

CBSE Class 12 Physics Question Paper 2017 with Solution PDF-Set 55/3

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are **26** questions in all.
- (ii) This question paper has **five** sections : Section A, Section B, Section C, Section D and Section E.
- (iii) Section A contains **five** questions of **one** mark each, Section B contains **five** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains one value based question of **four** marks and Section E contains **three** questions of **five** marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in **one** question of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron} = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. How does the angle of minimum deviation of a glass prism vary, if the incident violet light is replaced by red light? Give reason. [1 mark]

Solution:

Concept:

- In normal dispersion, glass has a smaller refractive index for red light than for violet light: $\mu_r < \mu_v$.
- For prism angle A and minimum deviation δ_m , $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$.

Step 1: Compare the refractive indices Red light has a longer wavelength than violet light. The refractive index of the same glass is therefore lower for red light, so the prism bends it less strongly.

Step 2: Relate the index to minimum deviation The prism angle A remains fixed. From the prism formula, a smaller value of μ gives a smaller value of $(A + \delta_m)/2$, and hence a smaller minimum deviation.

Step 3: Conclusion The angle of minimum deviation decreases on replacing violet light by red light, because $\mu_r < \mu_v$.

Quick Tip to Solve: For the same glass prism, remember that red is deviated least and violet most.

2. Name the phenomenon which shows the quantum nature of electromagnetic radiation. [1 mark]

Solution:

Concept:

- Electromagnetic radiation exchanges energy in discrete packets called photons, each with energy $E = h\nu$, where h is Planck's constant and ν is frequency.
- The photoelectric effect is the emission of electrons from a metal surface when radiation of sufficient frequency falls on it.

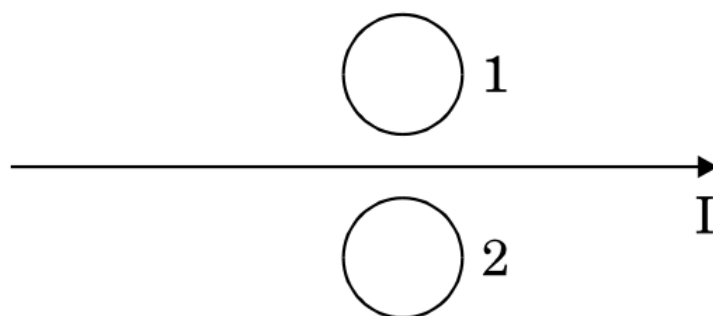
Step 1: Identify the relevant observation In photoelectric emission, electrons are emitted only above a threshold frequency. Increasing intensity at a frequency below this threshold cannot produce emission.

Step 2: Explain the photon interpretation One electron absorbs one photon of energy $h\nu$. This energy must overcome the metal's work function ϕ , with any remainder appearing as kinetic energy: $K_{\max} = h\nu - \phi$. Discrete photon energy accounts for the threshold.

Step 3: Conclusion The **photoelectric effect** demonstrates the quantum nature of electromagnetic radiation.

Quick Tip to Solve: Associate a threshold frequency and electron emission with the photon, or quantum, picture of light.

3. What is the direction of induced currents in metal rings 1 and 2 when current I in the wire is increasing steadily? [1 mark]



Solution:

Concept:

- The right-hand thumb rule gives the magnetic field around a straight current-carrying wire: point the thumb along the current and follow the curled fingers.
- Lenz's law states that induced current produces a magnetic field opposing the change in magnetic flux through the ring.

Step 1: Find the changing flux through each ring The wire current points to the right. Its magnetic field is out of the page at ring 1 above the wire and into the page at ring 2 below it. Both flux magnitudes increase as I increases.

Step 2: Oppose the increase in flux Ring 1 must produce a field into the page, requiring clockwise current. Ring 2 must produce a field out of the page, requiring anticlockwise current. These directions are as viewed in the printed figure.

Step 3: Conclusion The induced current is **clockwise in ring 1** and **anticlockwise in ring 2**, as viewed on the page.

Quick Tip to Solve: First find the wire's field on each side; then oppose its increase before assigning the ring current.

4. In which directions do the electric and magnetic field vectors oscillate in an electromagnetic wave propagating along the x-axis? [1 mark]

Solution:

Concept:

- An electromagnetic wave is transverse: both $\vec{E}$ and $\vec{B}$ are perpendicular to the direction of propagation.
- The electric and magnetic fields are mutually perpendicular, and $\vec{E} \times \vec{B}$ points along the direction of propagation.

Step 1: Use the transverse nature of the wave Propagation along the x-axis requires both fields to oscillate in the plane perpendicular to that axis, namely the yz-plane. Neither field has an x-component for the transverse wave.

Quick Tip to Solve: Use the cyclic order x, y, z to check that the cross product of the field directions gives propagation.

Step 2: Choose mutually perpendicular field axes One may choose the electric field along the y-axis and the magnetic field along the z-axis. For propagation along $+x$, their relative signs satisfy $\hat{y} \times \hat{z} = \hat{x}$. Any corresponding rotation of these transverse axes is also possible.

Step 3: Conclusion The fields oscillate perpendicular to each other and to the x-axis; for example, $\vec{E}$ along y and $\vec{B}$ along z.

Quick Tip to Solve: Use the cyclic order x, y, z to check that the cross product of the field directions gives propagation.

5. Nichrome and copper wires of same length and same radius are connected in series. Current I is passed through them. Which wire gets heated up more? Justify your answer. [1 mark]

Solution:

Concept:

- Joule's law gives the heat produced in time t as $H = I^2 R t$, where I is current and R is resistance.
- A wire of length L , radius r , and resistivity ρ has resistance $R = \rho L / (\pi r^2)$.

Step 1: Compare the wire resistances Both wires have the same length and cross-sectional area. Nichrome has a much greater resistivity than copper, so its resistance is greater: $R_{\text{NiCr}} > R_{\text{Cu}}$.

Step 2: Apply Joule's law for a series circuit The same current passes through both wires for the same time. Hence their heat production is proportional to resistance:

$$\frac{H_{\text{NiCr}}}{H_{\text{Cu}}} = \frac{R_{\text{NiCr}}}{R_{\text{Cu}}} > 1.$$

Step 3: Conclusion The **nichrome wire** gets heated more because its greater resistance produces more heat at the same current.

Quick Tip to Solve: For wires in series use $I^2 R$: at the shared current, the larger resistance dissipates more power.

Section B (10 marks)

6. Write two properties of a material suitable for making (a) a permanent magnet, and (b) an electromagnet. [2 marks]

Solution:

Concept:

- Retentivity is the ability of a magnetic material to retain magnetisation after the magnetising field is removed.
- Coercivity measures the reverse magnetic field required to remove the residual magnetisation. A large value means strong resistance to demagnetisation.
- Magnetic permeability measures how readily a material develops magnetic flux density in response

Quick Tip to Solve: Match the properties to the job: a permanent magnet must retain magnetism, while an electromagnet must switch on strongly and off readily.

to an applied magnetising field.

Step 1: Select properties for a permanent magnet (a) The material should have **high retentivity**, so that it retains strong magnetisation when the external magnetising field is removed. It should also have **high coercivity**, so that stray or reverse magnetic fields do not demagnetise it easily. Hardened steel is a familiar example.

Step 2: Select properties for an electromagnet (b) The core should have **high permeability**, so that the coil's current produces a strong magnetic field in it. It should have **low retentivity**, so that little magnetisation remains when the current is switched off. Soft iron is suitable; its low coercivity also permits easy magnetisation and demagnetisation.

Step 3: Conclusion A permanent magnet requires **high retentivity and high coercivity**. An electromagnet core requires **high permeability and low retentivity**.

Quick Tip to Solve: Match the properties to the job: a permanent magnet must retain magnetism, while an electromagnet must switch on strongly and off readily.

7. Draw the intensity pattern for single slit diffraction and double slit interference. Hence, state two differences between interference and diffraction patterns. [2 marks]

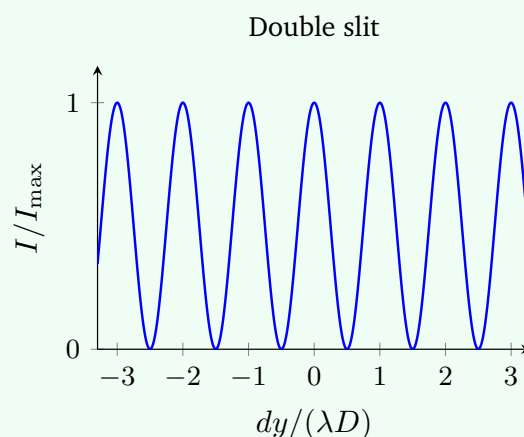
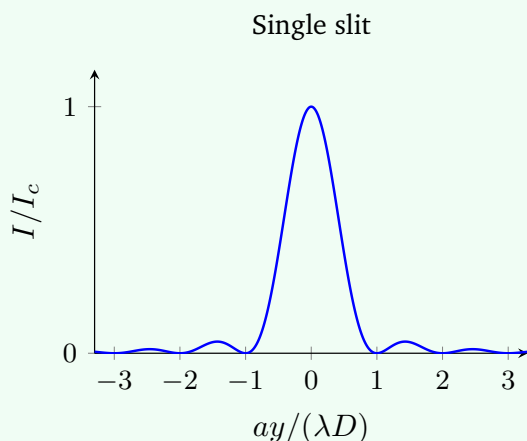
Solution:

Concept:

- Interference is the superposition of light from two coherent slits; diffraction is the superposition of secondary wavelets from different parts of a single slit.
- For slit width a , wavelength λ , screen distance D and transverse position y , single-slit intensity is $I = I_c(\sin \beta / \beta)^2$, where $\beta = \pi ay / (\lambda D)$. The limiting value at $y = 0$ is I_c .

Step 1: Sketch the intensity distributions The diffraction curve has a strong, broad central maximum and much weaker secondary maxima on both sides. The double-slit curve shown is the ideal interference pattern for two equal, very narrow slits, for which $I = I_{\max} \cos^2[\pi dy / (\lambda D)]$, with d the slit separation. Its bright maxima have equal heights. The horizontal coordinates below are scaled separately for the two arrangements.

Quick Tip to Solve: In a diffraction sketch, locate the first minima symmetrically before drawing the broad central peak and small side peaks.



Normalised intensity patterns: single-slit diffraction and ideal double-slit interference.

Step 2: Compare width and intensity **Width:** In interference, successive bright fringes have equal width $\lambda D/d$. In single-slit diffraction, the central maximum extends between the first minima at $y = \pm \lambda D/a$, so its width is $2\lambda D/a$, twice the width between consecutive minima bounding a secondary maximum.

Intensity: Ideal interference maxima have equal intensity for equal illuminating slits. Diffraction has the brightest central maximum; the intensities of successive secondary maxima decrease away from the centre.

Step 3: Conclusion Interference has equally spaced, equally intense bright fringes in the ideal case. Diffraction has a central maximum twice as wide as the secondary maxima, whose intensities fall rapidly outwards.

Quick Tip to Solve: In a diffraction sketch, locate the first minima symmetrically before drawing the broad central peak and small side peaks.

OR

7. Unpolarised light is passed through a polaroid P_1 . When this polarised beam passes through another polaroid P_2 and if the pass axis of P_2 makes angle θ with the pass axis of P_1 , then write the expression for the polarised beam passing through P_2 . Draw a plot showing the variation of intensity when θ varies from 0 to 2π . [2 marks]

Solution:

Concept:

- An ideal polaroid transmits half the intensity of incident unpolarised light and produces plane-polarised light along its pass axis.
- Malus' law states $I = I_1 \cos^2 \theta$, where I_1 is the intensity incident on the analyser and θ is the angle between the two pass axes.

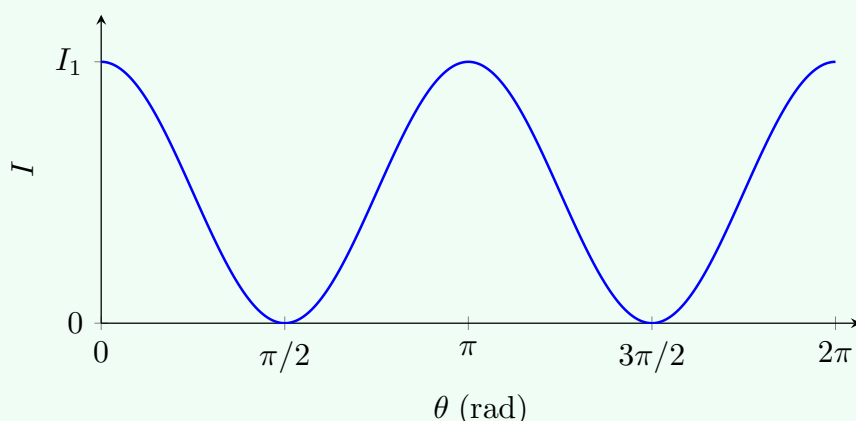
Step 1: Apply the two transmission rules Let the incident unpolarised intensity be I_u . After the first

Quick Tip to Solve: Define whether the given intensity is before or after the first polaroid; the factor of one-half applies only to the unpolarised input.

ideal polaroid, $I_1 = I_u/2$. The second polaroid transmits the component of electric-field amplitude along its pass axis, so the transmitted amplitude is proportional to $\cos \theta$. Since intensity is proportional to the square of amplitude,

$$I(\theta) = I_1 \cos^2 \theta = \frac{I_u}{2} \cos^2 \theta.$$

Step 2: Locate maxima and zeros for the plot The intensity is maximum, equal to I_1 , when the pass axes are parallel or antiparallel: $\theta = 0, \pi, 2\pi$. It vanishes for crossed axes at $\theta = \pi/2$ and $3\pi/2$. Between these points it varies smoothly as $\cos^2 \theta$, remains nonnegative, and repeats after an angular change of π .



