

CBSE Class 12 Physics Question Paper 2017 with Solution PDF-Delhi Set 3

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are **26** questions in all.
- (ii) This question paper has **five** sections : **Section A, Section B, Section C, Section D** and **Section E**.
- (iii) Section A contains **five** questions of **one** mark each, Section B contains **five** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains **one** value based question of **four** marks and Section E contains **three** questions of **five** marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in **one** question of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron} = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Section A (5 marks)

1. A long straight current carrying wire passes normally through the centre of circular loop. If the current through the wire increases, will there be an induced emf in the loop? Justify. [1 mark]

Solution:

Concept:

- Faraday's law gives the induced emf as $\mathcal{E} = -d\Phi_B/dt$, where Φ_B is the magnetic flux through the loop.
- Magnetic flux is $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$; only the field component normal to the loop contributes.

Step 1: Identify the magnetic-field direction The straight wire is normal to the plane of the circular loop. Its magnetic field follows circles centred on the wire, so the field lies in the plane of the loop and is perpendicular to the loop's area vector.

Step 2: Check the change in flux Hence $\mathbf{B} \cdot d\mathbf{A} = 0$ everywhere on the spanning surface and $\Phi_B = 0$. Increasing the current increases the in-plane field but does not create any normal component, so the flux remains zero.

Step 3: Conclusion No emf is induced in the loop, since $\mathcal{E} = -d\Phi_B/dt = 0$.

Quick Tip to Solve: Check the angle between the magnetic field and the area vector before deciding whether a changing current induces emf.

2. How is the speed of em-waves in vacuum determined by the electric and magnetic fields? [1 mark]

Solution:

Concept:

- In an electromagnetic wave in vacuum, the electric and magnetic fields are mutually perpendicular and oscillate in phase.
- Their amplitudes obey $E_0 = cB_0$, where E_0 is the electric-field amplitude, B_0 the magnetic-field amplitude and c the speed of light in vacuum.

Step 1: Relate the field amplitudes Maxwell's electromagnetic-wave relation in vacuum connects the magnitudes by $E_0 = cB_0$. Thus the electric-field amplitude divided by the magnetic-field amplitude has the dimensions of speed.

Step 2: Express the wave speed Rearranging the field relation gives

$$c = \frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

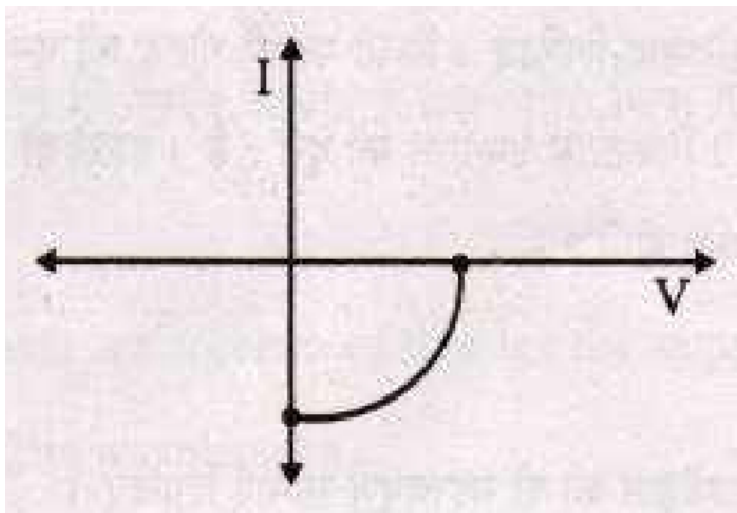
The same ratio E/B applies to nonzero instantaneous field magnitudes measured at the same position and time.

Step 3: Conclusion The speed is the ratio of electric- to magnetic-field amplitudes: $c = E_0/B_0 \simeq 3 \times 10^8 \text{ m s}^{-1}$.

Quick Tip to Solve: Use the two amplitudes in SI units: electric field in volts per metre and magnetic field in tesla.

3. Name the junction diode whose I-V characteristics are drawn below:

[1 mark]



Solution:

Concept:

- A solar cell is an illuminated p-n junction that converts light energy into electrical energy by the photovoltaic effect.
- With the usual diode sign convention, a solar cell supplies power when its terminal voltage is positive and its current is negative, placing its characteristic in the fourth quadrant.

Step 1: Read the signs on the characteristic The plotted curve lies below the voltage axis and to the right of the current axis. It shows a nonzero negative current at zero voltage and a positive voltage at zero current.

Step 2: Identify the physical device These intercepts represent the short-circuit photocurrent and the open-circuit photovoltage of an illuminated junction. The fourth-quadrant operating region indicates that the device can deliver electrical power to an external load.

Step 3: Conclusion The junction diode is a **solar cell**.

Quick Tip to Solve: Identify a solar cell by its fourth-quadrant I-V curve and its nonzero current and voltage intercepts under illumination.

4. At a place, the horizontal component of earth's magnetic field is B and angle of dip is 60° . What is the value of horizontal component of the earth's magnetic field at equator?

[1 mark]

Solution:

Concept:

- The horizontal component of the earth's magnetic field is $B_H = B_E \cos \delta$, where B_E is the total field magnitude and δ is the angle of dip.
- At the magnetic equator the dip is zero, so the field is horizontal and its horizontal component equals the total field there.

Quick Tip to Solve: First recover the total field by dividing the given horizontal component by the cosine of the dip; then use zero dip at the equator.

Step 1: Find the total field from the given component At the specified place, $B_H = B$ and $\delta = 60^\circ$. Hence

$$B = B_E \cos 60^\circ = \frac{B_E}{2}, \quad B_E = 2B.$$

Step 2: Apply the equatorial dip condition Using the usual assumption implicit in this question that the total field magnitude is unchanged in this comparison, set $\delta = 0^\circ$:

$$B_{H,\text{eq}} = B_E \cos 0^\circ = B_E.$$

In a real geographical comparison the total field can vary with location, so this assumption is needed to infer an equatorial value from the given data.

Step 3: Conclusion Under the intended constant-total-field assumption, the horizontal component at the equator is $2B$.

Quick Tip to Solve: First recover the total field by dividing the given horizontal component by the cosine of the dip; then use zero dip at the equator.

5. Does the charge given to a metallic sphere depend on whether it is hollow or solid? Give reason for your answer. [1 mark]

Solution:

Concept:

- In electrostatic equilibrium the electric field inside the material of a conductor is zero, and any excess charge on an isolated conductor resides on its outer surface.
- An isolated conducting sphere of outer radius R in air or vacuum has capacitance $C = 4\pi\epsilon_0 R$, whether it is hollow or solid.

Step 1: Locate the excess charge On giving charge to a metallic sphere, the free charges redistribute until the conductor is equipotential. The excess charge remains on the outer surface; filling the interior with metal does not provide a different place for equilibrium excess charge.

Step 2: Compare equal-sized spheres For a hollow sphere with no charge inside its cavity and a solid sphere of the same outer radius, the capacitance is the same. Thus at the same potential V , each carries $Q = CV = 4\pi\epsilon_0 RV$, independently of the amount of metal in the interior.

Step 3: Conclusion No. Whether the metallic sphere is hollow or solid does not affect the charge it can hold at a given potential for the same outer radius, since the excess charge resides on the outer surface.

Quick Tip to Solve: Compare the outer radius and potential of conducting spheres; hollowness alone does not change their capacitance.

Section B (10 marks)

6. Define the distance of closest approach. An α -particle of kinetic energy ' K ' is bombarded on a thin gold foil. The distance of the closest approach is ' r '. What will be the distance of closest approach for an α -particle of double the kinetic energy? [2 marks]

Solution:

Concept:

- The distance of closest approach is the minimum separation between an incident α -particle and the centre of the target nucleus in a head-on collision, where the particle momentarily comes to rest before returning.
- For Coulomb repulsion, conservation of energy converts the initial kinetic energy into electrostatic potential energy at the turning point: $K = k(2e)(Ze)/r$, where $k = 1/(4\pi\epsilon_0)$ and Ze is the nuclear charge.

Step 1: Express the turning-point condition Take the electrostatic potential energy to be zero far from the nucleus. As the positively charged α -particle approaches the positively charged gold nucleus head-on, repulsion reduces its kinetic energy. At separation r , its kinetic energy is zero, so

$$K = \frac{2kZe^2}{r}, \quad r = \frac{2kZe^2}{K}.$$

This is the standard heavy-nucleus approximation, with the nucleus treated as stationary.

Step 2: Double the incident kinetic energy The nuclear charge and the charge of the projectile remain unchanged. If the new closest distance is r' , energy conservation gives

$$2K = \frac{2kZe^2}{r'}, \quad \frac{r'}{r} = \frac{K}{2K} = \frac{1}{2}.$$

A more energetic particle therefore approaches more closely before its energy is completely stored as electric potential energy.

Step 3: Conclusion The closest-approach distance is the minimum centre-to-centre separation in a head-on encounter. For double the kinetic energy, the new distance is $r' = r/2$.

Quick Tip to Solve: For the same projectile and nucleus in a head-on encounter, use $rK = \text{constant}$: doubling the energy halves the closest distance.

OR

6. Write two important limitations of Rutherford nuclear model of the atom. [2 marks]

Solution:

Concept:

- Rutherford's model has electrons revolving around a small, positively charged nucleus under electrostatic attraction.

Quick Tip to Solve: Recall two observations: atoms persist and spectra have separate lines. State how continuous radiation in Rutherford's model contradicts each.

- According to classical electrodynamics, an accelerating charge radiates electromagnetic energy; an electron in a circular orbit has centripetal acceleration even at constant speed.

Step 1: Examine the stability of the atom A revolving electron must continuously radiate energy. As its mechanical energy decreases, its orbit should shrink and it should spiral into the nucleus. Thus Rutherford's classical model predicts collapse of the atom in a very short time, whereas ordinary atoms are stable. It gives no mechanism that prevents this loss of energy.

Step 2: Examine the atomic line spectrum As the electron spirals inward, its orbital frequency changes continuously. The classical model would therefore allow radiation over a continuous range of frequencies. It cannot explain why atoms emit or absorb only distinct spectral lines of definite frequencies, because it does not specify discrete allowed electron energies.

Step 3: Conclusion The two important limitations are failure to explain **atomic stability** and failure to explain **discrete atomic line spectra**.

Quick Tip to Solve: Recall two observations: atoms persist and spectra have separate lines. State how continuous radiation in Rutherford's model contradicts each.

7. When are two objects just resolved? Explain. How can the resolving power of a compound microscope be increased? Use relevant formula to support your answer. [2 marks]

Solution:

Concept:

- Diffraction makes the image of a point object a central bright disc surrounded by rings, rather than a geometrical point.
- For a microscope, the minimum resolvable separation is $d_{\min} = 0.61\lambda/(\mu \sin \theta)$, where λ is the vacuum wavelength, μ the medium's refractive index, and θ the half-angle subtended by the objective at the object.

Step 1: Apply the criterion for just resolution Two nearby point objects are just resolved when the central maximum of the diffraction pattern of one coincides with the first minimum of the other. This is Rayleigh's criterion. Their diffraction patterns overlap, but a small dip between their intensity maxima permits them to be distinguished as two separate objects.

Step 2: Relate resolving power to wavelength and aperture The resolving power is the reciprocal of the smallest distinguishable separation:

$$\text{RP} = \frac{1}{d_{\min}} = \frac{\mu \sin \theta}{0.61\lambda} = \frac{2\mu \sin \theta}{1.22\lambda}.$$

Hence it increases by using shorter-wavelength illumination and by increasing the numerical aperture $\mu \sin \theta$. A larger objective angular aperture increases $\sin \theta$; an immersion medium of higher refractive index increases μ .

