

CBSE Class 12 Physics Question Paper 2017 with Solution PDF-Delhi Set 2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are **26** questions in all.
- (ii) This question paper has **five** sections : Section A, Section B, Section C, Section D and Section E.
- (iii) Section A contains **five** questions of **one** mark each, Section B contains **five** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains **one** value based question of **four** marks and Section E contains **three** questions of **five** marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in **one** question of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron} = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Section A (5 marks)

1. At a place, the horizontal component of earth's magnetic field is B and angle of dip is 60° . What is the value of horizontal component of the earth's magnetic field at equator? [1 mark]

Solution:

Concept:

- The horizontal component is $B_H = B_E \cos \delta$, where B_E is the total earth's magnetic field and δ is the angle of dip.
- At the magnetic equator, $\delta = 0^\circ$, so the field is entirely horizontal.

Step 1: Recover the total field At the given place, $B_H = B$ and $\delta = 60^\circ$. Hence

$$B = B_E \cos 60^\circ = \frac{B_E}{2}.$$

The given symbol B denotes the horizontal component, not the total field.

Step 2: Use zero dip at the equator In the standard approximation implicit in this question, the same total field magnitude is used at the two locations. At the equator,

$$B_{H,\text{eq}} = B_E \cos 0^\circ = B_E.$$

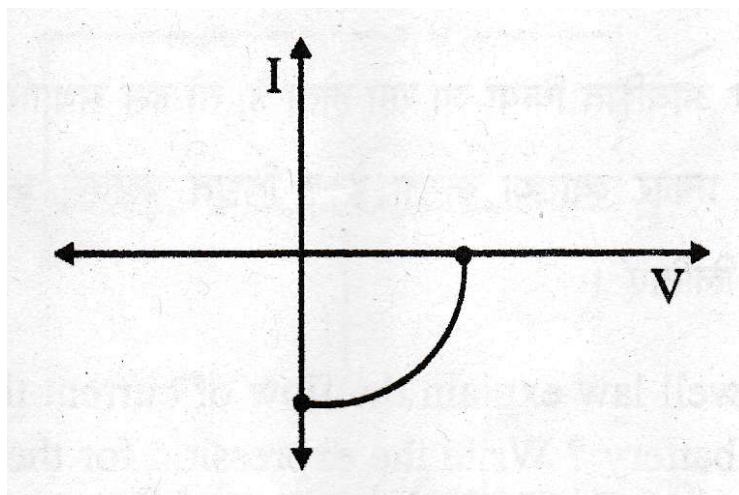
Thus the horizontal component there equals the total field obtained from the first relation.

Step 3: Conclusion The horizontal component at the equator is $2B$, using the assumed unchanged total field magnitude.

Quick Tip to Solve: First distinguish total field from horizontal component; divide by $\cos 60^\circ$ before using zero dip.

2. Name the junction diode whose I-V characteristics are drawn below:

[1 mark]



Solution:

Concept:

- A solar cell is an illuminated p-n junction that converts light energy into electrical energy through the photovoltaic effect.
- In the conventional diode sign convention, the power-generating portion of its characteristic lies in the fourth quadrant: $V > 0$ and $I < 0$.

Step 1: Read the intercepts of the characteristic The printed curve has a negative current at zero terminal voltage and reaches zero current at a positive voltage. These are the short-circuit photocurrent and the open-circuit photovoltage, respectively.

Step 2: Identify the operating mode Between these intercepts the junction delivers electrical power to a load. This is the characteristic photovoltaic operating region of an illuminated junction, unlike the first-quadrant forward characteristic of an ordinary unilluminated diode.

Step 3: Conclusion The junction device is a solar cell.

Quick Tip to Solve: A diode curve joining negative current to positive voltage in the fourth quadrant identifies a solar cell.

3. Does the charge given to a metallic sphere depend on whether it is hollow or solid? Give reason for your answer.

[1 mark]

Solution:

Concept:

- In electrostatic equilibrium, the electric field inside the material of a conductor is zero, and excess charge resides on its surface.
- For an isolated conducting sphere of outer radius R , $C = 4\pi\epsilon_0 R$; capacitance depends on the outer geometry and surrounding medium.

Step 1: Locate the excess charge Free charges move until electrostatic equilibrium is reached. For a sphere with no charge enclosed in any cavity, all the supplied excess charge lies on the outer surface.

Quick Tip to Solve: Compare outer radius and potential, not the amount of metal inside a charged conductor.

There is no excess charge stored throughout the metal's volume.

Step 2: Compare equal outer radii A hollow and a solid metallic sphere with the same outer radius have the same capacitance. At the same potential V , both therefore carry $Q = CV = 4\pi\epsilon_0 RV$. Removing metal from the interior does not change this external electrostatic relation.

Step 3: Conclusion No. The charge does not depend on whether the metallic sphere is hollow or solid, because excess charge resides on its outer surface.

Quick Tip to Solve: Compare outer radius and potential, not the amount of metal inside a charged conductor.

4. How is the speed of em-waves in vacuum determined by the electric and magnetic fields? [1 mark]

Solution:

Concept:

- The electric and magnetic fields in a plane electromagnetic wave in vacuum oscillate in phase and are mutually perpendicular.
- Their amplitudes satisfy $E_0 = cB_0$, where c is the speed of propagation, E_0 is the electric-field amplitude and B_0 is the magnetic-field amplitude.

Step 1: Relate the two field amplitudes Maxwell's equations for a travelling wave in vacuum give

$$\frac{E_0}{B_0} = \frac{\omega}{k},$$

where ω is the angular frequency and k is the wave number. The ratio ω/k is the propagation speed.

Step 2: Express the speed using the fields Dividing the electric-field amplitude by the magnetic-field amplitude therefore determines the speed. Use SI units, with E_0 in V m^{-1} and B_0 in tesla, so that the ratio is in m s^{-1} .

Step 3: Conclusion The speed is $c = E_0/B_0 = 3.0 \times 10^8 \text{ m s}^{-1}$. Equivalently, $c = E/B$ for corresponding nonzero instantaneous field magnitudes.

Quick Tip to Solve: Use electric field divided by magnetic field; the amplitude ratio avoids the zero-field instants.

5. A long straight current carrying wire passes normally through the centre of circular loop. If the current through the wire increases, will there be an induced emf in the loop? Justify. [1 mark]

Solution:

Concept:

- Faraday's law gives the induced emf around a stationary closed loop as $\mathcal{E} = -d\Phi_B/dt$, where $\Phi_B = \int \vec{B} \cdot d\vec{A}$.
- The magnetic field of a long straight current-carrying wire forms circles in planes perpendicular to the wire.

Quick Tip to Solve: Check the component of magnetic field normal to the loop before differentiating the flux.

Step 1: Determine the field direction relative to the loop The wire is normal to the circular loop. Hence its magnetic field is tangent to circles lying in the plane of the loop. The loop's area vector is parallel to the wire, so the magnetic field is perpendicular to this area vector.

Step 2: Evaluate the change in magnetic flux At every point of the loop's spanning surface away from the ideal wire axis, $\vec{B} \cdot d\vec{A} = 0$. Thus the magnetic flux through the loop is zero. Increasing the current changes the field's magnitude but not its direction, so this flux remains zero and $d\Phi_B/dt = 0$.

Step 3: Conclusion No induced emf appears around the loop: $\mathcal{E} = 0$, because the magnetic flux through it remains zero.

Quick Tip to Solve: Check the component of magnetic field normal to the loop before differentiating the flux.

Section B (10 marks)

6. Find the wavelength of the electron orbiting in the first excited state in hydrogen atom. [2 marks]

Solution:

Concept:

- The de Broglie wavelength of an electron is $\lambda = h/p$, where h is Planck's constant and p is the electron's momentum.
- A Bohr orbit contains an integral number of matter wavelengths: $2\pi r_n = n\lambda_n$. For hydrogen, $r_n = n^2 a_0$, where $a_0 = 5.29 \times 10^{-11}$ m.

Step 1: Identify the orbit and its radius The ground state has principal quantum number $n = 1$, so the first excited state has $n = 2$. Its radius is

$$r_2 = 2^2 a_0 = 4(5.29 \times 10^{-11}) = 2.116 \times 10^{-10} \text{ m}.$$

The phrase "wavelength of the electron" asks for its matter wavelength in this orbit, not the wavelength of light emitted during a transition.

Step 2: Apply the standing-wave condition From Bohr's angular-momentum condition, $pr_n = n\hbar = nh/(2\pi)$. On using $\lambda_n = h/p$, this becomes $2\pi r_n = n\lambda_n$. Thus two complete electron wavelengths fit around the first excited orbit:

$$\lambda_2 = \frac{2\pi r_2}{2} = \pi(2.116 \times 10^{-10}) \text{ m} = 6.65 \times 10^{-10} \text{ m}.$$

Equivalently, the radius formula gives $\lambda_n = 2\pi n a_0$, which produces the same result at $n = 2$.

Step 3: Conclusion The required electron wavelength is

$$\lambda_2 = 6.65 \times 10^{-10} \text{ m} = 0.665 \text{ nm}.$$

Quick Tip to Solve: First excited means $n = 2$; divide the orbit circumference by two to obtain the electron wavelength.

7. Distinguish between a transducer and a repeater.

[2 marks]

Solution:

Concept:

- A transducer converts one form of energy into another. In a communication system, an input transducer converts a physical signal into a corresponding electrical signal.
- A repeater is a combination of a receiver and a transmitter that receives a weakened communication signal and retransmits it after amplification or regeneration.

Step 1: Identify the function of a transducer A transducer changes the form in which information is carried. For example, a microphone converts sound pressure variations into corresponding electrical variations, allowing speech to enter an electronic communication system. Its defining function is energy or signal-form conversion, rather than extending the transmission range.

Step 2: Identify the function of a repeater A repeater is placed along the communication path between the original transmitter and the final receiver. It receives the signal weakened during propagation, amplifies or regenerates it, and transmits it onwards. This compensates for attenuation and increases the communication range; a communication satellite can perform this role.

Step 3: Conclusion A transducer converts the form of energy or signal, whereas a repeater receives, strengthens and retransmits an existing communication signal to extend its range.

Quick Tip to Solve: Use a microphone to remember conversion by a transducer, and a relay station to remember retransmission by a repeater.

8. How does Ampere-Maxwell law explain the flow of current through a capacitor when it is being charged by a battery? Write the expression for the displacement current in terms of the rate of change of electric flux.

[2 marks]

Solution:

Concept:

- The Ampere-Maxwell law includes conduction current I_c and displacement current I_d :

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(I_c + I_d), \quad I_d = \epsilon_0 \frac{d\Phi_E}{dt}.$$

- Here $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$ is electric flux. A time-varying electric field produces the same magnetic effect as a current, even in a region with no transport of free charge.

Step 1: Distinguish the wire current from the current between the plates While the battery charges the capacitor, conduction current flows through the connecting wires and charge accumulates on the plates. Free charge does not cross the insulating gap of an ideal capacitor. However, the electric field and hence the electric flux between the plates increase with time. Maxwell's displacement-current term accounts for this changing field and makes the current description continuous through the circuit.