Intensity transmitted by the second polaroid, with $I_1 = I_u/2$.

Step 3: Conclusion The intensity after P_2 is $I = I_1 \cos^2 \theta = (I_u/2) \cos^2 \theta$. The graph has three maximum points and two zero points over the closed interval $0 \leq \theta \leq 2\pi$.

Quick Tip to Solve: Define whether the given intensity is before or after the first polaroid; the factor of one-half applies only to the unpolarised input.

8. Why does current in a steady state not flow in a capacitor connected across a battery? However momentary current does flow during charging or discharging of the capacitor. Explain. [2 marks]

Solution:

Concept:

- An ideal capacitor consists of two conducting plates separated by an insulator; conduction charge cannot cross the insulating gap.
- The plate charge is $Q = CV_C$, and the external current is $I = dQ/dt = C dV_C/dt$ for constant capacitance C , where V_C is the potential difference between the plates.

Step 1: Explain the steady state When a capacitor connected to a battery of emf V is fully charged, its potential difference equals the battery emf. The plate charge has the constant value $Q = CV$. Hence $dQ/dt = 0$ and no current flows in the external circuit. The capacitor therefore behaves as an open circuit for steady direct current; its insulating gap also prevents conduction from one plate to the other.

Quick Tip to Solve: Use $I = dQ/dt$: constant plate charge means zero current, whereas changing plate charge means transient current.

Step 2: Explain charging and discharging currents Initially, the capacitor voltage is smaller than the battery emf. The battery drives electrons through the external wires, removing them from one plate and depositing them on the other. The changing plate charge produces a transient current, which decreases as the capacitor voltage rises towards V .

When the charged capacitor is disconnected from the battery and its plates are joined through a conducting path, the stored charge flows through that path in the discharge direction. Its plate charges and potential difference decrease, so a transient discharge current flows until the plates are neutral. Neither process requires conduction through the dielectric; the changing electric field in the gap corresponds to displacement current.

Step 3: Conclusion Steady current is zero because the capacitor charge and voltage are constant. A momentary current flows in the external circuit during charging or discharging because the charge on its plates changes with time.

Quick Tip to Solve: Use $I = dQ/dt$: constant plate charge means zero current, whereas changing plate charge means transient current.

9. The ground state energy of hydrogen atom is -13.6 eV. If an electron makes a transition from an energy level -1.51 eV to -3.4 eV, calculate the wavelength of the spectral line emitted and name the series of hydrogen spectrum to which it belongs. [2 marks]

Solution:

Concept:

- In the Bohr model, the energy of the hydrogen atom in level n is $E_n = -13.6/n^2$ eV, where n is the principal quantum number.
- A downward transition emits a photon with energy $E_\gamma = E_i - E_f = hc/\lambda$, where h is Planck's constant, c is the speed of light and λ is the emitted wavelength.

Step 1: Identify the initial and final levels Compare each supplied energy with the ground-state energy:

$$n_i^2 = \frac{13.6}{1.51} \simeq 9, \quad n_f^2 = \frac{13.6}{3.4} = 4.$$

Thus the electron falls from $n_i = 3$ to $n_f = 2$. All transitions terminating at the second level belong to the Balmer series.

Step 2: Calculate the photon energy and wavelength The emitted energy is positive even though both bound-state energies are negative:

$$E_\gamma = (-1.51) - (-3.4) = 1.89 \text{ eV} = 3.024 \times 10^{-19} \text{ J}.$$

Using $h = 6.63 \times 10^{-34} \text{ J s}$ and $c = 3 \times 10^8 \text{ m s}^{-1}$,

$$\lambda = \frac{hc}{E_\gamma} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{3.024 \times 10^{-19}} \simeq 6.58 \times 10^{-7} \text{ m}.$$

Step 3: Conclusion The wavelength is approximately $6.58 \times 10^{-7} \text{ m} = 658 \text{ nm}$, using the rounded

Quick Tip to Solve: Identify the series from the final level; subtract signed energies before converting eV to joules.

energies and constants supplied. It is the $3 \rightarrow 2$ line of the **Balmer series**.

Quick Tip to Solve: Identify the series from the final level; subtract signed energies before converting eV to joules.

10. Find the condition under which the charged particles moving with different speeds in the presence of electric and magnetic field vectors can be used to select charged particles of a particular speed.

[2 marks]

Solution:

Concept:

- A particle of charge q moving with velocity $\vec{v}$ in electric and magnetic fields experiences the Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$.
- A velocity selector uses crossed electric and magnetic fields whose forces cancel for one speed. For motion perpendicular to $\vec{B}$, the force magnitudes are $|q|E$ and $|q|vB$.

Step 1: Arrange mutually perpendicular fields Send a collimated beam perpendicular to both uniform fields, with $\vec{E} \perp \vec{B}$. Choose their directions so that $\vec{E}$ is opposite to $\vec{v} \times \vec{B}$; the electric and magnetic forces then oppose each other. For example, if the beam travels along $+x$, choose $\vec{E}$ along $+y$ and $\vec{B}$ along $+z$.

Step 2: Balance the electric and magnetic forces For an undeflected particle, the net transverse force must vanish:

$$q(\vec{E} + \vec{v} \times \vec{B}) = 0, \quad |q|E = |q|vB.$$

Since the particle is charged, division by $|q|B$ gives $v = E/B$. Particles with smaller or larger speeds have unequal electric and magnetic forces and are deflected away from the exit slit. The selected speed is independent of the particle's mass and the magnitude or sign of its charge.

Step 3: Conclusion Use mutually perpendicular $\vec{E}$, $\vec{B}$ and beam velocity, with opposing electric and magnetic forces. Only particles of speed $v = E/B$ pass undeflected through the aligned exit slit.

Quick Tip to Solve: Write the vector force balance first: crossed fields alone are insufficient unless the two forces oppose.

Section C (36 marks)

11. (a) Monochromatic light of wavelength 589 nm is incident from air on a water surface. If μ for water is 1.33, find the wavelength, frequency and speed of the refracted light.

(b) A double convex lens is made of a glass of refractive index 1.55, with both faces of the same radius of curvature. Find the radius of curvature required, if the focal length is 20 cm. [3 marks]

Solution:

Concept:

- Refraction changes the speed and wavelength of light but preserves its frequency. For refractive index μ , $v = c/\mu$, $\nu = c/\lambda_{\text{air}}$, and $\lambda_{\text{water}} = \lambda_{\text{air}}/\mu$, taking air's index as unity.
- For a thin lens in air, the lens maker's formula is $1/f = (\mu_g - 1)(1/R_1 - 1/R_2)$. Here f is the focal length and R_1, R_2 are the signed surface radii.

Step 1: (a) Calculate the frequency and speed in water The incident wavelength is $589 \text{ nm} = 589 \times 10^{-9} \text{ m}$. The source fixes the frequency, which remains unchanged at a stationary interface:

$$\nu = \frac{3 \times 10^8}{589 \times 10^{-9}} \simeq 5.09 \times 10^{14} \text{ Hz.}$$

Water has refractive index 1.33, so its optical speed is

$$v = \frac{3 \times 10^8}{1.33} \simeq 2.26 \times 10^8 \text{ m s}^{-1}.$$

Step 2: (a) Find the refracted wavelength Since $v = \nu\lambda$ and frequency stays constant, the reduced speed makes the wavelength shorter by the same factor:

$$\lambda_{\text{water}} = \frac{589}{1.33} \text{ nm} \simeq 443 \text{ nm} = 4.43 \times 10^{-7} \text{ m.}$$

The frequency must not be divided by the refractive index; that factor applies only to speed and wavelength here.

Step 3: (b) Apply the lens maker's formula with signed radii Let each surface have radius magnitude R . With light travelling left to right, a double convex lens has $R_1 = +R$ and $R_2 = -R$. In air its positive focal length is $f = 20 \text{ cm}$. Therefore,

$$\frac{1}{20} = (1.55 - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{1.10}{R}.$$

Multiplying by $20R$, the radius magnitude required for either face is $R = 1.10 \times 20 = 22 \text{ cm}$.

Step 4: Conclusion (a) In water, $\lambda \simeq 443 \text{ nm}$, $\nu \simeq 5.09 \times 10^{14} \text{ Hz}$, and $v \simeq 2.26 \times 10^8 \text{ m s}^{-1}$. (b) Each lens surface must have radius magnitude 22 cm.

Quick Tip to Solve: Keep frequency unchanged on refraction; for equal convex faces use opposite signed radii in the lens maker's formula.

12. Define mutual inductance between a pair of coils. Derive an expression for the mutual inductance of two long coaxial solenoids of same length wound one over the other. [3 marks]

Solution:

Concept:

- Mutual inductance M is the flux linkage of one coil per unit current in the other: $N_2\Phi_{21} = MI_1$. Here Φ_{21} is the flux through one turn of coil 2 due to current I_1 in coil 1.
- The induced emf is $\varepsilon_2 = -M dI_1/dt$. Thus M also equals the magnitude of emf induced in one coil per unit rate of change of current in the other. Its SI unit is henry (H).
- Neglecting end effects, a long air-core solenoid with N_1 turns and length l produces the uniform internal field $B_1 = \mu_0 N_1 I_1 / l$, where μ_0 is the permeability of free space.

Step 1: Set up the two coaxial solenoids Consider two long solenoids of equal length l , with N_1 and N_2 total turns, wound closely one over the other. Let A be the area of their common magnetic flux region, namely the inner solenoid's cross-sectional area if their radii differ. Assume an air core and negligible flux leakage and end effects. Pass current I_1 through the inner solenoid 1.

Step 2: Calculate the flux linked with the second solenoid The magnetic field of solenoid 1 is parallel to the common axis, so it is perpendicular to each turn's plane. The flux through each turn of solenoid 2 is

$$\Phi_{21} = B_1 A = \frac{\mu_0 N_1 I_1 A}{l}.$$

All N_2 turns link this flux, giving total flux linkage

$$N_2\Phi_{21} = \frac{\mu_0 N_1 N_2 A}{l} I_1.$$

Step 3: Compare the flux linkage with its definition Comparing with $N_2\Phi_{21} = MI_1$ yields

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 A l, \quad n_1 = \frac{N_1}{l}, \quad n_2 = \frac{N_2}{l}.$$

The expression is symmetric in the two turn numbers: reversing which coil carries the changing current gives the same mutual inductance. If the common core is a linear magnetic material of permeability μ , replace μ_0 by μ .

Step 4: Conclusion Mutual inductance is flux linkage in one coil per unit current in the other. For two long coaxial air-core solenoids, $M = \mu_0 N_1 N_2 A / l = \mu_0 n_1 n_2 A l$, measured in henries.

Quick Tip to Solve: Multiply flux per turn by the receiving coil's turn count before dividing by the source current; use the common flux area.

12. Define self-inductance of a coil. Obtain the expression for the energy stored in an inductor L connected across a source of emf. [3 marks]

Solution:

Concept:

- Self-inductance L is the flux linkage per unit current: $L = N\Phi/I$, where $N\Phi$ is the total magnetic flux linked with the coil.
- A changing current produces an opposing self-induced emf $\varepsilon_L = -L di/dt$. Work done against this back emf is stored as magnetic energy.

Step 1: Define self-inductance and its unit For a coil with fixed geometry in a linear magnetic medium, its flux linkage is proportional to the current through it:

$$N\Phi = LI.$$

Thus self-inductance is numerically equal to the flux linkage when unit current flows through the coil. Equivalently, it equals the magnitude of induced emf when the current changes at unit rate. Its SI unit is henry: a coil has inductance 1 H if a current change of 1 A s^{-1} induces an emf of 1 V.

Step 2: Find the work done against the back emf Connect the inductor to a source and let its current rise from zero to a final value I . At an intermediate instant the current is i , and the opposing emf has magnitude $L di/dt$. The source must supply an equal voltage across the ideal inductance. Hence the power supplied to build its magnetic field is

$$P = vi = Li \frac{di}{dt}.$$

In time dt , the energy stored is therefore $dU = P dt = Li di$. If the coil also has resistance, the source additionally supplies Joule heat; that heat is not part of the magnetic energy.

Step 3: Integrate as the current builds up Taking L to be constant and the initial magnetic energy to be zero, add the small amounts of work from zero current to I :

$$U = \int_0^I Li di = L \left[\frac{i^2}{2} \right]_0^I.$$

The energy resides in the magnetic field of the coil and can be returned to the circuit when the current decreases.

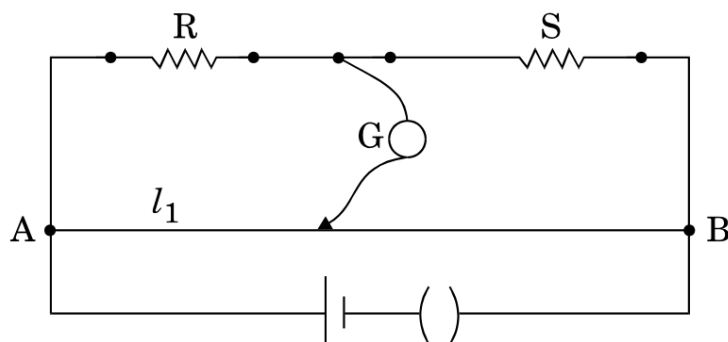
Step 4: Conclusion Self-inductance is $L = N\Phi/I$. The magnetic energy stored when current I flows is $U = \frac{1}{2}LI^2$, in joules when L is in henries and I in amperes.

