

CBSE Class 12 Physics Question Paper 2017 with Solution PDF-Delhi Set 1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 26 questions in all.
- (ii) This question paper has five sections : Section A, Section B, Section C, Section D and Section E.
- (iii) Section A contains five questions of one mark each, Section B contains five questions of two marks each, Section C contains twelve questions of three marks each, Section D contains one value based question of four marks and Section E contains three questions of five marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all the three questions of five marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron} = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Does the charge given to a metallic sphere depend on whether it is hollow or solid? Give reason for your answer. [1 mark]

Solution:

Concept:

- In electrostatic equilibrium, excess charge on an isolated conductor resides on its outer surface.
- For an isolated spherical conductor of outer radius R , its capacitance is $C = 4\pi\epsilon_0 R$, where ϵ_0 is the permittivity of free space.

Step 1: Locate the excess charge Free charges move until the electric field inside the conducting material vanishes. With no charge placed inside a cavity, the supplied excess charge remains on the outer surface, whether the sphere is hollow or solid.

Step 2: Compare spheres of equal outer radius At a given potential V , the charge is $Q = CV = 4\pi\epsilon_0 RV$. This expression contains the outer radius, but contains neither the metal thickness nor the amount of metal inside the sphere.

Step 3: Conclusion No. For the same outer radius and potential, a hollow and a solid metallic sphere hold the same charge because excess charge resides on the outer surface.

Quick Tip to Solve: Compare outer radii and potentials; the metal filling inside does not enter spherical capacitance.

2. A long straight current carrying wire passes normally through the centre of circular loop. If the current through the wire increases, will there be an induced emf in the loop? Justify. [1 mark]

Solution:

Concept:

- Faraday's law relates induced emf to changing magnetic flux: $\mathcal{E} = -d\Phi_B/dt$.
- Magnetic flux is $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$, where $d\mathbf{A}$ is normal to the loop plane.

Step 1: Determine the magnetic-field direction The straight wire is perpendicular to the circular loop. Its magnetic field forms concentric circles around the wire, so the field at every point of the loop surface lies in the plane of the loop.

Step 2: Evaluate the changing flux The field is perpendicular to the surface normal, giving $\mathbf{B} \cdot d\mathbf{A} = 0$. Thus $\Phi_B = 0$ at all times. Increasing current increases the field strength but cannot create a normal field component; hence $d\Phi_B/dt = 0$.

Step 3: Conclusion No emf is induced in the loop, since the magnetic flux through it remains zero even when the wire current increases.

Quick Tip to Solve: Before applying Faraday's law, check the field component perpendicular to the loop surface.

3. At a place, the horizontal component of earth's magnetic field is B and angle of dip is 60° . What is the value of horizontal component of the earth's magnetic field at equator? [1 mark]

Solution:

Concept:

- The horizontal component of the earth's field is $B_H = B_E \cos \delta$, where B_E is the resultant field and δ is the angle of dip.
- At the magnetic equator, the dip is zero, so the entire field is horizontal.

Step 1: Recover the resultant field The given horizontal component is B at dip 60° . Therefore

$$B = B_E \cos 60^\circ = \frac{B_E}{2}, \quad B_E = 2B.$$

Step 2: Use the equatorial dip Using the same resultant field magnitude, as intended in this elementary component comparison, the equatorial horizontal component is

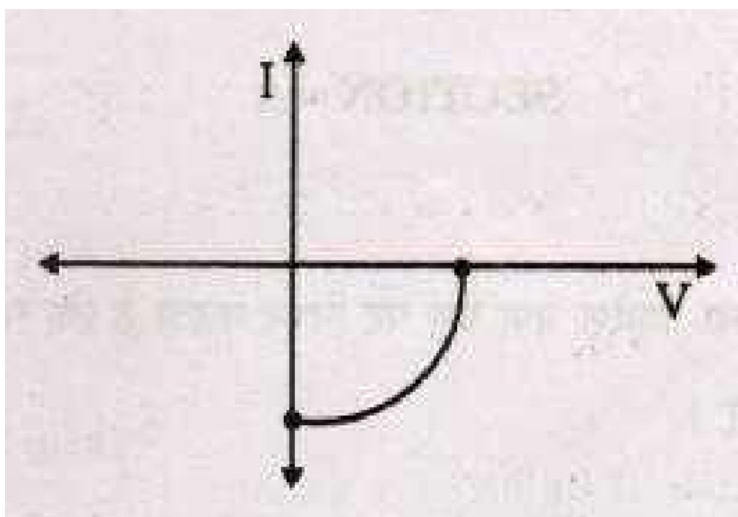
$$B_{H,eq} = B_E \cos 0^\circ = B_E.$$

The dip vanishes at the equator, so there is no vertical component.

Step 3: Conclusion The expected horizontal component at the equator is $2B$, taking the resultant field magnitude to be unchanged in this comparison.

Quick Tip to Solve: First divide the given horizontal field by $\cos$ of the dip; then use zero dip at the equator.

4. Name the junction diode whose I-V characteristics are drawn below: [1 mark]



Solution:

Concept:

- An illuminated solar cell converts light energy directly into electrical energy through the photo-voltaic effect.

Quick Tip to Solve: A fourth-quadrant I-V curve with both current and voltage intercepts identifies a solar cell delivering power.

- With the usual diode sign convention, a delivering solar cell has positive terminal voltage and negative current.

Step 1: Read the characteristic The curve lies in the fourth quadrant: the voltage is positive while the current is negative. It has a nonzero negative current at zero voltage and reaches zero current at a positive voltage.

Step 2: Identify the operating device These intercepts correspond to the short-circuit current and the open-circuit voltage of an illuminated solar cell. Its fourth-quadrant operation indicates delivery of electrical power to an external circuit, rather than ordinary forward conduction.

Step 3: Conclusion The junction diode shown is a solar cell.

Quick Tip to Solve: A fourth-quadrant I-V curve with both current and voltage intercepts identifies a solar cell delivering power.

5. How is the speed of em-waves in vacuum determined by the electric and magnetic fields? [1 mark]

Solution:

Concept:

- For a plane electromagnetic wave in vacuum, the electric and magnetic fields are perpendicular to each other and to propagation.
- Their amplitudes obey $E_0 = cB_0$, where c is the wave speed in vacuum.

Step 1: Relate the field strengths Maxwell's equations give the ratio of the electric-field amplitude to the magnetic-field amplitude as the propagation speed. Here E_0 is measured in V m^{-1} and B_0 in tesla.

Step 2: Express the speed Therefore

$$c = \frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

For corresponding nonzero instantaneous fields of a plane wave, the same relation is $c = E/B$, since the fields oscillate in phase.

Step 3: Conclusion The speed is the ratio of electric to magnetic field amplitudes:

$$c = \frac{E_0}{B_0} \approx 3.0 \times 10^8 \text{ m s}^{-1}$$

Quick Tip to Solve: Use corresponding electric and magnetic amplitudes, never their product, to obtain the wave speed.

Section B (10 marks)

6. How does Ampere-Maxwell law explain the flow of current through a capacitor when it is being charged by a battery? Write the expression for the displacement current in terms of the rate of change of electric flux. [2 marks]

Solution:

Concept:

- Ampere-Maxwell law is $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(I_c + I_d)$, where I_c is conduction current and I_d is displacement current.
- The displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, with $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$ the electric flux.

Step 1: Describe charging in the wires and gap A battery supplies conduction current in the connecting wires, depositing equal and opposite charges on the capacitor plates. No conduction charge crosses an ideal insulating gap. However, the growing plate charge produces a changing electric field and hence a changing electric flux between the plates.

Step 2: Apply Maxwell's correction The changing flux acts as displacement current in the gap and produces magnetic field just as conduction current does. Including this term makes the magnetic circulation independent of whether a surface bounded by the same loop cuts the wire or passes between the plates.

Step 3: Show continuity of current For parallel plates of area A , neglecting fringing, $E = Q/(\epsilon_0 A)$, so $\Phi_E = EA = Q/\epsilon_0$. Therefore

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} = I_c.$$

Thus total current remains continuous during charging even though carriers do not cross the dielectric.

Step 4: Conclusion Ampere-Maxwell law accounts for current continuity through the changing electric field in the capacitor gap. The required expression is $I_d = \epsilon_0 d\Phi_E/dt$, equal to the wire current during charging.

Quick Tip to Solve: In a capacitor gap replace conduction current with epsilon-zero times the changing electric flux.

7. Define the distance of closest approach. An α -particle of kinetic energy ' K ' is bombarded on a thin gold foil. The distance of the closest approach is ' r '. What will be the distance of closest approach for an α -particle of double the kinetic energy? [2 marks]

Solution:

Concept:

- Distance of closest approach is the minimum separation between an alpha-particle and the nucleus in a head-on collision, where the particle momentarily stops before reversing its motion.
- For charges $+2e$ and $+Ze$, electrostatic potential energy is $U = k 2Ze^2/r$, where $k = 1/(4\pi\epsilon_0)$.

Step 1: Apply conservation of energy Take the initial potential energy far from the nucleus as zero

Quick Tip to Solve: For an unchanged alpha-particle and nucleus, use $Kr = \text{constant}$: doubling the energy halves the closest approach.

and neglect recoil of the heavy gold nucleus. At closest approach, the alpha-particle's kinetic energy is zero; its initial kinetic energy has become electrostatic potential energy:

$$K = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{r}.$$

The nucleus and alpha-particle are both positively charged, so their repulsion slows the approaching particle. The minimum head-on separation occurs at its turning point.

Step 2: Compare the two kinetic energies Let r' denote the closest approach for incident energy $2K$. Neither charge changes, so

$$2K = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{r'}, \quad \frac{2K}{K} = \frac{r}{r'}.$$

The charge product cancels in this ratio. A more energetic particle can approach closer against the same repulsive force, consistent with the inverse dependence of closest approach on incident energy.

Step 3: Conclusion Closest approach is the minimum head-on separation at which the alpha-particle momentarily comes to rest. Doubling the kinetic energy gives $r' = r/2$.

Quick Tip to Solve: For an unchanged alpha-particle and nucleus, use $Kr = \text{constant}$: doubling the energy halves the closest approach.

OR

7. Write two important limitations of Rutherford nuclear model of the atom.

[2 marks]

Solution:

Concept:

- Rutherford's model places most atomic mass and positive charge in a tiny nucleus, with electrons revolving around it under electrostatic attraction.
- Classical electromagnetic theory predicts that an accelerating electric charge radiates energy. Discrete atomic spectral lines require particular allowed energy differences.

Step 1: Explain the failure to account for stability An electron moving around the nucleus has centripetal acceleration even if its speed remains constant. According to classical electromagnetic theory, this accelerating charge must continuously radiate energy. As it loses orbital energy, its orbit should shrink and it should spiral into the nucleus. Rutherford's model therefore predicts collapse rather than the experimentally observed stability of atoms; it provides no rule to prevent this radiative loss.

Step 2: Explain the failure to account for line spectra Rutherford's model gives no restriction on the permitted electron orbits or energies. A continuously shrinking classical orbit would radiate over continuously changing frequencies. It cannot explain why an element such as hydrogen emits or absorbs only certain discrete frequencies, forming a characteristic line spectrum. The model lacks quantised energy levels whose differences could determine those frequencies through $h\nu = E_i - E_f$.

