

CBSE Class 12 Physics Question Paper 2018 with Solution PDF-Set 55/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are **26** questions in all.
- (ii) This question paper has **five** sections : Section A, Section B, Section C, Section D and Section E.
- (iii) Section A contains **five** questions of **one** mark each, Section B contains **five** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains one value based question of **four** marks and Section E contains **three** questions of **five** marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in **one** question of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. A proton and an electron travelling along parallel paths enter a region of uniform magnetic field, acting perpendicular to their paths. Which of them will move in a circular path with higher frequency ? [1 mark]

Solution:

Concept:

- The magnetic force provides the centripetal force: $|q|vB = mv^2/r$, where m is the particle mass and B is the magnetic field.
- The frequency of circular motion is $f = |q|B/(2\pi m)$; its value depends on charge magnitude and mass.

Step 1: Find the frequency From $r = mv/(|q|B)$, the period is $T = 2\pi r/v = 2\pi m/(|q|B)$. Therefore $f = 1/T = |q|B/(2\pi m)$. The speed cancels, so equal speeds are not required for this comparison.

Step 2: Compare the two particles A proton and an electron have the same charge magnitude e and experience the same field B . Thus $f_e/f_p = m_p/m_e \approx 1836$. Their opposite charge signs reverse the directions of rotation but do not change the frequency magnitude.

Step 3: Conclusion The electron moves in a circular path with the higher frequency, approximately 1836 times that of the proton.

Quick Tip to Solve: For the same field and charge magnitude, compare masses: the lighter particle has the higher cyclotron frequency.

2. Name the electromagnetic radiations used for (a) water purification, and (b) eye surgery. [1 mark]

Solution:

Concept:

- Ultraviolet radiation has a frequency above violet visible light and is used for germicidal applications.
- Ultraviolet laser radiation can remove a controlled thin layer of corneal tissue in corrective eye surgery.

Step 1: Identify the water-purification radiation Water purification uses ultraviolet (UV) radiation. Its germicidal action damages the genetic material of microorganisms and prevents their reproduction, thereby disinfecting the water.

Step 2: Identify the eye-surgery radiation Corrective eye surgery uses ultraviolet radiation from an excimer laser. A precisely controlled UV beam reshapes the cornea by removing a very thin layer of tissue. The question asks for the electromagnetic radiation, so the required name is ultraviolet rays.

Step 3: Conclusion (a) Ultraviolet rays are used for water purification. (b) Ultraviolet rays are used for eye surgery.

Quick Tip to Solve: Remember that UV has both germicidal use in water treatment and precise tissue-removal use in corneal surgery.

3. Draw graphs showing variation of photoelectric current with applied voltage for two incident radiations of equal frequency and different intensities. Mark the graph for the radiation of higher intensity. [1 mark]

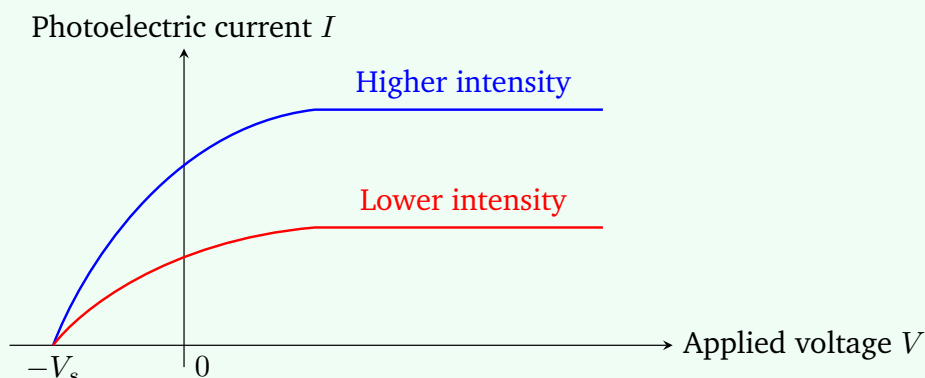
Solution:

Concept:

- At fixed frequency, greater intensity supplies more photons per second and hence produces a larger saturation photoelectric current.
- The stopping potential satisfies $eV_s = h\nu - \phi$, where ν is frequency and ϕ is the work function; intensity does not change V_s .

Step 1: Compare the stopping potentials The two radiations have equal frequency and illuminate the same photosensitive surface. Their emitted electrons therefore have the same maximum kinetic energy. Both current-voltage curves meet the voltage axis at the same negative voltage $-V_s$.

Step 2: Draw and identify the curves As the collector voltage becomes positive, more emitted electrons are collected and the current approaches saturation. The upper curve corresponds to the higher incident intensity because more electrons are emitted per second. Both curves share the stopping-potential intercept, while their saturation currents differ.



Photoelectric current for equal frequencies and different intensities

Step 3: Conclusion The upper curve is for higher intensity; both curves have the same stopping potential V_s .

Quick Tip to Solve: Keep the negative-voltage intercept identical and raise only the saturation level when increasing intensity at fixed frequency.

4. Four nuclei of an element undergo fusion to form a heavier nucleus, with release of energy. Which of the two — the parent or the daughter nucleus — would have higher binding energy per nucleon ?

[1 mark]

Solution:

Concept:

- Binding energy is the energy required to separate a nucleus into its constituent nucleons. Binding energy per nucleon measures the average strength of binding.
- Energy released in fusion is the increase in the total binding energy: $Q = B_d - \sum B_p > 0$.

Step 1: Use the released energy The fusion reaction releases energy, so the products have lower rest energy and greater total binding energy than the original parent nuclei. If each parent has binding energy B_p , the daughter has $B_d > 4B_p$.

Step 2: Compare binding energy per nucleon Let each parent contain A nucleons. The daughter formed by their fusion contains $4A$ nucleons, because nucleon number is conserved. Dividing $B_d > 4B_p$ by $4A$ gives $B_d/(4A) > B_p/A$. Thus its average binding per nucleon is larger.

Step 3: Conclusion The daughter nucleus has the higher binding energy per nucleon.

Quick Tip to Solve: For an energy-releasing fusion reaction, conserved nucleon number lets you compare total binding energies per nucleon directly.