Quick Tip to Solve: Use the instantaneous current in $dU = Li di$, then integrate from zero to the final current; using the final current throughout loses the factor $1/2$.

13. (a) Write the principle of working of a metre bridge.

(b) In a metre bridge, the balance point is found at a distance l_1 with resistances R and S as shown in the figure.

An unknown resistance X is now connected in parallel to the resistance S and the balance point is found at a distance l_2 . Obtain a formula for X in terms of l_1 , l_2 and S . [3 marks]



Solution:

Concept:

- A metre bridge works on the Wheatstone-bridge principle: at balance, the galvanometer carries no current and the ratios of resistances in the two arms are equal.
- A uniform wire has resistance proportional to length. Resistances S and X in parallel have equivalent resistance $S' = SX/(S + X)$.

Step 1: State the principle and the initial balance equation At the balance point the potential of the junction of R and S equals that of the jockey on the metre wire. There is no potential difference across the galvanometer, so it gives a null deflection. For a uniform wire, the Wheatstone-bridge balance condition becomes

$$\frac{R}{S} = \frac{l_1}{100 - l_1}.$$

Here l_1 is measured from the left end A , and all wire lengths in this derivation are in centimetres. The total wire length is 100 cm.

Step 2: Write the balance condition after adding the parallel resistance Connecting X in parallel with S changes the right-hand gap resistance to $SX/(S + X)$, while R remains unchanged. At the new balance point,

$$\frac{R}{SX/(S + X)} = \frac{l_2}{100 - l_2}.$$

Dividing this equation by the initial balance equation eliminates the unknown value of R :

$$\frac{S + X}{X} = \frac{l_2(100 - l_1)}{l_1(100 - l_2)}.$$

Quick Tip to Solve: Divide the second balance equation by the first to eliminate R . Check $l_2 > l_1$, because a shunt across the right-gap resistance shifts balance to the right.

Step 3: Isolate the unknown resistance Subtract unity from both sides and simplify the numerator:

$$\begin{aligned}\frac{S}{X} &= \frac{l_2(100 - l_1) - l_1(100 - l_2)}{l_1(100 - l_2)} \\ &= \frac{100(l_2 - l_1)}{l_1(100 - l_2)}.\end{aligned}$$

Adding a positive resistance in parallel decreases the right-gap resistance, so R/S' increases. Consequently the balance point moves towards B , giving $l_2 > l_1$; this ensures a positive value of X .

Step 4: Conclusion The metre bridge uses the Wheatstone-bridge null principle. The required resistance is

$$X = \frac{S l_1(100 - l_2)}{100(l_2 - l_1)}$$

with l_1, l_2 in centimetres; X has the same unit as S .

Quick Tip to Solve: Divide the second balance equation by the first to eliminate R . Check $l_2 > l_1$, because a shunt across the right-gap resistance shifts balance to the right.

14. (a) Draw the circuit diagram of an n-p-n transistor amplifier in common emitter configuration.
(b) Derive an expression for voltage gain of the amplifier and hence show that the output voltage is in opposite phase with the input voltage. [3 marks]

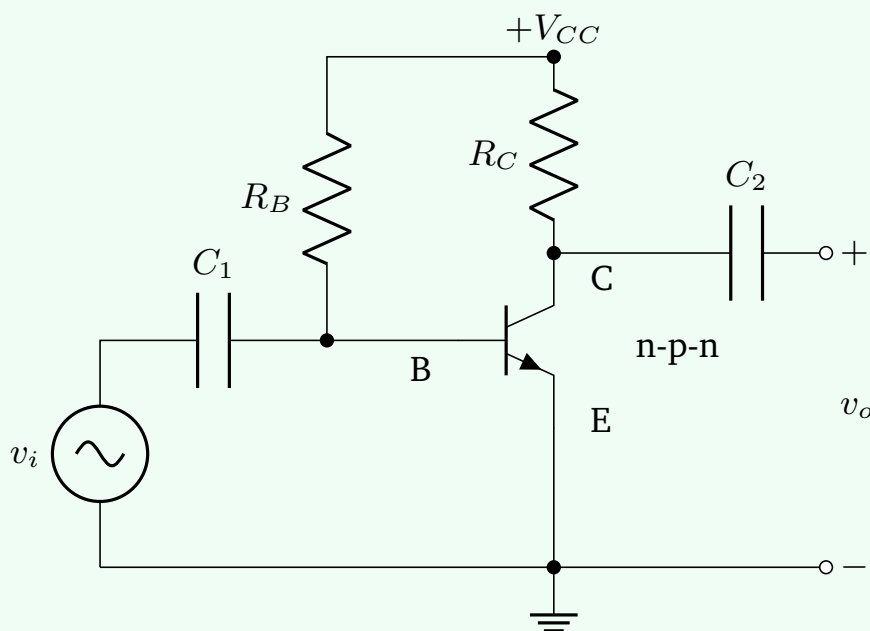
Solution:

Concept:

- In common-emitter configuration, the emitter is common to the input (base-emitter) and output (collector-emitter) circuits. The transistor is biased in its active region.
- For small signals, the current gain is $\beta_{ac} = \Delta I_C / \Delta I_B$, and the input resistance is $r_i = \Delta V_{BE} / \Delta I_B$.

Step 1: Draw and explain the common-emitter circuit The circuit below uses an n-p-n transistor. The base resistor R_B supplies the dc bias, and R_C is the collector load. The emitter is grounded and is common to both signal circuits. The coupling capacitors C_1 and C_2 pass the ac signal and block dc. Bias is selected so the base-emitter junction is forward biased and the collector-base junction is reverse biased, with the signal sufficiently small to avoid cutoff or saturation.

Quick Tip to Solve: Start from $V_{CE} = V_{CC} - I_C R_C$: increasing collector current lowers collector voltage, so the common-emitter voltage gain must be negative.



Common-emitter n-p-n amplifier; input and output voltages are measured relative to the grounded emitter.

Step 2: Relate the output voltage change to the input change At frequencies for which the coupling capacitors have negligible reactance, the ac input is $v_i = \Delta V_{BE} = r_i \Delta I_B$. With negligible current drawn by the output measuring device, the collector circuit obeys

$$V_{CE} = V_{CC} - I_C R_C.$$

Since the supply V_{CC} is constant, a small change in collector current gives the output change

$$v_o = \Delta V_{CE} = -R_C \Delta I_C = -\beta_{ac} R_C \Delta I_B.$$

The negative sign comes from the increased voltage drop across the collector resistor when the collector current increases.

Step 3: Obtain the gain and establish the phase relation Divide the output change by the input change to obtain

$$A_v = \frac{v_o}{v_i} = \frac{-\beta_{ac} R_C \Delta I_B}{r_i \Delta I_B} = -\frac{\beta_{ac} R_C}{r_i}.$$

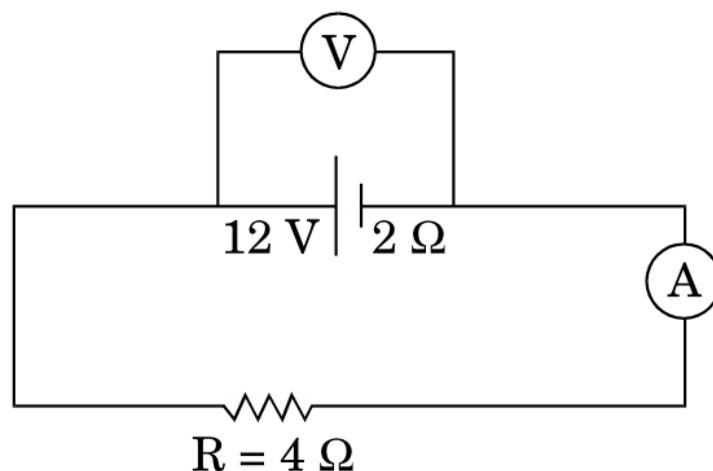
A positive input half-cycle increases base current and hence collector current. The increased drop across R_C lowers the collector voltage, producing a negative output half-cycle. During the negative input half-cycle the changes reverse. Thus a sinusoidal output is inverted relative to the input. If an external load R_L is connected through C_2 , replace R_C by $R_C \parallel R_L$ in this small-signal expression.

Step 4: Conclusion The common-emitter voltage gain is $A_v = -\beta_{ac} R_C / r_i$ for the unloaded circuit, with magnitude $\beta_{ac} R_C / r_i$. The minus sign shows a phase difference of 180° between input and output.

Quick Tip to Solve: Start from $V_{CE} = V_{CC} - I_C R_C$: increasing collector current lowers collector voltage, so the common-emitter voltage gain must be negative.

15. (a) The potential difference applied across a given resistor is altered so that the heat produced per second increases by a factor of 9. By what factor does the applied potential difference change?
- (b) In the figure shown, an ammeter A and a resistor of $4\ \Omega$ are connected to the terminals of the source. The emf of the source is 12 V having an internal resistance of $2\ \Omega$. Calculate the voltmeter and ammeter readings.

[3 marks]



Solution:

Concept:

- Joule's law gives the heat produced per second (electric power) in a resistor as $P = V^2/R$, where V is the potential difference and R is its resistance.
- For a cell delivering current, $I = \mathcal{E}/(R + r)$ and the terminal voltage is $V_{\text{term}} = \mathcal{E} - Ir = IR$. Here $\mathcal{E}$ is emf and r is internal resistance.

Step 1: (a) Compare the initial and final powers Let the original potential difference and power be V and P , and the altered values be V' and P' . The given resistor has the same resistance in both cases. Since heat per second is power, the condition is $P' = 9P$. Thus

$$\frac{P'}{P} = \frac{V'^2/R}{V^2/R} = \left(\frac{V'}{V}\right)^2 = 9.$$

Taking the positive square root for the ratio of voltage magnitudes gives $|V'|/|V| = 3$. The square in Joule's law is essential: multiplying voltage by nine would multiply the heating rate by eighty-one.

Step 2: (b) Calculate the current through the ammeter Treat the meters as ideal: the ammeter has negligible resistance and the voltmeter draws negligible current. The external resistor and the internal resistance are in series in the current path, so

$$R_{\text{total}} = 4 + 2 = 6\ \Omega, \quad I = \frac{12}{6} = 2\text{ A}.$$

The ammeter is in series with the external resistor and therefore measures this same current. Including the internal resistance is necessary because part of the source emf is lost inside the source.

Step 3: (b) Calculate the voltage across the source terminals The voltmeter is connected across the loaded source terminals, so it measures terminal potential difference rather than the open-circuit emf.

Quick Tip to Solve: For fixed resistance, take the square root of the heating-power factor to find the voltage factor. For a loaded cell, use total resistance to find current, then subtract Ir from emf.

The internal voltage drop is

$$Ir = (2)(2) = 4 \text{ V.}$$

Consequently,

$$V_{\text{term}} = \mathcal{E} - Ir = 12 - 4 = 8 \text{ V.}$$

As a check, the external resistor has potential difference $IR = (2)(4) = 8 \text{ V}$, equal to the terminal voltage because the ideal ammeter introduces no voltage drop.

Step 4: Conclusion (a) The applied potential difference increases by a factor of $\boxed{3}$. (b) The voltmeter reads $\boxed{8 \text{ V}}$ and the ammeter reads $\boxed{2 \text{ A}}$.

Quick Tip to Solve: For fixed resistance, take the square root of the heating-power factor to find the voltage factor. For a loaded cell, use total resistance to find current, then subtract Ir from emf.

16. Draw a block diagram of a generalized communication system. Write the functions of each of the following:

(a) Transmitter

(b) Channel

(c) Receiver

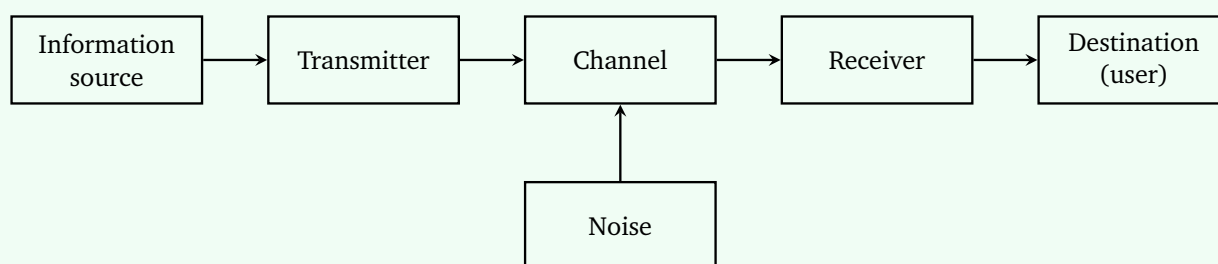
[3 marks]

Solution:

Concept:

- A communication system transfers information from a source to a destination. Its three essential functional elements are the transmitter, communication channel and receiver.
- A signal is the electrical representation of the message. Noise is an unwanted disturbance that may become superposed on the useful signal during transmission.