Step 3: Conclusion The two important limitations are its inability to explain **atomic stability** and its

Quick Tip to Solve: Link orbital acceleration to radiative collapse for stability, and unrestricted classical energies to the missing explanation of line spectra.

inability to explain the **discrete line spectra of atoms**.

Quick Tip to Solve: Link orbital acceleration to radiative collapse for stability, and unrestricted classical energies to the missing explanation of line spectra.

8. Find out the wavelength of the electron orbiting in the ground state of hydrogen atom. [2 marks]

Solution:

Concept:

- The electron's de Broglie wavelength is $\lambda = h/p = h/(m_e v)$, where h is Planck's constant and p is its momentum.
- Bohr's allowed circular orbit obeys $m_e v r_n = n\hbar$, equivalently $2\pi r_n = n\lambda$. The hydrogen ground state has $n = 1$ and $r_1 = 5.29 \times 10^{-11} \text{ m}$.

Step 1: Relate wavelength to the allowed orbit Use angular-momentum quantisation to express the electron momentum:

$$m_e v = \frac{nh}{2\pi r_n}, \quad \lambda = \frac{h}{m_e v} = \frac{2\pi r_n}{n}.$$

Thus the circumference of an allowed orbit contains an integer number of matter wavelengths. In the ground state, one complete wavelength fits around the orbit because $n = 1$.

Step 2: Substitute the ground-state radius For hydrogen, the Bohr radius is $r_1 = 0.529 \text{ angstrom} = 5.29 \times 10^{-11} \text{ m}$. Hence,

$$\begin{aligned}\lambda_1 &= 2\pi r_1 \\ &= 2\pi(5.29 \times 10^{-11}) \text{ m} \\ &= 3.32 \times 10^{-10} \text{ m}.\end{aligned}$$

This is the matter wavelength associated with the orbiting electron, rather than the wavelength of a photon emitted in an atomic transition.

Step 3: Conclusion The ground-state electron has de Broglie wavelength

$$\lambda_1 \simeq 3.32 \times 10^{-10} \text{ m} = 0.332 \text{ nm}.$$

Quick Tip to Solve: For a Bohr orbit use $n\lambda = 2\pi r_n$; in the ground state the wavelength equals the orbit's circumference.

9. Define the magnifying power of a compound microscope when the final image is formed at infinity. Why must both the objective and the eyepiece of a compound microscope have short focal lengths? Explain. [2 marks]

Solution:

Concept:

- Magnifying power is angular magnification: $M = \beta/\alpha$, where β is the angle subtended by the final image at the eye and α is the angle subtended by the object viewed directly at the least distance of distinct vision D .
- For a compound microscope in normal adjustment, the final image is at infinity. The magnitude of total magnification is $|M| = |m_o|m_e$, with $m_e = D/f_e$ and approximately $|m_o| = L/f_o$.

Step 1: Define the magnifying power in normal adjustment With the final image at infinity, the eye receives parallel rays and views the image without accommodation. Magnifying power compares its apparent angular size with that of the same object viewed directly at D , conventionally 25 cm. The objective first produces a real, inverted, magnified intermediate image. This image lies at the focal plane of the eyepiece, which acts as a simple magnifier and produces the final image at infinity.

Step 2: Relate magnification to both focal lengths For objective focal length f_o , eyepiece focal length f_e , and effective tube length L , the usual microscope approximation gives

$$|M| \simeq \frac{L}{f_o} \frac{D}{f_e}.$$

At fixed tube length, a short f_o increases the objective's linear magnification. A short f_e increases the eyepiece's angular magnification. Since their effects multiply, both focal lengths must be short to obtain a large overall magnifying power. If signed magnification is used, the negative sign indicates that the final image is inverted relative to the object.

Step 3: Conclusion Magnifying power is the ratio β/α defined above. With the final image at infinity, its magnitude is approximately $(L/f_o)(D/f_e)$; short focal lengths of both lenses produce high magnification.

Quick Tip to Solve: Remember the two factors separately: the objective contributes L/f_o , while the eyepiece at infinity contributes D/f_e .

10. Which basic mode of communication is used in satellite communication? What type of wave propagation is used in this mode? Write, giving reason, the frequency range used in this mode of propagation. [2 marks]

Solution:

Concept:

- In point-to-point communication a signal links one transmitter to one receiver; a satellite acts as a relay between ground stations.
- Space-wave propagation follows a line-of-sight path. Microwaves can pass through the ionosphere instead of being returned to the Earth.

Quick Tip to Solve: Distinguish the basic mode (point-to-point) from the propagation mechanism (space wave); mention penetration of the ionosphere as the reason.

Step 1: Identify the communication mode and propagation The basic mode used is point-to-point communication. An uplink carries the signal from a transmitting ground station to the satellite, and a downlink carries the relayed signal to a receiving ground station. These links use space-wave propagation, with the antennas directed toward the satellite.

Step 2: State the frequency range and its reason Space-wave propagation is used for frequencies above approximately 30 MHz. Satellite links normally use the microwave portion of this range, of order gigahertz. At these high frequencies the ionosphere does not reflect the waves back toward the ground, so they can reach the satellite and return to Earth. Their short wavelengths also permit highly directional antennas and large bandwidths. The 30 MHz value is the conventional lower boundary for space-wave communication, rather than a single satellite channel frequency.

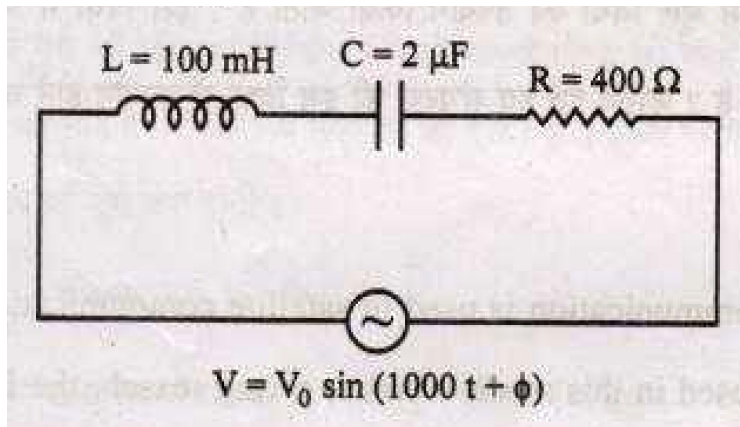
Step 3: Conclusion Satellite communication uses point-to-point mode with space-wave propagation, at frequencies above 30 MHz, normally microwaves, because such waves pass through the ionosphere.

Quick Tip to Solve: Distinguish the basic mode (point-to-point) from the propagation mechanism (space wave); mention penetration of the ionosphere as the reason.

Section C (36 marks)

11. (i) Find the value of the phase difference between the current and the voltage in the series LCR circuit shown below. Which one leads in phase: current or voltage?

(ii) Without making any other change, find the value of the additional capacitor C_1 , to be connected in parallel with the capacitor C , in order to make the power factor of the circuit unity. [3 marks]



Solution:

Concept:

- For a series LCR circuit, $X_L = \omega L$ and $X_C = 1/(\omega C)$, where ω is the angular frequency.
- Defining θ as the phase of voltage relative to current, $\tan \theta = (X_L - X_C)/R$. A negative value means that current leads voltage.
- The power factor is unity at resonance, where $X_L = X_C$. Capacitors in parallel have equivalent capacitance $C_{eq} = C + C_1$.

Quick Tip to Solve: Compute the required total capacitance at resonance first, then subtract the existing capacitor because the new capacitor is in parallel.

Step 1: (i) Calculate the reactances from the printed circuit The source is $V = V_0 \sin(1000t + \phi)$, so $\omega = 1000 \text{ rad s}^{-1}$. The circuit has $L = 100 \text{ mH} = 0.100 \text{ H}$, $C = 2 \mu\text{F}$ and $R = 400 \Omega$. Thus

$$X_L = 1000(0.100) = 100 \Omega, \quad X_C = \frac{1}{1000(2 \times 10^{-6})} = 500 \Omega.$$

The net reactance is $X_L - X_C = -400 \Omega$, so the circuit is capacitive.

Step 2: (i) Determine the magnitude and direction of the phase difference Using the voltage-relative-to-current convention,

$$\tan \theta = \frac{100 - 500}{400} = -1, \quad \theta = -45^\circ = -\frac{\pi}{4}.$$

Therefore the magnitude of the phase difference is 45° , with current leading the applied voltage. The phase constant ϕ in the source shifts both waveforms relative to the time origin and does not change this relative phase.

Step 3: (ii) Find the additional parallel capacitance Unity power factor requires zero net reactance at the same frequency. Adding C_1 in parallel with C gives

$$\omega L = \frac{1}{\omega(C + C_1)}, \quad C + C_1 = \frac{1}{\omega^2 L} = \frac{1}{(1000)^2(0.100)} = 10 \mu\text{F}.$$

Hence $C_1 = 10 - 2 = 8 \mu\text{F}$. This larger equivalent capacitance lowers the capacitive reactance from 500Ω to 100Ω , exactly matching the unchanged inductive reactance.

Step 4: Conclusion (i) Current leads voltage by 45° ($\pi/4$ rad). (ii) An additional capacitor $C_1 = 8 \mu\text{F}$ connected in parallel with C makes the power factor unity.

Quick Tip to Solve: Compute the required total capacitance at resonance first, then subtract the existing capacitor because the new capacitor is in parallel.

12. Write the two processes that take place in the formation of a p-n junction. Explain with the help of a diagram, the formation of depletion region and barrier potential in a p-n junction. [3 marks]

Solution:

Concept:

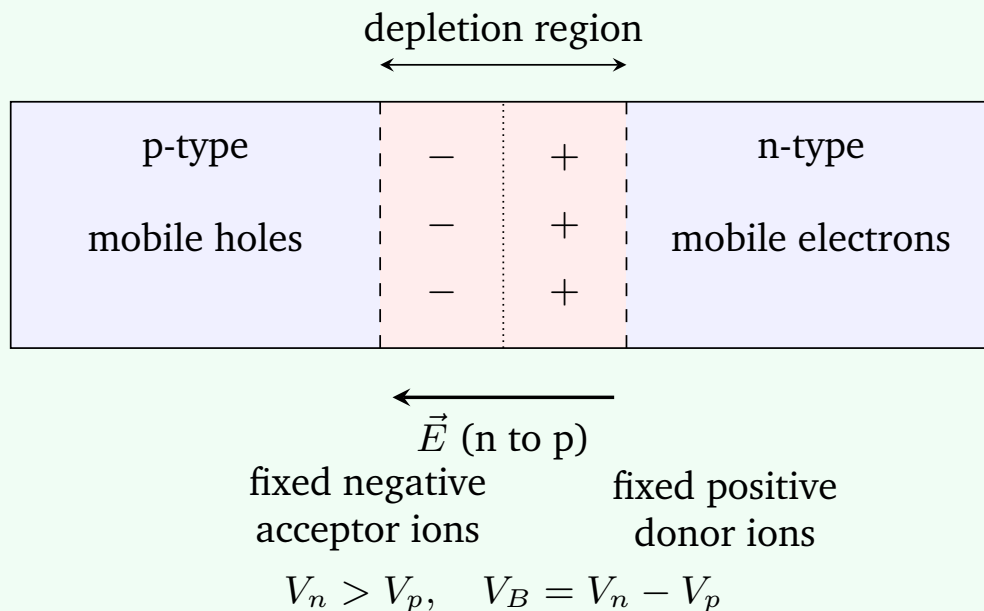
- Diffusion is carrier motion caused by a concentration difference; drift is carrier motion caused by an electric field.
- A p-n junction has holes as majority carriers on the p-side and electrons as majority carriers on the n-side. Ionised acceptors and donors are fixed in the lattice.
- The internal field of the uncovered ions produces a barrier potential that opposes further majority-carrier diffusion.

