1 = r_2$. For light travelling from a denser to a rarer medium, grazing emergence corresponds to the critical angle of incidence.

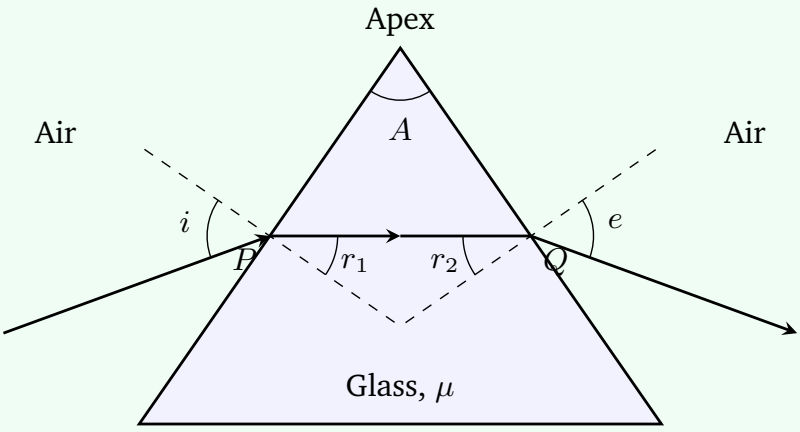
Step 1: (a) Draw the prism ray diagram and establish its angle relations Let the ray enter the first face at P and leave the second at Q. The dashed lines through these points are normals to the respective faces. The ray bends towards the normal at entry and away from it at exit. In the diagram, the internal path is shown for the symmetric, minimum-deviation position. Angles i, e, r_1, r_2 are all measured relative to the appropriate normal. The angle between the inward normals is $180^\circ - A$. The quadrilateral formed by the two refracting faces and these normals therefore gives

$$r_1 + r_2 = A.$$

The change in ray direction at the first face is $i - r_1$, and at the second face it is $e - r_2$. Thus the total deviation is

$$\delta = (i - r_1) + (e - r_2) = i + e - A.$$

Quick Tip to Solve: At minimum deviation, first write $r = A/2$ and $i = (A + \delta_m)/2$; for a grazing refracted ray, set its angle to the normal equal to 90°.



Symmetric ray path through a prism at minimum deviation. Dashed lines are normals; $i = e$, $r_1 = r_2 = A/2$, and $\delta_m = i + e - A$.

Step 2: (a) Use the minimum-deviation condition and Snell's law As the prism is rotated, the deviation decreases to its minimum value δ_m and then increases. At this position, reversibility of the symmetric ray path gives $i = e$. Applying Snell's law at the two faces also gives $\sin i = \mu \sin r_1$ and $\sin e = \mu \sin r_2$. Hence $r_1 = r_2$ for the physical internal angles. Consequently,

$$r_1 = r_2 = \frac{A}{2}, \qquad \delta_m = 2i - A, \qquad i = \frac{A + \delta_m}{2}.$$

Here μ denotes the refractive index of the glass relative to air. Substitution into Snell's law at the first face yields

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}.$$

Step 3: (b) Apply the grazing-emergence condition The incident medium has $\mu_1 = \sqrt{2}$, while the second has $\mu_2 = 1$; therefore the ray travels from denser to rarer medium. A refracted ray just grazing the boundary has refraction angle $r = 90^\circ$ relative to the normal. Snell's law gives

$$\begin{aligned} \mu_1 \sin i_c &= \mu_2 \sin 90^\circ, \\ \sin i_c &= \frac{1}{\sqrt{2}}, \\ i_c &= 45^\circ. \end{aligned}$$

This is the critical angle. At a smaller incidence angle the ray enters the second medium at an angle less than 90°; at a larger one, total internal reflection occurs. The requested incidence angle is measured from the normal to the boundary.

Step 4: Conclusion (a) For a glass prism in air, $\mu = \dfrac{\sin[(A + \delta_m)/2]}{\sin(A/2)}$, where A is the prism angle and δ_m is the minimum deviation. (b) The incidence angle for grazing emergence is $\boxed{45^\circ}$.