Step 1: Draw the generalized signal path The information flows in the direction of the arrows shown below. An input transducer, where needed, converts the source message into an electrical signal before transmission; for example, a microphone converts speech into an electrical signal. At the receiving end, an output transducer can convert the recovered electrical signal into a form understood by the user. The noise arrow indicates an unwanted signal entering the transmission path.



Generalized communication system; the noise arrow joins the signal path at the channel.

Step 2: (a) State the function of the transmitter The transmitter processes the electrical message signal and converts it into a form suitable for transmission through the chosen channel. Depending on the system, this involves amplification, encoding and modulation. For example, in radio communication

Quick Tip to Solve: Draw arrows from source to destination and a separate noise arrow into the channel. Remember the three functions in order: prepare, carry and recover the information signal.

the low-frequency message is used to modulate a high-frequency carrier, and the resulting signal is amplified and fed to the transmitting antenna.

Step 3: (b) State the function of the channel The channel is the physical medium or link through which the transmitted signal travels from transmitter to receiver. Examples include a pair of conducting wires, an optical fibre and free space. During propagation the useful signal may be attenuated or distorted, and unwanted noise may be added; the channel carries the signal but does not itself recover the original message.

Step 4: (c) State the function of the receiver The receiver accepts the signal arriving from the channel and extracts the original information from it. It selects the desired signal and performs appropriate amplification, demodulation or decoding, substantially reversing the processing done at transmission. The recovered message is then supplied to the destination, using an output transducer when required, such as a loudspeaker for speech.

Step 5: Conclusion The generalized arrangement is source, transmitter, channel, receiver and destination in sequence. The transmitter prepares the signal for transmission, the channel carries it, and the receiver recovers the message for the user.

Quick Tip to Solve: Draw arrows from source to destination and a separate noise arrow into the channel. Remember the three functions in order: prepare, carry and recover the information signal.

17. (a) Draw a ray diagram for the formation of image by a compound microscope.

(b) You are given the following three lenses. Which two lenses will you use as an eyepiece and as an objective to construct a compound microscope?

Lenses	Power (D)	Aperture (cm)
L_1	3	8
L_2	6	1
L_3	10	1

(c) Define resolving power of a microscope and write one factor on which it depends.

[3 marks]

Solution:

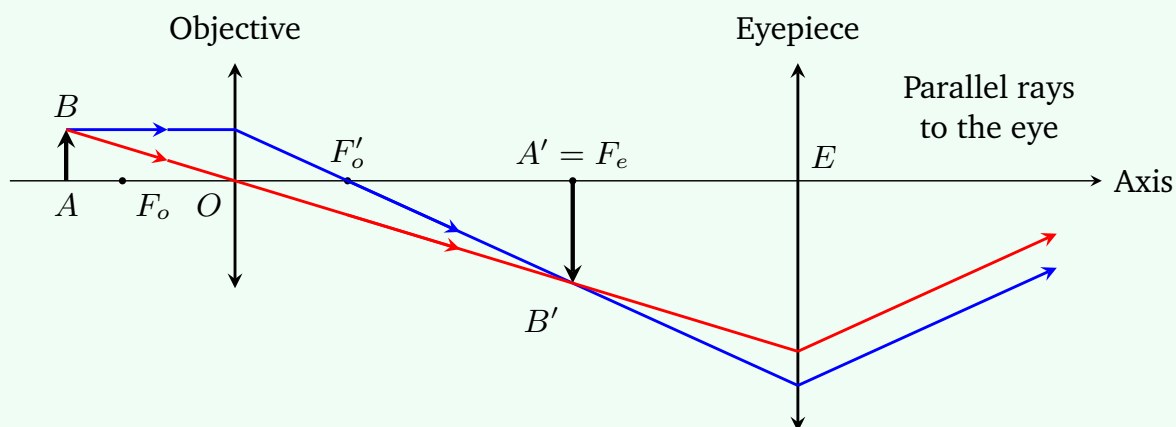
Concept:

- A compound microscope uses two converging lenses. The objective forms a real, inverted, enlarged intermediate image; the eyepiece acts as a magnifying glass for this image.
- Lens power is $P = 1/f$, where f is in metres and P in dioptres. The objective should have a short focal length and small aperture; the eyepiece has a larger aperture and longer focal length.
- Resolving power is the reciprocal of the minimum separation $d_{\min}$ of two nearby object points that can be distinguished as separate. For a microscope, $RP = 2\mu \sin \theta / (1.22\lambda)$, where λ is the vacuum wavelength and $\mu \sin \theta$ is the numerical aperture.

Step 1: (a) Trace image formation in normal adjustment Place the object AB just beyond the first focal point F_o of the objective. A ray from B parallel to the principal axis passes through the second focal point F'_o after refraction; a second ray through the optical centre O is undeviated. They meet at

Quick Tip to Solve: Choose the highest-power small-aperture lens as the objective; for the eyepiece, look for a larger aperture and lower power.

B' , forming the real, inverted, magnified image $A'B'$. Adjust the eyepiece so that $A'B'$ lies in its first focal plane, with $A' = F_e$. Rays from B' then emerge parallel, giving a final virtual image at infinity. This is normal adjustment and gives a relaxed-eye view; the final image remains inverted relative to AB .



Compound microscope in normal adjustment (schematic; final image at infinity).

Step 2: (b) Select the two lenses Convert the given powers into focal lengths:

$$f_1 = \frac{1}{3} \text{ m} = 33.3 \text{ cm}, \quad f_2 = \frac{1}{6} \text{ m} = 16.7 \text{ cm}, \quad f_3 = \frac{1}{10} \text{ m} = 10 \text{ cm}.$$

Choose L_3 as the objective because it has the shortest focal length and a small aperture of 1 cm. Choose L_1 as the eyepiece because it has a longer focal length and the largest aperture, 8 cm, allowing a convenient field of view. Thus L_2 is not used.

Step 3: (c) Define resolving power and its dependence Resolving power measures the ability to distinguish two very close object points separately, rather than merely enlarge a blurred image. Quantitatively,

$$\text{RP} = \frac{1}{d_{\min}}, \quad d_{\min} = \frac{1.22\lambda}{2\mu \sin \theta}.$$

Here θ is the semi-angle of the cone of light accepted by the objective. One factor is the wavelength of illumination: at fixed numerical aperture, shorter wavelength gives smaller $d_{\min}$ and hence greater resolving power.

Step 4: Conclusion The objective is L_3 and the eyepiece is L_1 . The ray diagram shows a real intermediate image and a final virtual image at infinity. Resolving power is $1/d_{\min}$, and increases when the illumination wavelength decreases.

Quick Tip to Solve: Choose the highest-power small-aperture lens as the objective; for the eyepiece, look for a larger aperture and lower power.

18. The following graph shows the variation of photocurrent for a photosensitive metal:

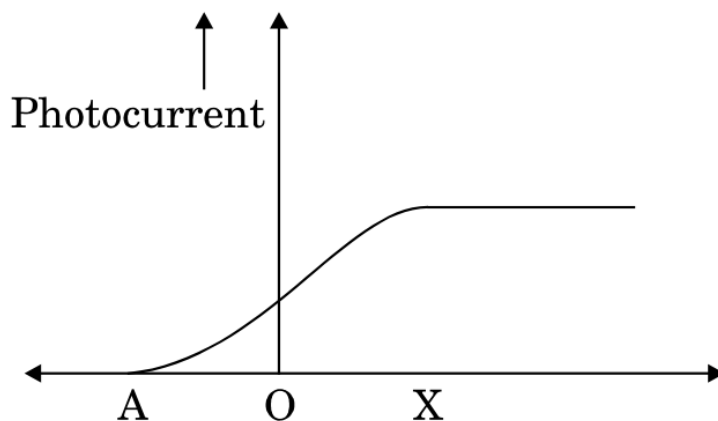
(a) Identify the variable X on the horizontal axis.

(b) What does the point A on the horizontal axis represent?

(c) Draw this graph for three different values of frequencies of incident radiation ν_1 , ν_2 and ν_3 ($\nu_1 > \nu_2 > \nu_3$) for same intensity.

(d) Draw this graph for three different values of intensities of incident radiation I_1 , I_2 and I_3 ($I_1 > I_2 > I_3$) having same frequency.

[3 marks]



Solution:

Concept:

- In the photoelectric effect, Einstein's equation is $K_{\max} = h\nu - \phi = eV_s$, where ϕ is the work function and V_s is the positive magnitude of the stopping potential.
- Increasing positive collector potential collects more emitted electrons until the saturation photocurrent is reached. In the standard school-level comparison, saturation current depends on incident intensity, whereas stopping potential depends on frequency.

Step 1: (a) Identify the horizontal-axis variable The horizontal axis represents the potential difference V of the collector (anode) relative to the emitting metal (cathode), measured in volts. For positive V , the field attracts photoelectrons and the current approaches saturation. For negative V , it retards the electrons, and only electrons with sufficient kinetic energy reach the collector.

Step 2: (b) Interpret point A Point A is the negative-potential intercept, where the photocurrent just becomes zero. Its coordinate is $-V_s$, where V_s is the stopping-potential magnitude. At this retarding voltage, even the fastest emitted photoelectrons fail to reach the collector:

$$eV_s = K_{\max} = h\nu - \phi.$$

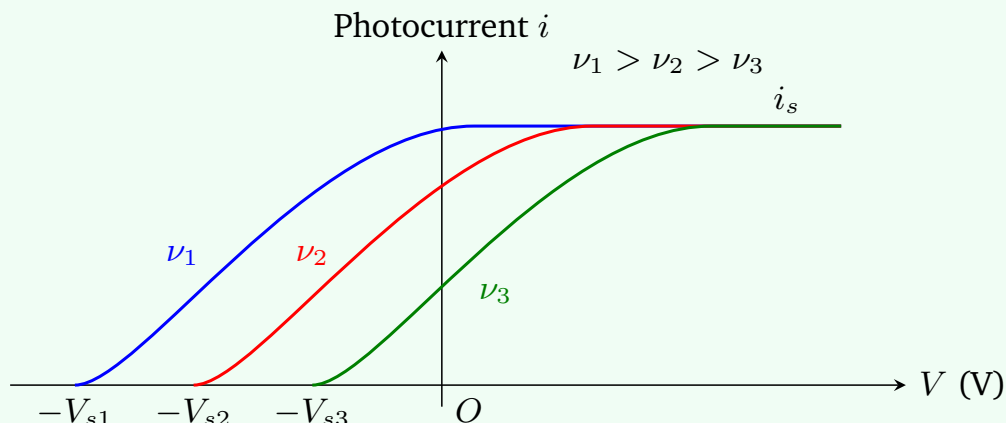
Thus the sign of the plotted intercept is negative, while the quoted stopping-potential magnitude is positive.

Step 3: (c) Compare three frequencies at the same intensity For the same photosensitive metal, ϕ is fixed. Therefore $\nu_1 > \nu_2 > \nu_3$ implies

$$V_{s1} > V_{s2} > V_{s3}, \quad -V_{s1} < -V_{s2} < -V_{s3}.$$

Quick Tip to Solve: Read frequency from the negative voltage intercept and intensity from the saturation plateau; label stopping potentials with their negative axis signs.

The highest-frequency curve cuts the voltage axis farthest to the left. With the same incident intensity, the standard photocurrent comparison shows the same saturation level for all three curves. The graph assumes that all three frequencies exceed the threshold frequency of the metal.

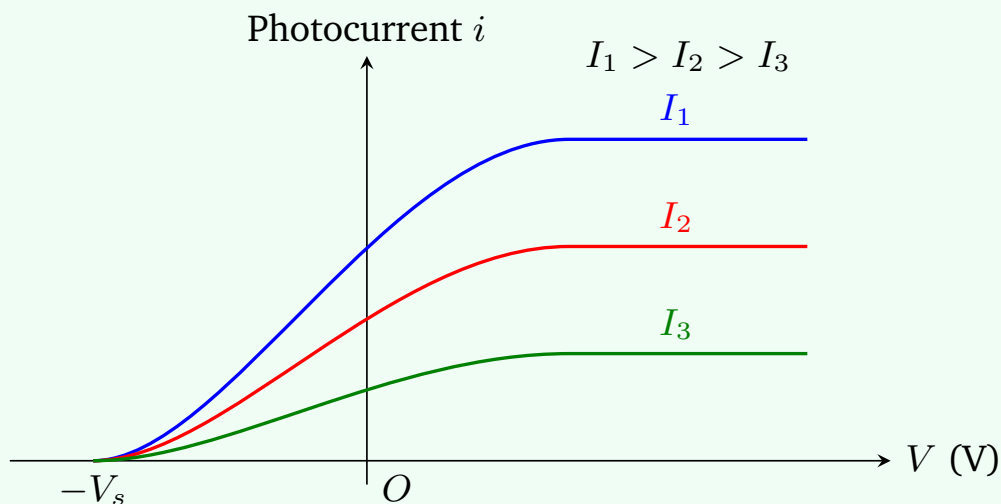


Same intensity: equal saturation photocurrent, different stopping potentials.

Step 4: (d) Compare three intensities at the same frequency At fixed frequency, the photon energy and maximum photoelectron kinetic energy are unchanged. All three curves therefore meet the voltage axis at the same stopping potential $-V_s$. Greater intensity emits more photoelectrons per second, so the saturation currents obey

$$i_{s1} > i_{s2} > i_{s3} \quad \text{for} \quad I_1 > I_2 > I_3.$$

The higher-intensity curve lies above the lower-intensity curve and reaches a higher saturation plateau.