Step 1: Identify the two processes and the initial diffusion The two processes are diffusion and drift. When p-type and n-type regions form a junction, electrons diffuse from the n-side to the p-side because their concentration is greater on the n-side. Similarly, holes diffuse from the p-side to the

Quick Tip to Solve: Label the depleted p-side with negative ions and the depleted n-side with positive ions; the built-in electric field must point from n to p.

n-side. Electrons and holes recombine near the junction, removing mobile majority carriers from that region.

Step 2: Explain the formation of the depletion region On the n-side, electrons that leave uncover positively charged donor ions. On the p-side, holes that disappear uncover negatively charged acceptor ions. These ions cannot move because they are fixed in the crystal lattice. The thin region on either side of the junction consequently contains very few mobile carriers and is called the depletion region. It contains fixed space charge, so depleted does not mean that it contains no charge.



Depletion region and built-in field of an unbiased p-n junction; the ions shown are immobile.

Step 3: Explain the barrier potential and equilibrium The positive donor ions on the n-side and negative acceptor ions on the p-side establish an electric field directed from n to p. The potential is therefore higher on the n-side; the built-in barrier is $V_B = V_n - V_p > 0$. This field exerts a force opposing diffusion of majority electrons from n to p and majority holes from p to n. It also produces drift of carriers in the direction opposite to their diffusion. As the space charge builds, the drift current increases until it balances the diffusion current. At thermal equilibrium, without an external bias, the net current is zero although the barrier and depletion region remain.

Step 4: Conclusion Diffusion and drift form the p-n junction. Recombination uncovers fixed negative acceptor ions on the p-side and positive donor ions on the n-side, creating the depletion region and an internal barrier potential that opposes further majority-carrier diffusion.

Quick Tip to Solve: Label the depleted p-side with negative ions and the depleted n-side with positive ions; the built-in electric field must point from n to p.

13. (i) Obtain the expression for the cyclotron frequency.

(ii) A deuteron and a proton are accelerated by the cyclotron. Can both be accelerated with the same oscillator frequency? Give reason to justify your answer. [3 marks]

Solution:

Concept:

- Magnetic force qvB supplies the centripetal force for a particle moving perpendicular to a uniform magnetic field B . Here q is the charge magnitude and m is the particle mass.
- The alternating electric field must reverse between successive gap crossings. Cyclotron resonance assumes nonrelativistic motion, with constant mass.

Step 1: (i) Derive the revolution period For speed v and circular orbit radius r ,

$$qvB = \frac{mv^2}{r}, \quad r = \frac{mv}{qB}.$$

The distance travelled during a revolution is $2\pi r$, so

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}.$$

The speed cancels: the particle's orbit grows as it gains energy, but the period remains constant in the nonrelativistic approximation. The magnetic field changes the direction of motion; acceleration in energy occurs in the electric field across the gap.

Step 2: Obtain the oscillator frequency Successive crossings of the gap are separated by a semicircle, taking $T/2$. The alternating voltage must reverse polarity in that same time to accelerate the particle on its next crossing. Its full period is therefore T , giving

$$f = \frac{1}{T} = \frac{qB}{2\pi m}, \quad \omega = \frac{qB}{m}.$$

Thus the required frequency depends on the magnetic field and charge-to-mass ratio, independently of the orbit radius and speed within this approximation.

Step 3: (ii) Compare the two particles A proton has charge $+e$ and mass m_p ; a deuteron has the same charge but approximately twice that mass. In the same magnetic field,

$$f_p = \frac{eB}{2\pi m_p}, \quad f_d \simeq \frac{eB}{4\pi m_p} = \frac{f_p}{2}.$$

Their different charge-to-mass ratios give different revolution periods. A frequency resonant with one particle fails to maintain the required gap-crossing synchronism for the other.

Step 4: Conclusion $f = qB/(2\pi m)$. Both particles cannot be accelerated at the same oscillator frequency in the same magnetic field: the deuteron needs approximately half the proton's frequency.

Quick Tip to Solve: Compare charge-to-mass ratios before comparing frequencies: equal charges alone do not ensure cyclotron resonance at the same frequency.

14. (i) How does one explain the emission of electrons from a photosensitive surface with the help of Einstein's photoelectric equation?

(ii) The work function of the following metals is given: Na = 2.75 eV, K = 2.3 eV, Mo = 4.17 eV and Ni = 5.15 eV. Which of these metals will not cause photoelectric emission for radiation of wavelength 3300 Å from a laser source placed 1 m away from these metals? What happens if the laser source is brought nearer and placed 50 cm away?

[3 marks]

Solution:

Concept:

- In Einstein's model, one electron absorbs one photon of energy $h\nu$. The work function ϕ is the minimum energy needed to escape the metal surface.
- The maximum photoelectron kinetic energy is $K_{\max} = h\nu - \phi$. Emission requires $h\nu \geq \phi$, irrespective of the intensity.

Step 1: (i) Explain electron emission using energy conservation An incident photon transfers its energy to an electron. Part of this energy frees the electron from the metal and the remainder appears as kinetic energy. For the most energetic emitted electrons,

$$h\nu = \phi + \frac{1}{2}m_e v_{\max}^2 = \phi + K_{\max}.$$

If $h\nu < \phi$, the photon cannot supply the escape energy, so no photoelectric emission occurs. The threshold frequency is $\nu_0 = \phi/h$. Above threshold, increasing frequency increases the maximum kinetic energy; increasing intensity at fixed frequency supplies more photons and can increase the emission rate.

Step 2: (ii) Calculate the laser photon energy Convert the wavelength before calculating:

$$\lambda = 3300 \text{ Å} = 330 \text{ nm} = 3.30 \times 10^{-7} \text{ m}.$$

Using $hc \simeq 1240 \text{ eV nm}$,

$$E_\gamma = \frac{hc}{\lambda} = \frac{1240}{330} \text{ eV} \simeq 3.76 \text{ eV}.$$

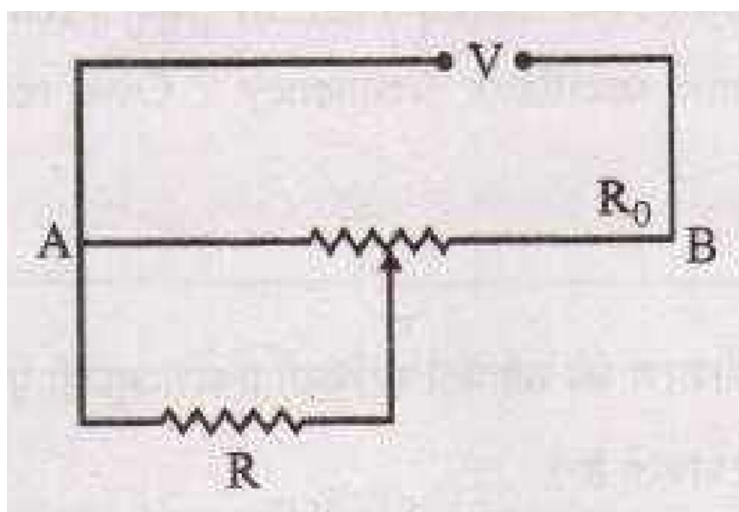
This exceeds the work functions of Na (2.75 eV) and K (2.3 eV), but is smaller than those of Mo (4.17 eV) and Ni (5.15 eV). Thus only the first two metals satisfy the photon-energy threshold.

Step 3: Assess moving the source to 50 cm Bringing the source nearer does not change its wavelength or the energy of an individual photon. Hence it cannot overcome the work functions of Mo and Ni. If bringing the source nearer increases the incident intensity, the number of emitted electrons per second from Na and K increases, while their maximum kinetic energies remain unchanged: approximately 1.01 eV for Na and 1.46 eV for K. No inverse-square intensity factor is required, since the question specifies a laser without its beam geometry.

Step 4: Conclusion Mo and Ni do not show photoelectric emission at either distance. Na and K emit; a higher incident intensity after moving the source nearer increases their emission rate but does not change $K_{\max}$.

Quick Tip to Solve: Compare photon energy in eV with each work function first; changing source distance cannot change the threshold condition at fixed wavelength.

15. A resistance of R draws current from a potentiometer. The potentiometer wire, AB, has a total resistance of R_0 . A voltage V is supplied to the potentiometer. Derive an expression for the voltage across R when the sliding contact is in the middle of potentiometer wire. [3 marks]



Solution:

Concept:

- A uniform potentiometer wire has resistance proportional to its length. At the midpoint C, each half has resistance $R_0/2$.
- Resistors connected between the same two nodes are in parallel; their equivalent resistance satisfies $1/R_p = 1/R_1 + 1/R_2$. Series voltage division uses the actual equivalent resistances after loading.

Step 1: Identify the loaded half of the wire In the printed circuit, the external resistance R is connected from A to the sliding contact C. With C at the midpoint, it is in parallel with the half-wire AC, whose resistance is $R_0/2$. The other half, CB, also has resistance $R_0/2$ and is in series with that parallel combination. Assume the stated supply maintains voltage V across AB and the wire is uniform.

Step 2: Calculate the parallel and total resistances The equivalent resistance between A and C is

$$R_p = \frac{R(R_0/2)}{R + R_0/2} = \frac{RR_0}{2R + R_0}.$$

Consequently the supply sees total resistance

$$R_{eq} = \frac{R_0}{2} + \frac{RR_0}{2R + R_0} = \frac{R_0(4R + R_0)}{2(2R + R_0)}.$$

The supply current flows through CB and then divides between AC and the external resistor; the two half-wire currents therefore need not be equal.

Step 3: Apply voltage division to the loaded circuit The voltage across R equals the voltage between A and C, because the resistor and AC share their end nodes. Using the current through the series

Quick Tip to Solve: Replace the loaded half-wire and external resistor by their parallel equivalent before using the voltage-divider formula.

combination,

$$\begin{aligned}V_R &= \frac{V}{R_{\text{eq}}} R_p \\&= V \frac{RR_0/(2R + R_0)}{R_0(4R + R_0)/[2(2R + R_0)]} \\&= \frac{2RV}{4R + R_0}.\end{aligned}$$

The loading lowers the midpoint voltage below $V/2$ for finite positive R . As $R \rightarrow \infty$, the external branch draws negligible current and this expression tends to $V/2$. As $R \rightarrow 0$, it tends to zero, as expected for a short circuit across AC. These limits check both the circuit interpretation and the algebra.

Step 4: Conclusion The voltage across the external resistance is $V_R = 2RV/(4R + R_0)$. The midpoint setting does not give exactly $V/2$ under a finite load.

Quick Tip to Solve: Replace the loaded half-wire and external resistor by their parallel equivalent before using the voltage-divider formula.

16. Define the term ‘amplitude modulation’. Explain any two factors which justify the need for modulating a low frequency base-band signal. [3 marks]

Solution:

Concept:

- A base-band signal is the original information signal. A carrier is a high-frequency sinusoidal wave used to transport that information.
- For electromagnetic waves, $\lambda = c/f$, where c is the speed of light. A practical transmitting antenna has a length of the order of $\lambda/4$.