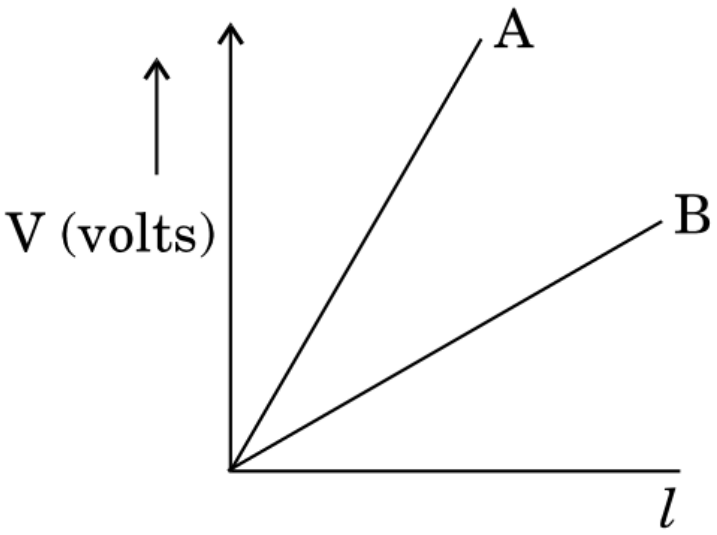
Quick Tip to Solve: At minimum deviation, first write $r = A/2$ and $i = (A + \delta_m)/2$; for a grazing refracted ray, set its angle to the normal equal to 90°.

25. (a) (i) State the principle on which a potentiometer works. How can a given potentiometer be made more sensitive?

(ii) In the graph shown below for two potentiometers, state with reason which of the two potentiometers, A or B, is more sensitive.

(b) Two metallic wires, P_1 and P_2 of the same material and same length but different cross-sectional areas, A_1 and A_2 are joined together and connected to a source of emf. Find the ratio of the drift velocities of free electrons in the two wires when they are connected (i) in series, and (ii) in parallel.

[5 marks]



Solution:

Concept:

- A uniform potentiometer wire carrying a steady current has a constant potential gradient $k = V/L$, where V is the potential difference across its total length L. Thus the potential drop across length l is kl.
- At the null point, the emf under measurement satisfies $\varepsilon = kl$. A smaller potential gradient gives a larger balancing length and allows smaller potential differences to be resolved.
- The current magnitude in a metallic wire is $I = neAv_d$, where n is the free-electron number density, e the elementary charge, A the cross-sectional area and v_d the drift-speed magnitude. Also, $R = \rho L/A$, where ρ is resistivity.

Step 1: State the potentiometer principle and increase sensitivity (a)(i) For a uniform wire of constant cross-sectional area, its resistance per unit length is constant. When a steady current flows through it, Ohm's law therefore makes the potential difference between two points directly proportional to their separation along the wire:

$$V_l = I\frac{\rho l}{A} = kl.$$

This is the working principle of a potentiometer. At balance no current flows through the galvanometer branch, so the measured emf equals this potential drop without drawing current from the test cell.

For a given potentiometer, decrease its potential gradient by decreasing the current in its wire, for example by increasing the series rheostat resistance in the primary circuit. The supply must still

Quick Tip to Solve: On a potentiometer V-versus-l graph, the smaller slope means greater sensitivity. For drift speed, first identify what is common: current in series or voltage in parallel.

provide enough total potential drop to balance the emf being measured. Since $l = \varepsilon/k$, a small change $\Delta\varepsilon$ then produces a larger displacement $\Delta l = \Delta\varepsilon/k$, making the null point easier to distinguish. A longer wire at the same total potential drop also reduces the gradient when choosing the instrument.

Step 2: Compare the slopes of the two graphs (a)(ii) The ordinate is potential difference V and the abscissa is balancing length l. Therefore the slope V/l represents the potential gradient, rather than sensitivity itself. The straight line for B is less steep than that for A, so $k_B < k_A$. For the same emf, the balancing length is consequently greater on B. Thus B can detect a smaller voltage change for the same readable length change.