5. Which mode of propagation is used by short wave broadcast services ?

[1 mark]

Solution:

Concept:

- Sky-wave propagation uses the ionosphere to return radio waves towards the Earth, allowing reception beyond the horizon.
- Short-wave radio lies in the high-frequency range, approximately 3 to 30 MHz, suited to ionospheric propagation.

Step 1: Identify the propagation path Short-wave broadcasts travel upwards towards the ionosphere and are refracted back towards the Earth. Repeated paths between the ionosphere and the Earth can carry these broadcasts over large distances.

Step 2: Relate the path to the named mode This ionosphere-assisted transmission is called sky-wave propagation. Ground-wave transmission is mainly useful at lower frequencies, whereas space-wave transmission normally follows a line-of-sight path and is limited by the horizon. The short-wave service therefore uses the ionospheric path.

Step 3: Conclusion Short-wave broadcast services use sky-wave propagation.

Quick Tip to Solve: Associate short-wave broadcasting with the ionosphere; this immediately identifies sky-wave propagation.

Section B (10 marks)

6. Two electric bulbs P and Q have their resistances in the ratio of 1 : 2. They are connected in series across a battery. Find the ratio of the power dissipation in these bulbs. [2 marks]

Solution:

Concept:

- In a series circuit the same current I flows through every component, because there is only one path for charge.
- The electrical power converted into heat in a resistance R is $P = I^2 R$. Thus, at a common current, power is directly proportional to resistance.

Step 1: Identify the common current Let the resistances of bulbs P and Q be $R_P = R$ and $R_Q = 2R$. Since the bulbs are connected in series, the current through P is equal to the current through Q. The battery voltage need not be known to obtain the requested ratio.

Step 2: Compare the powers using the series condition The dissipated powers are $P_P = I^2 R$ and $P_Q = I^2(2R)$. Hence

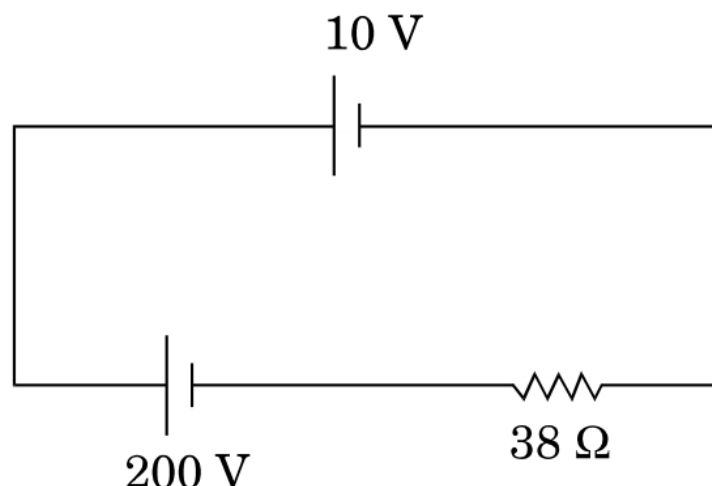
$$\frac{P_P}{P_Q} = \frac{I^2 R}{I^2(2R)} = \frac{1}{2}.$$

The common factor I^2 cancels. The formula $P = V^2/R$ would give an inverse ratio only if the two bulbs had equal voltages across them; series bulbs have equal currents instead.

Step 3: Conclusion The required ratio of power dissipation is $P_P : P_Q = 1 : 2$.

Quick Tip to Solve: Before comparing bulb powers, identify what is common: use $I^2 R$ for series bulbs and V^2/R for parallel bulbs.

7. A 10 V cell of negligible internal resistance is connected in parallel across a battery of emf 200 V and internal resistance $38\ \Omega$ as shown in the figure. Find the value of current in the circuit. [2 marks]



Solution:

Concept:

- Kirchhoff's loop rule states that the algebraic sum of potential changes around a closed loop is zero.
- An ideal cell fixes its terminal potential difference at its emf. For a battery delivering current I , its terminal voltage is $V = E - Ir$, where r is its internal resistance.

Step 1: Read the polarities in the printed circuit The long plate denotes the positive terminal. Both sources have their positive terminals connected to the left junction, so their emfs oppose each other when traversing the loop. The ideal 10 V cell maintains the left junction 10 V above the right junction. The 200 V battery supplies current into the positive terminal of the smaller cell.

Step 2: Apply the loop equation Take the current through the lower battery branch from right to left and through the upper ideal-cell branch from left to right. Traversing the circuit in this direction gives

$$200 - 38I - 10 = 0.$$

Equivalently, the terminal voltage of the large battery is $200 - 38I = 10$. Therefore

$$I = \frac{200 - 10}{38} = \frac{190}{38} = 5 \text{ A}.$$

The ideal cell has negligible internal resistance, so no additional resistive voltage drop is included.

Step 3: Conclusion The circuit current is 5 A, flowing from left to right through the 10 V cell; the larger battery charges the smaller cell.

Quick Tip to Solve: Use the long plates to determine polarities first: here the emfs oppose, so the driving emf is $200 - 10$, not their sum.

7. In a potentiometer arrangement for determining the emf of a cell, the balance point of the cell in open circuit is 350 cm. When a resistance of $9\ \Omega$ is used in the external circuit of the cell, the balance point shifts to 300 cm. Determine the internal resistance of the cell. [2 marks]

Solution:

Concept:

- At potentiometer balance, the measured potential difference is proportional to the balance length: $E = kl_1$ and $V = kl_2$, with unchanged potential gradient k .
- For external resistance R and internal resistance r , $E/V = (R + r)/R$, so $r = R(l_1 - l_2)/l_2$.

Step 1: Compare open-circuit emf with loaded voltage When the cell is open-circuited, no current is drawn from it and the balance length $l_1 = 350$ cm measures its emf E . With $R = 9\ \Omega$ connected across the cell, it supplies current and its terminal voltage falls to V , corresponding to $l_2 = 300$ cm. With the same potentiometer gradient, $E/V = l_1/l_2 = 350/300$.