Step 1: Define amplitude modulation Amplitude modulation is the process in which the amplitude of a high-frequency carrier wave is varied in accordance with the instantaneous value of a low-frequency message signal, while the carrier frequency remains constant. For a sinusoidal message, the modulated wave may be written as

$$s(t) = A_c[1 + \mu \cos(\omega_m t)] \cos(\omega_c t),$$

where A_c is the unmodulated carrier amplitude, μ is the modulation index, and ω_m and ω_c are the message and carrier angular frequencies. The envelope carries the information.

Step 2: Obtain a practical antenna size Direct transmission of a low-frequency signal requires an impractically long antenna. For example, at $f = 1$ kHz, the wavelength is 3×10^5 m, so a quarter-wave antenna would be 75 km long. If the information modulates a 1 MHz carrier, its wavelength is about 300 m, and the corresponding antenna length is about 75 m. Thus modulation shifts transmission to a frequency where antenna dimensions are manageable.

Step 3: Increase the efficiency of radiation The power radiated by a short antenna of fixed length l is proportional to $(l/\lambda)^2$. At low message frequencies, λ is very large, so l/λ is tiny and the radiated power is small. A high-frequency carrier has a much shorter wavelength and therefore enables more

Quick Tip to Solve: Use $\lambda = c/f$ to connect both antenna length and radiation efficiency directly to the higher carrier frequency.

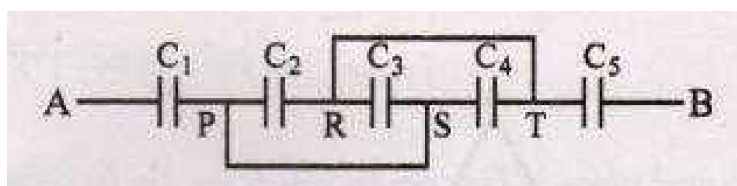
efficient radiation from the same antenna. This allows the information to be transmitted over useful distances.

Step 4: Conclusion Amplitude modulation varies the carrier amplitude with the message signal. Two reasons for using modulation are a practical antenna length and efficient electromagnetic radiation.

Quick Tip to Solve: Use $\lambda = c/f$ to connect both antenna length and radiation efficiency directly to the higher carrier frequency.

17. (i) Find equivalent capacitance between A and B in the combination given below. Each capacitor is of $2\ \mu\text{F}$ capacitance.

(ii) If a dc source of 7 V is connected across AB, how much charge is drawn from the source and what is the energy stored in the network? [3 marks]



Solution:

Concept:

- Ideal connecting wires join points at the same potential. Capacitors are parallel when both their terminals connect to the same pair of nodes.
- Parallel capacitances add; for series capacitors, $1/C_{\text{eq}} = \sum 1/C_i$. The source supplies charge $Q = C_{\text{eq}}V$, and stored energy is $U = \frac{1}{2}C_{\text{eq}}V^2$.

Step 1: Identify the nodes and simplify the middle capacitors In the printed network, the lower connecting wire joins P to S , while the upper connecting wire joins R to T . Thus P and S are one node, and R and T are another. Capacitor C_2 is between P and R , C_3 is between R and S , and C_4 is between S and T . All three therefore connect across the same two nodes, regardless of their appearance along the horizontal line. Their equivalent capacitance is

$$C_m = C_2 + C_3 + C_4 = 2 + 2 + 2 = 6\ \mu\text{F}.$$

Step 2: Calculate the equivalent capacitance between A and B The reduced network consists of $C_1 = 2\ \mu\text{F}$, $C_m = 6\ \mu\text{F}$, and $C_5 = 2\ \mu\text{F}$ connected in series. Hence

$$\frac{1}{C_{AB}} = \frac{1}{2} + \frac{1}{6} + \frac{1}{2} = \frac{7}{6}\ (\mu\text{F})^{-1}, \quad C_{AB} = \frac{6}{7}\ \mu\text{F}.$$

The wires do not short any of these three reduced series elements; each remains between distinct successive nodes.

Step 3: Calculate the charge supplied and energy stored On connecting the 7V dc source, the

Quick Tip to Solve: First merge points joined by wires: $P = S$ and $R = T$. This reveals the three middle capacitors as a parallel group.

magnitude of charge delivered from its positive terminal is

$$Q = C_{AB}V = \frac{6}{7} \times 10^{-6} \times 7 = 6 \times 10^{-6} \text{ C.}$$

The total electrostatic energy in the network is

$$U = \frac{1}{2}C_{AB}V^2 = \frac{1}{2} \times \frac{6}{7} \times 10^{-6} \times 49 = 21 \times 10^{-6} \text{ J.}$$

As a check, the two end capacitors each have a 3 V potential difference and the middle parallel group has 1 V; these add to the source voltage.

Step 4: Conclusion $C_{AB} = \frac{6}{7} \mu\text{F}$, charge drawn from the source $Q = 6 \mu\text{C}$, and total stored energy $U = 21 \mu\text{J}$.

Quick Tip to Solve: First merge points joined by wires: $P = S$ and $R = T$. This reveals the three middle capacitors as a parallel group.

18. (i) Derive the expression for electric field at a point on the equatorial line of an electric dipole.
(ii) Depict the orientation of the dipole in (i) stable, (ii) unstable equilibrium in a uniform electric field. [3 marks]

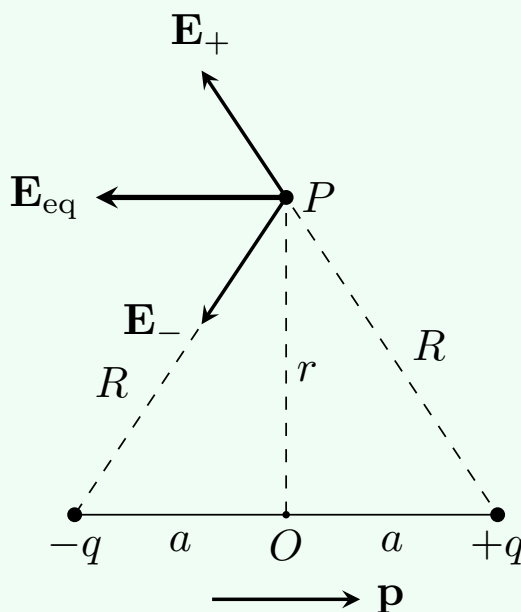
Solution:

Concept:

- An electric dipole consists of charges $-q$ and $+q$, separated by $2a$. Its moment $\mathbf{p}$ points from negative to positive charge and has magnitude $p = 2aq$.
- In a uniform field, $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$ and $U = -\mathbf{p} \cdot \mathbf{E} = -pE \cos \theta$. Stable equilibrium has minimum potential energy.

Step 1: Resolve the fields on the equatorial line Place the charges at $x = -a$ and $x = +a$, and let point P lie a distance r from the midpoint O on the perpendicular bisector. The distance of P from either charge is $R = \sqrt{r^2 + a^2}$. Each field has magnitude kq/R^2 , where $k = 1/(4\pi\epsilon_0)$. The field from $+q$ is directed away from it; the field from $-q$ is directed toward it. Their components along OP cancel, while their components parallel to the dipole axis add toward the negative charge. Each axial component has magnitude $(kq/R^2)(a/R)$.

Quick Tip to Solve: On the equatorial line cancel the perpendicular components first; classify equilibrium using the minimum or maximum of $-pE \cos \theta$.



Field components at an equatorial point; the resultant points opposite to the dipole moment.

Step 2: Obtain the exact field and the distant-point limit Adding the two axial components gives

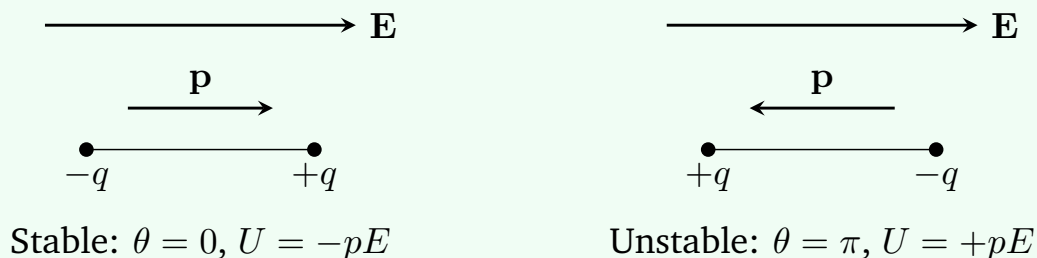
$$E_{\text{eq}} = \frac{2kqa}{R^3} = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2 + a^2)^{3/2}}.$$

The direction is opposite to $\mathbf{p}$, so the vector expression is

$$\mathbf{E}_{\text{eq}} = -\frac{1}{4\pi\epsilon_0} \frac{\mathbf{p}}{(r^2 + a^2)^{3/2}}.$$

For a short dipole observed far away, $r \gg a$, this becomes $\mathbf{E}_{\text{eq}} \simeq -\mathbf{p}/(4\pi\epsilon_0 r^3)$. The approximation must not replace the exact expression when the finite separation matters.

Step 3: Depict and explain both equilibrium orientations Equilibrium requires zero torque, so $\theta = 0$ or π . When $\mathbf{p}$ is parallel to $\mathbf{E}$, the energy is minimum, $U = -pE$. A small rotation produces a restoring torque, so this is stable equilibrium. When $\mathbf{p}$ is antiparallel to $\mathbf{E}$, the energy is maximum, $U = +pE$. A small rotation causes the dipole to move farther from that orientation, so the equilibrium is unstable. In a uniform field the opposite forces also give zero net force.



Stable and unstable equilibrium of a dipole in a uniform electric field.

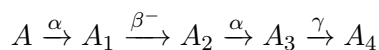
Step 4: Conclusion The equatorial field is opposite to the dipole moment, with magnitude $p/[4\pi\epsilon_0(r^2 +$

Quick Tip to Solve: On the equatorial line cancel the perpendicular components first; classify equilibrium using the minimum or maximum of $-pE \cos \theta$.

$a^2)^{3/2}]$. Parallel orientation is stable and antiparallel orientation is unstable.

Quick Tip to Solve: On the equatorial line cancel the perpendicular components first; classify equilibrium using the minimum or maximum of $-pE \cos \theta$.

19. (i) A radioactive nucleus 'A' undergoes a series of decays as given below:



The mass number and atomic number of A_2 are 176 and 71 respectively. Determine the mass and atomic numbers of A_4 and A .

(ii) Write the basic nuclear processes underlying β^+ and β^- decays.

[3 marks]

Solution:

Concept:

- In alpha decay a nucleus emits ${}^4_2\text{He}$, so its mass number M decreases by 4 and atomic number Z decreases by 2.
- In beta-minus decay a neutron becomes a proton; M is unchanged and Z increases by 1. Gamma emission only removes excitation energy, leaving both numbers unchanged.

Step 1: (i) Work forward from the known nucleus The given nucleus is $A_2 = {}^{176}_{71}\text{A}_2$. Its next decay is alpha emission, hence

$$M_3 = 176 - 4 = 172, \quad Z_3 = 71 - 2 = 69.$$

The subsequent gamma emission changes neither the number of nucleons nor the nuclear charge. Therefore $M_4 = 172$ and $Z_4 = 69$.