Step 3: Conclusion Just resolution occurs at Rayleigh's limit. A compound microscope's resolving power increases with numerical aperture and decreases with wavelength, so shorter-wavelength light and a larger numerical aperture improve resolution.

Quick Tip to Solve: Use $\text{RP} \propto \mu \sin \theta / \lambda$: increase the numerator or decrease the wavelength. Extra magnification alone does not separate unresolved details.

8. (i) What is the line of sight communication?

(ii) Why is it not possible to use sky waves for transmission of TV signals? Upto what distance can a signal be transmitted using an antenna of height ' h '? [2 marks]

Solution:

Concept:

- In line-of-sight communication, a radio wave travels directly from the transmitting antenna to the receiving antenna along an unobstructed path.
- Sky-wave communication requires the ionosphere to return the wave towards Earth. The geometric horizon limits direct transmission around Earth's curved surface.

Step 1: Explain line-of-sight communication and TV transmission The transmitting and receiving antennas must be within each other's radio line of sight so that the signal can reach the receiver directly. Television signals use carrier frequencies much higher than the range effectively returned by the ionosphere (typically above 40 MHz). Such waves penetrate the ionosphere and escape into space instead of returning to Earth, so sky-wave propagation cannot be used for TV transmission.

Step 2: Find the range of the antenna Let R be Earth's radius and h the antenna height. At the limiting visible point on Earth's surface, the signal path is tangent to Earth. The right triangle formed with Earth's centre gives the horizon distance d :

$$d^2 = (R + h)^2 - R^2 = 2Rh + h^2.$$

Since an ordinary antenna has $h \ll R$, the h^2 term is negligible and $d \simeq \sqrt{2Rh}$. This is the range to a receiver effectively at ground level; if its height is significant, its own horizon distance is added.

Step 3: Conclusion Line-of-sight communication uses a direct unobstructed antenna-to-antenna path. TV frequencies are not returned by the ionosphere. An antenna of height h has a horizon range $d \simeq \sqrt{2Rh}$, where R is Earth's radius.

Quick Tip to Solve: Use Earth's radius and antenna height in the same length unit before taking $\sqrt{2Rh}$; quadrupling antenna height only doubles its horizon range.

9. An α -particle and a proton are accelerated through the same potential difference. Find the ratio of their de Broglie wavelengths. [2 marks]

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h_P/p$, where h_P is Planck's constant and p is the particle's momentum.
- A particle of charge q , accelerated from rest through potential difference V , gains kinetic energy $K = qV$. For non-relativistic motion, $p = \sqrt{2mK}$, so $\lambda = h_P/\sqrt{2mqV}$.

Step 1: Identify the masses and charges For a proton, the mass is m_p and the charge is $+e$. An

Quick Tip to Solve: For equal accelerating voltage, compare mq , since $\lambda \propto 1/\sqrt{mq}$. Include both the alpha particle's fourfold mass and double charge.

alpha particle is a helium nucleus, so its mass is approximately $4m_p$ and its charge is $+2e$. Although both particles cross the same potential difference, their kinetic energies are different: $K_p = eV$ and $K_\alpha = 2eV$.

Step 2: Form the wavelength ratio Use the acceleration formula for each particle. Planck's constant, the common voltage and the factor of two cancel in the ratio:

$$\begin{aligned}\frac{\lambda_\alpha}{\lambda_p} &= \sqrt{\frac{m_p e}{m_\alpha q_\alpha}} \\ &= \sqrt{\frac{m_p e}{(4m_p)(2e)}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}.\end{aligned}$$

The heavier, doubly charged alpha particle acquires the larger momentum and therefore has the shorter wavelength.

Step 3: Conclusion In the order alpha particle to proton, the ratio is $\lambda_\alpha : \lambda_p = 1 : 2\sqrt{2}$.

Quick Tip to Solve: For equal accelerating voltage, compare mq , since $\lambda \propto 1/\sqrt{mq}$. Include both the alpha particle's fourfold mass and double charge.

10. How does Ampere-Maxwell law explain the flow of current through a capacitor when it is being charged by a battery? Write the expression for the displacement current in terms of the rate of change of electric flux. [2 marks]

Solution:

Concept:

- A time-varying electric flux produces a magnetic field. Maxwell's displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, where $\Phi_E = \int \vec{E} \cdot d\vec{A}$ is the electric flux.
- Ampere-Maxwell law is $\oint \vec{B} \cdot d\vec{l} = \mu_0(I_c + I_d)$, where I_c is the conduction current through the chosen surface and I_d is its displacement current.

Step 1: Identify the currents while charging The battery drives conduction current through the connecting wires, depositing equal and opposite charges on the capacitor plates. No conduction current crosses the insulating gap. However, the increasing plate charge produces a changing electric field, and hence a changing electric flux, between the plates. This gives a displacement current in the gap.

Step 2: Apply Ampere-Maxwell law consistently For an ideal parallel-plate capacitor with air or vacuum between its plates, neglecting edge effects, $E = Q/(\epsilon_0 A)$. Therefore

$$\Phi_E = EA = \frac{Q}{\epsilon_0}, \quad I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} = I_c.$$

A surface cutting the wire includes conduction current; one passing through the capacitor gap includes the equal displacement current. Both give the same circulation of the magnetic field. Thus the generalized current remains continuous during charging even though charge does not cross the dielectric gap.

Quick Tip to Solve: Distinguish charge transport in the wires from changing electric flux in the capacitor gap; equate their currents using $\Phi_E = Q/\epsilon_0$.

Step 3: Conclusion The changing electric field between the charging capacitor's plates supplies displacement current $I_d = \epsilon_0 d\Phi_E/dt$. Its magnitude equals the conduction current in the wires, resolving the apparent discontinuity in Ampere's law.

Quick Tip to Solve: Distinguish charge transport in the wires from changing electric flux in the capacitor gap; equate their currents using $\Phi_E = Q/\epsilon_0$.

Section C (36 marks)

11. Describe the working principle of a moving coil galvanometer. Why is it necessary to use (i) a radial magnetic field and (ii) a cylindrical soft iron core in a galvanometer? Write the expression for current sensitivity of the galvanometer. Can a galvanometer as such be used for measuring the current? Explain. [3 marks]

Solution:

Concept:

- A current-carrying coil in a magnetic field experiences a torque $\tau_m = NIAB \sin \alpha$, where N is its number of turns, A its area and α the angle between the coil's normal and $\vec{B}$.
- The suspension or spring supplies an opposing restoring torque $\tau_r = k\theta$, where k is the torsional constant and θ is the angular deflection.

Step 1: Explain the principle and equilibrium A light rectangular coil carrying current is placed in the magnetic field of a permanent magnet. Magnetic forces on its opposite sides form a couple and turn the coil. As it rotates, the suspension or spring develops an opposing torque. The pointer settles when the two torques balance. In the radial field used in the galvanometer, $\alpha = 90^\circ$, so

$$NIAB = k\theta, \quad \theta = \frac{NAB}{k}I.$$

Thus the deflection is proportional to the current, and reversing the current reverses the deflection.

Step 2: Explain the radial field and soft iron core (i) In a radial magnetic field, the field remains parallel to the plane of the coil throughout its working range. Its normal therefore remains perpendicular to the field, making the magnetic torque $NIAB$ independent of the coil's angular position. This gives a uniform, linear current scale.

(ii) The cylindrical soft iron core has high magnetic permeability. It strengthens the field and, together with the concave pole pieces, makes it radial. The stronger field increases the torque and the deflection for a given small current, improving sensitivity.

Step 3: State sensitivity and the limitation on current measurement Current sensitivity is the deflection per unit current:

$$S_I = \frac{\theta}{I} = \frac{NAB}{k},$$

with unit rad A^{-1} . An uncalibrated galvanometer mainly indicates the presence and direction of a small current. It cannot as such serve as an ordinary ammeter: its appreciable resistance alters the circuit current, and a large current may damage its delicate coil. To measure a useful current range,

Quick Tip to Solve: In the radial field the coil's plane, rather than its normal, is parallel to the field; use this to set the torque factor to unity.

connect a suitable low-resistance shunt in parallel so that most current bypasses the coil, and calibrate the scale in amperes. A calibrated galvanometer can itself measure small currents within its safe range.

Step 4: Conclusion The galvanometer works by balancing magnetic and restoring torques. A radial field gives $\theta \propto I$; the soft iron core strengthens and shapes that field. Its current sensitivity is $\boxed{NAB/k}$. A shunt and calibration are needed to use it as an ordinary ammeter.

Quick Tip to Solve: In the radial field the coil's plane, rather than its normal, is parallel to the field; use this to set the torque factor to unity.

OR

11. (a) Define the term 'self-inductance' and write its S.I. unit.

(b) Obtain the expression for the mutual inductance of two long co-axial solenoids S_1 and S_2 wound one over the other, each of length L and radii r_1 and r_2 and n_1 and n_2 number of turns per unit length, when a current I is set up in the outer solenoid S_2 . [3 marks]

Solution:

Concept:

- Self-inductance is flux linkage per unit current: $\mathcal{L} = N\Phi/I$ for a linear magnetic medium. The induced emf is $\varepsilon = -\mathcal{L} dI/dt$.
- Mutual inductance relates the flux linkage in one coil to the current in the other: $N_1\Phi_{12} = MI_2$. Inside an ideal long air-core solenoid, $B = \mu_0 nI$, with n turns per unit length.

Step 1: (a) Define self-inductance and its unit The self-inductance of a coil is the magnetic flux linked with all its turns per unit current flowing through that coil. Equivalently, it equals the magnitude of the induced emf when the current changes at a unit rate. Its S.I. unit is the henry, symbol H:

$$1 \text{ H} = 1 \text{ Wb A}^{-1} = 1 \text{ V s A}^{-1}.$$

Thus a coil has self-inductance 1 H if a current changing at 1 A s^{-1} induces an emf of magnitude 1 V. The negative sign in the emf law expresses opposition to the change of current.

Step 2: (b) Find the field produced by the outer solenoid Let S_2 be the outer solenoid carrying the given current I , and let S_1 be the inner solenoid, so $r_1 < r_2$. For long coaxial solenoids in air, neglecting end effects, the field throughout the interior of S_2 is uniform and parallel to the common axis:

$$B_2 = \mu_0 n_2 I.$$

Each turn of the inner solenoid encloses area $A_1 = \pi r_1^2$. Hence the flux through each inner turn due to the outer solenoid is

$$\Phi_{12} = B_2 A_1 = \mu_0 n_2 I \pi r_1^2.$$

Step 3: Use the total flux linkage of the inner solenoid The number of turns in S_1 is $N_1 = n_1 L$, where

Quick Tip to Solve: For nested long solenoids, multiply the turns per unit length of both coils by their common length and the smaller cross-sectional area, then by μ_0 .

L denotes the common physical length, not the self-inductance $\mathcal{L}$. Therefore

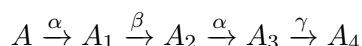
$$N_1 \Phi_{12} = \mu_0 n_1 n_2 L \pi r_1^2 I.$$

By the definition $N_1 \Phi_{12} = MI$, division by the current gives the mutual inductance. The relevant area is that enclosed by the inner turns, even though the magnetic field also occupies the remaining interior of the outer solenoid.

Step 4: Conclusion Self-inductance is $\mathcal{L} = N\Phi/I$, measured in henry. The mutual inductance of the two long air-core coaxial solenoids is $M = \mu_0 n_1 n_2 \pi r_1^2 L$, using the inner solenoid's cross-sectional area.

Quick Tip to Solve: For nested long solenoids, multiply the turns per unit length of both coils by their common length and the smaller cross-sectional area, then by μ_0 .

12. (i) A radioactive nucleus 'A' undergoes a series of decays as given below:



The mass number and atomic number of A_2 are 176 and 71 respectively. Determine the mass and atomic numbers of A_4 and A .