Step 2: Relate displacement current to charging current For parallel plates of area A with vacuum or

Quick Tip to Solve: For a charging ideal capacitor, use $\Phi_E = Q/\epsilon_0$ to show immediately that $I_d = dQ/dt = I_c$.

air between them, neglecting edge effects, $E = Q/(\epsilon_0 A)$. Thus

$$\Phi_E = EA = \frac{Q}{\epsilon_0}, \quad I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} = I_c.$$

A surface cutting the wire encloses conduction current, while one passing between the plates encloses displacement current of the same magnitude. Both therefore give the same magnetic circulation in the Ampere-Maxwell law.

Step 3: Conclusion The changing electric field between the plates provides a displacement current equal to the charging current, although no free charge crosses the gap:

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}.$$

Quick Tip to Solve: For a charging ideal capacitor, use $\Phi_E = Q/\epsilon_0$ to show immediately that $I_d = dQ/dt = I_c$.

9. Define the distance of closest approach. An α -particle of kinetic energy ' K ' is bombarded on a thin gold foil. The distance of the closest approach is ' r '. What will be the distance of closest approach for an α -particle of double the kinetic energy? [2 marks]

Solution:

Concept:

- The distance of closest approach is the minimum separation between the centres of an incident α -particle and the target nucleus during a head-on encounter. At this turning point, the particle momentarily comes to rest before being repelled.
- For a heavy nucleus of charge Ze , neglecting recoil, energy conservation gives $K = (1/4\pi\epsilon_0)(2Ze^2/r)$. The α -particle has charge $+2e$, and its initial kinetic energy is converted into electrostatic potential energy.

Step 1: Apply energy conservation at the turning point Take the electrostatic potential energy to be zero far from the gold nucleus. In a head-on approach, the repulsive force slows the positive α -particle until its kinetic energy becomes zero at separation r . Therefore

$$K = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{r}.$$

The same gold nucleus is used in both cases, so $Z = 79$ and the charge product remain unchanged.

Step 2: Compare the two incident energies Let the new distance of closest approach be r' . For initial kinetic energy $2K$, the corresponding energy equation is

$$2K = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{r'}.$$

Dividing this equation by the original one gives $2 = r/r'$. Thus closest approach varies inversely

Quick Tip to Solve: For the same target and projectile, keep Kr constant: doubling the kinetic energy halves the closest-approach distance.

with the incident kinetic energy: a more energetic particle penetrates farther against the Coulomb repulsion.

Step 3: Conclusion The distance of closest approach is the minimum centre-to-centre separation in a head-on encounter. On doubling the incident kinetic energy, it becomes $r' = r/2$.

Quick Tip to Solve: For the same target and projectile, keep Kr constant: doubling the kinetic energy halves the closest-approach distance.

OR

9. Write two important limitations of Rutherford nuclear model of the atom.

[2 marks]

Solution:

Concept:

- Rutherford's model places a small, massive, positively charged nucleus at the centre, with electrons revolving around it under electrostatic attraction.
- According to classical electrodynamics, an accelerating charge radiates energy. Atomic stability and the discrete line spectra of atoms must both be explained by a successful atomic model.

Step 1: Examine the predicted stability of an atom An electron moving around the nucleus is accelerated because its velocity continuously changes direction. Classical theory therefore requires it to radiate electromagnetic energy. As it loses energy, its orbit should shrink and it should spiral into the nucleus. Rutherford's model provides no permitted non-radiating orbits to prevent this collapse, so it cannot explain the observed stability of atoms.

Step 2: Compare the prediction with atomic line spectra In the classical picture, the electron's orbital radius and frequency would change continuously as it lost energy. Its emitted radiation would consequently cover a continuous range of frequencies. Actual atoms show distinct spectral lines at definite frequencies. Rutherford's model specifies no discrete allowed energy levels or transitions and therefore cannot explain these characteristic line spectra.

Step 3: Conclusion The two important limitations are its failure to explain **atomic stability** and its failure to explain the **discrete line spectra** of atoms.

Quick Tip to Solve: Remember the two tests: why does the orbiting electron not collapse, and why does an atom emit only selected frequencies?

10. Why should the objective of a telescope have large focal length and large aperture? Justify your answer. [2 marks]

Solution:

Concept:

- For an astronomical telescope in normal adjustment, the magnitude of angular magnification is $|M| = f_o/f_e$, where f_o and f_e are the objective and eyepiece focal lengths.
- Light collected by a circular objective is proportional to its area, $\pi D^2/4$, where D is its aperture diameter. Its diffraction-limited angular resolution is $\theta_{\min} = 1.22\lambda/D$.

Step 1: Explain the large focal length For a given eyepiece, increasing the objective focal length increases f_o/f_e and hence the angular magnification. The objective forms a larger real image of a distant object in its focal plane, which the eyepiece magnifies. Thus a large f_o helps the telescope show small angular details at a greater apparent size.

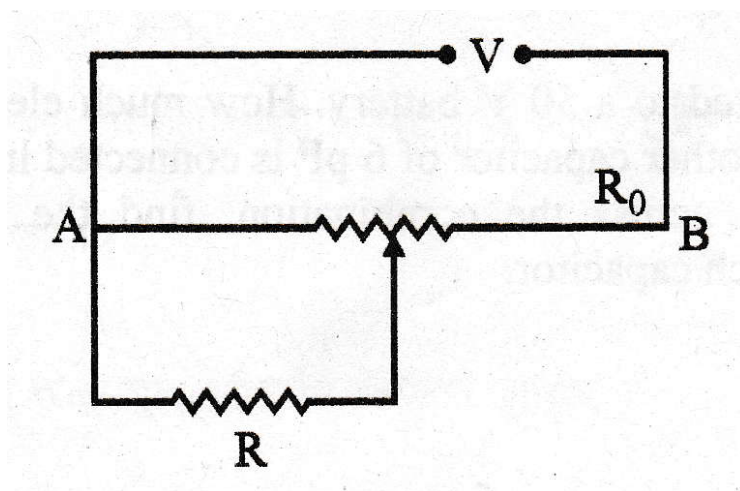
Step 2: Explain the large aperture A large diameter objective collects more light from faint distant stars, producing a brighter image. It also decreases the diffraction angle $1.22\lambda/D$, allowing two closely spaced distant objects to be distinguished. Therefore increasing aperture improves both light-gathering power and resolving power; magnification alone cannot reveal detail that the objective does not resolve.

Step 3: Conclusion A large objective focal length provides high magnifying power, while a large aperture provides greater brightness and higher resolving power.

Quick Tip to Solve: Link focal length to f_o/f_e , but link aperture to collected light and D/λ ; they serve different purposes.

Section C (36 marks)

11. A resistance of R draws current from a potentiometer. The potentiometer wire, AB, has a total resistance of R_0 . A voltage V is supplied to the potentiometer. Derive an expression for the voltage across R when the sliding contact is in the middle of potentiometer wire. [3 marks]



Solution:

Concept:

- A uniform potentiometer wire has resistance proportional to length. With slider C at its midpoint, $R_{AC} = R_{CB} = R_0/2$.
- A load connected across one segment is in parallel with that segment. For two resistances, $R_p = R_1 R_2 / (R_1 + R_2)$, and the voltage division rule applies to the resulting series combination.

Step 1: Identify the loaded half of the wire In the printed circuit, the resistance R is connected between end A and the sliding contact C . Consequently R and the left half of the wire have the same potential difference. Their equivalent resistance is

$$R_p = \frac{R(R_0/2)}{R + R_0/2} = \frac{RR_0}{2R + R_0}.$$

The right half CB , of resistance $R_0/2$, remains in series with this parallel combination. Loading therefore destroys the equal voltage division between the two equal lengths.

Step 2: Calculate the total current The voltage V is maintained across ends A and B . The equivalent resistance seen by the supply is

$$R_{eq} = \frac{R_0}{2} + \frac{RR_0}{2R + R_0} = \frac{R_0(4R + R_0)}{2(2R + R_0)}.$$

Thus the total supply current is

$$I = \frac{V}{R_{eq}} = \frac{2V(2R + R_0)}{R_0(4R + R_0)}.$$

This current flows through CB , then divides between the two parallel paths from C to A .

Step 3: Obtain the voltage across the load Multiplying the supply current by the equivalent resistance

Quick Tip to Solve: At the midpoint, first combine the load with $R_0/2$ in parallel; using $V/2$ directly ignores the current drawn by the load.

of the parallel portion gives its voltage, which is also the load voltage:

$$V_R = IR_p = \frac{2V(2R + R_0)}{R_0(4R + R_0)} \frac{RR_0}{2R + R_0} = \frac{2VR}{4R + R_0}.$$

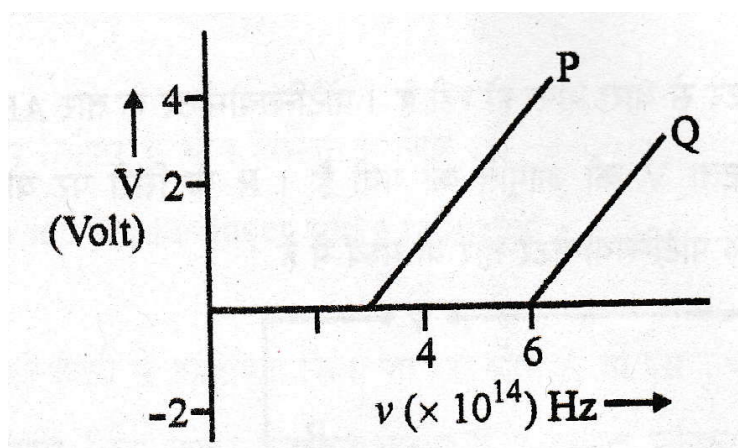
For a finite positive load resistance this is less than $V/2$. In the unloaded limit $R \rightarrow \infty$, it tends to $V/2$, as expected for the midpoint of a uniform wire.

Step 4: Conclusion The potential difference across the resistance is $V_R = \frac{2VR}{4R + R_0}$, where V is the supply voltage and R_0 is the total wire resistance.

Quick Tip to Solve: At the midpoint, first combine the load with $R_0/2$ in parallel; using $V/2$ directly ignores the current drawn by the load.

12. In the study of a photoelectric effect the graph between the stopping potential V and frequency ν of the incident radiation on two different metals P and Q is shown below:

- Which one of the two metals has higher threshold frequency?
 - Determine the work function of the metal which has greater value.
 - Find the maximum kinetic energy of electron emitted by light of frequency 8×10^{14} Hz for this metal.
- [3 marks]



Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = eV_s$, where ϕ is the work function, ν is the incident frequency and V_s is the stopping potential.
- At threshold frequency ν_0 , $K_{\max} = V_s = 0$, so $\phi = h\nu_0$. Thus the frequency-axis intercept of a stopping-potential graph gives the threshold frequency directly.

Step 1: Compare the threshold frequencies Read the intercepts where the plotted stopping potential becomes zero. The line for Q cuts the frequency axis at 6×10^{14} Hz, to the right of the intercept of P. Therefore Q has the higher threshold frequency. Since Planck's constant is the same for both metals, Q also has the greater work function.