Step 2: (i) Work backward to the parent nucleus The preceding beta-minus decay increases the atomic number by one without changing the mass number. Thus A_1 has $M_1 = 176$ and $Z_1 = 71 - 1 = 70$. Since A loses an alpha particle to form A_1 , its original numbers are

$$M_A = 176 + 4 = 180, \quad Z_A = 70 + 2 = 72.$$

As a check, the whole chain contains two alpha decays and one beta-minus decay, giving net changes $\Delta M = -8$ and $\Delta Z = -3$.

Step 3: (ii) State the elementary nuclear processes The beta-plus process converts a proton into a neutron with emission of a positron and an electron neutrino:

$$p \longrightarrow n + e^+ + \nu_e.$$

The beta-minus process converts a neutron into a proton with emission of an electron and an electron antineutrino:

$$n \longrightarrow p + e^- + \bar{\nu}_e.$$

These processes conserve electric charge and lepton number. In both cases the nucleon remains in the nucleus, so the mass number is unchanged; the atomic number decreases by one for beta-plus and increases by one for beta-minus.

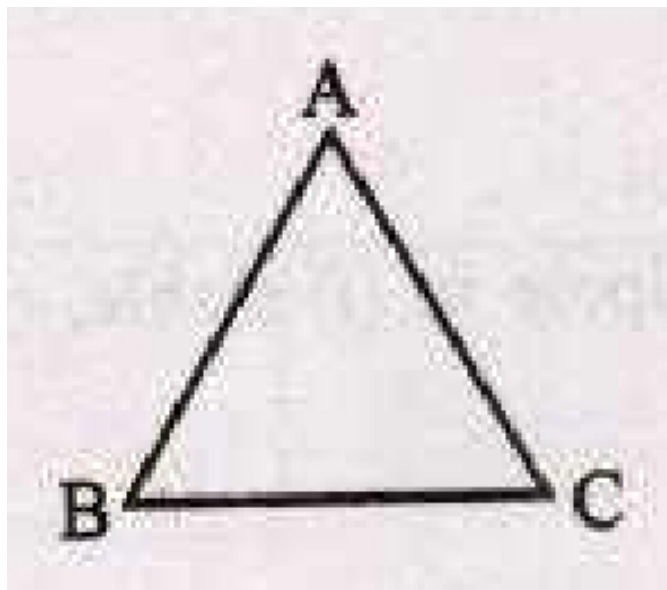
Quick Tip to Solve: When tracing backward, reverse each decay change; gamma emission never changes the mass or atomic number.

Step 4: Conclusion A_4 : mass number 172, atomic number 69. Parent A : mass number 180, atomic number 72. Beta-plus: $p \rightarrow n + e^+ + \nu_e$; beta-minus: $n \rightarrow p + e^- + \bar{\nu}_e$.

Quick Tip to Solve: When tracing backward, reverse each decay change; gamma emission never changes the mass or atomic number.

20. (i) A ray of light incident on face AB of an equilateral glass prism, shows minimum deviation of 30° . Calculate the speed of light through the prism.

(ii) Find the angle of incidence at face AB so that the emergent ray grazes along the face AC. [3 marks]



Solution:

Concept:

- For a prism of angle A , minimum deviation δ_m gives refractive index $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$. The speed in glass is $v = c/\mu$.
- Inside the prism the refraction angles satisfy $r_1 + r_2 = A$. Grazing emergence in air means $e = 90^\circ$, so $r_2 = C$, where the critical angle obeys $\sin C = 1/\mu$.

Step 1: (i) Calculate the refractive index An equilateral prism has prism angle $A = 60^\circ$. Using its measured minimum deviation $\delta_m = 30^\circ$,

$$\mu = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}.$$

At this symmetric setting the angles of incidence and emergence are equal, while both internal refraction angles are 30° . This symmetry is used to determine the material index.

Step 2: (i) Calculate the speed through glass Taking the speed in air to be $c = 3.0 \times 10^8 \text{ m s}^{-1}$,

$$v = \frac{c}{\mu} = \frac{3.0 \times 10^8}{\sqrt{2}} = 2.12 \times 10^8 \text{ m s}^{-1}.$$

Quick Tip to Solve: First obtain the material index from minimum deviation; then use the critical angle for the separate grazing-emergence condition.

The speed is smaller than the vacuum value because the prism has refractive index greater than unity.

Step 3: (ii) Apply the grazing-emergence condition For the new incidence condition, the emergent ray lies along face AC, so its angle to the normal is 90° . Applying Snell's law at AC gives $\mu \sin r_2 = \sin 90^\circ = 1$, hence $r_2 = 45^\circ$. Therefore $r_1 = 60^\circ - 45^\circ = 15^\circ$. At the entry face AB,

$$\begin{aligned}\sin i &= \mu \sin r_1 = \sqrt{2} \sin 15^\circ \\ &= \sqrt{2} \frac{\sqrt{6} - \sqrt{2}}{4} = \frac{\sqrt{3} - 1}{2}.\end{aligned}$$

Thus $i = \sin^{-1}(0.3660) \simeq 21.47^\circ$, measured from the normal to AB. The minimum-deviation setting and the grazing-emergence setting are different ray paths through the same material.

Step 4: Conclusion The speed of light in the prism is $2.12 \times 10^8 \text{ m s}^{-1}$. Grazing emergence along AC requires incidence at AB of approximately 21.5° to the normal.

Quick Tip to Solve: First obtain the material index from minimum deviation; then use the critical angle for the separate grazing-emergence condition.

21. For a CE-transistor amplifier, the audio signal voltage across the collector resistance of $2 \text{ k}\Omega$ is 2 V . Given the current amplification factor of the transistor is 100 , find the input signal voltage and base current, if the base resistance is $1 \text{ k}\Omega$. [3 marks]

Solution:

Concept:

- For the signal component in a CE amplifier, the current gain is $\beta = I_c/I_b$, where I_c and I_b denote collector and base signal-current amplitudes.
- Ohm's law gives output signal magnitude $V_o = I_c R_C$ and input signal magnitude $V_i = I_b R_B$. Hence the voltage-gain magnitude is $\beta R_C/R_B$.

Step 1: Convert the data and obtain the collector signal current The collector resistance is $R_C = 2 \text{ k}\Omega = 2.0 \times 10^3 \Omega$, and the audio signal voltage across it is $V_o = 2 \text{ V}$. Therefore

$$I_c = \frac{V_o}{R_C} = \frac{2}{2.0 \times 10^3} = 1.0 \times 10^{-3} \text{ A} = 1 \text{ mA}.$$

This is the signal current associated with the stated output voltage, rather than the total steady collector current.

Step 2: Use current amplification to calculate base signal current The current amplification factor is $\beta = 100$. Since $I_c = \beta I_b$,

$$I_b = \frac{I_c}{\beta} = \frac{1.0 \times 10^{-3}}{100} = 1.0 \times 10^{-5} \text{ A} = 10 \mu\text{A}.$$

The base signal current is smaller than the collector signal current by a factor of 100 , as required for current amplification.

Quick Tip to Solve: Calculate collector current from the output voltage first, divide by current gain, then multiply by base resistance.

Step 3: Calculate the input signal voltage and check the gain The base resistance is $R_B = 1\text{ k}\Omega = 1.0 \times 10^3\ \Omega$. Hence

$$V_i = I_b R_B = (1.0 \times 10^{-5})(1.0 \times 10^3) = 1.0 \times 10^{-2}\text{ V} = 10\text{ mV}.$$

Independently, the voltage-gain magnitude is $\beta R_C / R_B = 100(2/1) = 200$, so $V_o / V_i = 2/0.01 = 200$, confirming the calculation. A CE amplifier inverts the output signal relative to the input; the question asks for signal magnitudes, so this phase reversal does not introduce a negative amplitude.

Step 4: Conclusion The input signal voltage is $0.010\text{ V} = 10\text{ mV}$, and the base signal current is $1.0 \times 10^{-5}\text{ A} = 10\ \mu\text{A}$.

Quick Tip to Solve: Calculate collector current from the output voltage first, divide by current gain, then multiply by base resistance.

22. Describe the working principle of a moving coil galvanometer. Why is it necessary to use (i) a radial magnetic field and (ii) a cylindrical soft iron core in a galvanometer? Write the expression for current sensitivity of the galvanometer. Can a galvanometer as such be used for measuring the current? Explain. [3 marks]

Solution:

Concept:

- A current-carrying coil experiences magnetic torque $\tau_m = NIAB \sin \alpha$, where N is the number of turns, A the coil area, B the field, and α the angle between its normal and the field.
- A spring supplies restoring torque $\tau_r = k\theta$, where k is its torsional constant and θ the deflection. Current sensitivity is deflection per unit current.

Step 1: Principle and equilibrium Current through the suspended coil produces a magnetic torque that rotates it. The spring opposes rotation until its restoring torque balances the magnetic torque. In a radial field, the magnetic field remains parallel to the plane of the coil, so its normal makes 90° with the field. Hence,

$$NIAB = k\theta, \quad \theta = \frac{NAB}{k}I.$$

Deflection is proportional to current; reversing the current reverses the torque and deflection.

Step 2: Need for the radial field and cylindrical soft iron core (i) The radial field ensures $\sin \alpha = 1$ at every position of the coil. Thus the magnetic torque is independent of deflection and gives a linear current-deflection relation and a uniform scale.

(ii) Soft iron has high magnetic permeability. Its cylindrical core concentrates and strengthens the magnetic field, increasing the torque for a given small current and thereby improving sensitivity. Together with the shaped pole pieces, the cylindrical core also helps establish the radial field. Its low retentivity allows it to be readily magnetised without retaining appreciable magnetism.

Step 3: Sensitivity and measurement of current The equilibrium equation gives current sensitivity

$$S_I = \frac{\theta}{I} = \frac{NAB}{k}.$$

Quick Tip to Solve: Equate magnetic torque to spring torque in the radial field; the coefficient of current in the deflection formula is the current sensitivity.

A galvanometer as such is unsuitable as an ordinary ammeter: its delicate coil safely carries only small currents, and its appreciable resistance would alter the current in the circuit. It detects small currents and their direction. To measure ordinary larger currents, a low resistance shunt is connected in parallel with the coil, allowing most current to bypass it; the resulting instrument is calibrated as an ammeter. A calibrated galvanometer can measure small currents within its safe range.

Step 4: Conclusion Magnetic torque balanced by spring torque gives $\theta = NIAB/k$. A radial field gives a uniform scale, a soft iron core increases sensitivity, and $S_I = NAB/k$. A suitable shunt converts the galvanometer into an ammeter.

Quick Tip to Solve: Equate magnetic torque to spring torque in the radial field; the coefficient of current in the deflection formula is the current sensitivity.

OR

22. (a) Define the term ‘self-inductance’ and write its S.I. unit.

(b) Obtain the expression for the mutual inductance of two long co-axial solenoids S_1 and S_2 wound one over the other, each of length L and radii r_1 and r_2 and n_1 and n_2 number of turns per unit length, when a current I is set up in the outer solenoid S_2 . [3 marks]

Solution:

Concept:

- Self-inductance relates a coil's flux linkage to its own current: $N\Phi = \mathcal{L}I$. The induced emf is $\varepsilon = -\mathcal{L} dI/dt$ for constant inductance $\mathcal{L}$.
- Mutual inductance relates the flux linkage of one coil to current in another. For long air-core solenoids, the internal magnetic field is $B = \mu_0 nI$, where n is turns per unit length; end effects are neglected.