(ii) Write the basic nuclear processes underlying β^+ and β^- decays.

[3 marks]

Solution:

Concept:

- An alpha decay reduces the mass number M by 4 and the atomic number Z by 2: $(M, Z) \rightarrow (M - 4, Z - 2)$.
- In the usual decay-series notation, an unqualified β denotes β^- decay: M is unchanged and Z increases by 1. Gamma emission changes neither number.
- Beta decay converts a neutron into a proton or a proton into a neutron inside the nucleus, with the appropriate electron or positron and neutrino emitted.

Step 1: (i) Follow the chain forward from the known nucleus The given nucleus A_2 has $(M, Z) = (176, 71)$. Its alpha decay produces A_3 , so

$$M_3 = 176 - 4 = 172, \quad Z_3 = 71 - 2 = 69.$$

Gamma emission removes excitation energy without changing either number, so A_4 retains $(172, 69)$.

Step 2: (i) Work backwards to the initial nucleus The printed beta arrow is interpreted as the conventional beta-minus decay. Since $A_1 \rightarrow A_2$ raises the atomic number by 1 without changing mass number,

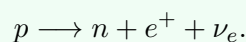
$$M_1 = 176, \quad Z_1 = 71 - 1 = 70.$$

The earlier alpha decay $A \rightarrow A_1$ removed four nucleons, including two protons. Restoring them gives

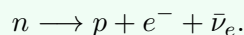
$$M_A = 176 + 4 = 180, \quad Z_A = 70 + 2 = 72.$$

Quick Tip to Solve: Track mass number and atomic number separately; reverse each change when tracing a decay chain backwards, and leave both unchanged at a gamma arrow.

Step 3: (ii) Write the underlying beta-decay processes In beta-plus decay, a proton in the nucleus converts into a neutron, emitting a positron and an electron neutrino:



In beta-minus decay, a neutron converts into a proton, emitting an electron and an electron antineutrino:



Both conserve charge and nucleon number; beta-plus reduces Z by 1 and beta-minus increases it by 1.

Step 4: Conclusion A_4 has mass number 172 and atomic number 69; the initial nucleus A has mass number 180 and atomic number 72. The basic processes are $p \rightarrow n + e^+ + \nu_e$ for β^+ , and $n \rightarrow p + e^- + \bar{\nu}_e$ for β^- .

Quick Tip to Solve: Track mass number and atomic number separately; reverse each change when tracing a decay chain backwards, and leave both unchanged at a gamma arrow.

13. For a CE-transistor amplifier, the audio signal voltage across the collector resistance of $2\text{ k}\Omega$ is 2 V . Given the current amplification factor of the transistor is 100, find the input signal voltage and base current, if the base resistance is $1\text{ k}\Omega$. [3 marks]

Solution:

Concept:

- For small alternating signals in a common-emitter amplifier, the current gain is $\beta = \Delta I_C / \Delta I_B$, where the changes represent collector and base signal-current magnitudes.
- Ohm's law gives the output signal magnitude $V_o = \Delta I_C R_C$ and input signal magnitude $V_i = \Delta I_B R_B$, where R_C and R_B are the given collector and base resistances.

Step 1: Find the collector signal current The audio signal of magnitude 2 V appears across the collector resistance $R_C = 2\text{ k}\Omega = 2000\ \Omega$. Therefore the corresponding signal-current magnitude is

$$\Delta I_C = \frac{V_o}{R_C} = \frac{2}{2000} = 1.0 \times 10^{-3}\text{ A}.$$

This is the alternating collector-current component, rather than the transistor's steady bias current.

Step 2: Use the current gain to obtain base current The transistor multiplies the small base-current variation by $\beta = 100$. Dividing the collector-current variation by this gain gives

$$\Delta I_B = \frac{\Delta I_C}{\beta} = \frac{1.0 \times 10^{-3}}{100} = 1.0 \times 10^{-5}\text{ A} = 10\ \mu\text{A}.$$

The requested base current is this signal component; a DC base current cannot be determined without bias-circuit information.

Step 3: Calculate the input signal voltage and check the gain With the stated base resistance

Quick Tip to Solve: First divide output voltage by collector resistance, then divide that current by beta; multiply the resulting base current by base resistance only at the end.

$R_B = 1 \text{ k}\Omega = 1000 \Omega$, the input signal voltage is

$$V_i = \Delta I_B R_B = (1.0 \times 10^{-5})(1000) = 1.0 \times 10^{-2} \text{ V}.$$

As a check, the magnitude of the voltage gain is

$$|A_v| = \beta \frac{R_C}{R_B} = 100 \frac{2}{1} = 200, \quad \frac{V_o}{|A_v|} = \frac{2}{200} = 0.01 \text{ V}.$$

A common-emitter output at the collector is inverted relative to the input. The problem asks for signal magnitudes, so the calculated voltage and current are stated as positive amplitudes.

Step 4: Conclusion The input signal voltage is $0.01 \text{ V} = 10 \text{ mV}$, and the base signal current is $10 \mu\text{A}$.

Quick Tip to Solve: First divide output voltage by collector resistance, then divide that current by beta; multiply the resulting base current by base resistance only at the end.

14. (i) State two important features of Einstein's photoelectric equation.

(ii) Radiation of frequency 10^{15} Hz is incident on two photosensitive surfaces P and Q. There is no photoemission from surface P. Photoemission occurs from surface Q but photoelectrons have zero kinetic energy. Explain these observations and find the value of work function for surface Q. [3 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where $h\nu$ is the incident photon energy and ϕ is the surface work function, the minimum energy needed to liberate an electron.
- The threshold frequency ν_0 is defined by $\phi = h\nu_0$. Emission requires $\nu \geq \nu_0$; at the threshold, the maximum kinetic energy is zero.

Step 1: (i) State two features of Einstein's equation First, for a given photosensitive surface, the maximum photoelectron kinetic energy increases linearly with incident frequency:

$$K_{\max} = h(\nu - \nu_0).$$

It depends on frequency and is independent of light intensity. Increasing intensity at a fixed frequency above threshold supplies more photons per second and can increase the photoelectric current, but does not increase energy per photon. Second, a definite minimum frequency $\nu_0 = \phi/h$ is required. If $\nu < \nu_0$, the energy of an individual photon is insufficient to overcome the work function, so there is no emission regardless of intensity in the usual photoelectric-effect regime.

Step 2: (ii) Explain the different responses of P and Q Both surfaces receive photons of the same energy because their incident frequency is the same, $\nu = 10^{15} \text{ Hz}$. The absence of emission from P means its work function exceeds that energy:

$$\phi_P > h\nu, \quad \nu_{0P} > 10^{15} \text{ Hz}.$$

For Q, electrons just escape with zero kinetic energy. The entire photon energy is used to overcome

Quick Tip to Solve: Zero photoelectron kinetic energy means the incident frequency equals the threshold: set the work function directly equal to the photon energy, h times frequency.

the work function:

$$0 = h\nu - \phi_Q, \quad \nu_{0Q} = 10^{15} \text{ Hz.}$$

Thus the observations arise from different surface work functions, with $\phi_P > \phi_Q$.

Step 3: Calculate the work function of Q Using $h = 6.63 \times 10^{-34} \text{ J s}$, the threshold relation gives

$$\phi_Q = h\nu = (6.63 \times 10^{-34})(10^{15}) = 6.63 \times 10^{-19} \text{ J.}$$

To express this energy in electron-volts, use $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$:

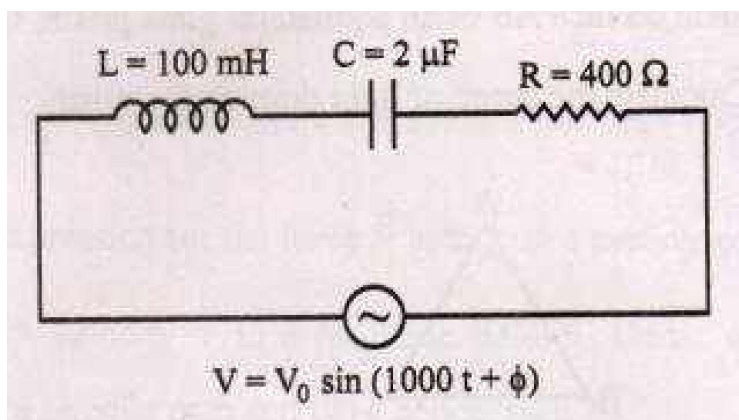
$$\phi_Q = \frac{6.63 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} \simeq 4.14 \text{ eV.}$$

Step 4: Conclusion Einstein's equation predicts a frequency-dependent maximum kinetic energy independent of intensity, and a threshold frequency. P has $\phi_P > h\nu$, whereas Q is exactly at threshold with $\phi_Q = 6.63 \times 10^{-19} \text{ J} \simeq 4.14 \text{ eV}$.

Quick Tip to Solve: Zero photoelectron kinetic energy means the incident frequency equals the threshold: set the work function directly equal to the photon energy, h times frequency.

15. (i) Find the value of the phase difference between the current and the voltage in the series LCR circuit shown below. Which one leads in phase: current or voltage?

(ii) Without making any other change, find the value of the additional capacitor C_1 , to be connected in parallel with the capacitor C , in order to make the power factor of the circuit unity. [3 marks]



Solution:

Concept:

- In a series LCR circuit, $X_L = \omega L$ and $X_C = 1/(\omega C)$, where ω is the angular frequency. The signed phase of voltage relative to current obeys $\tan \delta = (X_L - X_C)/R$.
- The power factor is $\cos \delta = R/Z$. It is unity at resonance, when $X_L = X_C$. Capacitors connected in parallel have equivalent capacitance $C_{eq} = C + C_1$.

Step 1: (i) Read the angular frequency and calculate the reactances The source is $V = V_0 \sin(1000t + \phi)$

Quick Tip to Solve: Read ω directly from the coefficient of time, and subtract the existing capacitance only after finding the resonant total capacitance.

ϕ), so $\omega = 1000 \text{ rad s}^{-1}$; this is angular frequency, not frequency in hertz. From the circuit, $L = 100 \text{ mH} = 0.1 \text{ H}$, $C = 2 \mu\text{F}$, and $R = 400 \Omega$. Hence

$$X_L = 1000(0.1) = 100 \Omega, \quad X_C = \frac{1}{1000(2 \times 10^{-6})} = 500 \Omega.$$

Step 2: Determine the phase relation Taking current as the phase reference,

$$\tan \delta = \frac{100 - 500}{400} = -1, \quad \delta = -\frac{\pi}{4}.$$

The negative sign means voltage lags current: the larger capacitive reactance makes the circuit net capacitive. The phase difference has magnitude $\pi/4$, and the current leads the applied voltage.

Step 3: (ii) Impose the condition for unity power factor With L and ω unchanged, the new capacitance must satisfy

$$\omega L = \frac{1}{\omega(C + C_1)}, \quad C + C_1 = \frac{1}{\omega^2 L} = \frac{1}{(1000)^2(0.1)} = 10 \mu\text{F}.$$

Since the added capacitor is in parallel, its capacitance is the difference between this required total and the original $2 \mu\text{F}$.

Step 4: Conclusion (i) The phase difference is $\pi/4 = 45^\circ$, with current leading voltage. (ii) The additional parallel capacitor is $C_1 = 10 - 2 = 8 \mu\text{F}$.

Quick Tip to Solve: Read ω directly from the coefficient of time, and subtract the existing capacitance only after finding the resonant total capacitance.

16. (i) Obtain the expression for the torque $\vec{\tau}$ experienced by an electric dipole of dipole moment $\vec{p}$ in a uniform electric field, $\vec{E}$.

(ii) What will happen if the field were not uniform?