Quick Tip to Solve: First read the horizontal intercept, including its 10^{14} factor; then use $\phi = h\nu_0$ and $K_{\max} = h(\nu - \nu_0)$.

Step 2: Calculate the greater work function Use the threshold frequency of Q and $h = 6.63 \times 10^{-34} \text{ J s}$:

$$\begin{aligned}\phi_Q &= h\nu_{0Q} \\ &= (6.63 \times 10^{-34})(6 \times 10^{14}) \\ &= 3.978 \times 10^{-19} \text{ J}.\end{aligned}$$

With $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$, this is $\phi_Q = 2.49 \text{ eV}$ approximately. The work function is the minimum energy needed to release an electron; at threshold there is no remaining kinetic energy.

Step 3: Find the maximum kinetic energy for Q For incident light of frequency $8 \times 10^{14} \text{ Hz}$, subtract the threshold frequency before multiplying by h :

$$\begin{aligned}K_{\max} &= h(\nu - \nu_{0Q}) \\ &= (6.63 \times 10^{-34})(8 - 6) \times 10^{14} \\ &= 1.326 \times 10^{-19} \text{ J} \\ &\approx 0.829 \text{ eV}.\end{aligned}$$

The incident frequency is above threshold, so emission occurs and this kinetic energy is positive. Use the labelled frequency intercept for the calculation; the sketch need not provide an accurately scaled stopping-potential value at every frequency.

Step 4: Conclusion (i) Metal Q has the higher threshold frequency. (ii) Its work function is $3.98 \times 10^{-19} \text{ J}$, or 2.49 eV . (iii) Its maximum emitted-electron kinetic energy is $1.33 \times 10^{-19} \text{ J}$, or 0.829 eV .

Quick Tip to Solve: First read the horizontal intercept, including its 10^{14} factor; then use $\phi = h\nu_0$ and $K_{\max} = h(\nu - \nu_0)$.

13. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor? If another capacitor of 6 pF is connected in series with it with the same battery connected across the combination, find the charge stored and potential difference across each capacitor.

[3 marks]

Solution:

Concept:

- A capacitor of capacitance C , charged to potential difference V , stores electrostatic energy $U = \frac{1}{2}CV^2$ and charge magnitude $Q = CV$.
- Capacitors in series carry equal charge magnitudes and their potential differences add to the battery voltage: $C_{\text{eq}} = C_1C_2/(C_1 + C_2)$, $V_i = Q/C_i$.

Step 1: Calculate the energy before adding the second capacitor Initially the 12 pF capacitor alone is connected across the 50 V battery. Convert picofarads to farads using $1 \text{ pF} = 10^{-12} \text{ F}$, then use the voltage across this single capacitor:

$$U = \frac{1}{2}(12 \times 10^{-12})(50)^2 = 1.5 \times 10^{-8} \text{ J}.$$

Quick Tip to Solve: For series capacitors, first find the common charge. Here the capacitance ratio is 2 : 1, so the voltage ratio must be 1 : 2, with a total of 50 V.

This energy refers to the original single-capacitor arrangement.

Step 2: Find the series capacitance and common charge In the new arrangement, the battery maintains 50 V across the entire series combination. The same charge magnitude appears on each capacitor, as for the usual series connection with an initially neutral intermediate conductor:

$$C_{\text{eq}} = \frac{12 \times 6}{12 + 6} \text{ pF} = 4 \text{ pF}, \quad Q = C_{\text{eq}} V = (4 \times 10^{-12})(50) = 2.0 \times 10^{-10} \text{ C}.$$

Thus each capacitor has opposite charges of magnitude 200 pC on its two plates.

Step 3: Divide the voltage between the capacitors With equal charge magnitudes, the smaller capacitance has the larger potential difference:

$$V_{12} = \frac{2.0 \times 10^{-10}}{12 \times 10^{-12}} = \frac{50}{3} \text{ V} \approx 16.7 \text{ V},$$
$$V_6 = \frac{2.0 \times 10^{-10}}{6 \times 10^{-12}} = \frac{100}{3} \text{ V} \approx 33.3 \text{ V}.$$

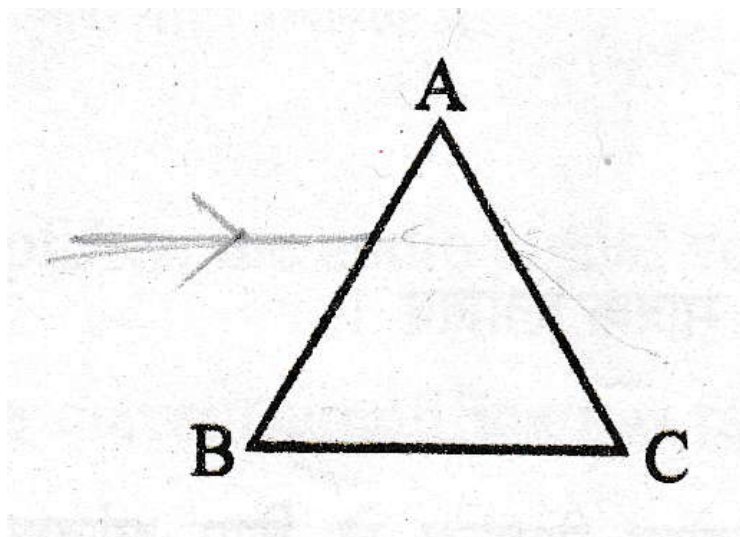
The check $V_{12} + V_6 = 50 \text{ V}$ confirms consistency with the battery voltage.

Step 4: Conclusion Initially, $U = 1.5 \times 10^{-8} \text{ J}$. In series, each capacitor stores charge magnitude $2.0 \times 10^{-10} \text{ C}$; the potential differences across the 12 pF and 6 pF capacitors are 16.7 V and 33.3 V, respectively.

Quick Tip to Solve: For series capacitors, first find the common charge. Here the capacitance ratio is 2 : 1, so the voltage ratio must be 1 : 2, with a total of 50 V.

14. (i) A ray of light incident on face AB of an equilateral glass prism, shows minimum deviation of 30° . Calculate the speed of light through the prism.

(ii) Find the angle of incidence at face AB so that the emergent ray grazes along the face AC. [3 marks]



Solution:**Concept:**

- At minimum deviation, a prism of angle A has refractive index $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$. Its light speed is $v = c/\mu$, where c is the speed in air.
- The internal angles satisfy $r_1 + r_2 = A$. Grazing emergence means an exit angle of 90° to the normal; hence $r_2 = C$, with critical angle $\sin C = 1/\mu$.

Step 1: Determine the index and speed (i) For an equilateral prism, $A = 60^\circ$. At minimum deviation, $r_1 = r_2 = 30^\circ$ and $i = e = (A + \delta_m)/2 = 45^\circ$. Therefore,

$$\mu = \frac{\sin 45^\circ}{\sin 30^\circ} = \sqrt{2}, \quad v = \frac{3.0 \times 10^8}{\sqrt{2}} = 2.12 \times 10^8 \text{ m s}^{-1}.$$

Step 2: Find the internal angles for grazing emergence (ii) The index remains $\sqrt{2}$, but this ray is not at minimum deviation. At AC, Snell's law gives $\mu \sin r_2 = \sin 90^\circ = 1$, so $r_2 = 45^\circ$. Thus the first internal angle is $r_1 = A - r_2 = 15^\circ$.

Step 3: Apply Snell's law at face AB For entry from air into glass, $\sin i = \mu \sin r_1$. Substitution gives

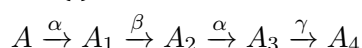
$$\sin i = \sqrt{2} \sin 15^\circ = \frac{\sqrt{3} - 1}{2} = 0.3660, \quad i = \sin^{-1}(0.3660) = 21.47^\circ.$$

All incidence and refraction angles here are measured from the respective face normals, not from the faces themselves.

Step 4: Conclusion (i) The speed through the prism is $2.12 \times 10^8 \text{ m s}^{-1}$. (ii) Grazing emergence along AC requires incidence $i \approx 21.5^\circ$ to the normal at AB.

Quick Tip to Solve: For grazing emergence, find the critical angle at exit first, subtract it from 60° , then apply Snell's law at entry.

15. (i) A radioactive nucleus 'A' undergoes a series of decays as given below:



The mass number and atomic number of A_2 are 176 and 71 respectively. Determine the mass and atomic numbers of A_4 and A.

(ii) Write the basic nuclear processes underlying β^+ and β^- decays.

[3 marks]

Solution:**Concept:**

- An alpha decay emits ${}_2^4\text{He}$, reducing the mass number by four and the atomic number by two. Gamma emission changes only the nuclear energy state, leaving both numbers unchanged.
- In beta-minus decay a neutron changes into a proton, so the mass number is unchanged and the atomic number increases by one. Beta-plus decay changes a proton into a neutron, reducing the atomic number by one.

Quick Tip to Solve: Track mass and atomic numbers separately: alpha changes them by $(-4, -2)$, beta-minus by $(0, +1)$, and gamma by $(0, 0)$. Reverse these changes when moving backward.

Step 1: Follow the chain forward from the known nucleus (i) The known nucleus A_2 has mass number 176 and atomic number 71. Its alpha decay produces A_3 , so

$$(\text{mass number, atomic number})_{A_3} = (176 - 4, 71 - 2) = (172, 69).$$

The subsequent gamma emission releases excitation energy without ejecting a nucleon or changing charge. Therefore A_4 also has mass number 172 and atomic number 69.

Step 2: Work backward to the original nucleus In the printed chain, the unqualified β denotes the usual beta-minus decay. Thus $A_1 \rightarrow A_2$ leaves the mass number unchanged and raises the atomic number by one. Working backward gives A_1 the numbers (176, 70). The preceding alpha decay $A \rightarrow A_1$ must have reduced these numbers by four and two, respectively. Consequently,

$$(\text{mass number, atomic number})_A = (176 + 4, 70 + 2) = (180, 72).$$

As a check, two alpha emissions and one beta-minus emission give net changes $\Delta M = -8$ and $\Delta Z = -3$, taking (180, 72) to (172, 69).

Step 3: Write the elementary beta-decay processes (ii) Inside a nucleus, the basic processes are

$$\begin{aligned}\beta^+ : \quad {}^1_1p &\longrightarrow {}^1_0n + {}^0_{+1}e + \nu_e, \\ \beta^- : \quad {}^1_0n &\longrightarrow {}^1_1p + {}^0_{-1}e + \bar{\nu}_e.\end{aligned}$$

Here ${}^0_{+1}e$ is a positron, ${}^0_{-1}e$ is an electron, ν_e is an electron neutrino and $\bar{\nu}_e$ is an electron antineutrino. The emitted leptons ensure conservation of lepton number as well as energy and momentum. The nucleon count is unchanged in either process; the charge is conserved when the emitted charged particle is included.

Step 4: Conclusion A_4 has mass number 172 and atomic number 69; A has mass number 180 and atomic number 72. The beta processes are $p \rightarrow n + e^+ + \nu_e$ and $n \rightarrow p + e^- + \bar{\nu}_e$, respectively.