Step 2: Calculate the internal resistance The current through the external resistance is $I = V/R$, and the internal voltage drop is $E - V = Ir$. Hence

$$\begin{aligned} r &= R \left(\frac{E}{V} - 1 \right) \\ &= 9 \left(\frac{350}{300} - 1 \right) \\ &= 9 \left(\frac{50}{300} \right) = 1.5\ \Omega. \end{aligned}$$

Both lengths are expressed in centimetres, so their ratio is dimensionless and no conversion to metres is needed.

Step 3: Conclusion The internal resistance of the cell is $r = 1.5\ \Omega$.

Quick Tip to Solve: Use the loaded balance length in the denominator of $r = R(l_1 - l_2)/l_2$; both lengths can remain in centimetres.

8. (a) Why are infra-red waves often called heat waves ? Explain.

(b) What do you understand by the statement, “Electromagnetic waves transport momentum” ?

[2 marks]

Solution:

Concept:

- Infra-red radiation is absorbed by matter and can increase the energy of molecular motions, thereby increasing the internal energy and temperature of the absorbing body.
- Electromagnetic radiation carries both energy and momentum. Radiation of energy U propagating in one direction carries momentum $p = U/c$, where c is the speed of light in vacuum.

Quick Tip to Solve: Distinguish the two effects: absorbed infra-red energy raises temperature, while transferred radiation momentum produces a force on a surface.

Step 1: (a) Explain the name heat waves Infra-red waves are readily absorbed by many materials, including water molecules present in bodies. Their absorption increases molecular vibrational and other thermal motions, producing a rise in temperature. Because of this prominent heating effect, infra-red waves are commonly called heat waves. Hot bodies also emit substantial infra-red radiation; the name refers to the heating produced on absorption.

Step 2: (b) Explain momentum transport An electromagnetic wave conveys momentum along its direction of propagation. When it strikes a surface, absorption or reflection changes the radiation momentum and transfers momentum to the surface. Consequently, the wave exerts a force and radiation pressure. For normally incident radiation carrying energy U , a perfectly absorbing surface receives momentum U/c ; a perfectly reflecting surface receives $2U/c$, because the momentum reverses direction.

Step 3: Conclusion (a) Infra-red waves are called heat waves because their absorption produces heating. (b) Electromagnetic waves carry momentum and can transfer it to matter, exerting force and radiation pressure.

Quick Tip to Solve: Distinguish the two effects: absorbed infra-red energy raises temperature, while transferred radiation momentum produces a force on a surface.

9. If light of wavelength 412.5 nm is incident on each of the metals given below, which ones will show photoelectric emission and why ? [2 marks]

Metal	Work Function (eV)
Na	1.92
K	2.15
Ca	3.20
Mo	4.17

Solution:

Concept:

- Photoelectric emission occurs when the energy of one incident photon satisfies $E_\gamma = h\nu = hc/\lambda \geq \phi$, where ϕ is the metal work function.
- Einstein's equation is $K_{\max} = E_\gamma - \phi$; a negative value means emission is energetically impossible.

Step 1: Calculate the photon energy For the incident wavelength $\lambda = 412.5 \text{ nm} = 412.5 \times 10^{-9} \text{ m}$, use the supplied constants:

$$E_\gamma = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{412.5 \times 10^{-9}} = 4.82 \times 10^{-19} \text{ J} = \frac{4.82 \times 10^{-19}}{1.60 \times 10^{-19}} \text{ eV} \approx 3.01 \text{ eV}.$$

Quick Tip to Solve: Compute photon energy once in eV, then compare it directly with every tabulated work function.

This is the energy available to each electron from absorption of one photon.

Step 2: Compare with each work function The printed table gives $\phi_{\text{Na}} = 1.92 \text{ eV}$, $\phi_{\text{K}} = 2.15 \text{ eV}$, $\phi_{\text{Ca}} = 3.20 \text{ eV}$ and $\phi_{\text{Mo}} = 4.17 \text{ eV}$. The photon energy exceeds the work functions of Na and K, leaving approximately 1.09 eV and 0.86 eV respectively as maximum kinetic energy. For Ca and Mo it falls short of the energy needed to release an electron, so neither emits photoelectrons at this wavelength.

Step 3: Conclusion Only sodium (Na) and potassium (K) show photoelectric emission, because $hc/\lambda \simeq 3.01 \text{ eV}$ exceeds their work functions; calcium and molybdenum do not.

Quick Tip to Solve: Compute photon energy once in eV, then compare it directly with every tabulated work function.

10. A carrier wave of peak voltage 15 V is used to transmit a message signal. Find the peak voltage of the modulating signal in order to have a modulation index of 60%. [2 marks]

Solution:

Concept:

- For amplitude modulation, the modulation index is $m = A_m/A_c$, where A_m and A_c are the peak message and carrier voltages.
- The percentage modulation is $100m$; use peak amplitudes consistently in the ratio.

Step 1: Express the given modulation as a ratio The carrier peak voltage is $A_c = 15 \text{ V}$. A modulation index of 60% means

$$m = \frac{60}{100} = 0.60.$$

This number is dimensionless. It measures the message amplitude as a fraction of the carrier amplitude, so it must first be converted from a percentage before substituting in the amplitude relation.

Step 2: Find the message peak voltage Rearranging $m = A_m/A_c$ gives

$$A_m = mA_c = (0.60)(15 \text{ V}) = 9 \text{ V}.$$

Both amplitudes here are peak voltages; no conversion to rms voltage or peak-to-peak voltage is needed. As a consistency check, $9/15 = 0.60$, reproducing the required modulation index. Since this value is below unity, the message does not cause overmodulation of the carrier.

Step 3: Conclusion The required peak voltage of the modulating signal is $\boxed{9 \text{ V}}$.

Quick Tip to Solve: Convert 60 percent to 0.60 and multiply by the carrier peak voltage; keep both amplitudes in the same convention.

