CBSE Class 12 Mathematics 2017 - Set 65-3 - Compartment

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 3 questions of four marks each and 3 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section A

1. Find: $\int \frac{3x}{3x-1}\,dx$.

2. If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 225$ and $|\vec{a}| = 5$, then write the value of $|\vec{b}|$.

3. If A is an invertible matrix of order 2 and $\det(A) = 4$, then write the value of $\det(A^{-1})$.

4. If the function $f(x)$ given by $f(x) = \begin{cases} \frac{\sin\frac{3x}{2}}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then write the value of k.

Section B

5. An aeroplane can carry a maximum of 250 passengers. A profit of Rs. 1,500 is made on each executive class ticket and a profit of Rs. 1,000 is made on each economy class ticket. The airline reserves at least 25 seats for executive class. However, at least 3 times as many passengers prefer to travel by economy class than by executive class. Frame the Linear Programming Problem to determine how many tickets of each type must be sold in order to maximize the profit for the airline.

6. Find the integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$.

7. Find: $\int \frac{x}{\sqrt{32 - x^2}}\, dx$.

8. If $f(x) = \sin 2x - \cos 2x$, find $f'\left(\frac{\pi}{6}\right)$.

9. Find the values of x and y from the following matrix equation: $2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$.

10. If the vectors $\hat{i} - \hat{j} + \hat{k}$, $3\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + \lambda\hat{j} - 3\hat{k}$ are coplanar, then find the value of λ.

11. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$, given that $y(0) = \sqrt{3}$.

12. A balloon, which always remains spherical, has a variable diameter $\frac{2}{3}(3x+1)$. Find the rate of change of its volume with respect to x.

Section C

13. A school wants to award its students for regularity and hardwork with a total cash award of Rs. 6,000. If three times the award money for hardwork added to that given for regularity amounts to Rs. 11,000, represent the above situation algebraically and find the award money for each value, using matrix method. Suggest two more values which the school must include for the award.

14(a). Bag A contains 3 red and 2 black balls, while bag B contains 2 red and 3 black balls. A ball drawn at random from bag A is transferred to bag B and then one ball is drawn at random from bag B. If this ball was found to be a red ball, find the probability that the ball drawn from bag A was red.

OR

14(b). If A and B are two independent events, then prove that the probability of occurrence of at least one of A and B is given by $1 - P(A')\,P(B')$.

15. Find the real solutions of the equation $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x$, $(x > 0)$.

16. Find the coordinates of the foot of perpendicular drawn from the point $A(1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$.

17. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, find the angles which the vector $2\vec{a} + \vec{b} + 2\vec{c}$ makes with the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

18. Find the intervals in which the function given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is (a) strictly increasing (b) strictly decreasing.

19(a). Find: $\int \frac{x^2 + x + 1}{(x+1)^2(x+2)}\,dx$.

OR

19(b). Find: $\int (x-3)\sqrt{3 - 2x - x^2}\,dx$.

20(a). If $y = (\cos x)^x + \sin^{-1}\sqrt{3x}$, find $\frac{dy}{dx}$.

OR

20(b). If $y = (\sec^{-1} x)^2$, then show that $x^2(x^2 - 1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} = 2$.

21. Solve the following linear programming problem graphically: find the maximum value of $Z = 105x + 90y$ subject to the constraints $x + y \leq 50$, $2x + y \leq 80$, $x \geq 20$, $x, y \geq 0$.

22. Find the particular solution of the differential equation $2xy + y^2 - 2x^2\frac{dy}{dx} = 0$; $y = 2$ when $x = 1$.

23. Evaluate: $\int_0^{\pi} \frac{x\,dx}{1 + \sin x}$.

Section D

24(a). Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are $A(1, -2)$, $B(3, 5)$ and $C(5, 2)$.

OR

24(b). Evaluate the following definite integral as the limit of sums: $\int_0^4 (3x^2 + 2x + 1)\, dx$.

25(a). Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

OR

25(b). Find the equation of the tangent to the curve $x = a\cos\theta + a\theta\sin\theta$, $y = a\sin\theta - a\theta\cos\theta$ at any point θ of the curve. Also show that at any point θ of the curve, the normal is at a constant distance from the origin.

26(a). Given a non-empty set X, consider the binary operation $* : P(X) \times P(X) \to P(X)$ given by $A * B = A \cap B$, $\forall A, B \in P(X)$, where $P(X)$ is the power set of X. Show that $*$ is commutative and associative, X is the identity element for this operation, and X is the only invertible element in $P(X)$ with respect to the operation $*$.

OR

26(b). Let $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x + 4}$. Show that f is a one-one function. Also check whether f is an onto function or not. Hence find f^{-1}, defined from the range of f to $\mathbb{R} - \left\{-\frac{4}{3}\right\}$.

27. If $a + b + c \neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then using properties of determinants, prove that $a = b = c$.

28. Find the coordinates of the point of intersection P of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane determined by the points $A(1, -2, 2)$, $B(4, 2, 3)$ and $C(3, 0, 2)$.

29. A biased die is such that $P(4) = \frac{1}{10}$ and the other scores are equally likely. The die is tossed twice. If X is the number of fours obtained, find the variance of X.

- End of Question Paper -