Quick Tip to Solve: Track mass and atomic numbers separately: alpha changes them by $(-4, -2)$, beta-minus by $(0, +1)$, and gamma by $(0, 0)$. Reverse these changes when moving backward.

16. Describe the working principle of a moving coil galvanometer. Why is it necessary to use (i) a radial magnetic field and (ii) a cylindrical soft iron core in a galvanometer? Write the expression for current sensitivity of the galvanometer. Can a galvanometer as such be used for measuring the current? Explain. [3 marks]

Solution:

Concept:

- A current-carrying coil experiences a magnetic torque. For N turns of area A in field B , its magnitude is $NIAB \sin \alpha$, where α is the angle between the coil's normal and the field.
- The suspension provides a restoring torque $C\theta$, where C is its torsional constant and θ is the angular deflection. Current sensitivity is deflection per unit current.

Quick Tip to Solve: In a radial field, set the angle between the coil's normal and the field to 90° , then balance $NABI$ with $C\theta$.

Step 1: Describe the working principle and radial field When current passes through the suspended coil, magnetic forces on its opposite sides produce a couple and rotate it. In a radial field, the plane of the coil remains parallel to the field throughout its motion; hence its normal is perpendicular to the field and $\sin \alpha = 1$. The magnetic torque is therefore $NAB I$, independent of the coil's deflection. The coil settles when the restoring torque balances this torque:

$$C\theta = NAB I, \quad \theta = \frac{NAB}{C} I.$$

Thus the deflection is proportional to the current, giving a uniform current scale.

Step 2: Explain the soft iron core and obtain sensitivity The cylindrical soft iron core has high magnetic permeability. It strengthens the field within the coil and, together with the concave pole pieces, helps produce the required radial field. A stronger field produces greater deflection for the same current. From the torque balance, the current sensitivity is

$$S_I = \frac{\theta}{I} = \frac{NAB}{C},$$

measured in rad A^{-1} when deflection is measured in radians.

Step 3: Explain the limitation for current measurement A galvanometer as such is not suitable as an ammeter for ordinary circuit currents. Its appreciable resistance changes the current when it is inserted in series, and its delicate coil can carry only a small current before reaching full-scale deflection or being damaged. Although a calibrated galvanometer can measure small currents within its range, general current measurement requires converting it into an ammeter by connecting a suitable low-resistance shunt in parallel. Most of the current then passes through the shunt and the instrument's effective resistance becomes small.

Step 4: Conclusion The galvanometer works by balancing magnetic and restoring torques. The radial field gives $\theta \propto I$; the soft iron core increases the field and helps make it radial. Its current sensitivity is NAB/C , and a parallel shunt is needed for use as an ordinary ammeter.

Quick Tip to Solve: In a radial field, set the angle between the coil's normal and the field to 90° , then balance $NAB I$ with $C\theta$.

OR

16. (a) Define the term 'self-inductance' and write its S.I. unit.

(b) Obtain the expression for the mutual inductance of two long co-axial solenoids S_1 and S_2 wound one over the other, each of length L and radii r_1 and r_2 and n_1 and n_2 number of turns per unit length, when a current I is set up in the outer solenoid S_2 . [3 marks]

Solution:

Concept:

- Self-inductance $\mathcal{L}$ relates a coil's flux linkage to its own current: $N\Phi = \mathcal{L}I$. Its induced emf is $\varepsilon = -\mathcal{L} dI/dt$.

Quick Tip to Solve: For coaxial solenoids, multiply the source field by the inner cross-sectional area and the receiving coil's total turns, then divide by the source current.

- Mutual inductance M is the flux linkage in one coil per unit current in the other. Inside a long air-core solenoid with n turns per unit length, $B = \mu_0 n I$, neglecting end effects.

Step 1: Define self-inductance and its unit (a) Self-inductance is the magnetic flux linkage with a coil per unit current flowing through the same coil:

$$\mathcal{L} = \frac{N\Phi}{I}.$$

Its S.I. unit is the henry (H): a coil has self-inductance 1 H if a current changing at 1 A s^{-1} induces an emf of magnitude 1 V. Here $\mathcal{L}$ denotes inductance and L denotes length.

Step 2: Find the flux through the inner solenoid (b) Since S_2 is the outer solenoid, $r_2 > r_1$. A current I in S_2 produces a nearly uniform magnetic field throughout the inner solenoid:

$$B_2 = \mu_0 n_2 I.$$

The field is along the common axis, perpendicular to the turns. Each turn of S_1 therefore links flux through its own area $A_1 = \pi r_1^2$:

$$\Phi_{12} = B_2 A_1 = \mu_0 n_2 I \pi r_1^2.$$

Only flux through the inner area links a turn of S_1 .

Step 3: Calculate the total linkage and mutual inductance The number of turns in the inner solenoid is $N_1 = n_1 L$. Hence its total flux linkage due to current in the outer solenoid is

$$\begin{aligned} N_1 \Phi_{12} &= (n_1 L)(\mu_0 n_2 I \pi r_1^2), \\ M &= \frac{N_1 \Phi_{12}}{I} = \mu_0 n_1 n_2 \pi r_1^2 L. \end{aligned}$$

The outer radius does not appear because the common flux-linking area is the smaller cross-section.

Step 4: Conclusion Self-inductance is flux linkage per unit current in the same coil, with S.I. unit henry. For the given coaxial solenoids, $M = \mu_0 n_1 n_2 \pi r_1^2 L$.

Quick Tip to Solve: For coaxial solenoids, multiply the source field by the inner cross-sectional area and the receiving coil's total turns, then divide by the source current.

17. For a CE-transistor amplifier, the audio signal voltage across the collector resistance of $2 \text{ k}\Omega$ is 2 V . Given the current amplification factor of the transistor is 100, find the input signal voltage and base current, if the base resistance is $1 \text{ k}\Omega$. [3 marks]

Solution:

Concept:

- For a common-emitter amplifier, the small-signal current gain is $\beta = \Delta I_C / \Delta I_B$, where ΔI_C and ΔI_B are the collector and base signal-current amplitudes.
- Ohm's law gives the output signal amplitude $V_o = \Delta I_C R_C$ across the collector resistance and input signal amplitude $V_i = \Delta I_B R_B$ across the base resistance.

Quick Tip to Solve: Work backwards from the output: divide by R_C , then by β , and finally multiply by R_B to obtain the input voltage.

Step 1: Obtain the collector signal current The stated audio signal voltage is the varying voltage across the collector resistance, so it determines the collector signal current. Convert the resistance to ohms before substitution:

$$R_C = 2 \text{ k}\Omega = 2 \times 10^3 \Omega.$$

The collector signal-current amplitude is

$$\Delta I_C = \frac{V_o}{R_C} = \frac{2}{2 \times 10^3} \text{ A} = 1.0 \times 10^{-3} \text{ A}.$$

Step 2: Use the current gain to find the base current Since the current amplification factor is 100, the base signal current is one hundredth of the collector signal current:

$$\Delta I_B = \frac{\Delta I_C}{\beta} = \frac{1.0 \times 10^{-3}}{100} \text{ A} = 1.0 \times 10^{-5} \text{ A} = 10 \mu\text{A}.$$

This is the varying base-current amplitude associated with the audio signal; the steady bias current is not required for this calculation.

Step 3: Calculate and check the input signal voltage With $R_B = 1 \text{ k}\Omega = 10^3 \Omega$, the input signal-voltage amplitude is

$$V_i = \Delta I_B R_B = (1.0 \times 10^{-5})(10^3) \text{ V} = 1.0 \times 10^{-2} \text{ V} = 10 \text{ mV}.$$

As a check, the magnitude of voltage gain is $\beta R_C / R_B = 100 \times 2 = 200$, consistent with $V_o / V_i = 2 / 0.01 = 200$. The CE output voltage is inverted relative to the input, but the requested signal voltages and currents are given as positive amplitudes.

Step 4: Conclusion The input signal voltage is $\boxed{10 \text{ mV}}$, and the base signal current is $\boxed{10 \mu\text{A}}$.

Quick Tip to Solve: Work backwards from the output: divide by R_C , then by β , and finally multiply by R_B to obtain the input voltage.

18. A zener diode is fabricated by heavily doping both p- and n- sides of the junction. Explain, why? Briefly explain the use of zener diode as a dc voltage regulator with the help of a circuit diagram. [3 marks]

Solution:

Concept:

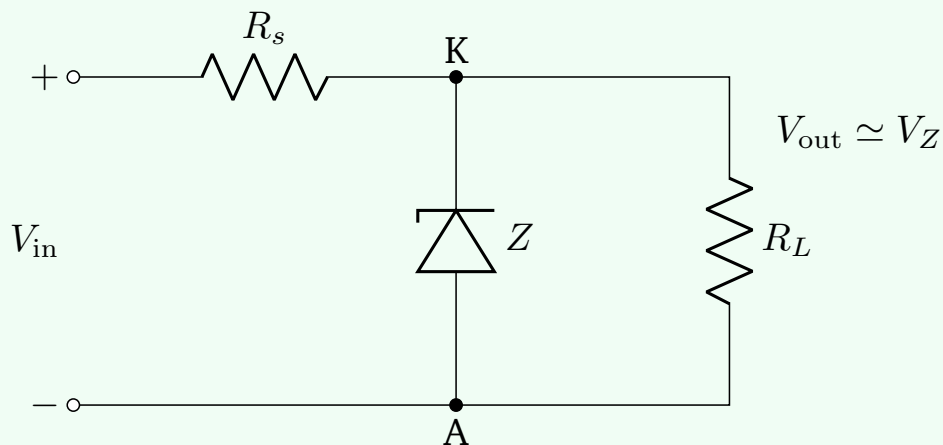
- Heavy doping makes the depletion layer very thin, producing a strong electric field even at a small reverse bias.
- In reverse breakdown a Zener diode maintains an almost constant voltage V_Z over a useful range of reverse current.
- A series resistor R_s limits current; the reverse-biased Zener is connected across the load R_L , so $V_{\text{out}} \simeq V_Z$.

Step 1: Explain the heavy doping Both sides of the junction are heavily doped so that the depletion region is extremely thin. Since the reverse voltage falls across this narrow region, the electric field

Quick Tip to Solve: Draw the Zener cathode at the positive output node; the load and Zener must share both terminals.

becomes very large at a relatively small reverse voltage. It can free electrons by breaking covalent bonds, causing a sharp rise in reverse current at the designed breakdown voltage. Thus the device provides a well-defined low reverse breakdown voltage suitable for regulation.

Step 2: Connect the regulator circuit Connect R_s in series with the unregulated dc supply, and connect the Zener diode in parallel with the load. Its cathode is connected to the positive output and its anode to the negative return, making it reverse biased. Choose the supply and R_s so that the diode operates in breakdown while its current remains within safe limits. The resistor takes up the excess supply voltage and prevents excessive diode current.



Zener shunt regulator: cathode K is at the positive output.