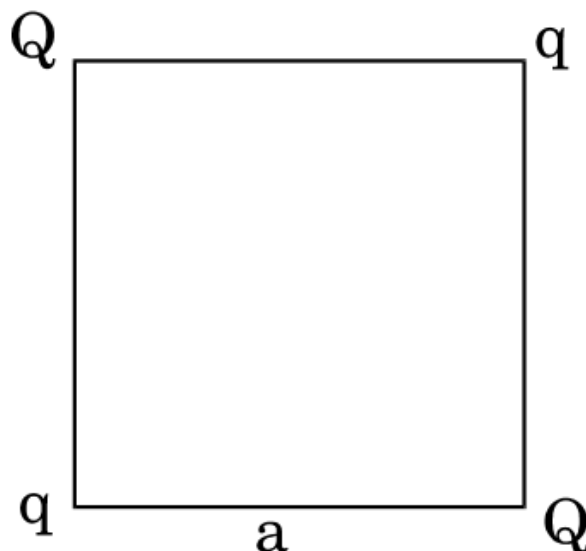
Section C (36 marks)

11. Four point charges Q, q, Q and q are placed at the corners of a square of side ' a ' as shown in the figure.

Find the

- (a) resultant electric force on a charge Q , and
 (b) potential energy of this system.

[3 marks]



Solution:

Concept:

- Coulomb's force obeys $\vec{F} = kQ_1Q_2\hat{r}/r^2$, where $k = 1/(4\pi\epsilon_0)$ and $\hat{r}$ points from the source to the selected charge.
- The electrostatic potential energy is $U = \sum_{i<j} kq_iq_j/r_{ij}$; count each unordered pair once.

Step 1: (a) Add forces on the upper-left charge Take the upper-left Q as the charge of interest; the other Q is diagonally opposite. Let $\hat{x}$ point right and $\hat{y}$ point upward. The adjacent q charges at the upper-right and lower-left corners contribute $-kQq\hat{x}/a^2$ and $+kQq\hat{y}/a^2$. The diagonal distance is $\sqrt{2}a$; hence the force due to the opposite Q has magnitude $kQ^2/(2a^2)$ and direction $(-\hat{x} + \hat{y})/\sqrt{2}$. Thus

$$\vec{F} = \frac{k}{a^2} \left(Qq + \frac{Q^2}{2\sqrt{2}} \right) (-\hat{x} + \hat{y}).$$

For positive Q, q , this points outward along the diagonal through the chosen corner. Its magnitude is $k(\sqrt{2}Qq + Q^2/2)/a^2$. The force on the opposite Q has the same magnitude and the opposite direction. If charges are treated as signed quantities, the vector expression determines the direction and its magnitude uses the absolute value of the coefficient.

Step 2: (b) Sum the six pair energies There are four edge pairs, each consisting of Q and q separated by a ; they contribute $4kQq/a$. The two diagonal pairs are Q, Q and q, q , each separated by $\sqrt{2}a$.

Quick Tip to Solve: For a square, count four edge pairs at distance a and two diagonal pairs at distance square root of two times a .

Therefore

$$U = \frac{4kQq}{a} + \frac{kQ^2}{\sqrt{2}a} + \frac{kq^2}{\sqrt{2}a}.$$

No extra factor of one-half is required because every pair has already been counted once, and the reference energy is zero when all charges are infinitely separated.

Step 3: Conclusion For positive charges, $F = \frac{1}{4\pi\epsilon_0 a^2}(\sqrt{2}Qq + Q^2/2)$, outward along the diagonal.

The system energy is $U = \frac{1}{4\pi\epsilon_0 a}[4Qq + (Q^2 + q^2)/\sqrt{2}]$.

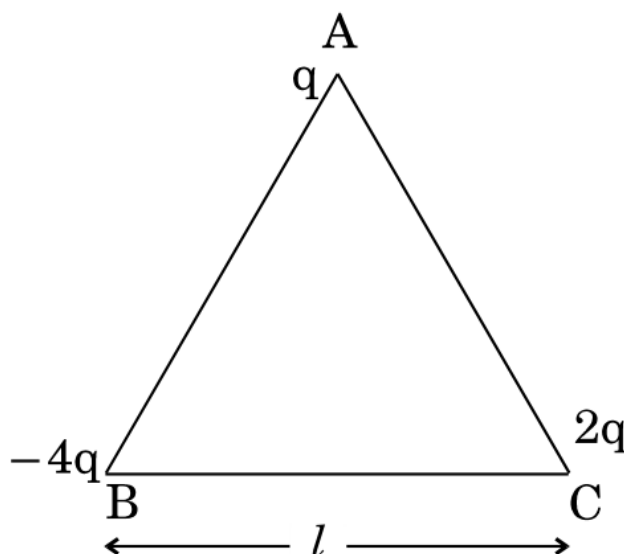
Quick Tip to Solve: For a square, count four edge pairs at distance a and two diagonal pairs at distance square root of two times a .

OR

11. (a) Three point charges q , $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side ' l ' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q .

(b) Find out the amount of the work done to separate the charges at infinite distance.

[3 marks]



Solution:

Concept:

- Coulomb's law gives force magnitude $F = k|q_1q_2|/r^2$, with $k = 1/(4\pi\epsilon_0)$; unlike charges attract and like charges repel.
- The external work for quasistatic separation is $W_{\text{ext}} = U_f - U_i$, with $U = \sum_{i<j} kq_iq_j/r_{ij}$.

Step 1: (a) Identify the two force directions The printed triangle has q at A , $-4q$ at B and $2q$ at C , with every side equal to l . The force on q due to $-4q$ attracts it towards B , along AB , and has

Quick Tip to Solve: At A , reverse AC for the repulsive force: the force angle is 120 degrees, although the triangle angle is 60 degrees.

magnitude $F_B = 4kq^2/l^2$. The force due to $2q$ repels it away from C , along the extension of CA beyond A , and has magnitude $F_C = 2kq^2/l^2$. The interior angle BAC is 60° , so the angle between the two actual force vectors is 120° , since the second force points opposite to AC .

Step 2: (a) Obtain the resultant magnitude Using vector addition and $\cos 120^\circ = -1/2$,

$$\begin{aligned} F^2 &= F_B^2 + F_C^2 + 2F_B F_C \cos 120^\circ \\ &= (16 + 4 - 8) \frac{k^2 q^4}{l^4} = 12 \frac{k^2 q^4}{l^4}. \end{aligned}$$

The square root gives $F = 2\sqrt{3}kq^2/l^2$. Using 60° between the forces would incorrectly treat the repulsive force as pointing towards C .