[3 marks]

Solution:

Concept:

- An electric dipole consists of charges $-q$ and $+q$ separated by distance $2a$. Its moment is $\vec{p} = q(2\vec{a})$, directed from the negative charge to the positive charge.
- A charge experiences force $\vec{F} = q\vec{E}$. Two equal and opposite forces with different lines of action form a couple; its torque is one force multiplied by the perpendicular separation of their lines of action.

Step 1: (i) Identify the two forces in a uniform field Let the dipole axis make angle θ with the uniform electric field $\vec{E}$. The force on $+q$ is $+q\vec{E}$, and that on $-q$ is $-q\vec{E}$. The forces are equal and opposite, so their vector sum is zero. They have different lines of action when the dipole is inclined to the field, and therefore exert a turning effect without producing net translational acceleration.

Step 2: Calculate the couple and give its vector form The perpendicular separation of the parallel

Quick Tip to Solve: Use the perpendicular distance $2a \sin \theta$ between the force lines, not simply the charge separation $2a$, when calculating the couple.

force lines is $2a \sin \theta$. Thus the torque magnitude is

$$\tau = (qE)(2a \sin \theta) = pE \sin \theta.$$

Equivalently, each force contributes $qEa \sin \theta$ about the midpoint, with both contributions in the same rotational sense. By the right-hand rule the torque direction is perpendicular to the plane of $\vec{p}$ and $\vec{E}$, giving

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

The torque tends to align the dipole moment with the field. It vanishes for parallel and antiparallel orientations, with the parallel orientation stable and the antiparallel orientation unstable.

Step 3: (ii) Consider a non-uniform electric field In a non-uniform field the field vectors at the two charges need not be equal. Consequently,

$$\vec{F}_{\text{net}} = q [\vec{E}(\vec{r}_+) - \vec{E}(\vec{r}_-)]$$

is generally nonzero. The dipole can therefore undergo translation as well as rotation. For a sufficiently small dipole, the torque about its centre is approximately $\vec{p} \times \vec{E}$, with the field evaluated at the centre. A net force or torque can vanish in special configurations; non-uniformity does not require both to be nonzero for every orientation.

Step 4: Conclusion In a uniform field, $\vec{\tau} = \vec{p} \times \vec{E}$ and $\tau = pE \sin \theta$, while the net force is zero. A non-uniform field generally produces a net force as well as a torque, allowing translation and rotation.

Quick Tip to Solve: Use the perpendicular distance $2a \sin \theta$ between the force lines, not simply the charge separation $2a$, when calculating the couple.

17. Explain briefly with the help of necessary diagrams, the forward and the reverse biasing of a p-n junction diode. Also draw their characteristic curves in the two cases. [3 marks]

Solution:

Concept:

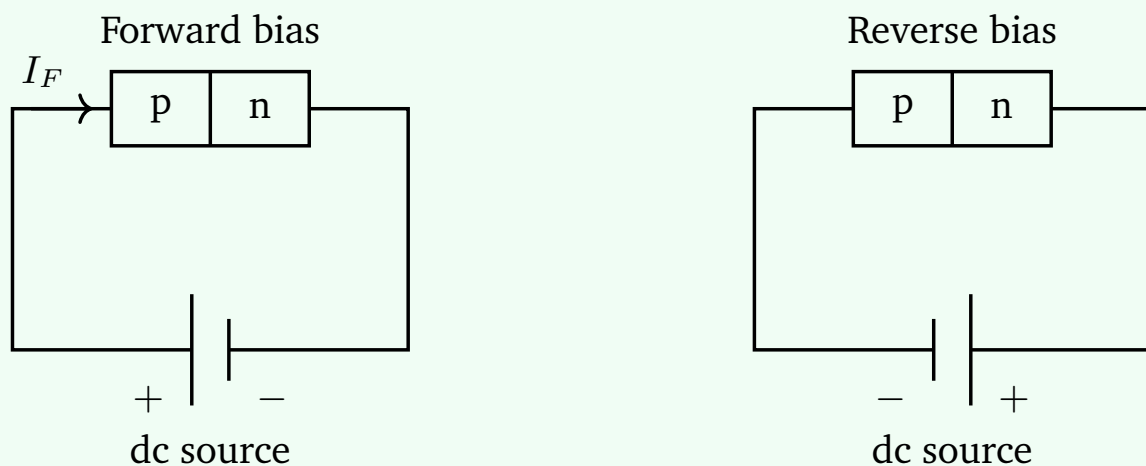
- A p-n junction has a depletion region and a built-in potential barrier. An applied voltage changes this barrier and hence the current through the junction.
- Forward current is mainly due to majority carriers; reverse current before breakdown is mainly due to thermally generated minority carriers. The characteristic curve plots diode current against diode voltage.

Step 1: Forward bias: connect p to positive and n to negative Connect the p-side to the positive

Quick Tip to Solve: Remember p-to-positive for forward bias; then draw the forward rise in the first quadrant and the reverse leakage in the third quadrant.

terminal and the n-side to the negative terminal of a dc source, as shown on the left. The applied field opposes the junction field, lowering the potential barrier and narrowing the depletion region. Majority carriers can cross the junction more easily: holes move from p to n and electrons from n to p. Thus a substantial conventional current flows through the junction from p to n once the applied voltage is sufficiently large.

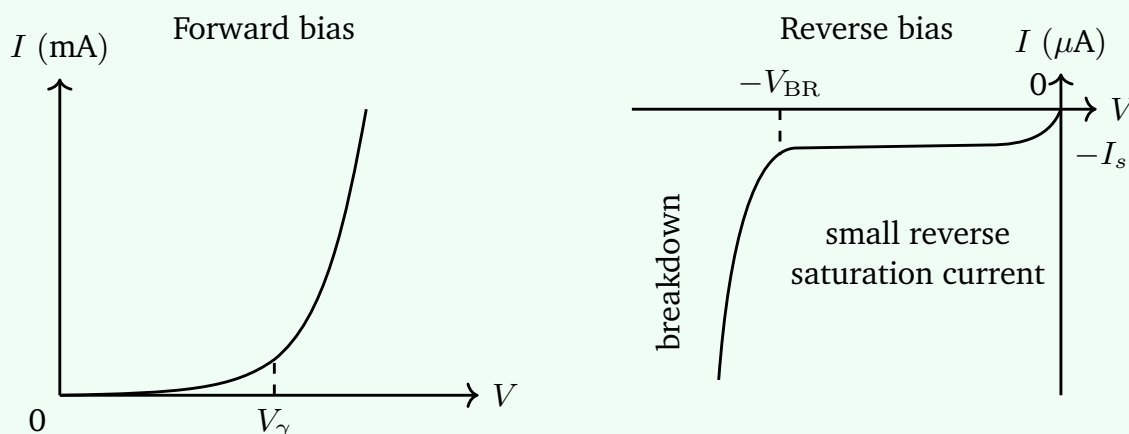
Step 2: Reverse bias: connect p to negative and n to positive Reverse the source terminals as shown on the right. The applied field reinforces the junction field; the potential barrier increases and the depletion region widens. Majority carriers move away from the junction, so their current is strongly suppressed. A small reverse current due to minority carriers still flows, opposite to the forward conventional-current direction. Before breakdown it is almost independent of the reverse voltage at a fixed temperature.



Bias connections of a p-n junction; the longer battery plate is positive.

Step 3: Compare the current-voltage characteristics In forward bias, current is initially small and then rises rapidly beyond the knee or cut-in voltage V_γ , approximately 0.7 V for silicon or 0.3 V for germanium. In reverse bias the small reverse saturation current I_s persists until the breakdown voltage magnitude V_{BR} is reached, after which reverse current increases sharply. In the graphs, forward current and voltage are positive; reverse current and voltage are negative. The two current scales differ because ordinary forward current is much larger than reverse saturation current. The curves are schematic.

Quick Tip to Solve: Remember p-to-positive for forward bias; then draw the forward rise in the first quadrant and the reverse leakage in the third quadrant.



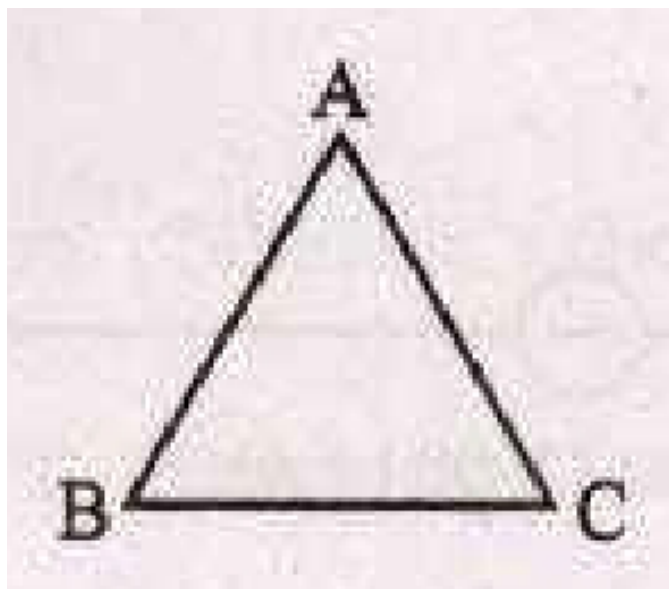
Schematic forward and reverse characteristics, with separate current scales.

Step 4: Conclusion Forward bias lowers the junction barrier and permits a large majority-carrier current; reverse bias raises the barrier and permits only a small minority-carrier current before breakdown. The corresponding circuits and characteristic curves are shown.

Quick Tip to Solve: Remember p-to-positive for forward bias; then draw the forward rise in the first quadrant and the reverse leakage in the third quadrant.

18. (i) A ray of light incident on face AB of an equilateral glass prism, shows minimum deviation of 30° . Calculate the speed of light through the prism.

(ii) Find the angle of incidence at face AB so that the emergent ray grazes along the face AC. [3 marks]



Solution:**Concept:**

- At minimum deviation, $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$, where A is the prism angle and δ_m is the minimum deviation. Light travels in glass at speed $v = c/\mu$.
- Inside the prism, $r_1 + r_2 = A$. For grazing emergence into air, $e = 90^\circ$ and the internal incidence equals the critical angle C , with $\sin C = 1/\mu$. Angles are measured from the normal.

Step 1: (i) Find the refractive index and speed An equilateral prism has $A = 60^\circ$. Using the given minimum deviation $\delta_m = 30^\circ$,

$$\mu = \frac{\sin[(60^\circ + 30^\circ)/2]}{\sin(60^\circ/2)} = \frac{\sin 45^\circ}{\sin 30^\circ} = \sqrt{2}.$$

Taking $c = 3.0 \times 10^8 \text{ m s}^{-1}$,

$$v = \frac{c}{\mu} = \frac{3.0 \times 10^8}{\sqrt{2}} = 2.12 \times 10^8 \text{ m s}^{-1}.$$

Step 2: (ii) Impose grazing emergence at AC At face AC, Snell's law gives $\mu \sin r_2 = \sin 90^\circ = 1$, so $r_2 = C = 45^\circ$. Thus the refraction angle at AB is $r_1 = A - r_2 = 60^\circ - 45^\circ = 15^\circ$. Applying Snell's law at entry from air into glass,

$$\sin i = \mu \sin r_1 = \sqrt{2} \sin 15^\circ = \frac{\sqrt{3} - 1}{2} = 0.3660.$$

Hence $i = \sin^{-1}(0.3660) = 21.47^\circ$. This is a different ray from the minimum-deviation ray, but the refractive index is unchanged.

Step 3: Conclusion (i) $v = 2.12 \times 10^8 \text{ m s}^{-1}$. (ii) The required incidence at AB is $i \approx 21.5^\circ$ from the normal.