Step 3: Explain the regulating action The current balance is $I_s = I_Z + I_L$, where

$$I_s = \frac{V_{in} - V_Z}{R_s}, \quad I_L \simeq \frac{V_Z}{R_L}.$$

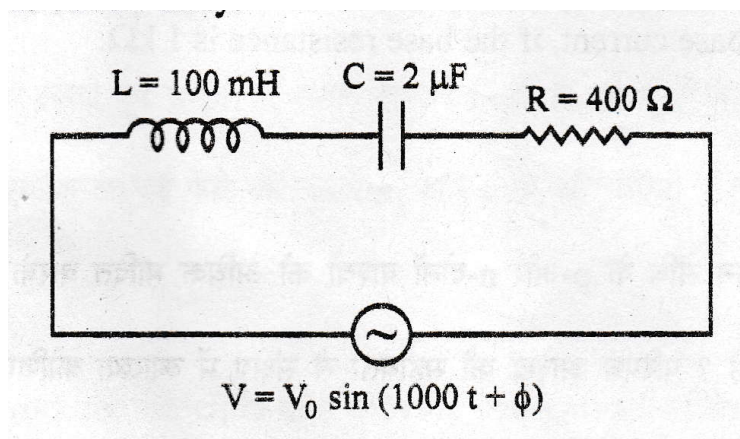
If the supply voltage rises, the extra current flows mainly through the Zener. The voltage drop across R_s increases, while the output stays nearly equal to V_Z . If the supply falls, the Zener current decreases and its voltage remains nearly constant. Similarly, an increase in load current is supplied by a decrease in Zener current, and a decrease in load current increases Zener current. Regulation holds while the diode remains in its permissible breakdown-current range.

Step 4: Conclusion Heavy doping produces a thin depletion layer and sharp reverse breakdown. A reverse-biased Zener connected across the load, with a series current-limiting resistor, maintains $V_{out} \simeq V_Z$.

Quick Tip to Solve: Draw the Zener cathode at the positive output node; the load and Zener must share both terminals.

19. (i) Find the value of the phase difference between the current and the voltage in the series LCR circuit shown below. Which one leads in phase: current or voltage?

(ii) Without making any other change, find the value of the additional capacitor C_1 , to be connected in parallel with the capacitor C , in order to make the power factor of the circuit unity. [3 marks]



Solution:

Concept:

- For a series LCR circuit, $X_L = \omega L$ and $X_C = 1/(\omega C)$, where ω is the angular frequency.
- Define $\theta = \phi_V - \phi_I$. Then $\tan \theta = (X_L - X_C)/R$; a negative value means current leads voltage.
- Unity power factor requires resonance, $X_L = X_C$. Capacitances in parallel add: $C_{\text{eq}} = C + C_1$.

Step 1: Read the component values and calculate reactances The printed circuit gives $L = 100 \text{ mH} = 0.100 \text{ H}$, $C = 2 \mu\text{F} = 2 \times 10^{-6} \text{ F}$, and $R = 400 \Omega$. From $V = V_0 \sin(1000t + \phi)$, the angular frequency is $\omega = 1000 \text{ rad s}^{-1}$. Hence

$$X_L = 1000(0.100) = 100 \Omega, \quad X_C = \frac{1}{1000(2 \times 10^{-6})} = 500 \Omega.$$

Step 2: Determine the phase difference and which quantity leads Substituting the reactances,

$$\tan \theta = \frac{100 - 500}{400} = -1, \quad \theta = -\frac{\pi}{4}.$$

The net reactance is capacitive because $X_C > X_L$. Consequently the current is ahead of the applied voltage in phase. In the chosen convention, $\phi_I = \phi_V + \pi/4$; the physical phase separation has magnitude $\pi/4$.

Step 3: Calculate the parallel capacitor needed for unity power factor At unity power factor the net reactance must vanish. With L and ω unchanged, the total capacitance must obey

$$\omega L = \frac{1}{\omega C_{\text{eq}}}, \quad C_{\text{eq}} = \frac{1}{\omega^2 L} = \frac{1}{(1000)^2(0.100)} = 10^{-5} \text{ F} = 10 \mu\text{F}.$$

Since the added capacitor is parallel to the existing one,

$$C_1 = C_{\text{eq}} - C = (10 - 2) \mu\text{F}.$$

Quick Tip to Solve: Read 1000 as angular frequency, and subtract the existing capacitance only after finding the resonance capacitance.

Increasing the capacitance reduces the capacitive reactance to match the unchanged inductive reactance; the resistance then provides the entire impedance.

Step 4: Conclusion (i) The phase difference is $45^\circ = \pi/4$ rad, with current leading voltage. (ii) The required additional parallel capacitor is $C_1 = 8 \mu\text{F}$.

Quick Tip to Solve: Read 1000 as angular frequency, and subtract the existing capacitance only after finding the resonance capacitance.

20. Define the term ‘amplitude modulation’. Explain any two factors which justify the need for modulating a low frequency base-band signal. [3 marks]

Solution:

Concept:

- Modulation impresses the information in a low-frequency message signal onto a high-frequency carrier suitable for transmission.
- The wavelength of an electromagnetic wave is $\lambda = c/f$, with c the speed of light and f its frequency.
- A practical transmitting antenna has a length comparable to a significant fraction of the wavelength, commonly $\lambda/4$.

Step 1: Define amplitude modulation Amplitude modulation is the process in which the amplitude of a high-frequency carrier wave is varied in accordance with the instantaneous value of the low-frequency message signal, while the carrier frequency and phase remain unchanged. For a single sinusoidal message, a representative expression is

$$s(t) = A_c [1 + \mu \cos(\omega_m t)] \cos(\omega_c t),$$

where A_c is the unmodulated carrier amplitude, μ the modulation index, and ω_m and ω_c the message and carrier angular frequencies, with $\omega_c \gg \omega_m$.

Step 2: First need: reduce the required antenna height Direct transmission of a low-frequency base-band signal would require an impractically large antenna because its wavelength is very large. For example, at $f = 1$ kHz,

$$\frac{\lambda}{4} = \frac{3 \times 10^8}{4 \times 10^3} = 7.5 \times 10^4 \text{ m} = 75 \text{ km}.$$

Modulation shifts transmission to a much higher carrier frequency. At 1 MHz, a quarter wavelength is only 75 m, making the antenna size much more practical.

Step 3: Second need: increase the efficiency of radiation A short antenna radiates very inefficiently when its length l is much smaller than the wavelength. For a given antenna length and comparable current, the radiated power varies approximately as $(l/\lambda)^2$. A low-frequency message therefore produces very little radiated power with a practical antenna. Using a high-frequency carrier reduces λ , increases l/λ , and allows more efficient radiation and useful transmission over a longer range. The information can be recovered from the modulated wave by demodulation at the receiver.

Step 4: Conclusion In amplitude modulation, the carrier amplitude follows the message signal.

Quick Tip to Solve: Keep the two reasons distinct: antenna height concerns physical size, while radiation efficiency concerns transmitted power.

Modulation is needed to obtain a practical antenna size and more efficient radiation of the transmitted signal.

Quick Tip to Solve: Keep the two reasons distinct: antenna height concerns physical size, while radiation efficiency concerns transmitted power.

21. An electron of mass m_e revolves around a nucleus of charge $+Ze$. Show that it behaves like a tiny magnetic dipole. Hence prove that the magnetic moment associated with it is expressed as $\vec{\mu} = -\frac{e}{2m_e}\vec{L}$, where $\vec{L}$ is the orbital angular momentum of the electron. Give the significance of negative sign. [3 marks]

Solution:

Concept:

- A charge completing one revolution in time T is equivalent to a current of magnitude $I = e/T$, where e is the positive magnitude of the electron charge.
- A small planar current loop has magnetic dipole moment $\vec{\mu} = I\vec{A}$, where $\vec{A}$ is the area vector given by the right-hand rule for conventional current.
- For a circular orbit of radius r and speed v , the orbital angular momentum has magnitude $L = m_evr$ and direction $\vec{r} \times \vec{v}$.

Step 1: Replace the orbiting electron by an equivalent current loop Let the electron move in a circular orbit of radius r with speed v about the nucleus. The time for one revolution and the magnitude of the associated current are

$$T = \frac{2\pi r}{v}, \quad I = \frac{e}{T} = \frac{ev}{2\pi r}.$$

Since the electron is negatively charged, conventional current circulates opposite to its motion. Thus the orbit is equivalent to a tiny closed current loop. Such a loop produces a dipolar magnetic field at distances large compared with its radius, and therefore behaves as a tiny magnetic dipole.

Step 2: Relate the moment magnitude to angular momentum The loop encloses area $A = \pi r^2$. Its magnetic moment magnitude is consequently

$$\mu = IA = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2}.$$

Since $L = m_evr$, eliminating vr gives

$$\mu = \frac{e}{2m_e}L.$$

The nuclear charge $+Ze$ supplies the central electric attraction that maintains the orbit. Its value affects the orbital speed, but cancels from the moment-to-angular-momentum ratio.

Step 3: Determine the vector direction and the negative sign The right-hand rule applied to the electron's motion gives the direction of $\vec{L}$, perpendicular to the orbital plane. The right-hand rule applied to conventional current gives $\vec{\mu}$. Because these two circulations are opposite, the two vectors are antiparallel. For example, if the electron moves anticlockwise as viewed from above, $\vec{L}$ points towards the viewer, while $\vec{\mu}$ points away.

Quick Tip to Solve: First derive the magnitudes using $I = e/T$; then insert the minus sign because conventional current flows opposite to electron motion.

Step 4: Conclusion The orbiting electron is equivalent to a tiny current loop, with orbital magnetic moment

$$\vec{\mu} = -\frac{e}{2m_e} \vec{L}.$$

The negative sign signifies that its magnetic moment is opposite to its orbital angular momentum, owing to the electron's negative charge.

Quick Tip to Solve: First derive the magnitudes using $I = e/T$; then insert the minus sign because conventional current flows opposite to electron motion.

22. (i) Derive the expression for the electric potential due to an electric dipole at a point on its axial line.

(ii) Depict the equipotential surfaces due to an electric dipole.

[3 marks]

Solution:

Concept:

- Potential is a scalar and obeys superposition: $V = \sum_i q_i / (4\pi\epsilon_0 r_i)$, with zero potential chosen at infinity.
- A dipole consists of charges $-q$ and $+q$ separated by $2a$. Its dipole moment $\vec{p}$ points from $-q$ to $+q$ and has magnitude $p = 2qa$.
- Every point on an equipotential surface has the same potential. The electric field is normal to that surface, so moving a test charge along it requires no electrostatic work.

Step 1: (i) Set up the axial geometry and add the potentials Place $-q$ at $x = -a$ and $+q$ at $x = +a$, with the origin at the midpoint. Choose an axial point P at $x = r > a$, on the positive-charge side. Its distances from $+q$ and $-q$ are respectively $r - a$ and $r + a$. Therefore,

$$V(P) = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r-a} - \frac{q}{r+a} \right).$$

The negative charge contributes a negative scalar potential; no vector addition of these potentials is needed.