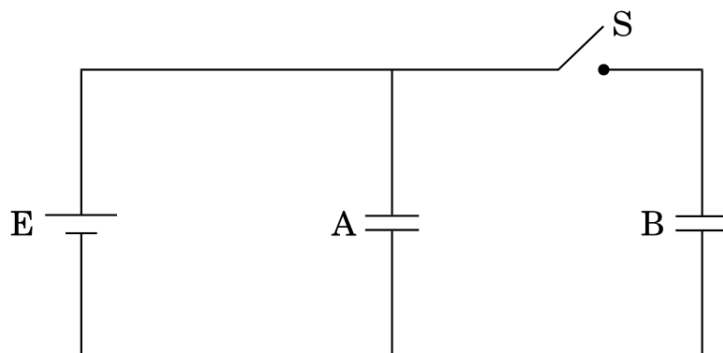


Same frequency: common stopping potential, different saturation photocurrents.

Step 5: Conclusion X is the collector potential relative to the emitter, and A denotes the stopping-potential intercept $-V_s$. Increasing frequency shifts this intercept left; increasing intensity raises the saturation current without shifting the intercept.

Quick Tip to Solve: Read frequency from the negative voltage intercept and intensity from the saturation plateau; label stopping potentials with their negative axis signs.

19. Two identical parallel plate capacitors A and B are connected to a battery of V volts with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K . Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. [3 marks]



Solution:

Concept:

- A capacitor of capacitance C , charge magnitude Q , and potential difference V stores energy $U = \frac{1}{2}CV^2 = Q^2/(2C)$.
- Completely filling its plate separation with dielectric constant K changes the capacitance from C to KC . A battery maintains voltage, whereas an isolated plate retains its charge.

Step 1: Find the total energy before insertion Let the initial capacitance of each identical capacitor be C . With switch S closed, both capacitors are connected in parallel across the battery. Hence each has voltage V , charge CV , and energy $\frac{1}{2}CV^2$. Opening the switch before inserting the dielectric does not immediately change these values. Thus

$$U_{\text{before}} = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2.$$

Step 2: Apply the correct constraint to each capacitor The diagram shows that opening S leaves capacitor A connected across the battery, so its voltage remains V . Its new capacitance is KC , and the battery supplies additional charge. Therefore

$$U'_A = \frac{1}{2}KCV^2.$$

The upper plate of capacitor B is disconnected, so its free charge remains $Q_B = CV$. Although its lower plate is still joined to the battery terminal, no charge can pass across its dielectric. Its new voltage is V/K , giving

$$U'_B = \frac{(CV)^2}{2KC} = \frac{CV^2}{2K}.$$

Step 3: Add the final energies and form the ratio The two constraints must be applied separately

Quick Tip to Solve: Trace the open switch first: voltage stays fixed for A , but charge stays fixed for B . Never apply the same energy scaling to both.

before adding the energies. The total final energy and the requested ratio are

$$U_{\text{after}} = \frac{CV^2}{2} \left(K + \frac{1}{K} \right),$$

$$\frac{U_{\text{before}}}{U_{\text{after}}} = \frac{2}{K + 1/K} = \frac{2K}{K^2 + 1}.$$

Step 4: Conclusion The ratio of total energy before to that after insertion is

$$U_{\text{before}} : U_{\text{after}} = 2K : (K^2 + 1).$$

Quick Tip to Solve: Trace the open switch first: voltage stays fixed for A, but charge stays fixed for B. Never apply the same energy scaling to both.

20. (a) How is amplitude modulation achieved?

(b) The frequencies of two side bands in an AM wave are 640 kHz and 660 kHz respectively. Find the frequencies of carrier and modulating signal. What is the bandwidth required for amplitude modulation? [3 marks]

Solution:

Concept:

- Amplitude modulation (AM) varies the amplitude of a high-frequency carrier in accordance with the instantaneous value of the message signal, keeping carrier frequency and phase unchanged.
- For a sinusoidal message of frequency f_m and carrier of frequency f_c , the sidebands occur at $f_L = f_c - f_m$ and $f_U = f_c + f_m$; their separation is the bandwidth $B = 2f_m$.

Step 1: Explain how amplitude modulation is produced In part (a), the low-frequency message and high-frequency carrier are combined in a modulator. A nonlinear element produces sum- and difference-frequency components, and a band-pass filter selects the carrier and its two sidebands. The resulting carrier amplitude follows the message signal. For a sinusoidal message, the AM wave is

$$v(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t),$$

where A_c is the unmodulated carrier amplitude and μ is the modulation index. For an undistorted envelope, $0 \leq \mu \leq 1$. The carrier oscillates rapidly while its envelope changes at the message frequency.

Step 2: Obtain the carrier and message frequencies In part (b), the lower and upper sideband frequencies are respectively 640 kHz and 660 kHz. Adding and subtracting the two sideband equations gives

$$f_c = \frac{f_U + f_L}{2} = \frac{660 + 640}{2} = 650 \text{ kHz},$$

$$f_m = \frac{f_U - f_L}{2} = \frac{660 - 640}{2} = 10 \text{ kHz}.$$

Quick Tip to Solve: Average the two AM sideband frequencies for the carrier; halve their difference for the message. Their full difference is the bandwidth.

Both sidebands lie 10 kHz from the carrier, as expected for a single-frequency modulating signal.

Step 3: Calculate the required bandwidth The transmission channel must accommodate the full range from the lower to the upper sideband. Therefore

$$B = f_U - f_L = 660 - 640 = 20 \text{ kHz} = 2f_m.$$

The carrier frequency sets the centre of this band; it is not the bandwidth.

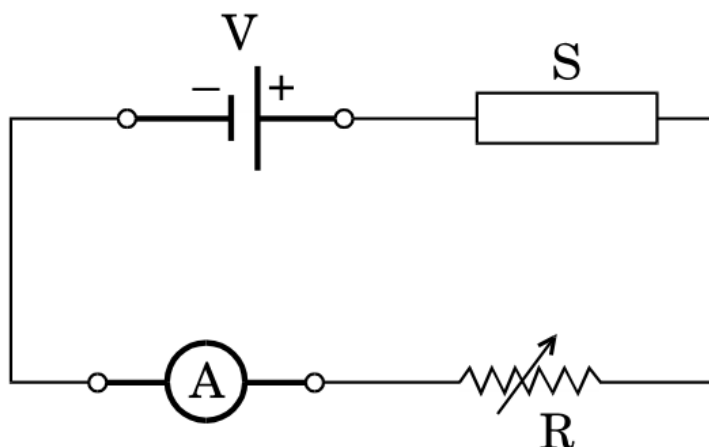
Step 4: Conclusion AM is achieved by varying carrier amplitude according to the message. The carrier frequency is 650 kHz, the modulating frequency is 10 kHz, and the required bandwidth is 20 kHz.

Quick Tip to Solve: Average the two AM sideband frequencies for the carrier; halve their difference for the message. Their full difference is the bandwidth.

21. (a) In the following diagram 'S' is a semiconductor. Would you increase or decrease the value of R to keep the reading of the ammeter A constant when S is heated? Give reason for your answer.

(b) Draw the circuit diagram of a photodiode and explain its working. Draw its $I - V$ characteristics.

[3 marks]



Solution:

Concept:

- Heating a semiconductor increases the concentration of mobile electrons and holes. Its resistance therefore decreases: it has a negative temperature coefficient of resistance.
- A photodiode is a light-sensitive p-n junction normally operated in reverse bias. Incident photons with energy $h\nu \geq E_g$, where E_g is the band gap, generate electron-hole pairs and increase reverse current.

Step 1: Keep the current constant when S is heated In part (a), the semiconductor and variable

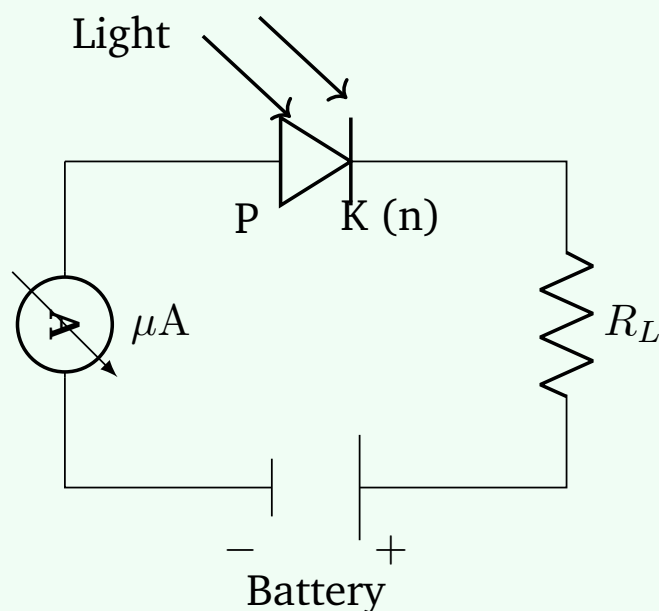
Quick Tip to Solve: For the resistor adjustment, keep the total series resistance fixed. For the photodiode sketch, put K at battery positive and draw light arrows towards the junction.

resistor are in series. Neglecting the ammeter resistance, the current is

$$I = \frac{V}{R + R_S},$$

where R_S is the resistance of the semiconductor. Heating increases its carrier concentration and reduces R_S , so the current would rise if R were unchanged. To keep the ammeter reading constant at fixed supply voltage, the total resistance $R + R_S$ must remain constant. Therefore increase R by the decrease in R_S .

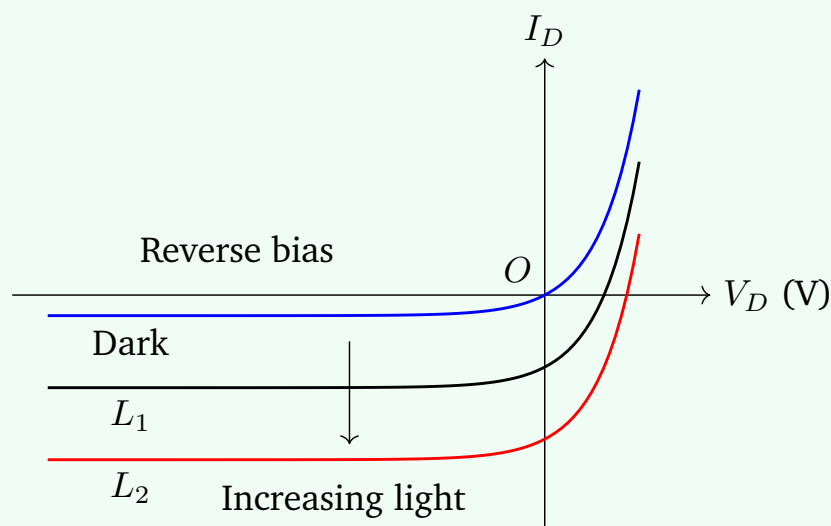
Step 2: Connect the photodiode in reverse bias In part (b), connect the n-side (cathode K) to the positive terminal of a battery and the p-side (anode P) to the negative terminal, through a microammeter and a current-limiting resistor. The arrows directed towards the junction represent incident light. The microammeter measures the reverse current.



Reverse-biased photodiode circuit; K is connected to positive supply.

Step 3: Explain light detection and the characteristics In darkness, thermally generated minority carriers produce a small reverse current called dark current. Light falling on the depletion region creates electron-hole pairs. The electric field sweeps electrons towards the n-side and holes towards the p-side, so they contribute to an additional reverse photocurrent. At a fixed reverse voltage, increasing illumination increases the number of carriers generated each second and hence the magnitude of the reverse current. Over the normal operating range below breakdown, this current is nearly independent of reverse voltage and is approximately proportional to incident optical power. The graph uses signed diode voltage $V_D = V_P - V_K$ and forward-positive current I_D , so reverse current is plotted below the horizontal axis. Curves for larger light intensity lie lower.

Quick Tip to Solve: For the resistor adjustment, keep the total series resistance fixed. For the photodiode sketch, put K at battery positive and draw light arrows towards the junction.



Schematic photodiode characteristics: illumination $L_2 > L_1 > 0$; breakdown is excluded.

Step 4: Conclusion (a) Increase R to compensate for the decrease in semiconductor resistance on heating. (b) A reverse-biased photodiode detects light through an increase in reverse-current magnitude; greater illumination produces a larger photocurrent.

Quick Tip to Solve: For the resistor adjustment, keep the total series resistance fixed. For the photodiode sketch, put K at battery positive and draw light arrows towards the junction.

22. (a) State Biot - Savart law and express this law in the vector form.

(b) Two identical circular coils, P and Q each of radius R , carrying currents 1 A and $\sqrt{3}\text{ A}$ respectively, are placed concentrically and perpendicular to each other lying in the XY and YZ planes. Find the magnitude and direction of the net magnetic field at the centre of the coils. [3 marks]

Solution:

Concept:

- Biot-Savart law gives the magnetic field of a current element: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$, where $d\vec{l}$ points along current and $\hat{r}$ points from the element to the observation point.
- A circular loop of radius R carrying current I produces a central field $B = \mu_0 I / (2R)$, normal to its plane. Fields add vectorially by superposition.

Step 1: (a) State the law and specify its direction The field magnitude due to a small current element is directly proportional to the current I , element length dl , and $\sin \alpha$, and inversely proportional to the square of its distance r from the point. Here α is the angle between $d\vec{l}$ and $\vec{r}$. Thus

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin \alpha}{r^2}, \quad d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}.$$

The field is perpendicular to the plane containing the current element and the position vector, with its sense given by the right-hand rule.