Quick Tip to Solve: For grazing emergence, set the internal exit angle to the critical angle and use $r_1 = A - C$ before applying Snell's law at entry.

19. Define the term 'amplitude modulation'. Explain any two factors which justify the need for modulating a low frequency base-band signal. [3 marks]

Solution:**Concept:**

- Modulation places a low-frequency message on a high-frequency carrier so that it can be transmitted effectively through a communication channel.
- Wavelength and frequency are related by $\lambda = c/f$, where c is the wave speed. A practical transmitting antenna commonly has length of the order of $\lambda/4$.
- For a fixed short transmitting antenna, radiation efficiency increases when the transmitted wavelength decreases; the radiated power scales as $(l/\lambda)^2$ under otherwise comparable conditions.

Quick Tip to Solve: For each reason, connect low frequency to large wavelength, then state how a high-frequency carrier removes the resulting difficulty.

Step 1: Define amplitude modulation Amplitude modulation is the process in which the amplitude of a high-frequency carrier wave is varied in accordance with the instantaneous value of the low-frequency message signal, while the carrier frequency and phase remain unchanged. For a sinusoidal message, the modulated wave can be written as

$$s(t) = A_c[1 + m \cos(2\pi f_m t)] \cos(2\pi f_c t),$$

where A_c is the unmodulated carrier amplitude, m is the modulation index, and $f_c \gg f_m$. The carrier's envelope carries the message information.

Step 2: Factor 1: reduce the required antenna size A base-band audio signal has a very long wavelength. For example, at $f = 1$ kHz,

$$\lambda = \frac{3 \times 10^8}{10^3} = 3 \times 10^5 \text{ m}, \quad \frac{\lambda}{4} = 75 \text{ km}.$$

Such an antenna is impractical. If the audio signal modulates a 1 MHz carrier, the carrier wavelength is 300 m, and the corresponding quarter-wave length is only 75 m. Modulation therefore permits manageable antenna dimensions.

Step 3: Factor 2: improve radiation of the signal A practical antenna is extremely short compared with the wavelength of a low-frequency base-band signal, so it radiates that signal very inefficiently. For a fixed short antenna of length l , the radiated power is proportional to $(l/\lambda)^2$ under comparable excitation. Using a high-frequency carrier reduces λ , increasing the radiation efficiency and the power effectively sent out as electromagnetic waves. Thus the message can be transmitted over useful distances.

Step 4: Conclusion Amplitude modulation varies a carrier's amplitude according to the message. Two reasons for modulation are practical antenna size and efficient radiation of power.

Quick Tip to Solve: For each reason, connect low frequency to large wavelength, then state how a high-frequency carrier removes the resulting difficulty.

20. Two identical capacitors of 12 pF each are connected in series across a battery of 50 V. How much electrostatic energy is stored in the combination? If these were connected in parallel across the same battery, how much energy will be stored in the combination now?

Also find the charge drawn from the battery in each case.

[3 marks]

Solution:

Concept:

- For two capacitors in series, $1/C_s = 1/C_1 + 1/C_2$; for a parallel combination, $C_p = C_1 + C_2$.
- An equivalent capacitance C_{eq} connected across a battery of voltage V stores energy $U = \frac{1}{2}C_{eq}V^2$ and draws charge $Q = C_{eq}V$, starting from an uncharged state.
- Series capacitors carry equal charge, whereas parallel capacitors have the same potential difference. The terminal charge of the combination determines the charge supplied by the battery.

Quick Tip to Solve: With the battery voltage fixed, calculate the equivalent capacitance first; both stored energy and battery charge are proportional to it.

Step 1: Find the series capacitance and energy Each capacitor has $C = 12 \text{ pF} = 12 \times 10^{-12} \text{ F}$, and the battery voltage is $V = 50 \text{ V}$. For identical capacitors in series,

$$C_s = \frac{C}{2} = 6 \times 10^{-12} \text{ F}.$$

Therefore the total stored electrostatic energy is

$$U_s = \frac{1}{2} C_s V^2 = \frac{1}{2} (6 \times 10^{-12}) (50)^2 = 7.5 \times 10^{-9} \text{ J}.$$

Step 2: Find the charge drawn in series The charge delivered through the battery to the series combination is

$$Q_s = C_s V = (6 \times 10^{-12}) (50) = 3.0 \times 10^{-10} \text{ C}.$$

This is also the magnitude of the charge on each capacitor. It is not doubled: the same charge passes through the battery's external circuit while equal and opposite charges are induced on the connected inner plates. Each identical capacitor has a potential difference of 25 V.

Step 3: Find the parallel energy and charge In parallel, both capacitors are connected directly across 50 V, so

$$C_p = 2C = 24 \times 10^{-12} \text{ F}.$$

The stored energy and the charge supplied by the battery are respectively

$$U_p = \frac{1}{2} C_p V^2 = \frac{1}{2} (24 \times 10^{-12}) (50)^2 = 3.0 \times 10^{-8} \text{ J},$$

$$Q_p = C_p V = (24 \times 10^{-12}) (50) = 1.2 \times 10^{-9} \text{ C}.$$

Each parallel branch draws $6.0 \times 10^{-10} \text{ C}$; their sum gives the total charge drawn. At the same battery voltage, both total charge and stored energy are four times their series values.

Step 4: Conclusion Series: $U_s = 7.5 \times 10^{-9} \text{ J}$ and $Q_s = 3.0 \times 10^{-10} \text{ C}$.

Parallel: $U_p = 3.0 \times 10^{-8} \text{ J}$ and $Q_p = 1.2 \times 10^{-9} \text{ C}$.

Quick Tip to Solve: With the battery voltage fixed, calculate the equivalent capacitance first; both stored energy and battery charge are proportional to it.

21. (a) Write the expression for the force $\vec{F}$ acting on a particle of mass m and charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$. Under what conditions will it move in (i) a circular path and (ii) a helical path?

(b) Show that the kinetic energy of the particle moving in magnetic field remains constant. [3 marks]

Solution:

Concept:

- The magnetic force on charge q with velocity $\vec{v}$ in magnetic field $\vec{B}$ is $\vec{F} = q(\vec{v} \times \vec{B})$. Its magnitude is $|q|vB \sin \theta$, where θ is the angle between velocity and field.

Quick Tip to Solve: Resolve velocity along and across the field first: the parallel component sets the pitch, the perpendicular component sets the radius, and neither changes the kinetic energy.

- Resolve velocity into $v_{\parallel} = v \cos \theta$ along the field and $v_{\perp} = v \sin \theta$ perpendicular to it. In a uniform field, only the perpendicular component experiences magnetic deflection.
- The rate of change of kinetic energy is the power $dK/dt = \vec{F} \cdot \vec{v}$; a force perpendicular to velocity does no work.

Step 1: (a) State the force and circular-motion condition For a charged particle moving under the magnetic force alone,

$$\vec{F} = q(\vec{v} \times \vec{B}).$$

In a uniform, steady magnetic field, if the initial velocity is perpendicular to $\vec{B}$, the force is always perpendicular to the velocity and has constant magnitude. It supplies the centripetal force:

$$|q|vB = \frac{mv^2}{r}, \quad r = \frac{mv}{|q|B}.$$

Thus the path is a circle in a plane perpendicular to the field. The sign of the charge reverses the sense of rotation, but not the radius.

Step 2: (a) State the helical-motion condition A helix results when both $v_{\parallel}$ and $v_{\perp}$ are nonzero in a uniform, steady field. The perpendicular component produces circular motion, while the parallel component stays constant because the magnetic force has no component along the field. Their superposition produces a helix whose axis is parallel to $\vec{B}$:

$$r = \frac{mv_{\perp}}{|q|B}, \quad T = \frac{2\pi m}{|q|B}, \quad p = |v_{\parallel}|T.$$

Here T is the period and p is the pitch magnitude. If the velocity is exactly parallel or antiparallel to the field, the magnetic force vanishes and the path is straight instead.

Step 3: (b) Show that kinetic energy is constant Since the cross product $\vec{v} \times \vec{B}$ is perpendicular to $\vec{v}$, the instantaneous power supplied by the magnetic force is

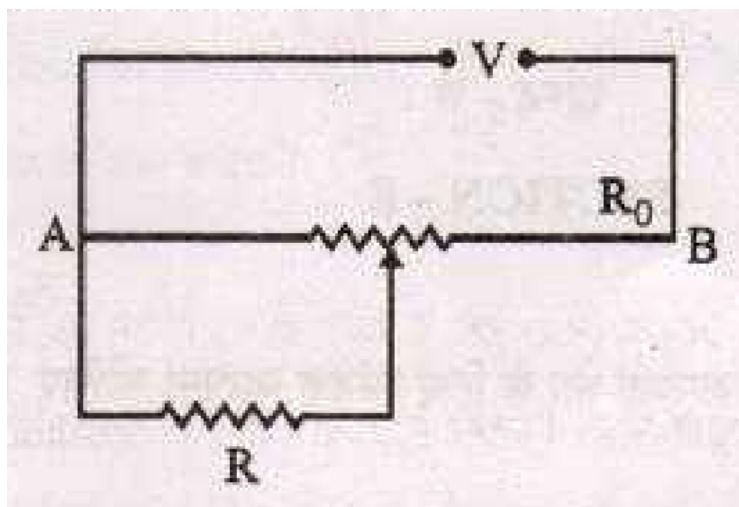
$$\frac{dK}{dt} = \vec{F} \cdot \vec{v} = q(\vec{v} \times \vec{B}) \cdot \vec{v} = 0.$$

Equivalently, the work in any infinitesimal displacement $d\vec{r} = \vec{v} dt$ is zero. Hence a magnetic force can change the direction of motion but cannot change the speed or kinetic energy of the particle.

Step 4: Conclusion $\vec{F} = q(\vec{v} \times \vec{B})$. In a uniform steady field, motion is circular for $\vec{v} \perp \vec{B}$, and helical when both parallel and perpendicular velocity components are nonzero. Under magnetic force alone, $K = \frac{1}{2}mv^2$ remains constant because $\vec{F} \cdot \vec{v} = 0$.

Quick Tip to Solve: Resolve velocity along and across the field first: the parallel component sets the pitch, the perpendicular component sets the radius, and neither changes the kinetic energy.

22. A resistance of R draws current from a potentiometer. The potentiometer wire, AB , has a total resistance of R_0 . A voltage V is supplied to the potentiometer. Derive an expression for the voltage across R when the sliding contact is in the middle of potentiometer wire. [3 marks]



Solution:

Concept:

- A uniform potentiometer wire has resistance proportional to its length. If its total resistance is R_0 , each half has resistance $R_0/2$.
- Resistors connected between the same two nodes are in parallel and have the same potential difference. Their equivalent resistance is $R_p = R_1 R_2 / (R_1 + R_2)$.
- For two series equivalent resistances supplied by voltage V , the voltage across one is its fraction of the total resistance multiplied by V .

Step 1: Identify the loaded half of the wire Call the sliding contact C . Since it is at the midpoint of uniform wire AB ,

$$R_{AC} = R_{CB} = \frac{R_0}{2}.$$

In the printed circuit the external resistance R is connected between A and C , so it is in parallel with the first half of the wire. Current through the second half CB is the total source current; at C it splits between the first half and the load. The load therefore changes the voltage division and the output cannot simply be taken as $V/2$.