Step 2: Simplify and obtain the short-dipole limit Combining the fractions and using $p = 2qa$,

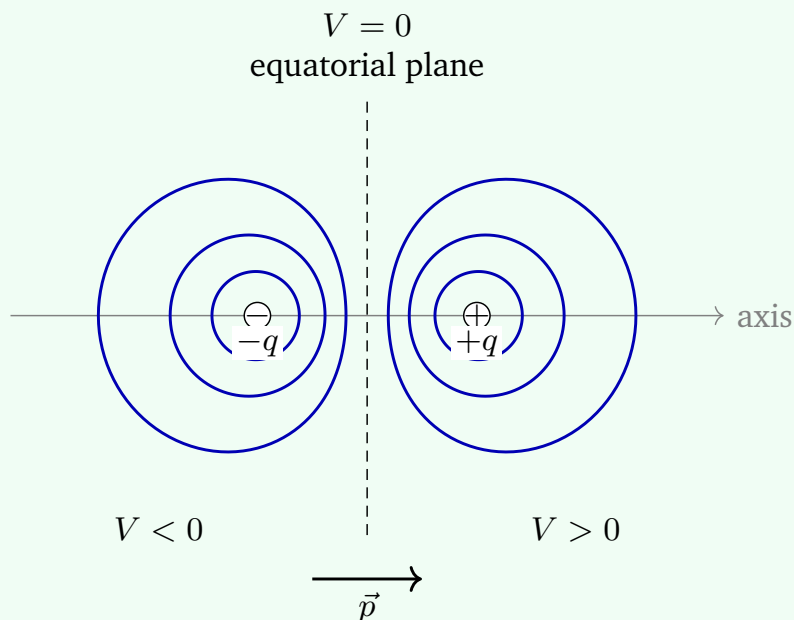
$$\begin{aligned} V(P) &= \frac{1}{4\pi\epsilon_0} \frac{q[(r+a) - (r-a)]}{(r-a)(r+a)} \\ &= \frac{1}{4\pi\epsilon_0} \frac{p}{r^2 - a^2}. \end{aligned}$$

For a short dipole observed far away, $r \gg a$, so a^2 can be neglected relative to r^2 , giving $V \simeq p / (4\pi\epsilon_0 r^2)$. At the corresponding point on the negative-charge side the potential has the same magnitude and the opposite sign.

Step 3: (ii) Depict and interpret the equipotential surfaces The figure shows a plane section through the dipole axis. Each closed curve is an equipotential contour; rotating it about the horizontal dipole

Quick Tip to Solve: Write the distances $r - a$ and $r + a$ before adding potentials; use the far-field approximation only after combining the fractions.

axis gives the corresponding three-dimensional surface. Positive-potential surfaces surround $+q$, and negative-potential surfaces surround $-q$. Close to either charge they are nearly spherical, and farther away they are distorted by the other charge. The perpendicular bisector represents the equatorial plane: every point there is equally distant from both charges, so their potentials cancel and $V = 0$. Different equipotential surfaces never intersect. The zero potential on this plane does not imply a zero electric field.



Plane section of dipole equipotentials; rotation about the dipole axis generates the surfaces.

Step 4: Conclusion On the axial line beyond $+q$,

$$V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2 - a^2} \simeq \frac{p}{4\pi\epsilon_0 r^2} \quad (r \gg a).$$

The sign reverses on the opposite side. The dipole has symmetric positive and negative equipotential surfaces separated by the zero-potential equatorial plane.

Quick Tip to Solve: Write the distances $r - a$ and $r + a$ before adding potentials; use the far-field approximation only after combining the fractions.

Section D (4 marks)

23. Mrs. Rashmi Singh broke her reading glasses. When she went to the shopkeeper to order new specs, he suggested that she should get spectacles with plastic lenses instead of glass lenses. On getting the new spectacles, she found that the new ones were thicker than the earlier ones. She asked this question to the shopkeeper but he could not offer satisfactory explanation for this. At home, Mrs. Singh raised the same question to her daughter Anuja who explained why plastic lenses were thicker.

(a) Write two qualities displayed each by Anuja and her mother.

(b) How do you explain this fact using lens maker's formula?

[4 marks]

Solution:

Concept:

- Everyday observations can demonstrate curiosity, scientific temper and helpfulness.
- For a thin lens in air, the lens maker's formula is $P = 1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where P is its power, f its focal length, μ the refractive index of its material, and R_1, R_2 its signed radii of curvature.
- Replacement reading glasses must provide the same prescribed positive power. For comparable lens diameter, greater convex surface curvature requires a greater centre-to-edge thickness difference.

Step 1: (a) Identify the qualities of Anuja and her mother Anuja displays **scientific understanding**, applying lens theory to an everyday observation, and **helpfulness**, sharing her knowledge with her mother. Her mother displays **curiosity**, noticing and questioning the changed thickness, and **an enquiring scientific attitude**, continuing to seek a satisfactory explanation.

Step 2: (b) Compare the lens maker's formula for the two materials Let p and g denote plastic and glass. The plastic in this comparison has a lower refractive index: $\mu_p < \mu_g$. The unchanged prescription requires equal powers. Writing $C = 1/R_1 - 1/R_2$ gives

$$P = (\mu_p - 1)C_p = (\mu_g - 1)C_g,$$
$$\frac{C_p}{C_g} = \frac{\mu_g - 1}{\mu_p - 1} > 1.$$

The smaller refractive-index contrast with air therefore requires a larger curvature factor. Keeping the old curvatures would instead reduce the power.

Step 3: Relate the greater curvature to lens thickness For illustration, consider a symmetric biconvex reading lens with $R_1 = R$ and $R_2 = -R$. The formula becomes

$$\frac{1}{f} = \frac{2(\mu - 1)}{R}, \quad R = 2f(\mu - 1).$$

At fixed f , smaller μ requires smaller R , hence more strongly curved surfaces. At equal diameter and comparable edge thickness, the central thickness increases. This comparison assumes lower-index plastic; high-index plastics need not be thicker than glass.

Step 4: Conclusion Anuja shows scientific understanding and helpfulness; her mother shows curiosity and an enquiring scientific attitude. The plastic lenses are thicker because their lower refractive index

Quick Tip to Solve: Keep the prescription fixed: a smaller $\mu - 1$ requires a larger curvature factor in the lens maker's formula, giving a thicker positive lens of comparable diameter.

requires greater surface curvature to produce the same prescribed power.

Quick Tip to Solve: Keep the prescription fixed: a smaller $\mu - 1$ requires a larger curvature factor in the lens maker's formula, giving a thicker positive lens of comparable diameter.

Section E (15 marks)

24. (a) Distinguish between unpolarized light and linearly polarized light. How does one get linearly polarised light with the help of a polaroid?

(b) A narrow beam of unpolarised light of intensity I_0 is incident on a polaroid P_1 . The light transmitted by it is then incident on a second polaroid P_2 with its pass axis making angle of 60° relative to the pass axis of P_1 . Find the intensity of the light transmitted by P_2 . [5 marks]

Solution:

Concept:

- Polarisation describes the direction of the electric-field vibrations of a transverse light wave. These vibrations are always perpendicular to its direction of propagation.
- An ideal polaroid transmits the component of the electric field parallel to its pass axis and absorbs the perpendicular component. Unpolarised light of intensity I_0 therefore emerges with intensity $I_0/2$.
- Malus's law is $I = I_{\text{in}} \cos^2 \theta$, where I_{in} is the intensity of linearly polarised light incident on an analyser and θ is the angle between the incident polarisation direction and the analyser's pass axis.

Step 1: (a) Distinguish the two kinds of light In unpolarised light, the electric-field vibrations have randomly changing directions in the plane perpendicular to propagation, with no preferred transverse direction. In linearly polarised light, the electric field vibrates along one fixed direction in that transverse plane; its magnitude and sign vary with time, but the line of vibration remains fixed.

Step 2: (a) Produce linearly polarised light using a polaroid Pass the unpolarised beam through a polaroid. Resolve each incident electric-field vibration into components parallel and perpendicular to its pass axis. The parallel component is transmitted and the perpendicular component is absorbed by selective absorption. All the transmitted vibrations are consequently along the same pass axis, so the emerging beam is linearly polarised. For an ideal polaroid the intensity is halved, since the uniformly distributed incident directions give $\langle \cos^2 \phi \rangle = 1/2$.

Step 3: (b) Find the intensity incident on the second polaroid The incident beam on P_1 is unpolarised. Its transmitted intensity is therefore

$$I_1 = \frac{I_0}{2}.$$

This beam is now linearly polarised along the pass axis of P_1 , so it is suitable for applying Malus's law at P_2 .

Step 4: (b) Apply Malus's law at the second polaroid The angle between the incident polarisation

Quick Tip to Solve: For unpolarised light through two ideal polaroids, halve the intensity at the first and then multiply by the squared cosine of the angle between their pass axes.

direction and the pass axis of P_2 is 60° . Hence

$$I_2 = I_1 \cos^2 60^\circ = \frac{I_0}{2} \left(\frac{1}{2}\right)^2.$$

The cosine is squared because intensity is proportional to the square of electric-field amplitude.

Step 5: Conclusion Unpolarised light has no fixed direction of electric-field vibration, whereas linearly polarised light has one fixed transverse vibration direction. A polaroid produces the latter by transmitting the electric-field component along its pass axis. The intensity transmitted by P_2 is $I_2 = I_0/8$, and this light is polarised along the pass axis of P_2 .

Quick Tip to Solve: For unpolarised light through two ideal polaroids, halve the intensity at the first and then multiply by the squared cosine of the angle between their pass axes.

OR

24. (a) Explain two features to distinguish between the interference pattern in Young's double slit experiment with the diffraction pattern obtained due to a single slit.

(b) A monochromatic light of wavelength 500 nm is incident normally on a single slit of width 0.2 mm to produce a diffraction pattern. Find the angular width of the central maximum obtained on the screen.

Estimate the number of fringes obtained in Young's double slit experiment with fringe width 0.5 mm, which can be accommodated within the region of total angular spread of the central maximum due to single slit. [5 marks]

Solution:

Concept:

- Young's interference involves two coherent slits; diffraction involves wavelets from different parts of one slit. Single-slit minima satisfy $a \sin \theta = m\lambda$, with slit width a and wavelength λ .
- The central diffraction maximum has angular width $\Delta\theta \simeq 2\lambda/a$. On a screen at distance D , its linear width is $W \simeq D\Delta\theta$; the number of fringe spacings of width β fitting within it is $N \simeq W/\beta$.

Step 1: (a) Compare intensities and widths **Intensity:** Ideal Young fringes from equally illuminated, narrow slits have equal maximum intensities. Single-slit diffraction has a brightest central maximum and successively weaker secondary maxima. Finite-width Young slits can additionally impose a diffraction envelope on their interference pattern.

Width: Young fringes have uniform width $\beta = \lambda D/d$, where d is slit separation. In single-slit diffraction the central band is twice as wide as a secondary bright band, measuring widths between adjacent minima in the small-angle region: the central band spans minima $m = -1$ to $m = +1$, while each secondary band spans consecutive minima on one side.