Quick Tip to Solve: First draw each field normal to its coil's plane. For perpendicular equal-radius coils, the field ratio equals the current ratio, so $1 : \sqrt{3}$ immediately gives the 30° – 60° direction triangle.

Step 2: (b) Find the two perpendicular central fields Coil P , in the XY plane, produces a field along the Z axis; coil Q , in the YZ plane, produces a field along the X axis. Using the stated currents,

$$B_P = \frac{\mu_0(1\text{ A})}{2R}, \quad B_Q = \frac{\sqrt{3}\mu_0(1\text{ A})}{2R}.$$

Since these fields are perpendicular, their magnitudes must be combined by the Pythagorean rule, rather than added directly:

$$B = \sqrt{B_P^2 + B_Q^2} = \frac{\mu_0(1\text{ A})}{2R} \sqrt{1+3} = \frac{\mu_0(1\text{ A})}{R}.$$

Step 3: Determine the resultant direction The resultant lies in the XZ plane. If θ is its angle with the field of coil P , measured towards the field of coil Q , then

$$\tan \theta = \frac{B_Q}{B_P} = \sqrt{3}, \quad \theta = 60^\circ.$$

It therefore makes 30° with the field of Q . For currents producing fields along $+Z$ and $+X$, respectively,

$$\vec{B} = \frac{\mu_0(1\text{ A})}{2R} (\sqrt{3}\hat{i} + \hat{k}).$$

The printed question does not specify the circulation senses of the currents. The signs of the X and Z components follow those senses by the right-hand rule; their magnitudes and the acute angles with the axes remain as above.

Step 4: Conclusion The net field has magnitude $B = \mu_0(1\text{ A})/R$, in tesla when R is in metres. It lies in the XZ plane, making 60° with $\vec{B}_P$ towards $\vec{B}_Q$, or 30° with $\vec{B}_Q$; the current senses determine its quadrant.

Quick Tip to Solve: First draw each field normal to its coil's plane. For perpendicular equal-radius coils, the field ratio equals the current ratio, so $1 : \sqrt{3}$ immediately gives the 30° – 60° direction triangle.

Section D (4 marks)

23. Asha's mother read an article in the newspaper about a disaster that took place at Chernobyl. She could not understand much from the article and asked a few questions from Asha regarding the article. Asha tried to answer her mother's questions based on what she learnt in Class XII Physics.

(a) What was the installation at Chernobyl where the disaster took place? What, according to you, was the cause of this disaster?

(b) Explain the process of release of energy in the installation at Chernobyl.

(c) What, according to you, were the values displayed by Asha and her mother?

[4 marks]

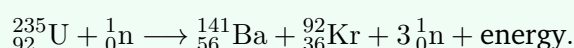
Solution:

Concept:

- Nuclear fission is the splitting of a heavy nucleus into lighter nuclei after neutron absorption, with release of energy and further neutrons.
- The energy released is $E = \Delta mc^2$, where Δm is the decrease in total rest mass and c is the speed of light. The fission products have greater average binding energy per nucleon than the original heavy nucleus.
- A nuclear reactor maintains a controlled, self-sustaining chain reaction. Neutron-absorbing control rods regulate fission, and coolant removes the heat.

Step 1: (a) Identify the installation and explain the disaster The installation at Chernobyl was a nuclear power station containing nuclear fission reactors. The disaster involved loss of safe control of a reactor: a rapid rise in power caused severe overheating, fuel damage and explosions, releasing radioactive material into the surroundings. At the Class XII level, the essential cause is an uncontrolled increase in the fission chain reaction and failure to keep the reactor operating safely. Deficiencies in reactor design and unsafe operating actions contributed to this loss of control.

Step 2: (b) Describe the fission chain reaction A fissile uranium-235 nucleus can absorb a slow neutron, forming an excited uranium-236 nucleus. This unstable nucleus splits into two lighter nuclei and releases neutrons and energy. One representative channel is



Both nucleon number and charge are conserved: $236 = 141 + 92 + 3$ and $92 = 56 + 36$. The emitted neutrons can induce fission in other uranium nuclei, sustaining a chain reaction. In normal operation, neutron absorption is regulated so that, on average, one neutron from each fission continues the reaction.

Step 3: Explain how nuclear energy becomes useful heat The total rest mass of the fission products and emitted particles is less than that of the initial nucleus and incident neutron. This mass decrease releases energy according to $E = \Delta mc^2$, about 200 MeV per uranium-235 fission. Much of this appears as kinetic energy of the fragments, which becomes heat as they slow down in the reactor. The heat produces steam that drives turbines coupled to electric generators. Controlled energy release and continuous heat removal are therefore essential for safe electricity production.

Step 4: (c) Relate the actions to the values displayed Asha displayed scientific understanding,

Quick Tip to Solve: Organise this answer as installation and cause, fission with a balanced example and $E = \Delta mc^2$, then values tied to each person's actual behaviour.

willingness to apply classroom knowledge, helpfulness, patience and respect for her mother while explaining the article. Her mother displayed curiosity, awareness of events, a desire to learn and openness to learning from a younger person. Their discussion reflects a scientific attitude: asking questions and seeking reasoned explanations.

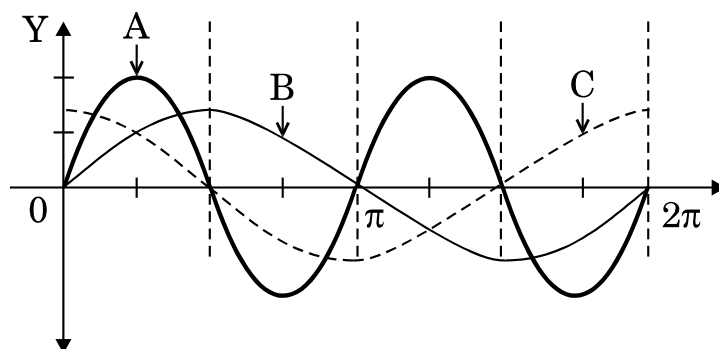
Step 5: Conclusion Chernobyl was a nuclear power station; loss of reactor control led to the disaster. Its normal energy source was a controlled nuclear fission chain reaction, with mass converted into energy according to $E = \Delta mc^2$. Asha showed knowledge, helpfulness and respect, while her mother showed curiosity and willingness to learn.

Quick Tip to Solve: Organise this answer as installation and cause, fission with a balanced example and $E = \Delta mc^2$, then values tied to each person's actual behaviour.

Section E (15 marks)

24. A device X is connected to an ac source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph:

- Identify the device ' X '.
- Which of the curves A , B and C represent the voltage, current and the power consumed in the circuit? Justify your answer.
- How does its impedance vary with frequency of the ac source? Show graphically.
- Obtain an expression for the current in the circuit and its phase relation with ac voltage. [5 marks]



Solution:

Concept:

- A capacitor of capacitance C obeys $q = CV$, where q is its charge and V is the applied voltage; the circuit current is $i = dq/dt$.
- Capacitive reactance is $X_C = 1/(\omega C) = 1/(2\pi fC)$, where f is the source frequency and $\omega = 2\pi f$. An ideal capacitor has impedance magnitude $Z = X_C$.
- Instantaneous power is $p = Vi$. When current and voltage differ in phase by $\pi/2$, the average power over a complete cycle is zero.

Step 1: (a) Identify the device from the phase difference The voltage is specified as $V = V_0 \sin \omega t$, so it must start from zero and initially increase. Curve B has this behaviour and completes one cycle

Quick Tip to Solve: First identify the given sine voltage by its zero crossing at the origin. A current maximum at that instant means a capacitor; the curve with twice the frequency is instantaneous power.

between 0 and 2π . Curve C is already at its positive maximum when the voltage is zero, and crosses zero a quarter cycle before the voltage does. Thus the current leads the voltage by $\pi/2$. This is the characteristic of an **ideal capacitor**, so device X is a capacitor. An ideal inductor would make the current lag the voltage, while a resistor would have current and voltage in phase.

Step 2: (b) Assign the voltage, current and power curves Curve B represents voltage and curve C represents current. In particular, their phase dependence is

$$V = V_0 \sin \omega t, \quad i = I_0 \cos \omega t.$$

Their product gives the instantaneous power:

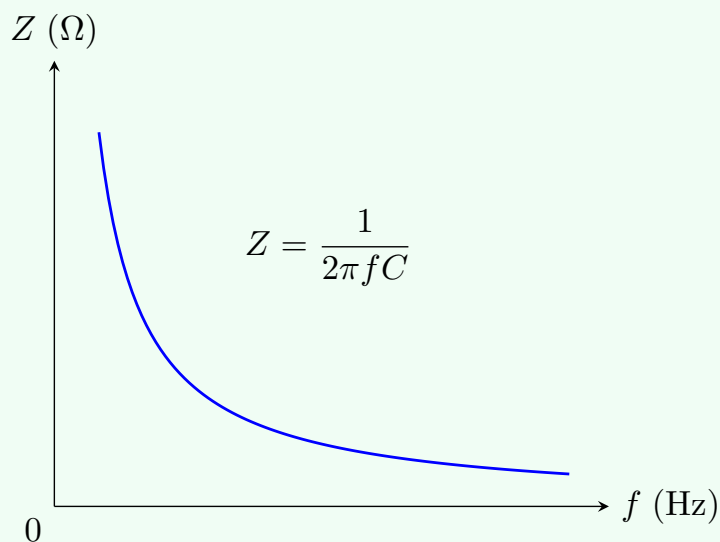
$$p = V_0 I_0 \sin \omega t \cos \omega t = \frac{V_0 I_0}{2} \sin 2\omega t.$$

Hence curve A , which completes two oscillations in one voltage cycle, represents instantaneous power. It is zero whenever either current or voltage is zero. Its positive and negative areas are equal: energy is alternately stored in the capacitor's electric field and returned to the source. Thus $P_{av} = 0$; the plotted alternating power is not a positive average power loss.

Step 3: (c) Obtain the impedance-frequency relation For a fixed capacitance, the impedance magnitude is

$$Z = X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

Therefore Z is inversely proportional to frequency. Doubling f halves Z . The graph in the positive f - Z plane is a rectangular hyperbola: Z tends to infinity as $f \rightarrow 0^+$, and tends to zero as $f \rightarrow \infty$. The curve approaches both axes asymptotically and does not meet them at any finite positive frequency.



Capacitive impedance decreases inversely with source frequency for fixed capacitance.

Step 4: (d) Derive the instantaneous current Using the capacitor relation, the charge varies as

$$q(t) = CV(t) = CV_0 \sin \omega t.$$

Quick Tip to Solve: First identify the given sine voltage by its zero crossing at the origin. A current maximum at that instant means a capacitor; the curve with twice the frequency is instantaneous power.

Differentiate with respect to time to obtain the current:

$$\begin{aligned}i(t) &= \frac{dq}{dt} = \omega C V_0 \cos \omega t \\&= I_0 \sin \left(\omega t + \frac{\pi}{2} \right), \\I_0 &= \omega C V_0 = \frac{V_0}{X_C}.\end{aligned}$$

The positive phase term $+\pi/2$ shows explicitly that the current leads the applied voltage by 90° , or one-quarter of a period. This also verifies the identification of curve C and the sign of curve A immediately after $t = 0$.

Step 5: Conclusion X is a capacitor. Curves A , B and C represent instantaneous power, voltage and current respectively. Its impedance magnitude is $Z = 1/(2\pi fC)$, a decreasing hyperbola with frequency. The current is $i = \omega C V_0 \cos \omega t$, leading the voltage by $\pi/2$; the average power is zero.

Quick Tip to Solve: First identify the given sine voltage by its zero crossing at the origin. A current maximum at that instant means a capacitor; the curve with twice the frequency is instantaneous power.

OR

24. (a) Draw a labelled diagram of an ac generator. Obtain the expression for the emf induced in the rotating coil of N turns each of cross-sectional area A , in the presence of a magnetic field $\vec{B}$.

(b) A horizontal conducting rod 10 m long extending from east to west is falling with a speed 5.0 m s^{-1} at right angles to the horizontal component of the Earth's magnetic field, $0.3 \times 10^{-4} \text{ Wb m}^{-2}$. Find the instantaneous value of the emf induced in the rod. [5 marks]

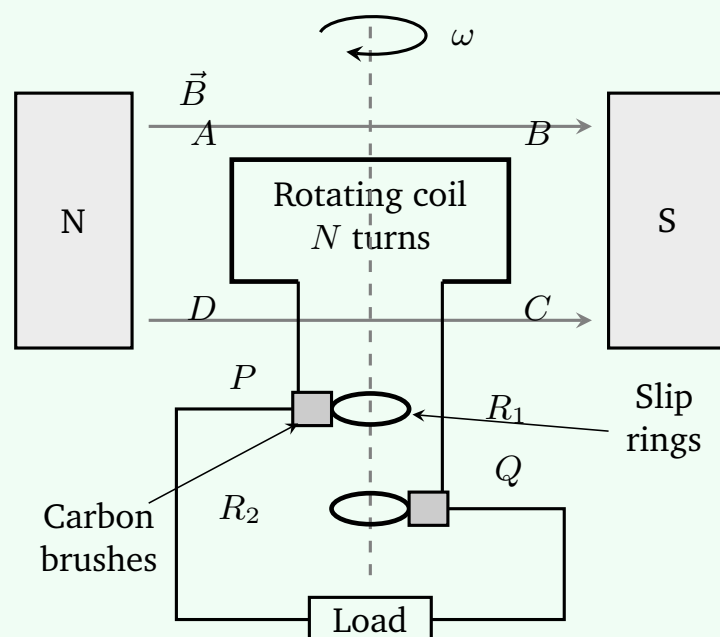
Solution:

Concept:

- Faraday's law gives the induced emf as $\varepsilon = -d(N\Phi)/dt$, where $N\Phi$ is the flux linkage of an N -turn coil.
- In a uniform magnetic field, the flux through one turn is $\Phi = BA \cos \theta$, where A is the area of the turn and θ is the angle between its normal and $\vec{B}$.
- For a straight conductor moving perpendicular to both its length and a magnetic field, the motional emf has magnitude $\varepsilon = Blv$, where l is its length and v its speed.