Step 3: Use the common current in series (b)(i) Both wires are made of the same material, so their free-electron number densities are equal. In a series combination the same current I passes through both. Applying $I = neAv_d$ to each wire gives

$$v_{d1} = \frac{I}{neA_1}, \qquad v_{d2} = \frac{I}{neA_2}, \qquad \frac{v_{d1}}{v_{d2}} = \frac{A_2}{A_1}.$$

A thinner wire has a greater drift speed because fewer charge carriers cross its section, while the same charge per second must pass through it.

Step 4: Use the common potential difference in parallel (b)(ii) Each branch has the same potential difference V. The equal length L and common resistivity give

$$I_1 = \frac{VA_1}{\rho L}, \qquad I_2 = \frac{VA_2}{\rho L}.$$

Substituting these currents in the drift-speed relation,

$$v_{d1} = \frac{V}{ne\rho L} = v_{d2}.$$

The greater current in the thicker parallel wire is exactly compensated by its greater area. Equivalently, both wires have the same electric field V/L, and electrons in the same material have the same drift-speed response to that field. The quoted ratios concern speed magnitudes; electron drift is opposite to conventional current.

Step 5: Conclusion The potentiometer works on $V_l \propto l$ for a uniform wire carrying steady current. Reduce its potential gradient to increase sensitivity; the smaller graph slope makes B more sensitive. The drift-speed ratios are $v_{d1} : v_{d2} = A_2 : A_1$ in series and $1 : 1$ in parallel.

Quick Tip to Solve: On a potentiometer V-versus-l graph, the smaller slope means greater sensitivity. For drift speed, first identify what is common: current in series or voltage in parallel.

OR

25. (a) Define the capacitance of a capacitor. Obtain the expression for the capacitance of a parallel plate capacitor in vacuum in terms of plate area A and separation d between the plates.

(b) A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor but has a thickness $\frac{3d}{4}$. Find the ratio of the capacitance with dielectric inside it to its capacitance without the dielectric. [5 marks]

Solution:

Concept:

- Capacitance is the ratio $C = Q/V$, where Q is the magnitude of the charge on either plate and V the potential difference between the plates. Its SI unit is the farad, F = C/V.
- For large, closely spaced parallel plates, edge effects may be neglected. The field between plates in vacuum is $E_0 = \sigma/\varepsilon_0$, where $\sigma = Q/A$ and ε_0 is vacuum permittivity.
- A dielectric of relative permittivity K reduces the field in its region to E_0/K for a given free surface charge. The total potential difference is the sum of the drops across the vacuum and dielectric regions.

Step 1: Define capacitance and find the vacuum field (a) If equal and opposite charges $+Q$ and $-Q$ on a capacitor produce a potential difference V, its capacitance is $C = Q/V$. It measures the charge stored per unit potential difference and depends on geometry and the intervening medium for an ideal capacitor.

Consider two parallel conducting plates, each of area A, separated by a vacuum gap d. Assume that d is small compared with the plate dimensions, so the central field is uniform and fringing is negligible. The surface charge-density magnitudes are $\sigma = Q/A$. Each large charged plate contributes a field $\sigma/(2\varepsilon_0)$; between the oppositely charged plates these contributions reinforce. Hence

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}.$$

The electric field points from the positive plate to the negative plate.

Step 2: Relate the potential drop to the plate charge For a uniform field, the magnitude of the potential difference across the gap is

$$V_0 = E_0 d = \frac{Qd}{\varepsilon_0 A}.$$

Therefore the vacuum capacitance is

$$C_0 = \frac{Q}{V_0} = \frac{\varepsilon_0 A}{d}.$$

The cancellation of Q shows that capacitance is fixed by plate area, separation and permittivity, rather than by the particular charge used to measure it. Increasing area increases charge storage, while increasing separation increases the potential difference needed for the same charge.