Step 2: (b) Find the angular width between the first minima Convert the data: $\lambda = 500 \times 10^{-9}$ m

Quick Tip to Solve: Double λ/a for the full angular width, then multiply by D before dividing by a linear fringe width.

and $a = 0.2 \times 10^{-3}$ m. For the first minimum,

$$\sin \theta_1 = \frac{\lambda}{a} = \frac{500 \times 10^{-9}}{0.2 \times 10^{-3}} = 2.5 \times 10^{-3}.$$

This angle is small, so $\theta_1 \simeq 2.5 \times 10^{-3}$ rad. The central maximum extends from $-\theta_1$ to $+\theta_1$; its full width is $2\theta_1$, not the half-width θ_1 .

Step 3: (b) Express the fringe count using the screen distance The given $\beta = 0.5$ mm $= 5.0 \times 10^{-4}$ m is a linear width. Convert the angular spread into a screen length before dividing:

$$N \simeq \frac{D\Delta\theta}{\beta} = \frac{(5.0 \times 10^{-3})D}{5.0 \times 10^{-4} \text{ m}} = 10 \frac{D}{1 \text{ m}}.$$

The printed question gives neither the screen distance D nor an equivalent angular fringe width. Thus a unique numerical count is underdetermined. If $D = 1$ m were additionally given, $W = 5$ mm would accommodate about ten fringe spacings; this is only a conditional example.

Step 4: Conclusion Ideal interference fringes have equal peak intensities and uniform widths; diffraction has weaker side maxima and a central band twice the secondary-band width. The full angular width is 5.0×10^{-3} rad. The estimate $N \simeq 10D/(1 \text{ m})$ requires the missing screen distance for a numerical value.

Quick Tip to Solve: Double λ/a for the full angular width, then multiply by D before dividing by a linear fringe width.

25. (i) Derive an expression for drift velocity of electrons in a conductor. Hence deduce Ohm's law.
(ii) A wire whose cross-sectional area is increasing linearly from its one end to the other, is connected across a battery of V volts. Which of the following quantities remain constant in the wire?

- (a) drift speed
(b) current density
(c) electric current
(d) electric field

Justify your answer.

[5 marks]

Solution:

Concept:

- An electron of mass m and charge $-e$ has acceleration $\vec{a} = -e\vec{E}/m$. Random thermal velocities average to zero; the applied field produces drift.
- With electron density n , area A and drift speed v_d , current is $I = neAv_d$. Charge conservation makes steady current equal through every cross-section.

Step 1: (i) Derive the drift velocity Let $\vec{u}$ be the random velocity just after a collision. After a

Quick Tip to Solve: Hold steady I constant, then use $J = I/A$, $v_d = J/(ne)$ and $E = \rho J$ to compare narrow and wide sections.

free-flight time t , $\vec{v} = \vec{u} - e\vec{E}t/m$. Averaging, with $\langle \vec{u} \rangle = 0$ and relaxation time $\langle t \rangle = \tau$, gives

$$\vec{v}_d = -\frac{e\tau}{m}\vec{E}, \quad v_d = \frac{eE\tau}{m}.$$

Here τ is the mean time between successive collisions. Electron drift is opposite to $\vec{E}$; conventional current is along it.

Step 2: (i) Deduce Ohm's law In time Δt , electrons within volume $Av_d\Delta t$ cross a section, carrying charge of magnitude $\Delta Q = neAv_d\Delta t$. Thus $I = \Delta Q/\Delta t = neAv_d$. For a uniform wire of length L , $E = V/L$, giving

$$I = \frac{ne^2\tau A}{mL}V, \quad V = IR, \quad R = \frac{mL}{ne^2\tau A} = \rho \frac{L}{A}.$$

The resistivity is $\rho = m/(ne^2\tau)$. At constant temperature and fixed dimensions, n and τ , hence R , are constant. Therefore current is proportional to potential difference, which is Ohm's law.

Step 3: (ii) Compare quantities in the tapered wire In steady conduction, charge entering any segment per second must equal charge leaving it; otherwise charge would continually accumulate. Hence I is constant throughout the wire. For the same material at uniform temperature, n and ρ are constant, so at local area $A(x)$,

$$v_d(x) = \frac{I}{neA(x)}, \quad J(x) = \frac{I}{A(x)}, \quad E(x) = \rho J(x) = \frac{\rho I}{A(x)}.$$

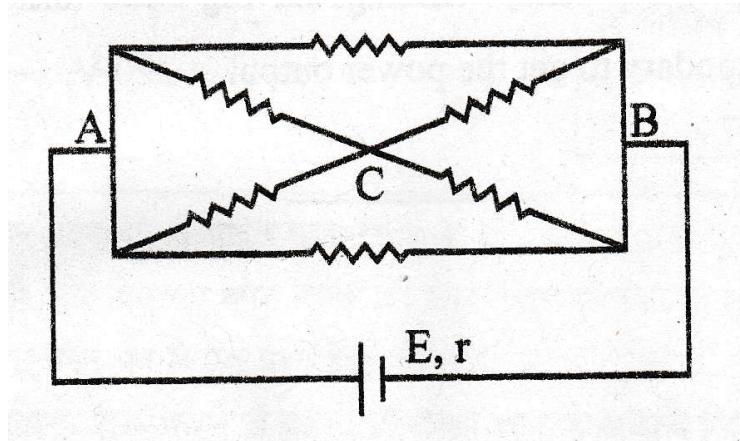
Thus (a) drift speed, (b) current density and (d) electric field all decrease as area increases; none is constant. Their inverse dependence on area applies even when the area increases linearly with position. The battery fixes the total voltage drop, not a uniform field.

Step 4: Conclusion $\vec{v}_d = -e\tau\vec{E}/m$ leads to $V = IR$, with $R = mL/(ne^2\tau A)$. Only (c) electric current remains constant in the tapered wire.

Quick Tip to Solve: Hold steady I constant, then use $J = I/A$, $v_d = J/(ne)$ and $E = \rho J$ to compare narrow and wide sections.

25. (i) State the two Kirchhoff's laws. Explain briefly how these rules are justified.

(ii) The current is drawn from a cell of emf E and internal resistance r connected to the network of resistors each of resistance r as shown in the figure. Obtain the expression for (i) the current drawn from the cell and (ii) the power consumed in the network. [5 marks]



Solution:

Concept:

- Kirchhoff's junction rule expresses conservation of charge: the sum of currents entering a junction equals the sum leaving it in steady state.
- Kirchhoff's loop rule expresses conservation of energy: the algebraic sum of potential changes around any closed conducting loop is zero.
- For resistors, series resistances add and parallel conductances add. The cell's emf is E , its internal resistance is r , and each of the six external resistors also has resistance r .

Step 1: (i) State and justify the junction rule At any junction, $\sum I_{\text{in}} = \sum I_{\text{out}}$, or equivalently $\sum I = 0$ with a consistent sign convention. If incoming and outgoing currents differed, charge would accumulate continuously at the junction. In a steady circuit there is no such accumulation; conservation of charge therefore requires their equality. For example, a current I splitting into two branches satisfies $I = I_1 + I_2$.

Step 2: (i) State and justify the loop rule Around any closed loop, $\sum \Delta V = 0$, often written $\sum \mathcal{E} - \sum IR = 0$ when all terms carry their appropriate signs. On crossing a cell from its negative to positive terminal, take a potential rise $+\mathcal{E}$; traversing a resistor in the direction of current gives a drop $-IR$. A charge completing a loop returns to its initial potential. Hence the energy supplied per unit charge equals the energy lost per unit charge, as required by conservation of energy. These rules apply here to the steady direct-current network.

Step 3: (ii) Reduce the external network The left vertical wire is one node A , and the right vertical wire is one node B . The two sloping resistors joining A to the central point C are parallel. Similarly, the two resistors joining C to B are parallel. Thus

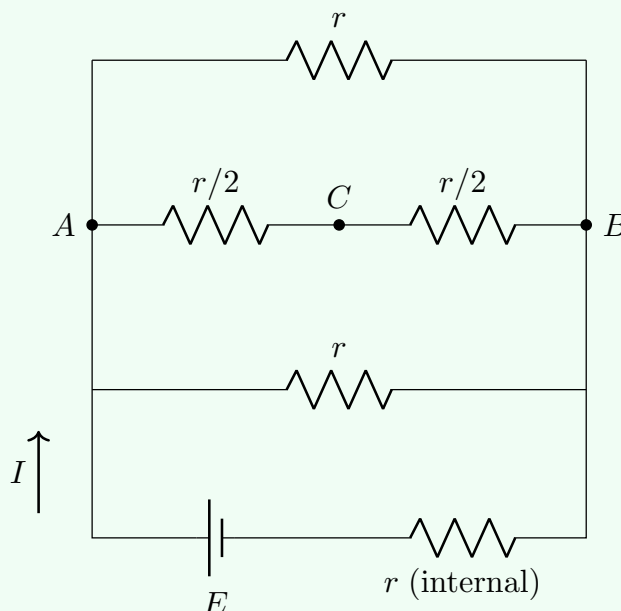
$$R_{AC} = r \parallel r = \frac{r}{2}, \quad R_{CB} = r \parallel r = \frac{r}{2}.$$

Quick Tip to Solve: First merge points joined by ideal wires into single nodes. Reduce the external network completely before adding the cell's internal resistance.

These two equivalent sections are in series, giving $R_{ACB} = r$. There are now three parallel branches between A and B : the top resistor r , this central equivalent branch r , and the bottom resistor r . Consequently,

$$\frac{1}{R_{\text{net}}} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{3}{r}, \quad R_{\text{net}} = \frac{r}{3}.$$

The figure below redraws this equivalent external network with the cell's internal resistance shown separately.



Equivalent network: each central half represents two original resistors in parallel.

Step 4: (ii) Calculate current drawn from the cell The internal resistance is in series with the whole external network, so the total resistance seen by the emf is

$$R_{\text{total}} = r + \frac{r}{3} = \frac{4r}{3}.$$

Applying the loop rule to the cell and its equivalent external resistance gives

$$E - Ir - I\frac{r}{3} = 0 \quad \Rightarrow \quad I = \frac{3E}{4r}.$$

The terminal voltage across A and B is $V_{AB} = E - Ir = E/4$. This also agrees with $IR_{\text{net}} = E/4$.

Step 5: (ii) Calculate power consumed in the external network The requested network power excludes the heat dissipated inside the cell. Therefore use the external equivalent resistance:

$$P_{\text{net}} = I^2 R_{\text{net}} = \left(\frac{3E}{4r}\right)^2 \frac{r}{3} = \frac{3E^2}{16r}.$$

As a check, each of the three equivalent branches carries $V_{AB}/r = E/(4r)$. Adding their dissipated powers gives $3V_{AB}^2/r = 3E^2/(16r)$, the same result. The larger quantity EI includes both external power and internal heating, so it is not the network power.

Quick Tip to Solve: First merge points joined by ideal wires into single nodes. Reduce the external network completely before adding the cell's internal resistance.

Step 6: Conclusion Kirchhoff's junction and loop rules follow from conservation of charge and energy, respectively. The network has equivalent resistance $r/3$; hence the current drawn is $I = 3E/(4r)$, and the power consumed in the network is $P_{\text{net}} = 3E^2/(16r)$.