Step 1: (a) Draw the ac generator and identify its parts A rectangular armature coil $ABCD$, having N turns, rotates between the north and south poles of a magnet. Its rotation axis lies in the plane of the coil and is perpendicular to the magnetic field. The two ends of the coil connect to separate slip rings R_1 and R_2 , which rotate with the coil and are insulated from each other and from the shaft. Stationary carbon brushes P and Q contact these rings and connect the rotating coil to the external load. An external mechanical drive maintains the rotation: the generator converts mechanical energy into electrical energy by electromagnetic induction.

Quick Tip to Solve: Measure the coil angle from its normal when writing $BA \cos \theta$. For the falling rod, use only B_H and check that field, velocity and rod are mutually perpendicular before applying Blv .



AC generator (schematic): rotating coil and separate slip rings, stationary brushes and external load.

Step 2: Express the changing magnetic flux Let the coil rotate with constant angular speed ω . Choose $t = 0$ when the normal to the coil is parallel to $\vec{B}$, so that $\theta = \omega t$. The magnetic flux through one turn and the total flux linkage are then

$$\Phi = BA \cos \omega t, \quad N\Phi = NBA \cos \omega t.$$

The flux changes because the angle changes, even though B , A and N remain constant. It is maximum when the coil plane is perpendicular to the field and zero when the coil plane is parallel to the field.

Step 3: Differentiate the flux linkage to obtain the emf Apply Faraday's law to the entire coil:

$$\begin{aligned} \varepsilon(t) &= -\frac{d}{dt}(NBA \cos \omega t) \\ &= NBA\omega \sin \omega t. \end{aligned}$$

Thus the peak emf is $\varepsilon_0 = NBA\omega$. The negative sign in Faraday's law expresses Lenz's law; the positive sine follows from our chosen flux and terminal convention. The emf reverses sign after every half-revolution, giving alternating emf at the brushes. It is zero when flux is extremal and has its greatest magnitude when flux passes through zero. A different initial orientation changes only the phase of the sine function.

Step 4: (b) Calculate the instantaneous motional emf The rod lies east–west, falls vertically, and is perpendicular to the horizontal north–south component of the Earth's field. Consequently, the rod, its velocity and B_H are mutually perpendicular. Magnetic force separates charges along the rod until the electric force balances it, giving $qE = qvB_H$ and $\varepsilon = El = B_Hlv$. Substituting the stated instantaneous speed,

$$\begin{aligned} \varepsilon &= (0.3 \times 10^{-4} \text{ T})(10 \text{ m})(5.0 \text{ m s}^{-1}) \\ &= 1.5 \times 10^{-3} \text{ V}. \end{aligned}$$

Quick Tip to Solve: Measure the coil angle from its normal when writing $BA \cos \theta$. For the falling rod, use only B_H and check that field, velocity and rod are mutually perpendicular before applying Blv .

The vertical component of the Earth's field contributes no motional emf because it is parallel to the falling velocity. The expression uses the speed at that instant, so the rod's acceleration need not be known.

Step 5: Conclusion (a) The ac generator produces $\varepsilon(t) = NBA\omega \sin \omega t$, with peak emf $\varepsilon_0 = NBA\omega$, for the stated initial orientation. (b) The instantaneous induced emf in the rod has magnitude $1.5 \times 10^{-3} \text{ V} = 1.5 \text{ mV}$.

Quick Tip to Solve: Measure the coil angle from its normal when writing $BA \cos \theta$. For the falling rod, use only B_H and check that field, velocity and rod are mutually perpendicular before applying Blv .

25. (a) Define wavefront. Use Huygens' principle to verify the laws of refraction.

(b) How is linearly polarised light obtained by the process of scattering of light? Find the Brewster angle for air - glass interface, when the refractive index of glass = 1.5. [5 marks]

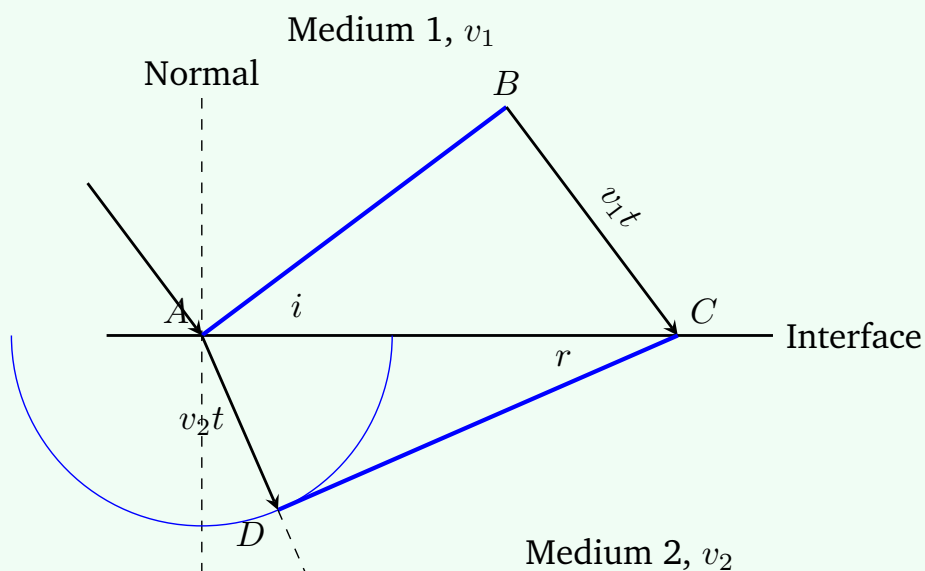
Solution:

Concept:

- A wavefront is the locus of points of a wave which vibrate in the same phase at a given instant. In an isotropic medium, a ray is normal to the wavefront.
- Huygens' principle: every point on a wavefront acts as a source of secondary wavelets; their forward common envelope gives the new wavefront. In a medium of refractive index n , the speed is $v = c/n$.
- Light scattered through 90° by small particles can be linearly polarised. Brewster's law for reflection at a dielectric interface is $\tan i_B = n_2/n_1$.

Step 1: (a) Construct the refracted wavefront Let a plane wavefront AB in medium 1 be incident on a plane interface, with A reaching the interface first. Let v_1 and v_2 be the wave speeds in media 1 and 2. During a time t , point B reaches the interface at C , so $BC = v_1 t$. Meanwhile, the secondary wavelet from A in medium 2 has radius $AD = v_2 t$. Draw the tangent CD from C to this wavelet. The forward envelope CD is the refracted wavefront, and AD , perpendicular to CD , gives a refracted ray. The construction shown is for $v_2 < v_1$.

Quick Tip to Solve: In the Huygens construction, divide the two sine relations to cancel the common hypotenuse and time. For Brewster's angle, use the refractive index of the second medium relative to the first.



Huygens' construction: AB is incident and CD refracted; $BC = v_1 t$, $AD = v_2 t$.

Step 2: (a) Verify Snell's law and coplanarity Let i and r be the angles the incident and refracted rays make with the normal to the interface. Since rays are perpendicular to their wavefronts, $\angle BAC = i$ and $\angle ACD = r$. The right triangles ABC and ADC have the same hypotenuse AC , hence

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{v_2 t}{AC},$$

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{n_2}{n_1}, \quad n_1 \sin i = n_2 \sin r.$$

This verifies the constant sine ratio for a given pair of media. In three dimensions, the incident plane wavefront and the plane interface intersect along a line perpendicular to the plane of incidence. The spherical wavelets have the same construction in every section perpendicular to this line, so their refracted envelope is also perpendicular to the plane of incidence. Its normal, the refracted ray, therefore lies in the plane containing the incident ray and the interface normal. This verifies the other law of refraction.

Step 3: (b) Obtain linear polarisation by scattering Pass an unpolarised beam through a medium containing small scattering particles, and observe the scattered light at right angles to the incident beam. For example, take the incident beam along the x -axis and observe along the y -axis. The incident electric field has components along y and z , which induce corresponding oscillations of charges in the particles. An oscillating electric dipole does not radiate along its own oscillation direction. Thus the y -directed oscillations send no radiation towards the observer along y ; only the z -directed component is seen. The scattered light is therefore linearly polarised, with its electric field perpendicular to the scattering plane (the xy -plane). For ideal single scattering by sufficiently small particles this polarisation is complete at 90° ; at general scattering angles it is partial.

Step 4: (b) Calculate the Brewster angle For light incident from air onto glass, $n_1 \simeq 1$ and $n_2 = 1.5$. At the polarising angle, the reflected and refracted rays are perpendicular. Substituting $r = 90^\circ - i_B$

Quick Tip to Solve: In the Huygens construction, divide the two sine relations to cancel the common hypotenuse and time. For Brewster's angle, use the refractive index of the second medium relative to the first.

in Snell's law gives

$$n_1 \sin i_B = n_2 \cos i_B, \quad \tan i_B = \frac{n_2}{n_1} = 1.5.$$

Thus $i_B = \tan^{-1}(1.5) = 56.31^\circ$, measured from the normal in air. This last calculation concerns polarisation by reflection at the interface; the scattering arrangement above is a separate way of producing polarised light.

Step 5: Conclusion A wavefront is a surface of constant phase. Huygens' construction gives both coplanarity of the incident ray, refracted ray and normal, and $n_1 \sin i = n_2 \sin r$. Light observed at 90° to an incident unpolarised beam is linearly polarised in ideal single scattering, perpendicular to the scattering plane. The air-glass Brewster angle is $i_B \approx 56.3^\circ$.

Quick Tip to Solve: In the Huygens construction, divide the two sine relations to cancel the common hypotenuse and time. For Brewster's angle, use the refractive index of the second medium relative to the first.

OR

25. (a) Draw a ray diagram to show the image formation by a combination of two thin convex lenses in contact. Obtain the expression for the power of this combination in terms of the focal lengths of the lenses.

(b) A ray of light passing from air through an equilateral glass prism undergoes minimum deviation when the angle of incidence is $\frac{3}{4}$ th of the angle of prism. Calculate the speed of light in the prism.

[5 marks]

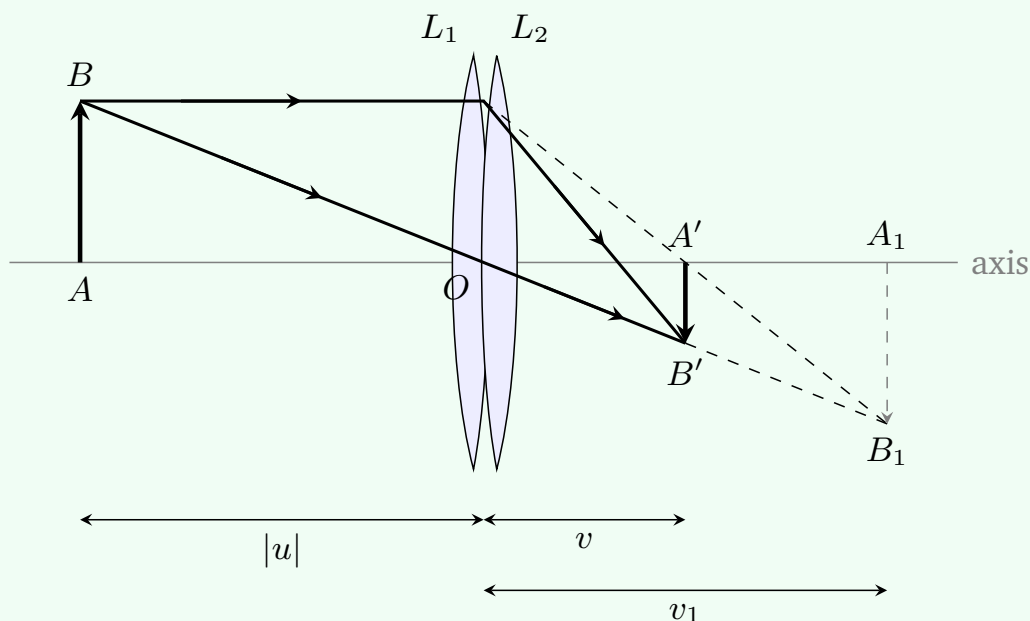
Solution:

Concept:

- For a thin lens in air, the Cartesian lens formula is $1/v - 1/u = 1/f$, where u , v and f are the signed object distance, image distance and focal length.
- The power of a lens is $P = 1/f$, with f in metres and P in dioptres. For lenses in contact their optical centres are treated as coincident.
- At minimum deviation through a prism, $i = e$ and $r_1 = r_2 = A/2$. Snell's law gives $\mu = \sin i / \sin r_1$, and the speed in glass is c/μ .