Step 2: Replace the parallel combination by its equivalent The equivalent resistance between A and C is

$$R_p = \frac{R(R_0/2)}{R + R_0/2} = \frac{RR_0}{2R + R_0}.$$

This combination is in series with $R_0/2$, the resistance of the other half. Thus the resistance across the

Quick Tip to Solve: First place the load in parallel with the half-wire it bridges, then apply the voltage-divider rule to the resulting two series resistances.

applied supply is

$$\begin{aligned} R_{\text{eq}} &= R_p + \frac{R_0}{2} \\ &= \frac{RR_0}{2R + R_0} + \frac{R_0}{2} \\ &= \frac{R_0(4R + R_0)}{2(2R + R_0)}. \end{aligned}$$

Step 3: Apply voltage division to the loaded half The voltage V_R across the external resistance equals the voltage across R_p , because their terminals are the same two nodes. Using the total source current $I = V/R_{\text{eq}}$,

$$\begin{aligned} V_R &= IR_p = V \frac{R_p}{R_{\text{eq}}} \\ &= V \frac{RR_0}{2R + R_0} \frac{2(2R + R_0)}{R_0(4R + R_0)} \\ &= \frac{2VR}{4R + R_0}. \end{aligned}$$

As a consistency check, when the load resistance becomes very large, the load draws negligible current and this expression tends to $V/2$. For finite positive R , it is below $V/2$, as expected from loading the first half of the divider.

Step 4: Conclusion With the sliding contact at the midpoint, the voltage across the load resistance is

$$V_R = \frac{2VR}{4R + R_0}.$$

Quick Tip to Solve: First place the load in parallel with the half-wire it bridges, then apply the voltage-divider rule to the resulting two series resistances.

Section D (4 marks)

23. Mrs. Rashmi Singh broke her reading glasses. When she went to the shopkeeper to order new specs, he suggested that she should get spectacles with plastic lenses instead of glass lenses. On getting the new spectacles, she found that the new ones were thicker than the earlier ones. She asked this question to the shopkeeper but he could not offer satisfactory explanation for this. At home, Mrs. Singh raised the same question to her daughter Anuja who explained why plastic lenses were thicker.

(a) Write two qualities displayed each by Anuja and her mother.

(b) How do you explain this fact using lens maker's formula?

[4 marks]

Solution:

Concept:

- A replacement reading lens must have the same prescribed power $P = 1/f$, where f is its focal length in metres. Reading glasses ordinarily use converging lenses.
- For a thin lens in air, the lens maker's formula is $P = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the refractive

Quick Tip to Solve: Keep the prescribed power fixed in the lens maker's formula: a smaller $\mu - 1$ must be compensated by a larger curvature factor, not by changing the reading prescription.

index of the lens material and R_1, R_2 are the signed radii of its surfaces.

- For the same aperture and edge thickness, stronger curvature makes a convex lens thicker at its centre. Here the ordinary plastic has a lower refractive index than the earlier glass.

Step 1: (a) Identify the qualities shown by each person Anuja shows **scientific understanding** by applying optics to daily life, and **helpfulness** by explaining it to her mother. Mrs. Singh shows **keen observation** by noticing the increased thickness, and **an enquiring attitude** by seeking a satisfactory explanation.

Step 2: (b) Compare the power and refractive indices The replacement spectacles have the same prescribed focal length and power. Denote the glass and plastic refractive indices by μ_g and μ_p . For the materials intended in the question, $\mu_p < \mu_g$. Define the curvature factor $C = 1/R_1 - 1/R_2$. Applying the lens maker's formula separately gives

$$P = (\mu_g - 1)C_g = (\mu_p - 1)C_p.$$

Consequently,

$$\frac{C_p}{C_g} = \frac{\mu_g - 1}{\mu_p - 1} > 1.$$

Plastic therefore needs greater combined curvature for the same power.

Step 3: Relate the greater curvature to the lens thickness For comparable symmetric biconvex lenses, $R_1 = +R$ and $R_2 = -R$, so

$$P = \frac{2(\mu - 1)}{R}, \quad R = 2f(\mu - 1).$$

At fixed f , lower refractive index requires a smaller R . With the same aperture and comparable edge thickness, the more curved surfaces bulge farther at the centre. Refractive index, rather than density, explains this change.

Step 4: Conclusion Anuja shows scientific understanding and helpfulness; her mother shows observation and enquiry. The plastic's lower refractive index requires stronger curvature for equal power, making the reading lenses thicker.

Quick Tip to Solve: Keep the prescribed power fixed in the lens maker's formula: a smaller $\mu - 1$ must be compensated by a larger curvature factor, not by changing the reading prescription.

Section E (15 marks)

24. (i) Derive an expression for drift velocity of electrons in a conductor. Hence deduce Ohm's law.
 (ii) A wire whose cross-sectional area is increasing linearly from its one end to the other, is connected across a battery of V volts. Which of the following quantities remain constant in the wire?

- (a) drift speed
- (b) current density
- (c) electric current
- (d) electric field

Justify your answer.

[5 marks]

Solution:

Concept:

- Drift velocity is the average directed velocity acquired by free electrons in an electric field. For an electron of charge $-e$ and mass m , $\vec{F} = -e\vec{E}$, where e is the positive magnitude of electronic charge.
- The relaxation time τ is the mean time between successive collisions. For electron number density n , cross-sectional area A and drift speed v_d , the current magnitude is $I = neAv_d$.
- In a steady current, charge conservation requires equal current through all cross sections of a wire. The local current density is $J = I/A$, and for a uniform material at constant temperature $J = \sigma E$, where σ is conductivity.

Step 1: (i) Average the electron motion between collisions Without an applied field, electrons move randomly and their mean velocity is zero. On applying $\vec{E}$, each electron has acceleration

$$\vec{a} = -\frac{e}{m}\vec{E}.$$

If its velocity just after a collision is $\vec{u}_i$, then after a free-flight time t_i its velocity is $\vec{v}_i = \vec{u}_i + \vec{a}t_i$. Collisions randomise the velocities, so $\langle \vec{u}_i \rangle = 0$. Taking the mean free-flight time as τ ,

$$\vec{v}_d = \langle \vec{v}_i \rangle = -\frac{e\tau}{m}\vec{E}, \quad v_d = \frac{eE\tau}{m}.$$

Thus electrons drift opposite to the electric field; conventional current is in the direction of the field.

Step 2: Deduce Ohm's law for a uniform conductor Consider a conductor of length l , uniform cross-sectional area A , and potential difference V . In time Δt , electrons in a volume $Av_d\Delta t$ cross a section, carrying charge magnitude $\Delta Q = neAv_d\Delta t$. Therefore, using $E = V/l$,

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} = neAv_d = \frac{ne^2\tau A}{m}E \\ &= \frac{ne^2\tau A}{ml}V, \\ V &= IR, \quad R = \frac{ml}{ne^2\tau A} = \rho \frac{l}{A}, \quad \rho = \frac{m}{ne^2\tau}. \end{aligned}$$

At constant temperature and unchanged physical conditions, n and τ are constant, so R is independent

Quick Tip to Solve: In a single wire with varying area, first keep I constant by charge conservation, then use $v_d = I/(neA)$, $J = I/A$ and $E = \rho J$ to compare the thinner and thicker portions.

of V . Hence I is proportional to V , which is Ohm's law.

Step 3: (ii) Apply charge conservation to the tapered wire Once steady current is established, take any two cross sections of the wire. If their currents differed, charge would continuously accumulate between them. Since there is no such accumulation in the steady state, the charge entering per second equals that leaving per second. Thus I has the same value at every cross section even though the area changes. This conclusion follows from continuity of current and does not require a uniform cross section.

Step 4: Examine the other three quantities locally Let x measure distance from the thinner end and let $A(x) = A_0 + kx$, with $k > 0$, describe the stated linear increase of area. For a homogeneous wire at constant temperature,

$$\begin{aligned}v_d(x) &= \frac{I}{neA(x)}, \\J(x) &= \frac{I}{A(x)}, \\E(x) &= \rho J(x) = \frac{\rho I}{A(x)}.\end{aligned}$$

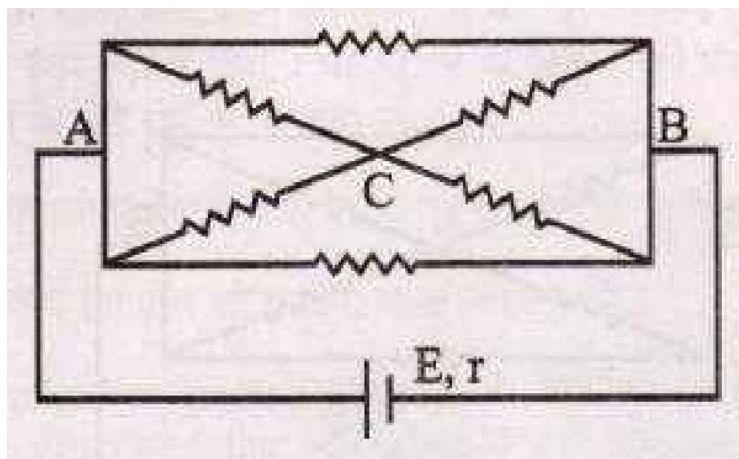
Thus drift speed, current density and electric field magnitude each decrease towards the thicker end. None of (a), (b) and (d) remains constant. A fixed battery voltage specifies the total potential drop along the wire; it does not make the local electric field uniform in a wire of varying area.

Step 5: Conclusion (i) $\vec{v}_d = -\frac{e\tau}{m}\vec{E}$, and $V = IR$ with $R = \frac{ml}{ne^2\tau A}$ for a uniform conductor under constant physical conditions. (ii) Only **(c) electric current** remains constant in the tapered wire; v_d , J and E vary inversely with its local cross-sectional area.

Quick Tip to Solve: In a single wire with varying area, first keep I constant by charge conservation, then use $v_d = I/(neA)$, $J = I/A$ and $E = \rho J$ to compare the thinner and thicker portions.

24. (i) State the two Kirchhoff's laws. Explain briefly how these rules are justified.

(ii) The current is drawn from a cell of emf E and internal resistance r connected to the network of resistors each of resistance r as shown in the figure. Obtain the expression for (i) the current drawn from the cell and (ii) the power consumed in the network. [5 marks]



Solution:

Concept:

- Kirchhoff's junction rule follows conservation of charge: the total current entering a junction equals the total current leaving it in a steady circuit.
- Kirchhoff's loop rule follows conservation of energy: the algebraic sum of potential changes around any closed loop is zero.
- Ideal wires join points of equal potential. For resistors in parallel, reciprocals add; for resistors in series, resistances add.
- A cell of emf E , internal resistance r , and external resistance R supplies current $I = E/(R + r)$. The external network dissipates power $P = I^2 R$.

Step 1: State and justify the junction rule At any junction, the algebraic sum of currents is zero. Taking currents entering the junction as positive and currents leaving it as negative gives

$$\sum I = 0, \quad \sum I_{\text{in}} = \sum I_{\text{out}}.$$

In a short time interval Δt , the entering charge is $(\sum I_{\text{in}})\Delta t$ and the leaving charge is $(\sum I_{\text{out}})\Delta t$. Charge cannot be created or destroyed. In steady state no charge accumulates at the junction, so these amounts are equal. This establishes Kirchhoff's first law from charge conservation.

Step 2: State and justify the loop rule Around any closed loop, the algebraic sum of emfs and resistor potential changes is zero:

$$\sum \Delta V = 0, \quad \sum \mathcal{E} - \sum IR = 0$$

with all terms assigned signs for the chosen traversal. Traversing a resistor in the current direction gives $-IR$; traversing it against the current gives $+IR$. Crossing a cell from its negative to positive terminal gives $+E$, and the reverse crossing gives $-E$. A charge completing a loop returns to its

Quick Tip to Solve: First merge points joined by plain wire: the two A-C resistors form one parallel pair and the two C-B resistors another. Add internal resistance only after reducing the external network.

starting potential, so its net change in electric potential energy is zero. The energy supplied by the cells equals the energy dissipated in the resistances. This is Kirchhoff's second law, justified by energy conservation for the steady circuit.