Step 3: (b) Calculate the work of separation All three initial separations are l . Sum the pair energies with their signs:

$$U_i = \frac{k}{l} [q(-4q) + (-4q)(2q) + (2q)q] = -\frac{10kq^2}{l}.$$

At infinite separation $U_f = 0$. Thus the work done by an external agent in separating the charges quasistatically is $W_{\text{ext}} = U_f - U_i = 10kq^2/l$. It is positive because the initially bound system must gain energy; the electric field itself does the negative of this work.

Step 4: Conclusion $F = \frac{2\sqrt{3}q^2}{4\pi\epsilon_0 l^2}$ and the required external work is $W = \frac{10q^2}{4\pi\epsilon_0 l}$.

Quick Tip to Solve: At A, reverse AC for the repulsive force: the force angle is 120 degrees, although the triangle angle is 60 degrees.

12. (a) Define the term ‘conductivity’ of a metallic wire. Write its SI unit.

(b) Using the concept of free electrons in a conductor, derive the expression for the conductivity of a wire in terms of number density and relaxation time. Hence obtain the relation between current density and the applied electric field $\vec{E}$. [3 marks]

Solution:

Concept:

- Conductivity σ measures the ability of a material to conduct electric current. It is the reciprocal of resistivity: $\sigma = 1/\rho$.
- For free electrons, e is the magnitude of electronic charge, m their mass, n their number per unit volume, and τ the mean relaxation time between successive collisions.

Step 1: Define conductivity and its unit The conductivity of a metallic wire is the current density produced per unit applied electric field in its material. For an isotropic ohmic conductor, $\sigma = J/E$. Since current density has unit A m^{-2} and electric field has unit V m^{-1} , the SI unit is $\text{A V}^{-1}\text{m}^{-1} = \text{S m}^{-1}$, equivalently $\Omega^{-1}\text{m}^{-1}$. Conductivity is a property of the material at the specified temperature, rather than a property determined by the wire length alone.

Step 2: Obtain the mean electron drift velocity Without an electric field, the random thermal velocities

Quick Tip to Solve: Keep electron drift and conventional current separate: multiplying the negative electron charge by the opposite drift direction makes current parallel to the field.

of free electrons have zero average, so there is no net current. An applied field $\vec{E}$ exerts force $-e\vec{E}$ on each electron, giving acceleration $\vec{a} = -e\vec{E}/m$. Collisions interrupt the acceleration and randomise the motion. Averaging over the time elapsed since the last collision, whose mean is the relaxation time τ , gives

$$\vec{v}_d = -\frac{e\tau}{m}\vec{E}.$$

The minus sign means that electron drift is opposite to the field.

Step 3: Derive conductivity and the current-density relation The charge density of the mobile electrons is $-ne$. Therefore conventional current density is $\vec{J} = (-ne)\vec{v}_d$, opposite to electron drift. Substituting the drift velocity,

$$\vec{J} = (-ne)\left(-\frac{e\tau}{m}\vec{E}\right) = \frac{ne^2\tau}{m}\vec{E}.$$

Comparing with the microscopic form of Ohm's law, $\vec{J} = \sigma\vec{E}$, identifies the conductivity. This derivation assumes the linear steady-state response of an isotropic metallic conductor.

Step 4: Conclusion

$$\sigma = \frac{ne^2\tau}{m}, \quad \vec{J} = \sigma\vec{E}.$$

Its SI unit is S m^{-1} .

Quick Tip to Solve: Keep electron drift and conventional current separate: multiplying the negative electron charge by the opposite drift direction makes current parallel to the field.

13. A bar magnet of magnetic moment 6 J/T is aligned at 60° with a uniform external magnetic field of 0.44 T. Calculate (a) the work done in turning the magnet to align its magnetic moment (i) normal to the magnetic field, (ii) opposite to the magnetic field, and (b) the torque on the magnet in the final orientation in case (ii). [3 marks]

Solution:

Concept:

- A magnetic dipole of moment M in a uniform field B has potential energy $U = -MB \cos \theta$, where θ is the angle between its moment and the field.
- For slow rotation without a change in kinetic energy, external work is $W_{\text{ext}} = U_f - U_i = MB(\cos \theta_i - \cos \theta_f)$. Torque magnitude is $\tau = MB \sin \theta$.

Step 1: Determine the initial energy Here $M = 6 \text{ J T}^{-1}$, $B = 0.44 \text{ T}$ and the initial angle is $\theta_i = 60^\circ$. Thus $MB = 6 \times 0.44 = 2.64 \text{ J}$, and

$$U_i = -2.64 \cos 60^\circ = -1.32 \text{ J}.$$

The work requested is the work done by the external agent turning the magnet against the magnetic torque. It equals the increase in magnetic potential energy for a slow rotation.

Step 2: Find the work for the normal orientation: (a)(i) Normal to the field means $\theta_f = 90^\circ$; hence

Quick Tip to Solve: Use cosine for the energy change and sine for torque; evaluate both rotations from the stated initial angle of 60 degrees.

$U_f = -MB \cos 90^\circ = 0$. Starting from the given initial orientation,

$$W_{\text{ext}} = 0 - (-1.32) = 1.32 \text{ J}.$$

The work is positive because moving the dipole away from its preferred parallel orientation raises its potential energy.

Step 3: Find the work for the opposite orientation: (a)(ii) Opposite to the field means $\theta_f = 180^\circ$, so $\cos 180^\circ = -1$ and $U_f = +2.64 \text{ J}$. Again taking the specified initial angle of 60° ,

$$W_{\text{ext}} = 2.64 - (-1.32) = 3.96 \text{ J}.$$

This is the total work from the initial orientation, not just the additional work from the normal orientation.