Step 3: Add the drops across the dielectric and vacuum (b) The printed slab thickness is $t = 3d/4$; therefore the total remaining vacuum thickness is $d - t = d/4$. Because the slab covers the full plate area, these layers lie successively along the field. For the same free charge Q, the field magnitude is

Quick Tip to Solve: For a slab that covers the whole plate area, replace the gap by the effective thickness $(d - t) + t/K$. Insert the printed $t = 3d/4$ before simplifying the ratio.

$Q/(\varepsilon_0 A)$ in vacuum and $Q/(K\varepsilon_0 A)$ in the dielectric. Thus

$$V = \frac{Q}{\varepsilon_0 A}(d-t) + \frac{Q}{K\varepsilon_0 A}t$$
$$= \frac{Qd}{4\varepsilon_0 A}\left(1 + \frac{3}{K}\right).$$

This also applies if the slab leaves vacuum gaps on both sides: only their combined thickness enters the sum. The dielectric is assumed to fill the area uniformly without a conducting path between the plates.

Step 4: Obtain the capacitance and compare it with vacuum Using $C = Q/V$, the effective capacitance is

$$C = \frac{4\varepsilon_0 A}{d(1+3/K)} = \frac{4K\varepsilon_0 A}{d(K+3)}.$$

Dividing by $C_0 = \varepsilon_0 A/d$ gives

$$\frac{C}{C_0} = \frac{4K}{K+3}.$$

As a consistency check, $K = 1$ reproduces the vacuum capacitor. For an ordinary dielectric with $K > 1$, the ratio exceeds unity. Even as K becomes very large it approaches 4, because the remaining vacuum gap still contributes a finite potential drop.

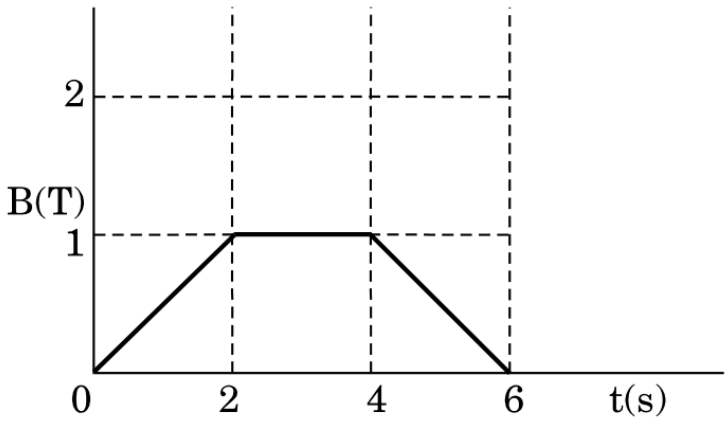
Step 5: Conclusion Capacitance is $C = Q/V$, measured in farads, and the parallel-plate vacuum value is $C_0 = \varepsilon_0 A/d$. With the printed dielectric thickness $3d/4$, the required ratio is $\boxed{C/C_0 = 4K/(K+3)}$.

Quick Tip to Solve: For a slab that covers the whole plate area, replace the gap by the effective thickness $(d-t) + t/K$. Insert the printed $t = 3d/4$ before simplifying the ratio.

26. (a) State Faraday's law of electromagnetic induction.

(b) The magnetic field through a circular loop of wire 12 cm in radius and $8 \cdot 5\ \Omega$ resistance, changes with time as shown in the figure. The magnetic field is perpendicular to the plane of the loop. Calculate the induced current in the loop and plot it as a function of time.

(c) Show that Lenz's law is a consequence of conservation of energy. [5 marks]



Solution:

Concept:

- Faraday's law: a changing magnetic flux through a circuit induces an emf. Its magnitude equals the rate of change of flux linkage, $\varepsilon = -N\, d\Phi_B/dt$, where N is the number of turns and Φ_B is the flux through one turn.
- For a uniform field perpendicular to a single circular loop, $\Phi_B = BA$, where $A = \pi r^2$. Neglecting the self-inductance of the wire, Ohm's law gives $I = \varepsilon/R = -(A/R)\, dB/dt$.
- Lenz's law fixes the direction: the induced current produces a magnetic effect that opposes the change in magnetic flux. The opposition ensures that energy dissipated in the loop is supplied by external work.