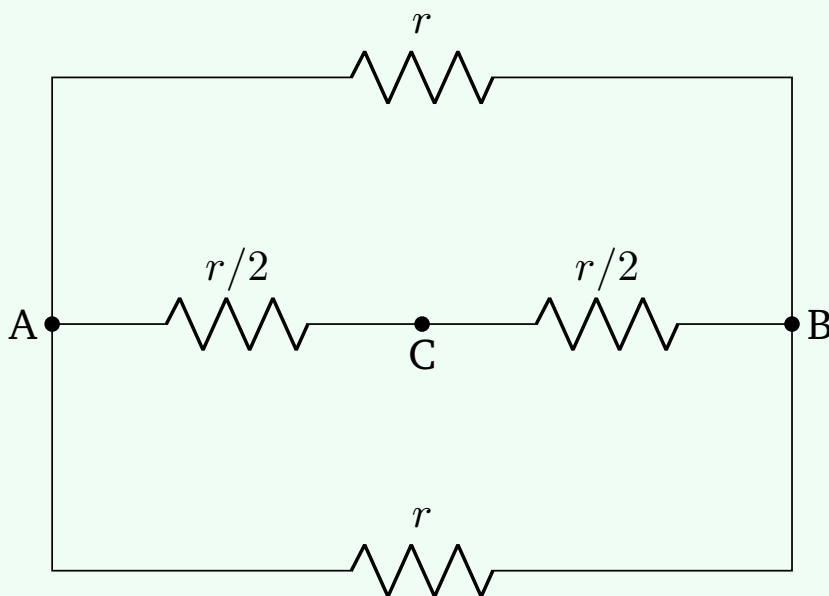
Step 3: Reduce the resistor network between A and B Both left corners are connected by an ideal wire to A, and both right corners are connected to B. Each of the six resistors has resistance r . The upper and lower horizontal resistors each connect directly between A and B. The two resistors joining A to C are parallel, and so are the two joining C to B:

$$R_{AC} = \frac{r r}{r + r} = \frac{r}{2}, \quad R_{CB} = \frac{r}{2}.$$

These two equivalent resistances are in series along the central route, giving $R_{AC} + R_{CB} = r$. Thus the external network is three equal branches of resistance r in parallel:

$$\frac{1}{R_{AB}} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{3}{r}, \quad R_{AB} = \frac{r}{3}.$$

The following electrically equivalent drawing makes the three routes explicit; the two central resistors represent the two parallel pairs in the printed network.



Equivalent external network: three A–B routes, each of resistance r .

Step 4: Include the internal resistance to find the cell current The cell's internal resistance is in series with the entire external network, not in parallel with an individual branch. Applying the loop rule to the cell and the equivalent external resistance gives

$$E - Ir - IR_{AB} = 0.$$

Therefore,

$$R_{\text{total}} = r + \frac{r}{3} = \frac{4r}{3}, \quad I = \frac{E}{4r/3} = \frac{3E}{4r}.$$

Quick Tip to Solve: First merge points joined by plain wire: the two A–C resistors form one parallel pair and the two C–B resistors another. Add internal resistance only after reducing the external network.

The terminal voltage across the network is consequently

$$V_{AB} = E - Ir = E - \frac{3E}{4} = \frac{E}{4}.$$

Step 5: Calculate the power consumed in the external network Use the network resistance $r/3$, because the requested power excludes heating within the cell:

$$P_{\text{network}} = I^2 R_{AB} = \left(\frac{3E}{4r}\right)^2 \frac{r}{3} = \frac{3E^2}{16r}.$$

As a check, the terminal voltage gives the same result:

$$P_{\text{network}} = \frac{V_{AB}^2}{R_{AB}} = \frac{(E/4)^2}{r/3} = \frac{3E^2}{16r}.$$

The total power delivered by the emf, EI , also includes $I^2 r$ dissipated inside the cell; it must not be used as the external-network power.

Step 6: Conclusion Kirchhoff's junction and loop rules express conservation of charge and energy, respectively. The external network has resistance $r/3$. The current drawn from the cell is $I = 3E/(4r)$, and the power consumed in the network is $P = 3E^2/(16r)$. For E in volts and r in ohms, these are in amperes and watts.

Quick Tip to Solve: First merge points joined by plain wire: the two A–C resistors form one parallel pair and the two C–B resistors another. Add internal resistance only after reducing the external network.

25. (a) Draw a labelled diagram of AC generator. Derive the expression for the instantaneous value of the emf induced in the coil.

(b) A circular coil of cross-sectional area 200 cm^2 and 20 turns is rotated about the vertical diameter with angular speed of 50 rad s^{-1} in a uniform magnetic field of magnitude $3.0 \times 10^{-2} \text{ T}$. Calculate the maximum value of the current in the coil. [5 marks]

Solution:

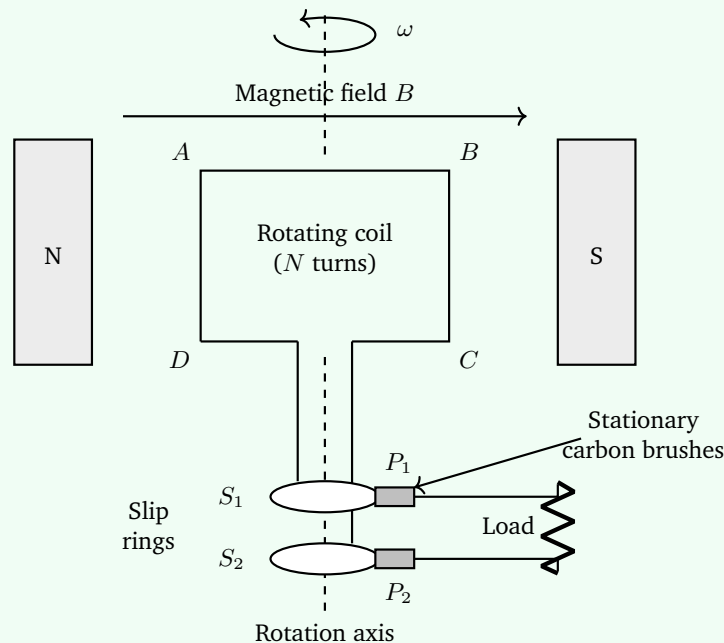
Concept:

- An AC generator converts mechanical energy into electrical energy by electromagnetic induction. Faraday's law gives $e = -d(N\Phi)/dt$, where $N\Phi$ is the flux linkage of the rotating coil.
- For a plane coil of area A in a uniform magnetic field B , $\Phi = BA \cos \theta$, where θ is the angle between the field and the normal to the coil.
- For uniform rotation at angular speed ω , the peak emf is $e_0 = NBA\omega$. In a purely resistive closed circuit of total resistance R , the peak current is $I_0 = e_0/R$.

Step 1: (a) Describe the labelled generator A rectangular armature coil $ABCD$, containing N turns, rotates mechanically between the north and south poles of a magnet. Its rotation axis lies in the coil's plane and is perpendicular to the magnetic field. The two ends of the coil connect to separate complete slip rings S_1 and S_2 , which rotate with the shaft and are insulated from one another. Stationary

Quick Tip to Solve: Convert cm^2 to m^2 using 10^{-4} , then find $NBA\omega$. This product has units of volts; divide by a specified resistance before reporting amperes.

carbon brushes P_1 and P_2 touch the respective rings and connect the rotating coil to the external load. Unlike a split-ring commutator, these rings preserve each coil end's connection as the induced polarity reverses.



Schematic AC generator: separate slip rings and stationary brushes connect the rotating armature to the load.

Step 2: Derive the instantaneous emf from flux linkage Choose $t = 0$ when the normal to the coil is parallel to the field. After time t , the normal has turned through $\theta = \omega t$. Hence the magnetic flux linked with all turns is

$$N\Phi(t) = NBA \cos(\omega t).$$

Applying Faraday's law with the positive circulation chosen consistently with this normal,

$$\begin{aligned} e(t) &= -\frac{d}{dt} [NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t) = e_0 \sin(\omega t). \end{aligned}$$

Here $e_0 = NBA\omega$. The emf changes sign after each half rotation, so the output is alternating. When the coil's normal is parallel to the field the flux is extreme and its rate of change is zero; when the normal is perpendicular to the field the emf magnitude is maximum. A different starting orientation changes only the phase of this sinusoidal expression.

Step 3: (b) Convert the area and calculate the peak emf The area of one turn, number of turns, magnetic field and angular speed are

$$\begin{aligned} A &= 200 \times 10^{-4} = 2.00 \times 10^{-2} \text{ m}^2, \\ N &= 20, \quad B = 3.0 \times 10^{-2} \text{ T}, \\ \omega &= 50 \text{ rad s}^{-1}. \end{aligned}$$

Quick Tip to Solve: Convert cm^2 to m^2 using 10^{-4} , then find $NBA\omega$. This product has units of volts; divide by a specified resistance before reporting amperes.

Taking the usual generator arrangement with the rotation axis perpendicular to the field,

$$e_0 = 20(3.0 \times 10^{-2})(2.00 \times 10^{-2})(50) = 0.60 \text{ V}.$$

Step 4: Check the data needed to find the current Current also depends on the resistance of the coil and any connected load. For a closed, purely resistive circuit, neglecting self-inductance,

$$I_0 = \frac{e_0}{R} = \frac{0.60 \text{ V}}{R}.$$

The printed question supplies no resistance or circuit impedance. Thus the numerical data determine the peak emf, but cannot determine a unique numerical peak current. No resistance value may be assumed. If R is specified in ohms, the peak current is $0.60/R$ amperes under the stated resistive approximation.

Step 5: Conclusion The generator produces $e(t) = NBA\omega \sin(\omega t)$. For part (b), $e_0 = 0.60 \text{ V}$; the requested peak current is $I_0 = (0.60 \text{ V})/R$ for a resistive circuit. A numerical current cannot be obtained because the printed question omits the resistance.

Quick Tip to Solve: Convert cm^2 to m^2 using 10^{-4} , then find $NBA\omega$. This product has units of volts; divide by a specified resistance before reporting amperes.

OR

25. (a) Draw a labelled diagram of a step-up transformer. Obtain the ratio of secondary to primary voltage in terms of number of turns and currents in the two coils.

(b) A power transmission line feeds input power at 2200 V to a step-down transformer with its primary windings having 3000 turns. Find the number of turns in the secondary to get the power output at 220 V. [5 marks]

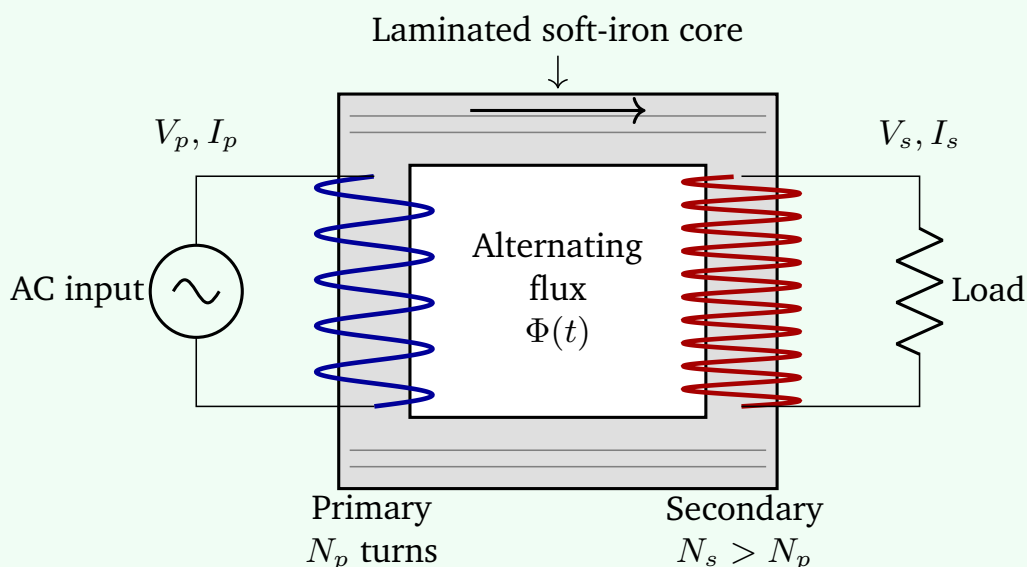
Solution:

Concept:

- A transformer works by mutual induction: alternating current in the primary coil produces changing magnetic flux in a common iron core, inducing an emf in the secondary coil.
- Faraday's law gives $e_p = -N_p d\Phi/dt$ and $e_s = -N_s d\Phi/dt$, where N_p, N_s are the primary and secondary turns and Φ is the flux through each turn.
- For an ideal transformer with negligible losses and flux leakage, the voltage ratio equals the turns ratio and input power equals output power. Here V_p, V_s, I_p, I_s denote rms voltages and currents.