Step 4: Calculate the final torque: (b) In case (ii), the moment is exactly antiparallel to the external field. Since $\vec{\tau} = \vec{M} \times \vec{B}$, its magnitude is

$$\tau = MB \sin 180^\circ = 2.64 \times 0 = 0 \text{ N m}.$$

Although the torque vanishes at this exact orientation, the equilibrium is unstable: a small angular displacement produces a torque tending to move the magnet toward alignment. Zero torque therefore does not imply minimum potential energy.

Step 5: Conclusion The external work is 1.32 J for (a)(i) and 3.96 J for (a)(ii). The final torque in (ii) is 0 N m .

Quick Tip to Solve: Use cosine for the energy change and sine for torque; evaluate both rotations from the stated initial angle of 60 degrees.

- 14. (a)** An iron ring of relative permeability μ_r has windings of insulated copper wire of n turns per metre. When the current in the windings is I , find the expression for the magnetic field in the ring.
- (b)** The susceptibility of a magnetic material is 0.9853. Identify the type of magnetic material. Draw the modification of the field pattern on keeping a piece of this material in a uniform magnetic field. [3 marks]

Solution:

Concept:

- In a linear magnetic medium, $B = \mu_0 \mu_r H$, with μ_r the relative permeability and H the magnetising field.
- Magnetic susceptibility relates magnetisation to the applied magnetising field: $M = \chi_m H$, and $\mu_r = 1 + \chi_m$. A positive susceptibility of ordinary magnitude identifies a paramagnetic material in the textbook classification.

Step 1: Find the magnetising field of the ring: (a) Treat the closely wound ring as an ideal toroidal solenoid. Take an Amperian circle inside the iron along its mean circumference ℓ . If there are N

Quick Tip to Solve: Check the sign of susceptibility first: positive susceptibility makes the specimen attract field lines, while negative susceptibility makes them spread away.

turns in total, the enclosed free current is NI . Symmetry makes the magnetising field tangential and approximately uniform along this path, so Ampere's law gives

$$H\ell = NI, \quad H = \frac{N}{\ell}I = nI.$$

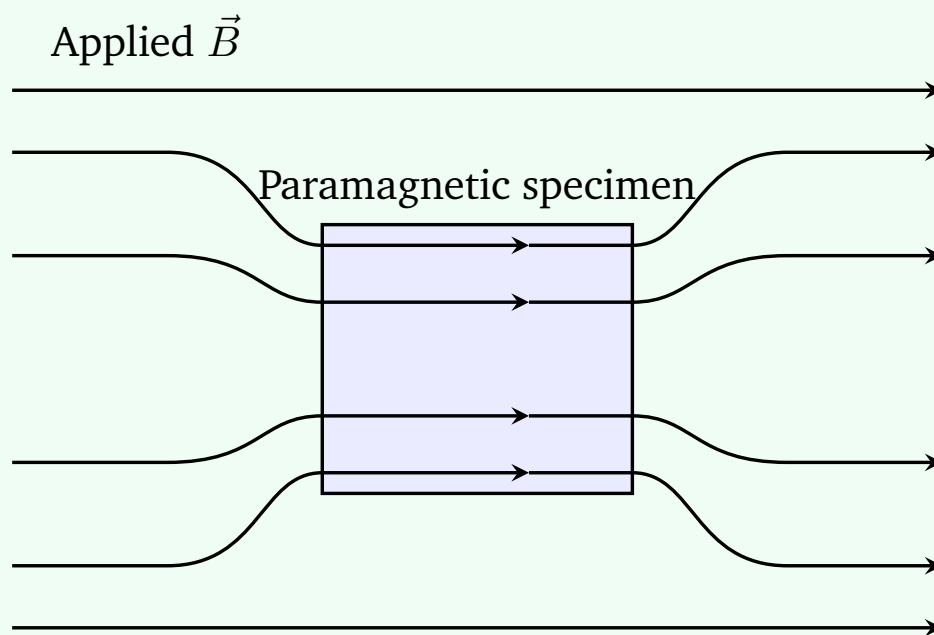
Here $n = N/\ell$ is the stated number of turns per metre.

Step 2: Include the permeability of iron The iron increases the flux density by its relative permeability. Substituting $H = nI$ into the constitutive relation yields

$$B = \mu_0\mu_r H = \mu_0\mu_r nI.$$

This is the magnetic field inside the ring under the ideal toroid approximation, using the given relative permeability at the operating field. Its direction runs around the ring and follows the right-hand rule for the winding current.

Step 3: Identify the material and explain its field pattern: (b) The susceptibility is $\chi_m = +0.9853$, so the material is paramagnetic in the standard classification: its magnetisation is along the applied field. Also $\mu_r = 1 + 0.9853 = 1.9853 > 1$. Field lines consequently bend toward the specimen and become denser within it than in the surrounding space. The drawing shows continuous field lines entering and leaving the sample; arrows retain the original applied-field direction. The enhanced internal density distinguishes this pattern from the expulsion of field lines by a diamagnetic specimen.



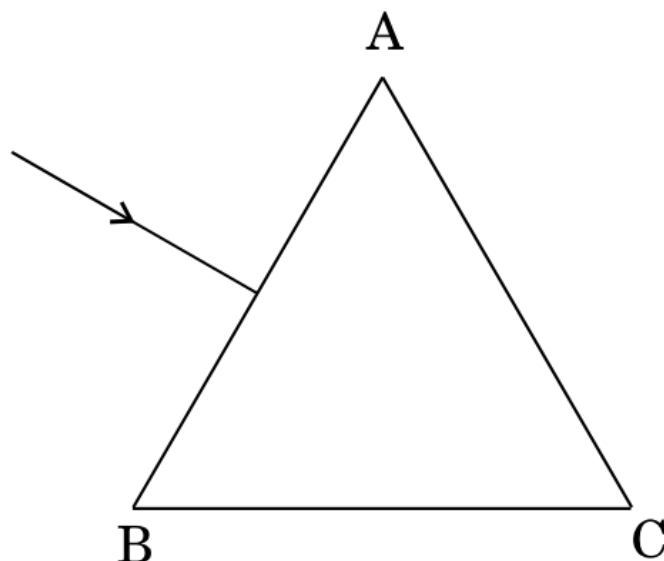
Field lines concentrate inside a paramagnetic specimen placed in a uniform field.