Step 1: (a) Define self-inductance and its unit The self-inductance of a coil is the magnetic flux linkage with the coil per unit current flowing through it:

$$\mathcal{L} = \frac{N\Phi}{I}.$$

Equivalently, its numerical value equals the magnitude of induced emf when the current changes at unit rate. The minus sign in $\varepsilon = -\mathcal{L} dI/dt$ represents Lenz's law: the induced emf opposes the change in current. The S.I. unit is the henry (H); $1 \text{ H} = 1 \text{ Wb A}^{-1} = 1 \text{ Vs A}^{-1}$.

Step 2: (b) Field due to current in the outer solenoid Since S_2 is the outer solenoid, $r_2 > r_1$. For two long coaxial solenoids in air, neglecting end effects, current I in S_2 produces a nearly uniform field throughout the inner solenoid:

$$B_2 = \mu_0 n_2 I.$$

The inner solenoid's cross-sectional area is $A_1 = \pi r_1^2$. The flux through each of its turns is therefore $\Phi_1 = B_2 A_1 = \mu_0 n_2 I \pi r_1^2$. The area to use is the smaller shared area, not the full area of the outer coil.

Step 3: Find the flux linkage and mutual inductance The inner solenoid has $N_1 = n_1 L$ turns, where

Quick Tip to Solve: For nested solenoids, multiply the outer solenoid's field by the inner cross-sectional area and the inner number of turns, then divide by current.

L is the common length. Its total flux linkage is

$$N_1 \Phi_1 = (n_1 L)(\mu_0 n_2 I \pi r_1^2).$$

By definition $N_1 \Phi_1 = M_{12} I$. Dividing by the current gives

$$M_{12} = \mu_0 n_1 n_2 L \pi r_1^2.$$

This is positive when the winding orientations are chosen alike. It depends on geometry and permeability, not on the magnitude of the exciting current. Reciprocity gives the same mutual inductance for the reverse excitation.

Step 4: Conclusion Self-inductance is flux linkage per unit current, with S.I. unit henry. The mutual inductance of the specified long air-core solenoids is $M = \mu_0 n_1 n_2 L \pi r_1^2$, where r_1 is the inner radius.

Quick Tip to Solve: For nested solenoids, multiply the outer solenoid's field by the inner cross-sectional area and the inner number of turns, then divide by current.

Section D (4 marks)

23. Mrs. Rashmi Singh broke her reading glasses. When she went to the shopkeeper to order new specs, he suggested that she should get spectacles with plastic lenses instead of glass lenses. On getting the new spectacles, she found that the new ones were thicker than the earlier ones. She asked this question to the shopkeeper but he could not offer satisfactory explanation for this. At home, Mrs. Singh raised the same question to her daughter Anuja who explained why plastic lenses were thicker.

(a) Write two qualities displayed each by Anuja and her mother.

(b) How do you explain this fact using lens maker's formula?

[4 marks]

Solution:

Concept:

- Scientific curiosity means noticing an observation and seeking its cause; scientific awareness means explaining it using a physical principle.
- For a thin lens in air, the lens maker's formula is $P = 1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where f is focal length, μ is refractive index and R_1, R_2 are signed radii of curvature.
- Replacement reading spectacles must have the same prescribed converging power. The plastic considered in this question has a lower refractive index than the earlier glass.

Step 1: (a) Identify two qualities of each person Mrs. Singh shows keen observation by noticing that her replacement lenses are thicker. She also shows scientific curiosity and a questioning attitude: she seeks an explanation and continues her enquiry when the shopkeeper cannot satisfy her. Anuja shows scientific awareness by explaining the observation with an appropriate physical law. She also shows helpfulness by patiently sharing her knowledge with her mother.

Step 2: (b) Compare the curvature required for the same power Since the prescription is unchanged, the focal length and power must remain the same. Let $C = 1/R_1 - 1/R_2$ denote the curvature factor.

Quick Tip to Solve: Keep the prescription fixed: in $P = (\mu - 1)C$, a lower refractive index demands a larger curvature factor.

With subscripts p and g for plastic and glass, respectively, the lens maker's formula gives

$$(\mu_p - 1)C_p = (\mu_g - 1)C_g = P, \quad \frac{C_p}{C_g} = \frac{\mu_g - 1}{\mu_p - 1} > 1.$$

The inequality follows from the lower refractive index of the plastic used here, $\mu_p < \mu_g$. Thus, the plastic lens needs a greater curvature factor. Its surfaces must be more strongly curved, so their radii of curvature have smaller magnitudes for comparable lens shapes.

Step 3: Explain why stronger curvature makes the lens thicker Reading lenses are convex. At the same aperture and comparable edge thickness, stronger curvature makes their central portions bulge out farther. For example, for a symmetric biconvex lens, $R_1 = R$ and $R_2 = -R$, hence $P = 2(\mu - 1)/R$. A lower value of μ requires a smaller radius R to maintain the prescribed power. For aperture radius a , the sag of a spherical surface is $s = R - \sqrt{R^2 - a^2}$. Decreasing R increases this sag, and the two surface sags increase the centre thickness. This comparison assumes similar aperture and edge thickness and the lower-index plastic stated by the intended explanation; not every commercial plastic has a lower index than every glass.

Step 4: Conclusion (a) Anuja shows scientific awareness and helpfulness; her mother shows keen observation and scientific curiosity. (b) Lower-index plastic needs stronger curvature for the same focal length, giving the convex reading lens a thicker centre at comparable aperture and edge thickness.

Quick Tip to Solve: Keep the prescription fixed: in $P = (\mu - 1)C$, a lower refractive index demands a larger curvature factor.

Section E (15 marks)

24. (a) Draw a labelled diagram of AC generator. Derive the expression for the instantaneous value of the emf induced in the coil.

(b) A circular coil of cross-sectional area 200 cm^2 and 20 turns is rotated about the vertical diameter with angular speed of 50 rad s^{-1} in a uniform magnetic field of magnitude $3.0 \times 10^{-2} \text{ T}$. Calculate the maximum value of the current in the coil. [5 marks]

Solution:

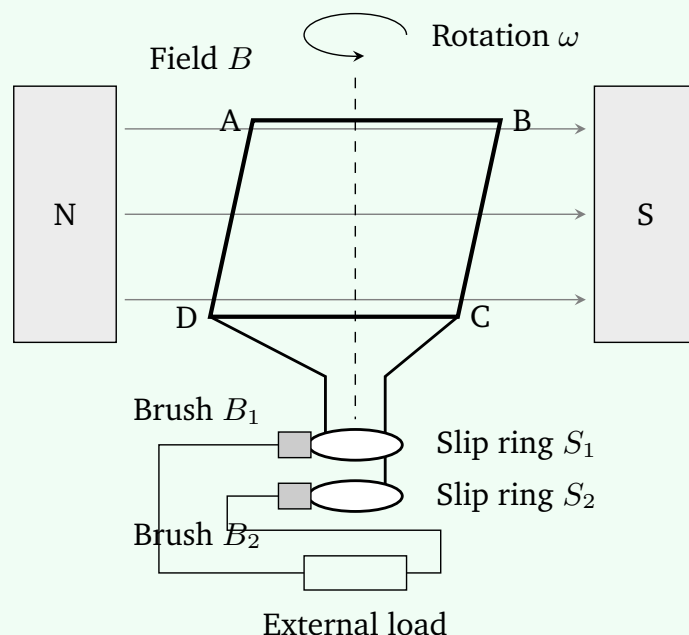
Concept:

- An AC generator converts mechanical energy into electrical energy through electromagnetic induction. Faraday's law gives $e = -d(N\Phi)/dt$, where N is the number of turns and Φ is the flux through one turn.
- For a plane coil of area A in uniform field B , $\Phi = BA \cos \theta$, where θ is the angle between the field and the normal to the coil. Uniform rotation gives $\theta = \omega t$ for a suitable initial orientation.
- For a purely resistive circuit of total resistance R , the peak current is $I_0 = e_0/R$. A numerical current requires the resistance, as well as the peak emf.

Step 1: (a) Construction and working of the AC generator A rectangular armature coil $ABCD$ rotates between the north and south poles of a magnet. Its two ends connect to separate slip rings S_1, S_2 , which rotate with the coil. Stationary carbon brushes B_1, B_2 touch these rings and connect the coil to

Quick Tip to Solve: Convert square centimetres using $1 \text{ cm}^2 = 10^{-4} \text{ m}^2$, and check that $NBA\omega$ gives volts; divide by the circuit resistance to obtain amperes.

an external load. A mechanical drive rotates the shaft. The brushes maintain electrical contact while the slip rings allow the coil to turn continuously. As the coil rotates, its magnetic flux changes; the induced emf reverses after each half-turn, producing an alternating output.



AC generator: rotating armature, separate slip rings and stationary brushes.

Step 2: (a) Derive the instantaneous induced emf Choose $t = 0$ when the coil normal is parallel to the magnetic field. After time t , the normal makes angle $\theta = \omega t$ with the field. For N turns the flux linkage is

$$N\Phi = NBA \cos(\omega t).$$

Applying Faraday's law and differentiating,

$$\begin{aligned} e(t) &= -\frac{d}{dt} [NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t) = e_0 \sin(\omega t), \\ e_0 &= NBA\omega. \end{aligned}$$

The sign depends on the chosen terminal polarity; the amplitude is $NBA\omega$. The emf is zero when flux is maximum and is greatest in magnitude when the coil normal is perpendicular to the field. Its frequency is $f = \omega/(2\pi)$. Thus uniform rotation produces a sinusoidal alternating emf.

Step 3: (b) Calculate the peak emf and determine what is needed for the current The area in SI units is

$$A = 200 \times 10^{-4} = 2.00 \times 10^{-2} \text{ m}^2.$$

For the standard generator orientation, with the magnetic field perpendicular to the rotation axis,

$$e_0 = 20(3.0 \times 10^{-2})(2.00 \times 10^{-2})(50) = 0.60 \text{ V}.$$

Quick Tip to Solve: Convert square centimetres using $1 \text{ cm}^2 = 10^{-4} \text{ m}^2$, and check that $NBA\omega$ gives volts; divide by the circuit resistance to obtain amperes.

Hence, if the coil and its circuit have total resistance R , neglecting inductive reactance,

$$I_0 = \frac{0.60 \text{ V}}{R}.$$

The printed question supplies no coil resistance or circuit impedance. Consequently it does not determine a unique numerical maximum current. The calculated 0.60 V is an emf, not a current, and a resistance value cannot be assumed without additional information.

Step 4: Conclusion (a) The induced emf is $e(t) = NBA\omega \sin(\omega t)$, with peak $e_0 = NBA\omega$. (b) The peak emf is 0.60 V ; for total resistance R , $I_0 = (0.60 \text{ V})/R$. A numerical peak current cannot be obtained because the printed question omits resistance or impedance.

Quick Tip to Solve: Convert square centimetres using $1 \text{ cm}^2 = 10^{-4} \text{ m}^2$, and check that $NBA\omega$ gives volts; divide by the circuit resistance to obtain amperes.

OR

24. (a) Draw a labelled diagram of a step-up transformer. Obtain the ratio of secondary to primary voltage in terms of number of turns and currents in the two coils.