Step 1: (a) Draw and explain the step-up transformer Wind two insulated coils on a common laminated soft-iron core. Connect the primary to an AC source and the secondary to a load. The secondary has more turns than the primary, $N_s > N_p$. The closed core links nearly the same changing flux with each turn of both coils; laminations reduce eddy-current loss. The alternating flux induces the secondary emf even though the two coils are electrically isolated.

Quick Tip to Solve: Match the ratios in the same order: secondary voltage over primary voltage equals secondary turns over primary turns. The current ratio is reversed; a tenfold voltage reduction needs a tenfold turns reduction.



Step-up transformer: the secondary has more turns than the primary.

Step 2: Derive the voltage and turns ratio Apply Faraday's law to the two coils, choosing corresponding winding senses. Since the flux per turn is common, their induced emfs are

$$e_p = -N_p \frac{d\Phi}{dt}, \quad e_s = -N_s \frac{d\Phi}{dt}.$$

Dividing gives $e_s/e_p = N_s/N_p$. Equivalently, for sinusoidal flux $\Phi = \Phi_m \sin \omega t$, the emf amplitudes are $E_{0p} = N_p \omega \Phi_m$ and $E_{0s} = N_s \omega \Phi_m$. Their rms values have the same ratio because both amplitudes are divided by $\sqrt{2}$. Neglecting resistance drops, terminal-voltage magnitudes equal the corresponding induced-emf magnitudes, so

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

Thus more secondary turns produce a larger secondary voltage for a step-up transformer. This derivation uses voltage magnitudes; the primary induced emf opposes the applied voltage by Lenz's law.

Step 3: Relate the voltage ratio to the currents In an ideal transformer, no energy is lost in the windings or core. For a resistive load, conservation of power gives

$$V_p I_p = V_s I_s.$$

Rearranging and combining with the turns ratio gives

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}.$$

Hence stepping up the voltage decreases the current: $V_s > V_p$ while $I_s < I_p$. A transformer changes voltage and current levels without creating power. The current relation neglects the small magnetising current of a practical transformer.

Quick Tip to Solve: Match the ratios in the same order: secondary voltage over primary voltage equals secondary turns over primary turns. The current ratio is reversed; a tenfold voltage reduction needs a tenfold turns reduction.

Step 4: (b) Substitute the data for the step-down transformer This part concerns a separate step-down transformer. The given values are $V_p = 2200\text{ V}$, $V_s = 220\text{ V}$ and $N_p = 3000$. Assuming ideal operation, rearrange the turns relation:

$$\begin{aligned} N_s &= N_p \frac{V_s}{V_p} \\ &= 3000 \times \frac{220}{2200} \\ &= 3000 \times \frac{1}{10}. \end{aligned}$$

The secondary needs one-tenth as many turns as the primary because the required output voltage is one-tenth of the input voltage. The numerical voltages describe the voltage levels at which power is supplied, rather than power measured in watts.

Step 5: Conclusion For the ideal transformer, $V_s/V_p = N_s/N_p = I_p/I_s$. A step-up transformer has $N_s > N_p$ and therefore $V_s > V_p$, with reduced secondary current. In part (b), the required secondary winding is $N_s = 300\text{ turns}$.

Quick Tip to Solve: Match the ratios in the same order: secondary voltage over primary voltage equals secondary turns over primary turns. The current ratio is reversed; a tenfold voltage reduction needs a tenfold turns reduction.

26. (a) Distinguish between unpolarized light and linearly polarized light. How does one get linearly polarised light with the help of a polaroid?

(b) A narrow beam of unpolarised light of intensity I_0 is incident on a polaroid P_1 . The light transmitted by it is then incident on a second polaroid P_2 with its pass axis making angle of 60° relative to the pass axis of P_1 . Find the intensity of the light transmitted by P_2 . [5 marks]

Solution:

Concept:

- Polarisation describes the orientation of the electric-field vibrations of a transverse light wave, in the plane perpendicular to its direction of propagation.
- An ideal polaroid transmits the electric-field component along its pass axis and absorbs the perpendicular component. Unpolarised incident light therefore emerges linearly polarised with half its original intensity.
- Malus's law is $I = I_{\text{in}} \cos^2 \theta$, where I_{in} is the intensity of linearly polarised light incident on an analyser and θ is the angle between its electric-field direction and the analyser's pass axis.

Step 1: (a) Distinguish the two types of light In unpolarised light, the electric vector changes direction randomly with time, with vibrations distributed over all directions perpendicular to propagation. In linearly polarised light, the electric vector oscillates along one fixed direction perpendicular to propagation. It therefore defines a fixed plane with the direction of propagation.

Step 2: (a) Explain how a polaroid produces linear polarisation Pass the unpolarised beam through a polaroid sheet. Resolve each incident electric vibration into components parallel and perpendicular to its pass axis. The parallel component is transmitted and the perpendicular component is selectively

Quick Tip to Solve: For unpolarised light passing through two ideal polaroids, halve the intensity at the first sheet, then multiply by the cosine squared of the angle between the pass axes at the second sheet.

absorbed. The transmitted vibrations are all parallel to the pass axis, so the emerging beam is linearly polarised. For an ideal polaroid, the randomly oriented incident vibrations give the average $\langle \cos^2 \phi \rangle = 1/2$, where ϕ is their angle to the pass axis. Hence transmission of unpolarised light reduces its intensity by a factor of two.

Step 3: (b) Find the intensity emerging from the first polaroid The beam incident on P_1 is unpolarised and has intensity I_0 . Thus the intensity incident on P_2 is

$$I_1 = \frac{I_0}{2}.$$

This beam is linearly polarised along the pass axis of P_1 , making 60° with the pass axis of P_2 .

Step 4: (b) Apply Malus's law to the second polaroid The electric-field projection along the second pass axis gives the transmitted amplitude. Squaring its factor gives Malus's law:

$$\begin{aligned} I_2 &= I_1 \cos^2 60^\circ \\ &= \frac{I_0}{2} \left(\frac{1}{2} \right)^2 = \frac{I_0}{8}. \end{aligned}$$

Step 5: Conclusion A polaroid converts unpolarised light into linearly polarised light by selective absorption. The final intensity is $I_2 = I_0/8$, and the emerging electric vibrations are parallel to the pass axis of P_2 .

Quick Tip to Solve: For unpolarised light passing through two ideal polaroids, halve the intensity at the first sheet, then multiply by the cosine squared of the angle between the pass axes at the second sheet.

OR

26. (a) Explain two features to distinguish between the interference pattern in Young's double slit experiment with the diffraction pattern obtained due to a single slit.

(b) A monochromatic light of wavelength 500 nm is incident normally on a single slit of width 0.2 mm to produce a diffraction pattern. Find the angular width of the central maximum obtained on the screen.

Estimate the number of fringes obtained in Young's double slit experiment with fringe width 0.5 mm, which can be accommodated within the region of total angular spread of the central maximum due to single slit. [5 marks]

Solution:

Concept:

- Interference in Young's experiment is the superposition of light from two coherent slits; single-slit diffraction is the superposition of secondary wavelets from different portions of one slit.
- For slit width a , diffraction minima satisfy $a \sin \theta = n\lambda$. The central maximum lies between the first minima on opposite sides, so its full angular width is $2 \sin^{-1}(\lambda/a) \simeq 2\lambda/a$.
- At screen distance D , a small angular span $\Delta\theta$ corresponds to linear width $W \simeq D\Delta\theta$. The

Quick Tip to Solve: Double λ/a for the full central angular width. Before counting fringes, convert that angle to a screen length using D ; both widths in the ratio must have the same units.

estimated number of fringe widths accommodated is $N \simeq W/\beta$, where β is the linear fringe width.

Step 1: (a) Compare the widths of the fringes In ideal Young's double-slit interference, the bright fringes are equally spaced and have equal widths; successive dark fringes are also equally spaced. The fringe width is $\beta = \lambda D/d$, where d is the slit separation. In single-slit diffraction, the central bright maximum is twice as wide as each adjacent secondary bright band, approximately. Its width between the first minima is $2\lambda D/a$, whereas the width between successive minima on either side is approximately $\lambda D/a$. Thus the central diffraction maximum has a distinctly larger width.

Step 2: (a) Compare the intensities of the maxima For two equally illuminated, very narrow coherent slits in the ideal Young's experiment, all bright interference fringes have the same maximum intensity. In single-slit diffraction, the central maximum is the brightest, and successive secondary maxima become much weaker as their distance from the centre increases. These comparisons describe the ideal interference pattern; real slits of finite width can impose a diffraction envelope on the interference fringes.

Step 3: (b) Locate the first diffraction minima Convert both supplied quantities to SI units before applying the minimum condition:

$$\lambda = 500 \text{ nm} = 5.00 \times 10^{-7} \text{ m}, \quad a = 0.2 \text{ mm} = 2.0 \times 10^{-4} \text{ m}.$$

The first minimum occurs at $\sin \theta_1 = \lambda/a$, giving

$$\sin \theta_1 = \frac{5.00 \times 10^{-7}}{2.0 \times 10^{-4}} = 2.5 \times 10^{-3}.$$

Since this angle is small, $\theta_1 \simeq 2.5 \times 10^{-3} \text{ rad}$. The first minima occur at $-\theta_1$ and $+\theta_1$, so the total angular spread is their separation:

$$\Delta\theta = 2\theta_1 \simeq 5.0 \times 10^{-3} \text{ rad} \simeq 0.286^\circ.$$

Step 4: (b) Relate the central width to the interference fringe width The supplied interference fringe width is a length, $\beta = 0.5 \text{ mm} = 5.0 \times 10^{-4} \text{ m}$, whereas the diffraction spread just calculated is an angle. A length cannot be divided directly into an angle. Let D be the distance from the single slit to the screen. Then

$$W \simeq D\Delta\theta = (5.0 \times 10^{-3})D, \quad N \simeq \frac{W}{\beta} = 10 \frac{D}{1 \text{ m}}.$$

The printed question does not supply D , nor an angular fringe width that could replace it. Consequently, a unique numerical fringe count cannot be obtained from the printed data. If a screen distance of 1 m were additionally specified, the central width would be 5 mm, accommodating about 10 fringe widths; this is a conditional example, not a supplied value.

Step 5: Conclusion Ideal interference fringes have equal widths and equal peak intensities; diffraction has a central maximum of twice the secondary-band width and progressively weaker secondary maxima. The full central angular width is $5.0 \times 10^{-3} \text{ rad}$. The fringe estimate is $N \simeq 10D/(1 \text{ m})$; the omitted screen distance prevents a unique numerical count.

Quick Tip to Solve: Double λ/a for the full central angular width. Before counting fringes, convert that angle to a screen length using D ; both widths in the ratio must have the same units.