Step 4: Conclusion $B = \mu_0\mu_r nI$. The material with $\chi_m = 0.9853$ is paramagnetic; magnetic field lines concentrate inside it.

Quick Tip to Solve: Check the sign of susceptibility first: positive susceptibility makes the specimen attract field lines, while negative susceptibility makes them spread away.

15. (a) Show using a proper diagram how unpolarised light can be linearly polarised by reflection from a transparent glass surface.

(b) The figure shows a ray of light falling normally on the face AB of an equilateral glass prism having refractive index $\frac{3}{2}$, placed in water of refractive index $\frac{4}{3}$. Will this ray suffer total internal reflection on striking the face AC ? Justify your answer. [3 marks]



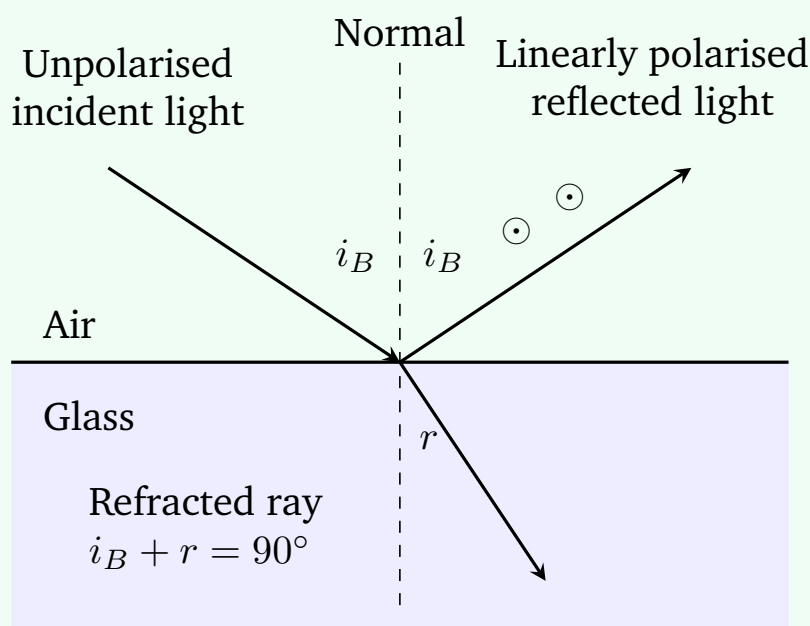
Solution:

Concept:

- At Brewster's angle i_B , $\tan i_B = n_g/n_a$; reflected light is completely linearly polarised perpendicular to the plane of incidence.
- Total internal reflection from glass into water requires $i > C$, where $\sin C = n_w/n_g$. All angles are measured from the normal.

Step 1: (a) Polarisation by reflection Let unpolarised light strike a plane transparent glass surface at the polarising angle i_B . Its electric field initially has components both parallel and perpendicular to the plane containing the incident ray and the normal. At this angle the reflected ray has only the perpendicular electric-field component, so it is linearly polarised. In the diagram, dots on the reflected ray represent vibrations perpendicular to the page, which is the plane of incidence. The refracted light is generally only partially polarised. Brewster's condition also gives $i_B + r = 90^\circ$, so reflected and refracted rays are perpendicular.

Quick Tip to Solve: For a prism immersed in liquid, use the liquid-to-glass index ratio when computing the critical angle.



Polarisation at Brewster's angle; dots show electric vibrations out of the plane.

Step 2: (b) Test the ray at face AC The ray enters AB normally, so it continues undeviated along the normal to AB. Since the angle between AB and AC is 60° , the acute angle between their normals, and hence the incidence angle at AC, is $i = 60^\circ$. For the glass-water boundary,

$$\sin C = \frac{4/3}{3/2} = \frac{8}{9}, \quad C = \sin^{-1}(8/9) \simeq 62.7^\circ.$$

Thus $60^\circ < C$. Equivalently Snell's law gives $\sin r = (9/8) \sin 60^\circ = 9\sqrt{3}/16 < 1$, so a refracted ray into water is possible.

Step 3: Conclusion Reflection at Brewster's angle produces light polarised perpendicular to the plane of incidence. The prism ray does not undergo total internal reflection at AC because $60^\circ < 62.7^\circ$.

Quick Tip to Solve: For a prism immersed in liquid, use the liquid-to-glass index ratio when computing the critical angle.

16. (a) If one of two identical slits producing interference in Young's experiment is covered with glass, so that the light intensity passing through it is reduced to 50%, find the ratio of the maximum and minimum intensity of the fringe in the interference pattern.

(b) What kind of fringes do you expect to observe if white light is used instead of monochromatic light ?

[3 marks]

Solution:

Concept:

- For coherent slit amplitudes a_1, a_2 , $I_{\max} = (a_1 + a_2)^2$ and $I_{\min} = (a_1 - a_2)^2$, with intensity proportional to amplitude squared.

Quick Tip to Solve: Convert intensity to amplitude by taking a square root before using the sum-and-difference formula.

- Fringe width is $\beta = \lambda D/d$, where D is screen distance and d is slit separation.

Step 1: (a) Find the amplitude ratio Let the intensity from each identical uncovered slit be I_0 , and its amplitude be a . After covering one slit, its intensity becomes $I_0/2$, whereas the other remains I_0 . Its amplitude is therefore $a/\sqrt{2}$, not $a/2$. A fixed phase shift introduced by the glass may shift the fringe positions but does not change these extreme intensities.