Step 1: (a) State Faraday's law Whenever the magnetic flux linked with a circuit changes, an emf is induced in it. The induced emf exists while the flux is changing, and its magnitude is equal to the rate of change of magnetic flux linkage:

$$\varepsilon = -\frac{d(N\Phi_B)}{dt}.$$

For a fixed number of turns this is $\varepsilon = -N\, d\Phi_B/dt$. The minus sign expresses Lenz's law. In the given single loop, $N = 1$.

Step 2: (b) Calculate the loop area and choose a sign convention The radius and resistance are $r = 12\,\text{cm} = 0.12\,\text{m}$ and $R = 8.5\,\Omega$. Hence

$$A = \pi(0.12)^2 = 0.04524\,\text{m}^2.$$

Choose the positive area normal along the direction of the plotted magnetic field, and choose positive current by the right-hand rule about that normal. For example, if this normal points out of the page, positive current is anticlockwise as seen by the reader. The field direction relative to the page is not supplied, so a clockwise direction cannot be assigned without this convention. Using $I = -(A/R)\, dB/dt$, it is the slope of the field-time graph that determines the current, rather than the value of the magnetic field.

Step 3: Find the current in each time interval From 0 to $2\,\text{s}$, the field rises uniformly from 0 to $1\,\text{T}$:

$$\frac{dB}{dt} = \frac{1-0}{2-0} = 0.5\,\text{T s}^{-1}, \qquad I = -\frac{0.04524 \times 0.5}{8.5} = -2.66 \times 10^{-3}\,\text{A}.$$

The induced field opposes the increasing applied field, so the current is negative under the chosen convention. From 2 to $4\,\text{s}$, the field is constant at $1\,\text{T}$. Thus $dB/dt = 0$, the flux is constant and $I = 0$, even though the field itself is nonzero. From 4 to $6\,\text{s}$, the field falls uniformly from $1\,\text{T}$ to zero:

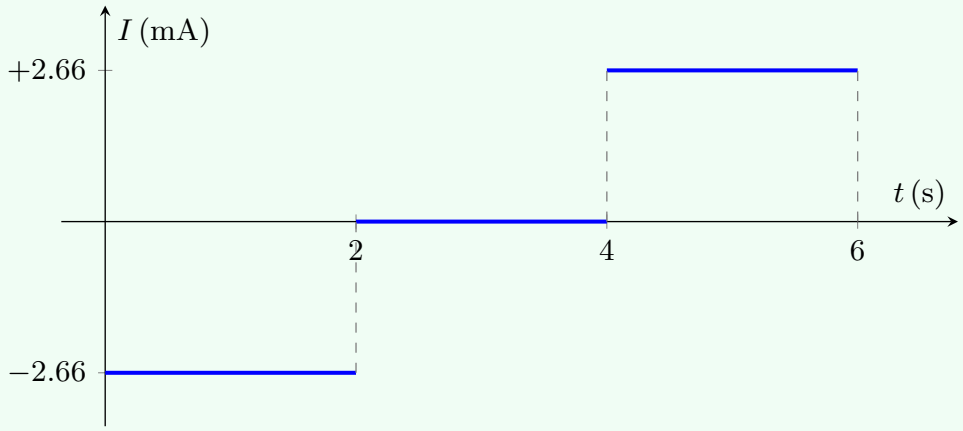
$$\frac{dB}{dt} = \frac{0-1}{6-4} = -0.5\,\text{T s}^{-1}, \qquad I = +2.66 \times 10^{-3}\,\text{A}.$$

Here the induced field acts in the direction of the applied field to oppose its decrease. Thus the two nonzero currents have equal magnitudes and opposite directions.