Step 1: (a) Trace the image through the two lenses Let two thin convex lenses L_1 and L_2 , of focal lengths f_1 and f_2 , be in contact. Place an object AB to the left of them. If L_1 acted alone, it would produce an image A_1B_1 at distance v_1 . The second lens intercepts this converging beam and produces the final image $A'B'$ at distance v . The diagram shows one representative real-image arrangement; the dashed ray shows the path with only the first lens present. The two lens outlines are enlarged horizontally for visibility, while refraction is represented at their common thin-lens plane O .

Quick Tip to Solve: For contact lenses, add reciprocal focal lengths, not focal lengths. For minimum deviation in an equilateral prism, put $r = 30^\circ$ before using Snell's law.



Two convex lenses in contact: solid rays form $A'B'$; the dashed path locates the image A_1B_1 due to L_1 alone.

Step 2: (a) Apply the lens formula successively For the first lens, using the Cartesian sign convention,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}.$$

The rays incident on the second lens are converging towards A_1B_1 , so this acts as its virtual object. Since the separation is negligible, its signed object distance is $u_2 = v_1$, which is positive for the arrangement shown. Therefore,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

Adding eliminates the intermediate-image distance:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Step 3: (a) Identify the equivalent power If the combination is replaced by a single thin lens of focal length F , it must form the same final image of the same object. Its lens formula is $1/v - 1/u = 1/F$. Comparing with the preceding equation gives

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}, \quad F = \frac{f_1 f_2}{f_1 + f_2}.$$

Thus the equivalent power is the sum of the individual powers when all focal lengths are expressed in metres. Both convex lenses have positive power, so their combination has a shorter focal length than either lens alone. This result assumes thin lenses in contact, in air, and paraxial rays.

Step 4: (b) Determine the angles inside the prism For an equilateral prism the prism angle is $A = 60^\circ$.

Quick Tip to Solve: For contact lenses, add reciprocal focal lengths, not focal lengths. For minimum deviation in an equilateral prism, put $r = 30^\circ$ before using Snell's law.

The given angle of incidence is

$$i = \frac{3}{4}A = 45^\circ.$$

At minimum deviation the path is symmetric, so $e = i = 45^\circ$, and

$$r_1 = r_2 = \frac{A}{2} = 30^\circ.$$

As a check, the minimum deviation is $\delta_m = 2i - A = 30^\circ$.

Step 5: (b) Apply Snell's law and obtain the speed Taking the refractive index of air as unity, Snell's law at the first face gives

$$\mu = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}.$$

Hence, using $c = 3.0 \times 10^8 \text{ m s}^{-1}$,

$$v_{\text{glass}} = \frac{c}{\mu} = \frac{3.0 \times 10^8}{\sqrt{2}} \text{ m s}^{-1}.$$

Step 6: Conclusion (a) The power of the combination is $P = P_1 + P_2 = 1/f_1 + 1/f_2$, in dioptres for focal lengths in metres. (b) The speed of light in the prism is $2.12 \times 10^8 \text{ m s}^{-1}$.

Quick Tip to Solve: For contact lenses, add reciprocal focal lengths, not focal lengths. For minimum deviation in an equilateral prism, put $r = 30^\circ$ before using Snell's law.

26. (a) Derive an expression for the electric field $\vec{E}$ due to a dipole of length ' $2a$ ' at a point distant r from the centre of the dipole on the axial line.

(b) Draw a graph of E versus r for $r \gg a$.

(c) If this dipole were kept in a uniform external electric field $\vec{E}_0$, diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases.

[5 marks]

Solution:

Concept:

- A dipole consists of charges $-q$ and $+q$, separated by $2a$. Its moment is $\vec{p} = 2aq\hat{p}$, directed from the negative to the positive charge.
- By Coulomb's law and superposition, the net electric field is the vector sum of the fields of the two charges; write $k = 1/(4\pi\epsilon_0)$.
- In a uniform external field, $\vec{\tau} = \vec{p} \times \vec{E}_0$ and $U = -pE_0 \cos \theta$, where θ is the angle between $\vec{p}$ and $\vec{E}_0$.

Step 1: (a) Set up the two axial fields Put the centre at O , with $-q$ at $x = -a$ and $+q$ at $x = a$. Take the observation point P at $x = r > a$, on the positive-charge side. The distances from $+q$ and $-q$ to P are $r - a$ and $r + a$, respectively. The positive charge produces a field along $\hat{p}$, while the negative

Quick Tip to Solve: Subtract the two axial field magnitudes before applying $r \gg a$. Zero torque alone does not establish stability: check whether $-pE_0 \cos \theta$ is a minimum or maximum.

charge produces a field opposite to $\hat{p}$. Therefore,

$$\vec{E}(P) = kq \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right] \hat{p}.$$

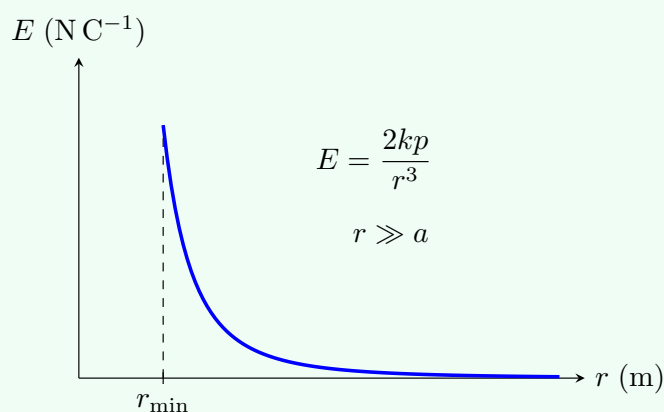
Step 2: Simplify and obtain the distant-point expression Combining the fractions and using $p = 2aq$,

$$\begin{aligned} \vec{E}(P) &= kq \frac{(r+a)^2 - (r-a)^2}{(r^2 - a^2)^2} \hat{p} \\ &= k \frac{4qar}{(r^2 - a^2)^2} \hat{p} = \frac{1}{4\pi\epsilon_0} \frac{2r\vec{p}}{(r^2 - a^2)^2}. \end{aligned}$$

The field on either exterior side of the dipole is parallel to $\vec{p}$. For $r \gg a$, neglect a^2 compared with r^2 , giving

$$\vec{E}_{\text{axial}} \simeq \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}.$$

Step 3: (b) Sketch the far-field variation For a fixed dipole moment, $E \propto r^{-3}$. Thus the positive field magnitude decreases rapidly with increasing distance and approaches zero asymptotically. Doubling r reduces E to one-eighth of its value. The sketch begins at an arbitrary $r_{\min} \gg a$: the distant-dipole approximation must not be extrapolated to the origin.



Axial field magnitude in the far-field region only; $r_{\min} \gg a$.

Step 4: (c) Identify stable equilibrium When $\vec{p}$ is parallel to $\vec{E}_0$, $\theta = 0$. The two equal and opposite forces are along the dipole axis, so the torque vanishes:

$$\vec{\tau} = \vec{p} \times \vec{E}_0 = \vec{0}, \quad \tau = pE_0 \sin 0 = 0.$$

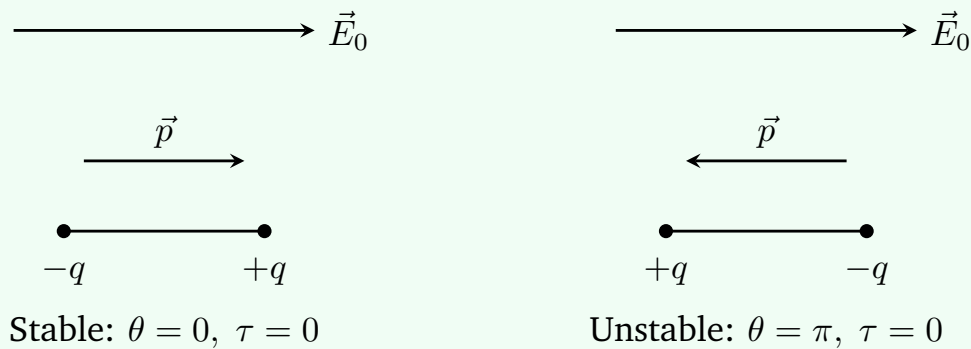
Here $U = -pE_0$ is minimum. A small angular displacement produces a restoring torque, making this equilibrium stable.

Step 5: Identify unstable equilibrium When $\vec{p}$ is antiparallel to $\vec{E}_0$, $\theta = \pi$. Again the forces have no turning effect at the exact equilibrium position:

$$\vec{\tau} = \vec{p} \times \vec{E}_0 = \vec{0}, \quad \tau = pE_0 \sin \pi = 0.$$

Quick Tip to Solve: Subtract the two axial field magnitudes before applying $r \gg a$. Zero torque alone does not establish stability: check whether $-pE_0 \cos \theta$ is a minimum or maximum.

Here $U = +pE_0$ is maximum. A small angular displacement produces a torque that turns the dipole farther from this position, so the equilibrium is unstable. The net force is zero in both orientations because the external field is uniform.



Dipole orientations in a uniform electric field directed to the right.

Step 6: Conclusion The axial field is $\vec{E} = k 2r\vec{p}/(r^2 - a^2)^2$, becoming $2k\vec{p}/r^3$ for $r \gg a$. Its magnitude falls as r^{-3} . Parallel alignment is stable and antiparallel alignment is unstable; the torque is zero at both exact equilibrium positions.

Quick Tip to Solve: Subtract the two axial field magnitudes before applying $r \gg a$. Zero torque alone does not establish stability: check whether $-pE_0 \cos \theta$ is a minimum or maximum.

OR

26. (a) Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge density σ .

(b) An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Obtain the expression for the amount of work done in bringing a point charge q from infinity to a point, distant r , in front of the charged plane sheet. [5 marks]

Solution:

Concept:

- Gauss's theorem states that $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the enclosed charge and ϵ_0 is the permittivity of free space.
- Translational and reflection symmetry make the field of a uniformly charged infinite sheet perpendicular to the sheet, equal in magnitude at equal distances on its two sides, and directed away from a positive sheet.
- For quasistatic motion, the external work is $W_{\text{ext}} = \Delta U = q\Delta V = -q \int \mathbf{E} \cdot d\mathbf{l}$. A zero of potential at infinity is permissible only if the potential difference from infinity converges.

Step 1: (a) Choose a Gaussian pillbox Consider a thin cylindrical Gaussian surface crossing the sheet, with its axis perpendicular to the sheet and its two flat faces, each of area A , equally far from it. The electric field is perpendicular to both flat faces and parallel to the curved surface. Thus the curved

Quick Tip to Solve: Before choosing zero potential at infinity, check whether the field integral converges. The constant field of an infinite sheet makes that reference invalid.

surface contributes zero flux. For a positive sheet, the field and outward area vector have the same direction on each face, giving EA through each face. The charge enclosed is $Q_{\text{enc}} = \sigma A$.

Step 2: (a) Apply Gauss's theorem Adding the two flat-face contributions gives

$$EA + EA = \frac{\sigma A}{\epsilon_0}, \quad E = \frac{\sigma}{2\epsilon_0} \quad (\sigma > 0).$$

The magnitude for either sign of charge density is $|\sigma|/(2\epsilon_0)$. The field points normally away from a positive sheet and towards a negative sheet. Since the distance of the pillbox faces does not enter the result, the field remains constant with distance; in particular, it does not decay at infinity. This is the field of a thin charged sheet in free space, with flux emerging on both sides.

Step 3: (b) First bring the charge from a finite distance Put the positive sheet at $x = 0$ and take the region in front of it as $x > 0$. There $\mathbf{E} = E\hat{x}$, with $E = \sigma/(2\epsilon_0)$. Start the point charge at $x = R$, where $R > r$, and bring it slowly to $x = r$. The potential change and the external work are

$$V(r) - V(R) = - \int_R^r E dx = E(R - r),$$

$$W_{\text{ext}}(R \rightarrow r) = qE(R - r) = \frac{q\sigma}{2\epsilon_0}(R - r).$$

The work done by the electric field is the negative of this expression. For $q > 0$, the external agent works against the repulsive force, so its work is positive.

Step 4: (b) Examine the printed limit of infinity The printed question specifies an infinite sheet and a starting point at infinity. Taking that limit in the finite-distance expression gives

$$W_{\text{ext}}(\infty \rightarrow r) = \lim_{R \rightarrow \infty} \frac{q\sigma}{2\epsilon_0}(R - r).$$

For nonzero q , this has no finite value: it tends to $+\infty$ for $q > 0$ and to $-\infty$ for $q < 0$. Thus setting $V(\infty) = 0$ is not a valid potential convention for the ideal infinite sheet. A finite reference can instead be used: choosing $V(0) = 0$ gives $V(r) = -\sigma r/(2\epsilon_0)$. Accordingly, $-q\sigma r/(2\epsilon_0)$ would describe the external work from the sheet to r , not from infinity. This distinction follows from the non-decaying field and is necessary for the printed wording.

Step 5: Conclusion (a) The field has magnitude $E = |\sigma|/(2\epsilon_0)$, normal to the sheet and independent of distance. (b) From a finite distance R , $W_{\text{ext}} = q\sigma(R - r)/(2\epsilon_0)$. From infinity the work is divergent for nonzero q ; it is $+\infty$ for a positive test charge. No finite work expression with infinity as reference exists for the ideal infinite sheet.

Quick Tip to Solve: Before choosing zero potential at infinity, check whether the field integral converges. The constant field of an infinite sheet makes that reference invalid.