(b) A power transmission line feeds input power at 2200 V to a step-down transformer with its primary windings having 3000 turns. Find the number of turns in the secondary to get the power output at 220 V. [5 marks]

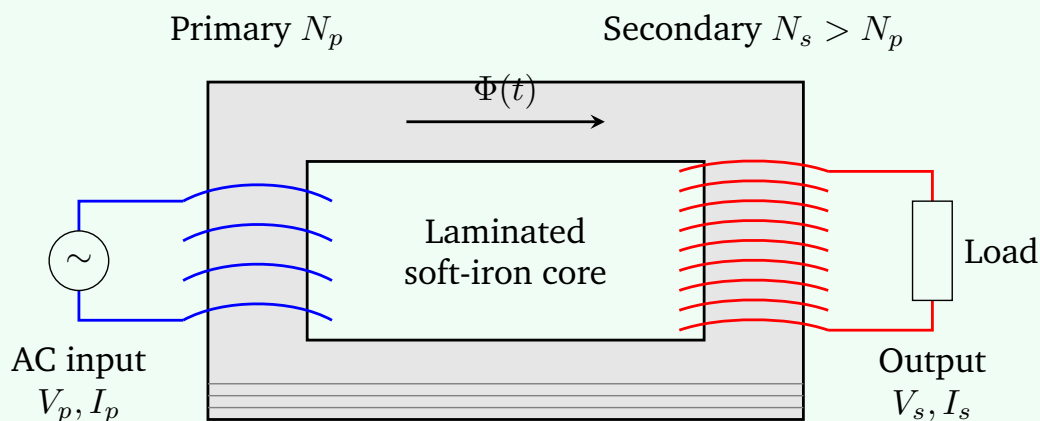
Solution:

Concept:

- A transformer works on mutual induction: alternating current in the primary produces changing magnetic flux $\Phi(t)$ in a laminated soft-iron core, linking both coils.
- Faraday's law gives induced emf $e = -N d\Phi/dt$, where N is the number of turns. For an ideal transformer both windings link the same flux and winding voltage drops are negligible.
- An ideal transformer conserves average power: $V_p I_p = V_s I_s$. Here V_p, V_s and I_p, I_s are rms voltages and currents; the usual current ratio assumes negligible losses and magnetising current.

Step 1: (a) Construction and operation of a step-up transformer The primary coil has N_p turns and is connected to an alternating-voltage source. The secondary coil has $N_s > N_p$ turns and supplies a load. Both insulated coils are wound on a common laminated soft-iron core. The core provides a low-reluctance path for magnetic flux; laminations reduce eddy-current losses. There is no direct electrical connection between the windings. Changing primary current produces changing core flux, which induces emf in the secondary by mutual induction. A steady dc supply cannot produce continuous transformer action.

Quick Tip to Solve: Write secondary over primary for both voltage and turns, but reverse the current ratio; a tenfold voltage decrease requires tenfold fewer turns.



Labelled step-up transformer: a common laminated core links a primary with fewer turns and a secondary with more turns.

Step 2: (a) Derive the voltage ratio from common magnetic flux Let the flux linking each turn be $\Phi(t)$. Taking the same reference winding sense, the induced emfs are

$$e_p = -N_p \frac{d\Phi}{dt}, \quad e_s = -N_s \frac{d\Phi}{dt}.$$

Thus their magnitudes have the constant ratio N_s/N_p . For example, if $\Phi = \Phi_m \sin \omega t$, the emf amplitudes are $N_p \omega \Phi_m$ and $N_s \omega \Phi_m$. Dividing both amplitudes by $\sqrt{2}$ gives the rms values. Neglecting winding resistance and leakage flux therefore gives

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

The primary induced emf balances the applied alternating voltage in the ideal limit; the secondary induced emf is available across the load. The negative sign in Faraday's law expresses Lenz's law and does not make the voltage magnitude ratio negative. Consequently a step-up transformer raises the voltage because its secondary has more turns than its primary.

Step 3: (a) Obtain the corresponding current ratio With negligible core and copper losses, input power equals output power. For the ideal transformer and the usual resistive-load treatment,

$$V_p I_p = V_s I_s \quad \Rightarrow \quad \frac{V_s}{V_p} = \frac{I_p}{I_s}.$$

Combining this with the turns ratio gives $V_s/V_p = N_s/N_p = I_p/I_s$. Increasing voltage therefore decreases the secondary current for the same transferred power. These are ratios of magnitudes; terminal phase depends on winding polarity.

Step 4: (b) Calculate the secondary turns of the step-down transformer The numerical part concerns a separate step-down transformer with $V_p = 2200 \text{ V}$, $V_s = 220 \text{ V}$, and $N_p = 3000$. Applying the ideal turns relation,

$$N_s = N_p \frac{V_s}{V_p} = 3000 \frac{220}{2200} = 300.$$

The voltage falls by a factor of ten, so the number of secondary turns must also be one tenth of the

Quick Tip to Solve: Write secondary over primary for both voltage and turns, but reverse the current ratio; a tenfold voltage decrease requires tenfold fewer turns.

primary turns. The computed value is dimensionless, as a turn count must be.

Step 5: Conclusion (a) For an ideal transformer, $V_s/V_p = N_s/N_p = I_p/I_s$; a step-up transformer has $N_s > N_p$. (b) The required secondary winding has 300 turns.

Quick Tip to Solve: Write secondary over primary for both voltage and turns, but reverse the current ratio; a tenfold voltage decrease requires tenfold fewer turns.

25. (a) Distinguish between unpolarized light and linearly polarized light. How does one get linearly polarised light with the help of a polaroid?

(b) A narrow beam of unpolarised light of intensity I_0 is incident on a polaroid P_1 . The light transmitted by it is then incident on a second polaroid P_2 with its pass axis making angle of 60° relative to the pass axis of P_1 . Find the intensity of the light transmitted by P_2 . [5 marks]

Solution:

Concept:

- Light is a transverse electromagnetic wave. Polarisation describes the directions in which its electric field oscillates, perpendicular to the direction of propagation.
- An ideal polaroid transmits the electric-field component parallel to its pass axis and absorbs the perpendicular component. For unpolarised incident light, the average transmitted intensity is half the incident intensity.
- Malus's law is $I = I_{\text{in}} \cos^2 \theta$, where I_{in} is the intensity of linearly polarised light incident on an analyser and θ is the angle between its polarisation direction and the analyser's pass axis.

Step 1: (a) Distinguish the two states of polarisation In unpolarised light, the electric-field vibrations occur in all possible directions in the plane perpendicular to the direction of propagation, with no fixed preferred direction. The vibration direction changes randomly with time. In linearly polarised light, the electric field vibrates along one fixed direction perpendicular to propagation; hence its vibration and the propagation direction define a fixed plane. The distinction concerns the electric-field direction, not the direction in which the beam travels.

Step 2: (a) Obtain linearly polarised light using a polaroid Pass the unpolarised beam through a polaroid. Resolve each incident electric-field vibration into components parallel and perpendicular to the polaroid's pass axis. The perpendicular component is selectively absorbed, while the parallel component is transmitted. Therefore the emerging electric field vibrates along the pass axis, producing linearly polarised light. Rotating the polaroid rotates this transmitted vibration direction. For an ideal polaroid, averaging over all incident vibration directions gives $\langle \cos^2 \phi \rangle = 1/2$, so the transmitted intensity is half the incident intensity.

Step 3: (b) Find the intensity leaving the first polaroid The incident light on P_1 is unpolarised and has intensity I_0 . Hence the intensity leaving the ideal first polaroid is

$$I_1 = \frac{I_0}{2}.$$

This emerging light is linearly polarised along the pass axis of P_1 , so it is a suitable incident beam for

Quick Tip to Solve: For unpolarised light through two ideal polaroids, first halve the incident intensity, then multiply by the squared cosine of the angle between their pass axes.

applying Malus's law at P_2 .

Step 4: (b) Apply Malus's law at the second polaroid The angle between the polarisation direction of the incident beam and the pass axis of P_2 is 60° . Thus

$$\begin{aligned} I_2 &= I_1 \cos^2 60^\circ \\ &= \frac{I_0}{2} \left(\frac{1}{2}\right)^2 \\ &= \frac{I_0}{8}. \end{aligned}$$

The second polaroid transmits one quarter of the intensity reaching it. This quarter must be applied to I_1 , after accounting for the reduction at the first polaroid.

Step 5: Conclusion Unpolarised light has electric-field vibrations in randomly varying transverse directions; linearly polarised light has vibrations along one fixed transverse direction. A polaroid produces the latter by transmitting the component along its pass axis. The intensity transmitted by P_2 is $I_2 = I_0/8$.

Quick Tip to Solve: For unpolarised light through two ideal polaroids, first halve the incident intensity, then multiply by the squared cosine of the angle between their pass axes.

OR

25. (a) Explain two features to distinguish between the interference pattern in Young's double slit experiment with the diffraction pattern obtained due to a single slit.

(b) A monochromatic light of wavelength 500 nm is incident normally on a single slit of width 0.2 mm to produce a diffraction pattern. Find the angular width of the central maximum obtained on the screen.

Estimate the number of fringes obtained in Young's double slit experiment with fringe width 0.5 mm, which can be accommodated within the region of total angular spread of the central maximum due to single slit.

[5 marks]

Solution:

Concept:

- Young's interference arises from superposition of light from two coherent slits; single-slit diffraction arises from superposition of secondary waves from different portions of one slit.
- For slit width a and wavelength λ , the first diffraction minima obey $a \sin \theta_1 = \lambda$. The central maximum extends from $-\theta_1$ to $+\theta_1$.
- A linear fringe width β must be compared with a linear spread on the same screen. If the slit-to-screen distance is D , the central spread is approximately $W = 2D\lambda/a$.

Step 1: (a) Compare fringe widths In Young's double slit experiment, under the usual small-angle conditions, successive bright fringes are equally spaced and the bright and dark bands have equal widths. Their common fringe width is $\beta = \lambda D/d$, where d is the separation of the slits. For single-slit

Quick Tip to Solve: First find the two first minima to obtain the full central width; convert the angular spread to a screen width before dividing by a fringe width in millimetres.

diffraction, the central bright maximum is twice as wide as each of the secondary bright bands, approximately. The first minima lie at $\pm\lambda/a$, whereas successive minima on either side are separated by approximately λ/a in angle.

Step 2: (a) Compare the intensities of maxima In ideal Young's interference with equally illuminated narrow slits, all bright fringes have the same maximum intensity, and destructive interference produces zero-intensity minima. In single-slit diffraction, the central maximum has the greatest intensity and the secondary maxima become progressively weaker as their distance from the centre increases. Thus uniform bright-fringe intensity and equal fringe widths distinguish the ideal interference pattern from the dominant, broader central peak and weaker secondary peaks of diffraction.

Step 3: (b) Calculate the total angular width Convert the supplied quantities to SI units:

$$\lambda = 500 \times 10^{-9} \text{ m}, \quad a = 0.2 \times 10^{-3} \text{ m}.$$

Therefore $\sin \theta_1 = \lambda/a = 2.5 \times 10^{-3}$. Since this angle is small, $\theta_1 \simeq 2.5 \times 10^{-3} \text{ rad}$. The requested width is the full angle between the first minima on opposite sides:

$$\Delta\theta = 2\theta_1 \simeq 5.0 \times 10^{-3} \text{ rad}.$$

Using $2 \arcsin(0.0025)$ instead gives the same result to the precision of the data.