Quick Tip to Solve: First merge points joined by ideal wires into single nodes. Reduce the external network completely before adding the cell's internal resistance.

26. (a) Draw a labelled diagram of AC generator. Derive the expression for the instantaneous value of the emf induced in the coil.

(b) A circular coil of cross-sectional area 200 cm^2 and 20 turns is rotated about the vertical diameter with angular speed of 50 rad s^{-1} in a uniform magnetic field of magnitude $3.0 \times 10^{-2} \text{ T}$. Calculate the maximum value of the current in the coil. [5 marks]

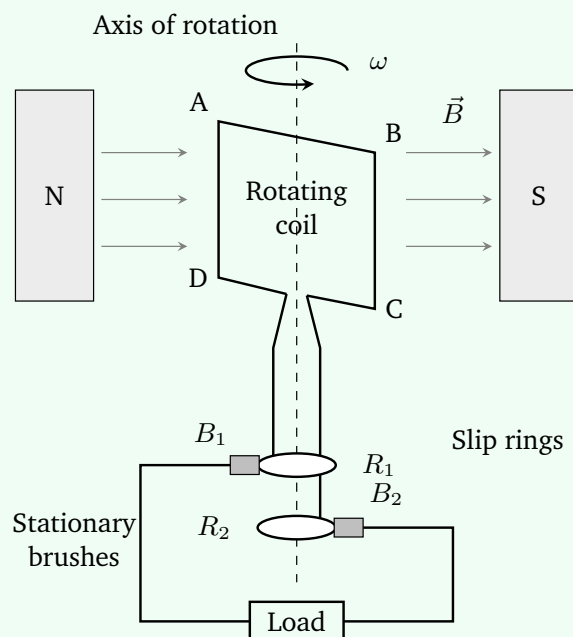
Solution:

Concept:

- An AC generator converts mechanical energy into electrical energy by electromagnetic induction. Faraday's law gives $\varepsilon = -d(N\Phi)/dt$, where N is the number of turns and Φ is the magnetic flux through one turn.
- For a coil of area A in a uniform magnetic field B , $\Phi = BA \cos \theta$, where θ is the angle between the field and the normal to the coil. Rotation at angular speed ω gives $\theta = \omega t$ for a suitable time origin.
- In a purely resistive closed circuit of total resistance R , the peak current is $I_0 = \varepsilon_0/R$. An emf alone does not determine a current without the circuit resistance (or, more generally, impedance).

Step 1: (a) Identify the parts and working of the generator A rectangular armature coil $ABCD$, having N insulated turns, rotates mechanically between the north and south poles of a magnet. Its axis is perpendicular to the magnetic field. The two ends of the coil connect to two separate, continuous slip rings R_1 and R_2 , which rotate with the coil. Stationary carbon brushes B_1 and B_2 make contact with the rings and connect the coil to the external load. As the coil rotates, its magnetic flux changes and an emf is induced. The slip rings maintain electrical contact without interchanging the coil connections; consequently, the load receives an alternating emf.

Quick Tip to Solve: Convert cm^2 to m^2 using 10^{-4} , then calculate $NBA\omega$. This gives volts; check that resistance is supplied before reporting amperes.



AC generator: rotating coil, separate slip rings, stationary brushes and external load (schematic).

Step 2: (a) Write the flux linkage and differentiate Choose $t = 0$ when the coil normal is parallel to the magnetic field. At time t , the normal has turned through $\theta = \omega t$. The flux linkage is therefore

$$N\Phi = NBA \cos(\omega t).$$

Applying Faraday's law to the complete coil gives

$$\begin{aligned} \varepsilon(t) &= -\frac{d}{dt}[NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t). \end{aligned}$$

Thus the amplitude is $\varepsilon_0 = NBA\omega$. The sign reverses after every half revolution, so the emf is alternating. It is zero when the normal is parallel to the field and largest in magnitude when the plane of the coil is parallel to the field. Choosing another initial orientation changes only the phase of the sinusoidal expression.

Step 3: (b) Convert the area and calculate the peak emf For the usual generator arrangement, the magnetic field is perpendicular to the axis of rotation. The given quantities are

$$\begin{aligned} A &= 200 \times 10^{-4} = 2.00 \times 10^{-2} \text{ m}^2, \\ N &= 20, \quad B = 3.0 \times 10^{-2} \text{ T}, \\ \omega &= 50 \text{ rad s}^{-1}. \end{aligned}$$

Substitution in the peak-emf formula yields

$$\varepsilon_0 = 20(3.0 \times 10^{-2})(2.00 \times 10^{-2})(50) = 0.60 \text{ V}.$$

Quick Tip to Solve: Convert cm^2 to m^2 using 10^{-4} , then calculate $NBA\omega$. This gives volts; check that resistance is supplied before reporting amperes.

Step 4: (b) Check the information required for a current The printed question asks for the maximum current, but supplies neither the coil resistance nor any total circuit resistance. Neglecting self-inductance and denoting the total resistance by R , Ohm's law gives

$$I_0 = \frac{\varepsilon_0}{R} = \frac{0.60 \text{ V}}{R}.$$

Hence the stated data determine the peak emf, but are insufficient for a numerical peak current. In particular, 0.60 V cannot be relabelled as amperes. Also, the numerical emf above assumes the standard perpendicular field orientation; if the field is inclined to the rotation axis, only its component perpendicular to that axis contributes to the changing flux.

Step 5: Conclusion (a) The instantaneous induced emf is $\varepsilon(t) = NBA\omega \sin(\omega t)$, with peak value $NBA\omega$. (b) In the standard generator orientation, $\varepsilon_0 = 0.60 \text{ V}$. The maximum current is $I_0 = (0.60 \text{ V})/R$ for a resistive circuit; a numerical current cannot be obtained because the printed question omits the resistance.

Quick Tip to Solve: Convert cm^2 to m^2 using 10^{-4} , then calculate $NBA\omega$. This gives volts; check that resistance is supplied before reporting amperes.

OR

26. (a) Draw a labelled diagram of a step-up transformer. Obtain the ratio of secondary to primary voltage in terms of number of turns and currents in the two coils.

(b) A power transmission line feeds input power at 2200 V to a step-down transformer with its primary windings having 3000 turns. Find the number of turns in the secondary to get the power output at 220 V. [5 marks]

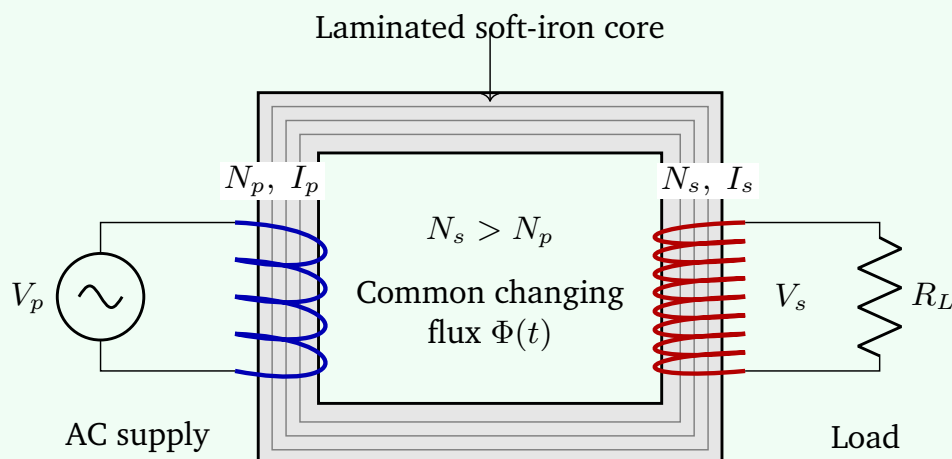
Solution:

Concept:

- A transformer works by mutual induction: alternating current in the primary produces a changing magnetic flux Φ through both windings on a common soft-iron core.
- Faraday's law gives $e = -N d\Phi/dt$, where N is the number of turns. For an ideal transformer, flux leakage and winding resistance are negligible.
- An ideal transformer conserves power: $V_p I_p = V_s I_s$, where the subscripts p and s denote primary and secondary; voltages and currents are rms values for a resistive load.

Step 1: (a) Identify the parts of the step-up transformer The primary winding is connected to an AC supply and the secondary winding is connected to the load. Both insulated windings surround the same closed laminated soft-iron core but have no electrical connection to each other. For a step-up transformer, the secondary has more turns than the primary, $N_s > N_p$. The core guides the changing flux through both coils; its laminations reduce eddy-current loss. The diagram represents a step-up transformer; part (b) separately concerns a step-down transformer.

Quick Tip to Solve: Match secondary with secondary in $V_s/V_p = N_s/N_p$; the current ratio is inverted. A tenfold voltage reduction requires tenfold fewer secondary turns.



Step-up transformer: common laminated core, AC input and load; $N_s > N_p$.

Step 2: (a) Derive the voltage ratio from the common flux Let the magnetic flux through each turn be $\Phi = \Phi_m \sin \omega t$. Applying Faraday's law to each winding gives

$$e_p = -N_p \omega \Phi_m \cos \omega t,$$

$$e_s = -N_s \omega \Phi_m \cos \omega t.$$

The minus signs express Lenz's law. Taking the induced-emf amplitudes and then their rms values gives

$$\frac{E_s}{E_p} = \frac{N_s \omega \Phi_m / \sqrt{2}}{N_p \omega \Phi_m / \sqrt{2}} = \frac{N_s}{N_p}.$$

With negligible winding voltage drops, the terminal voltage magnitudes equal the corresponding induced-emf magnitudes, so this is also the secondary-to-primary voltage ratio.

Step 3: (a) Relate the voltage ratio to the currents Neglecting all losses and the small magnetising current, the average input power equals the output power. For the ideal transformer with a resistive load,

$$V_p I_p = V_s I_s \quad \Rightarrow \quad \frac{V_s}{V_p} = \frac{I_p}{I_s}.$$

Thus increasing the voltage reduces the current in the same ratio. In a step-up transformer $V_s > V_p$ and $I_s < I_p$; the voltage increase does not create extra power. The frequency remains that of the AC supply.

Step 4: (b) Substitute the step-down transformer's data Here $V_p = 2200 \text{ V}$, $V_s = 220 \text{ V}$, and $N_p = 3000$. Rearranging the turns relation,

$$N_s = N_p \frac{V_s}{V_p} = 3000 \times \frac{220}{2200} = 3000 \times \frac{1}{10}.$$

The output voltage is one tenth of the input voltage, so the secondary must have one tenth as many turns, consistent with a step-down transformer.

Step 5: Conclusion For an ideal transformer, $V_s/V_p = N_s/N_p = I_p/I_s$. A step-up transformer has $N_s > N_p$. For the step-down transformer in part (b), the required secondary winding is

Quick Tip to Solve: Match secondary with secondary in $V_s/V_p = N_s/N_p$; the current ratio is inverted. A tenfold voltage reduction requires tenfold fewer secondary turns.

$$N_s = 300 \text{ turns}.$$

Quick Tip to Solve: Match secondary with secondary in $V_s/V_p = N_s/N_p$; the current ratio is inverted. A tenfold voltage reduction requires tenfold fewer secondary turns.