Step 4: Plot the signed current against time The current-time graph consists of three horizontal segments: $-2.66\,\text{mA}$ for $0 < t < 2\,\text{s}$, zero for $2 < t < 4\,\text{s}$, and $+2.66\,\text{mA}$ for $4 < t < 6\,\text{s}$. The ideal

Quick Tip to Solve: Read the slope of each straight segment of the field graph first; a horizontal field graph gives zero current, and equal opposite slopes give equal opposite currents.

graph changes level at the corners of the given field graph. Dashed vertical guides below indicate these transitions; they are not finite-duration current segments. The one-sided values specify the ideal current at each change of slope.



Signed induced current; positive direction is defined by the right-hand rule along the applied field.

Step 5: (c) Relate Lenz's law to conservation of energy Consider bringing the north pole of a magnet towards a conducting loop. By Lenz's law the near face of the loop becomes a north pole and repels the approaching magnet. An external agent must do positive work against this opposing force. That work supplies the electrical energy dissipated as heat in the loop, apart from any change in stored field energy. If the induced current instead assisted the change, the near face would become a south pole and attract the approaching north pole. The magnet would accelerate towards the loop while the current simultaneously generated heat. Both mechanical and electrical energy would then be produced without the necessary external energy supply. This contradicts conservation of energy. Similarly, on withdrawing the magnet, the induced current produces an attraction that opposes withdrawal, so external work is again needed. Therefore, the induced magnetic effect must oppose the change that produces it: Lenz's law is a consequence of conservation of energy.

Step 6: Conclusion Faraday's law is $\varepsilon = -N\, d\Phi_B/dt$. With positive current defined by the right-hand rule along the applied field, $I = -2.66\,\text{mA}$ during $0 < t < 2\,\text{s}$, $I = 0$ during $2 < t < 4\,\text{s}$, and $I = +2.66\,\text{mA}$ during $4 < t < 6\,\text{s}$. Lenz's opposition makes external work supply the induced electrical energy, preserving energy conservation.

Quick Tip to Solve: Read the slope of each straight segment of the field graph first; a horizontal field graph gives zero current, and equal opposite slopes give equal opposite currents.

OR

26. (a) Describe, with the help of a suitable diagram, the working principle of a step-up transformer. Obtain the relation between input and output voltages in terms of the number of turns of primary and secondary windings and the currents in the input and output circuits.

(b) Given the input current 15 A and the input voltage of 100 V for a step-up transformer having 90% efficiency, find the output power and the voltage in the secondary if the output current is 3 A. [5 marks]

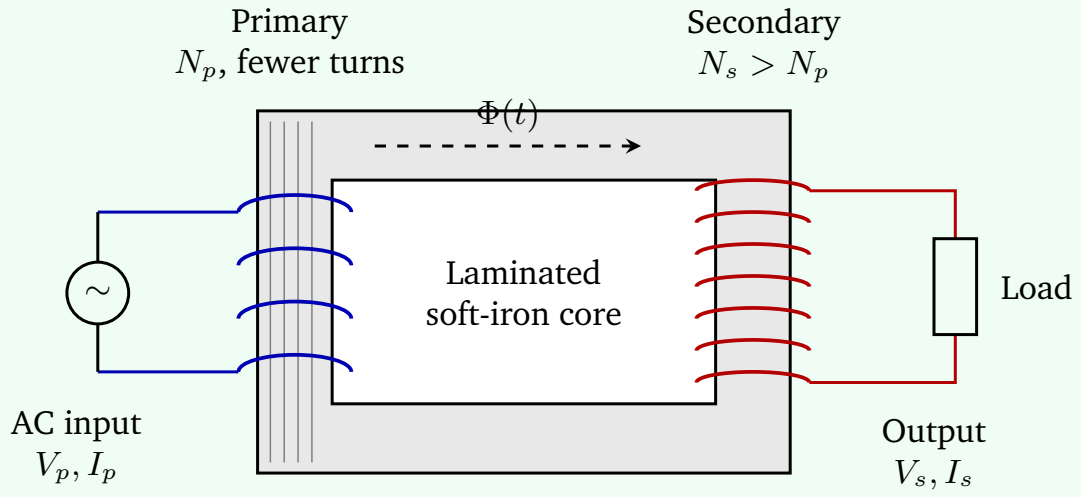
Solution:

Concept:

- A transformer works by mutual induction: alternating current in its primary winding produces changing magnetic flux, which induces an emf in the secondary winding.
- For primary and secondary turns N_p, N_s linking the same flux Φ, Faraday's law gives $e_p = -N_p\, d\Phi/dt$ and $e_s = -N_s\, d\Phi/dt$. A step-up transformer has $N_s > N_p$.
- For an ideal transformer, $V_p I_p = V_s I_s$. For a practical transformer, efficiency is $\eta = P_{\text{out}}/P_{\text{in}}$; the voltages and currents used for power here are rms values with the usual resistive-load assumption.

Step 1: (a) Construction and working principle Two electrically insulated coils are wound on a common laminated soft-iron core. The primary, with fewer turns N_p, is connected to an alternating-voltage source; the secondary, with more turns N_s, supplies the load. The core provides a path for magnetic flux linking both windings. Laminations reduce eddy-current losses, and soft iron reduces hysteresis loss.

An alternating primary current sets up alternating magnetic flux in the core. This changing flux induces an emf in each winding by Faraday's law. When the secondary circuit is closed, its induced emf drives a current through the load. Thus energy is transferred through the shared magnetic field without an electrical connection between the windings. A steady dc source cannot sustain this action because its steady flux has zero time derivative.



Step-up transformer: both windings link the alternating flux in a laminated soft-iron core; the secondary has more turns.

Step 2: (a) Obtain the voltage and current relations In the ideal approximation, all the flux links

Quick Tip to Solve: Apply the efficiency to input power first, then divide the resulting output power by the secondary current; do not apply the ideal inverse-current relation to this lossy transformer.

both coils and winding resistances are negligible. For their induced emfs,

$$e_p = -N_p \frac{d\Phi}{dt}, \qquad e_s = -N_s \frac{d\Phi}{dt}.$$

Dividing gives $e_s/e_p = N_s/N_p$. The same ratio holds for their rms magnitudes. Taking terminal voltages equal to the corresponding emf magnitudes gives

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} > 1.$$

Conservation of energy in an ideal transformer requires equal input and output powers:

$$V_p I_p = V_s I_s \quad \Longrightarrow \quad \frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}.$$

Hence raising the voltage reduces the current in the ideal case. With efficiency $\eta < 1$, the power relation becomes $V_s I_s = \eta V_p I_p$, so $I_s/I_p = \eta V_p/V_s$. The ideal inverse current relation must therefore be modified when using the stated efficiency.

Step 3: (b) Calculate input power and output power The input voltage is $V_p = 100\,\text{V}$, and the input current is $I_p = 15\,\text{A}$. Therefore,

$$P_{\text{in}} = V_p I_p = 100 \times 15 = 1500\,\text{W}.$$

The efficiency is $\eta = 90/100 = 0.90$, so

$$P_{\text{out}} = \eta P_{\text{in}} = 0.90 \times 1500 = 1350\,\text{W}.$$

The difference $1500 - 1350 = 150\,\text{W}$ represents the transformer losses; it must not be included in the power delivered to the secondary load.

Step 4: (b) Calculate the secondary voltage For the output current $I_s = 3\,\text{A}$, use the actual output power:

$$V_s = \frac{P_{\text{out}}}{I_s} = \frac{1350}{3} = 450\,\text{V}.$$

This value exceeds the input voltage, as expected for a step-up transformer. Using ideal power conservation directly would give $500\,\text{V}$, which incorrectly ignores the specified loss of energy.

Step 5: Conclusion A step-up transformer operates by mutual induction with $N_s > N_p$. Ideally, $V_s/V_p = N_s/N_p = I_p/I_s$. For the given practical transformer, the output power is $\boxed{1350\,\text{W}}$ and the secondary voltage is $\boxed{450\,\text{V}}$.

Quick Tip to Solve: Apply the efficiency to input power first, then divide the resulting output power by the secondary current; do not apply the ideal inverse-current relation to this lossy transformer.