Step 4: Estimate the number of Young's fringes and identify the missing datum The printed question supplies a linear fringe width $\beta = 0.5 \text{ mm} = 5.0 \times 10^{-4} \text{ m}$, but gives no slit-to-screen distance. An angular width cannot be divided directly by this linear width. For a screen at distance D , the region has linear width

$$W \simeq D\Delta\theta = 5.0 \times 10^{-3} D, \quad N \simeq \frac{W}{\beta} = 10 \frac{D}{1 \text{ m}}.$$

Here N estimates the number of fringe-width intervals that fit in the central diffraction region; detailed counting of bright peaks also depends on the endpoints. A unique numerical estimate requires D , or equivalently the angular fringe spacing. For example, if $D = 1 \text{ m}$ were supplied, about ten fringe widths would fit, but that distance is not stated in the paper and cannot be assumed as given.

Step 5: Conclusion Interference fringes have equal widths and, for equal slit intensities, equal peak intensities. Single-slit diffraction has a central maximum of twice the secondary-band width, with secondary maxima of decreasing intensity. The central angular width is $5.0 \times 10^{-3} \text{ rad}$. The fringe estimate is $N \simeq 10D/(1 \text{ m})$; the printed data do not determine a unique numerical count because the screen distance is missing.

Quick Tip to Solve: First find the two first minima to obtain the full central width; convert the angular spread to a screen width before dividing by a fringe width in millimetres.

26. (i) Derive an expression for drift velocity of electrons in a conductor. Hence deduce Ohm's law.
 (ii) A wire whose cross-sectional area is increasing linearly from its one end to the other, is connected across a battery of V volts. Which of the following quantities remain constant in the wire?
- (a) drift speed
 (b) current density
 (c) electric current
 (d) electric field

Justify your answer.

[5 marks]

Solution:

Concept:

- Drift velocity is the mean velocity acquired by conduction electrons in an applied electric field. Let $e > 0$ be the elementary charge, m the electron mass and τ the mean relaxation time between collisions.
- For electron number density n , cross-sectional area A and drift speed v_d , the conventional current magnitude is $I = neAv_d$; its density is $J = I/A$.
- For a homogeneous conductor at constant temperature, $\sigma = ne^2\tau/m$ is the conductivity and $\rho = 1/\sigma$ is the resistivity. Steady current obeys conservation of charge.

Step 1: (i) Obtain the electron drift velocity In the absence of an applied field, electrons have random thermal velocities, whose vector average is zero. In a field $\vec{E}$, the force on an electron is $-e\vec{E}$, so its acceleration between collisions is

$$\vec{a} = -\frac{e\vec{E}}{m}.$$

Collisions randomise the velocity; the mean time available for field-driven acceleration is the relaxation time τ . Averaging over the electrons therefore gives

$$\vec{v}_d = \vec{a}\tau = -\frac{e\tau}{m}\vec{E}, \quad v_d = \frac{eE\tau}{m}.$$

Thus electrons drift opposite to the electric field, whereas conventional current flows along it. The drift is superposed on the random thermal motion; it is the small mean velocity, not the thermal speed.

Step 2: (i) Deduce Ohm's law for a uniform conductor During a time dt , the electrons in a cylinder of length $v_d dt$ cross a section of area A . Their number is $nAv_d dt$, giving the current magnitude

$$I = \frac{neAv_d dt}{dt} = neAv_d = \frac{ne^2\tau A}{m}E.$$

For a uniform wire of length L , with potential difference V , $E = V/L$. Hence

$$I = \frac{ne^2\tau A}{mL}V, \quad V = IR, \quad R = \frac{mL}{ne^2\tau A} = \rho \frac{L}{A}.$$

At constant temperature the material parameters n and τ , and the dimensions L, A , are fixed. Therefore R is constant and I is proportional to V , which is Ohm's law. Locally the same derivation gives $\vec{J} = \sigma\vec{E}$.

Quick Tip to Solve: In a tapered wire, first use steady-state charge conservation to fix the current, then divide by the local area to compare drift speed, current density and field.

Step 3: (ii) Apply charge conservation to the non-uniform wire Consider the portion between any two cross-sections. If their currents were unequal, charge would accumulate in that portion, contradicting the steady state after connection to the battery. Thus the same electric current I passes through every cross-section, even though its area changes.

For the same homogeneous material at constant temperature, n and ρ are fixed. At position x ,

$$J(x) = \frac{I}{A(x)}, \quad v_d(x) = \frac{I}{neA(x)}, \quad E(x) = \rho J(x) = \frac{\rho I}{A(x)}.$$

As $A(x)$ increases, each of these three quantities decreases. Consequently (a) drift speed, (b) current density and (d) electric field are not constant. The fixed battery voltage constrains the total potential drop along the wire; it does not require the local field to be uniform in a wire of varying area.

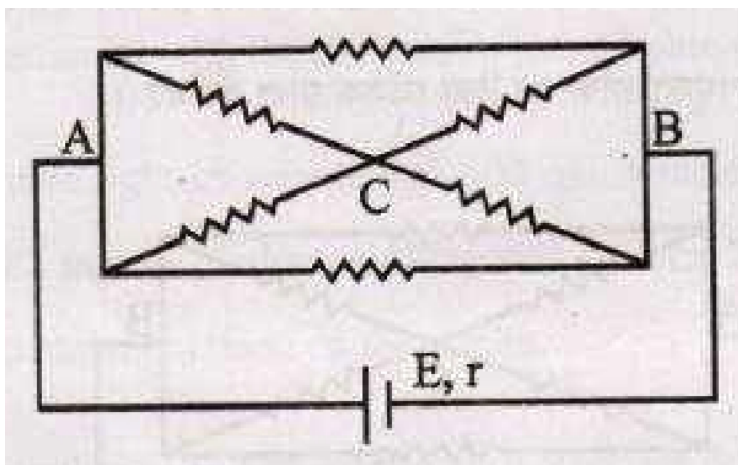
Step 4: Conclusion The electron drift velocity is $\vec{v}_d = -e\tau\vec{E}/m$. For a uniform conductor this yields $V = IR$, with $R = mL/(ne^2\tau A)$. In the non-uniform wire only (c) electric current remains constant; v_d , J and E vary inversely with the cross-sectional area.

Quick Tip to Solve: In a tapered wire, first use steady-state charge conservation to fix the current, then divide by the local area to compare drift speed, current density and field.

OR

26. (i) State the two Kirchhoff's laws. Explain briefly how these rules are justified.

(ii) The current is drawn from a cell of emf E and internal resistance r connected to the network of resistors each of resistance r as shown in the figure. Obtain the expression for (i) the current drawn from the cell and (ii) the power consumed in the network. [5 marks]



Solution:

Concept:

- Kirchhoff's junction rule expresses conservation of charge: the sum of currents entering a junction equals the sum leaving it in a steady circuit.
- Kirchhoff's loop rule expresses conservation of energy: the algebraic sum of potential changes

Quick Tip to Solve: Identify equipotential wire nodes first: combine the two A-C resistors and the two C-B resistors before adding the cell's internal resistance.

around a closed circuit is zero.

- Series resistances add, whereas parallel resistances satisfy $1/R = \sum_i 1/R_i$. The cell's internal resistance is in series with the external network.
- The current supplied by a cell is $I = E/(r + R)$, and the power dissipated in its external network is $P = I^2 R$, where R is the network resistance.

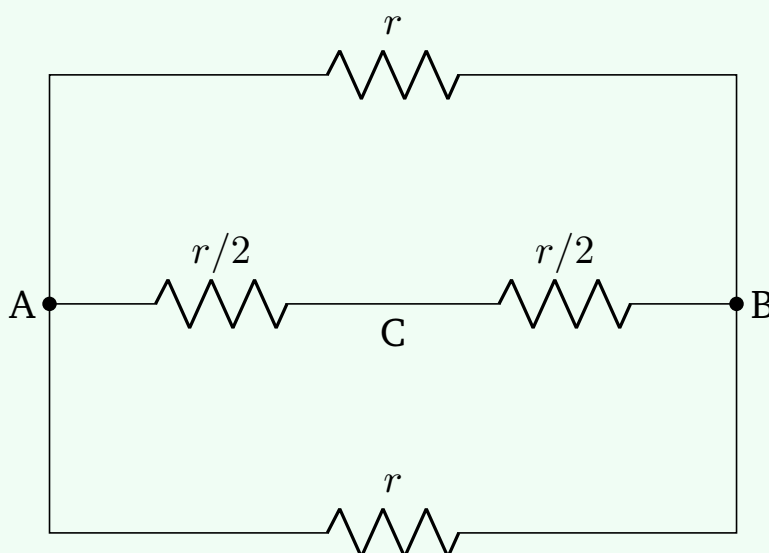
Step 1: (i) State and justify the junction rule At any junction, $\sum I_{\text{in}} = \sum I_{\text{out}}$, or equivalently $\sum I = 0$ when incoming and outgoing currents have opposite signs. In steady state, charge cannot accumulate at the junction. The amount of charge arriving in a time interval must therefore equal the amount leaving during that interval. Dividing by the interval gives the current rule. A persistent difference would cause charge accumulation and a changing junction potential, contrary to steady current.

Step 2: (i) State and justify the loop rule Around any closed loop, $\sum \Delta V = 0$. Equivalently, the algebraic sum of emfs equals the algebraic sum of resistive potential drops. On moving through a resistor in the current direction, the potential change is $-IR$; on crossing a cell from its negative to positive terminal, it is $+E$. A charge returning to its starting point has the same potential energy. Thus the energy gained from the sources equals the energy transferred to the resistors. This is conservation of energy for the steady circuit.

Step 3: (ii) Reduce the given network between A and B The entire left vertical wire is node A and the entire right vertical wire is node B. The two resistors from A to C are parallel, so $R_{AC} = r \parallel r = r/2$. Similarly, $R_{CB} = r/2$. Their series combination is therefore $R_{AC} + R_{CB} = r$. The top and bottom resistors each independently connect A to B and each has resistance r . Hence the network contains three parallel branches, each of effective resistance r :

$$\frac{1}{R_{\text{net}}} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r}, \quad R_{\text{net}} = \frac{r}{3}.$$

The diagram below redraws these branches after combining the two pairs incident on C. All original resistors have the same resistance; the vertical wires introduce no additional resistance.



Equivalent external network: three branches, each of total resistance r

Quick Tip to Solve: Identify equipotential wire nodes first: combine the two A-C resistors and the two C-B resistors before adding the cell's internal resistance.

Step 4: Find the current drawn from the cell The internal resistance r of the cell must be included in series with the external equivalent resistance. Thus

$$R_{\text{total}} = r + \frac{r}{3} = \frac{4r}{3}, \quad I = \frac{E}{R_{\text{total}}} = \frac{3E}{4r}.$$

The voltage across the external network is $V_{AB} = IR_{\text{net}} = E/4$; the remaining $3E/4$ is the potential drop across the cell's internal resistance.

Step 5: Find the power consumed in the external network Apply Joule's heating relation to the external equivalent resistance, using the total current entering the network:

$$P_{\text{net}} = I^2 R_{\text{net}} = \left(\frac{3E}{4r} \right)^2 \frac{r}{3} = \frac{3E^2}{16r}.$$

Independently, $V_{AB}I = (E/4)(3E/4r)$ gives the same result. This excludes the heating inside the cell; using EI would include that internal energy dissipation too.

Step 6: Conclusion The junction and loop laws follow from conservation of charge and energy, respectively. For the given network, $R_{\text{net}} = r/3$, the cell supplies $I = 3E/(4r)$, and the power consumed in the network is $P_{\text{net}} = 3E^2/(16r)$.

Quick Tip to Solve: Identify equipotential wire nodes first: combine the two A-C resistors and the two C-B resistors before adding the cell's internal resistance.