Step 2: Calculate the maximum-to-minimum ratio At constructive interference the amplitudes add; at destructive interference they subtract. Thus

$$I_{\max} = I_0(1 + 1/\sqrt{2})^2, \quad I_{\min} = I_0(1 - 1/\sqrt{2})^2.$$

Consequently,

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right)^2 = (3 + 2\sqrt{2})^2 = 17 + 12\sqrt{2} \simeq 33.97.$$

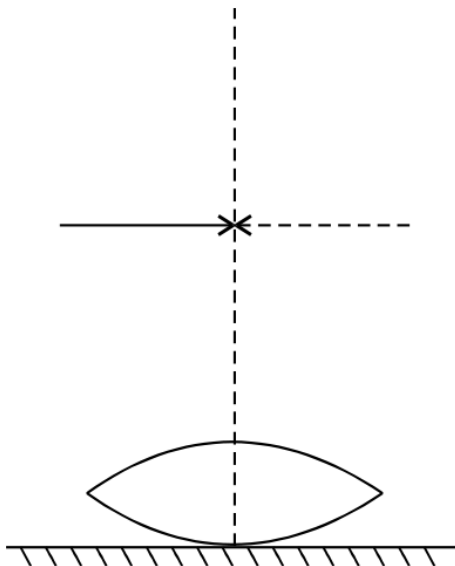
The minimum is nonzero because the slit amplitudes are unequal; their fields cannot cancel completely.

Step 3: (b) Describe white-light fringes In the usual white-light Young arrangement, the zero-path-difference central fringe is white because all wavelengths interfere constructively there. A few coloured fringes occur symmetrically on either side: violet fringes lie nearer the centre and red fringes farther away, since fringe width increases with wavelength. At larger distances different orders and colours overlap, making the fringes indistinct. If the glass plate of part (a) is retained, its optical path delay shifts the pattern; the white-fringe description refers to zero total optical path difference, when achievable.

Step 4: Conclusion The ratio is $I_{\max} : I_{\min} = (17 + 12\sqrt{2}) : 1$, approximately 34 : 1. White light gives a white central fringe and a few coloured side fringes that become indistinct through overlap.

Quick Tip to Solve: Convert intensity to amplitude by taking a square root before using the sum-and-difference formula.

17. A symmetric biconvex lens of radius of curvature R and made of glass of refractive index 1.5, is placed on a layer of liquid placed on top of a plane mirror as shown in the figure. An optical needle with its tip on the principal axis of the lens is moved along the axis until its real, inverted image coincides with the needle itself. The distance of the needle from the lens is measured to be x . On removing the liquid layer and repeating the experiment, the distance is found to be y . Obtain the expression for the refractive index of the liquid in terms of x and y . [3 marks]



Solution:

Concept:

- In autocollimation, an object at the focal point sends a parallel beam to the plane mirror; the returned beam forms an inverted image at the object position.
- Thin lenses in contact have additive powers: $1/F = 1/f_g + 1/f_l$. For a lens in air, $1/f = (\mu - 1)(1/R_1 - 1/R_2)$.

Step 1: Identify the measured focal lengths When the real inverted image coincides with the optical needle, the rays after passing through the lens system are parallel. The plane mirror reverses their direction and the lens system brings them back to the original focal plane. Therefore the distance y , measured without liquid, is the focal length f_g of the glass lens. With the liquid present, x is the focal length F of the glass-liquid combination. This is the thin-lens approximation used in the experiment.

Step 2: Find the glass lens radius and liquid lens power For the symmetric biconvex glass lens, take $R_1 = +R$, $R_2 = -R$ and $\mu_g = 1.5$. Hence

$$\frac{1}{y} = (1.5 - 1) \left(\frac{1}{R} + \frac{1}{R} \right) = \frac{1}{R}, \quad R = y.$$

The liquid between the convex underside of the glass and the plane mirror forms a plano-concave lens. For downward propagation its curved upper surface has radius $-R$, while its plane surface has

Quick Tip to Solve: The liquid layer acts as a plano-concave lens, so subtract its power; using a positive power reverses the result.

infinite radius. Thus, if its index is μ_l ,

$$\frac{1}{f_l} = (\mu_l - 1) \left(-\frac{1}{R} - 0 \right) = -\frac{\mu_l - 1}{R}.$$

The negative power is expected for a concave liquid lens.

Step 3: Combine the powers and isolate the index Adding the two thin-lens powers gives

$$\frac{1}{x} = \frac{1}{y} - \frac{\mu_l - 1}{R}.$$

Substituting $R = y$,

$$\frac{1}{x} = \frac{2 - \mu_l}{y}, \quad \mu_l = 2 - \frac{y}{x}.$$

For a liquid of index greater than unity but below 2, the combined positive power is smaller than the glass-lens power, so $x > y$, consistent with the result. The distances must be expressed in the same units.

Step 4: Conclusion The liquid refractive index is $\mu_l = 2 - y/x$, with x the coincidence distance with liquid and y that without liquid.

Quick Tip to Solve: The liquid layer acts as a plano-concave lens, so subtract its power; using a positive power reverses the result.

18. (a) State Bohr's postulate to define stable orbits in hydrogen atom. How does de Broglie's hypothesis explain the stability of these orbits ?

(b) A hydrogen atom initially in the ground state absorbs a photon which excites it to the $n = 4$ level. Estimate the frequency of the photon. [3 marks]

Solution:

Concept:

- Bohr's allowed stationary orbits have quantised electron angular momentum: $m_e v r = n \hbar = n h / (2\pi)$, where $n = 1, 2, 3, \dots$
- The electron's de Broglie wavelength is $\lambda = h / (m_e v)$. A stationary wave fits around the orbit when $2\pi r = n \lambda$.
- Hydrogen energy levels are $E_n = -13.6/n^2$ eV. Absorption requires photon energy $h\nu = E_f - E_i$.

Step 1: (a) State the condition for stationary orbits An electron can revolve only in certain permitted orbits without radiating energy. Its angular momentum in such an orbit is an integral multiple of $h/(2\pi)$:

$$m_e v r = \frac{n h}{2\pi}.$$

Each permitted orbit therefore has a definite energy. Radiation is absorbed or emitted when the electron changes between these stationary states, rather than continuously while it remains in an allowed orbit.

Quick Tip to Solve: For excitation from the hydrogen ground state, use the positive energy gap $13.6(1 - 1/n^2)$ eV, then convert eV to joules before dividing by h .

Step 2: Explain the condition using matter waves The electron is associated with a de Broglie wave. A stable orbit must accommodate a whole number of wavelengths around its circumference, so that the wave joins onto itself in phase and forms a standing wave:

$$2\pi r = n\lambda = n \frac{h}{m_e v} \implies m_e v r = \frac{nh}{2\pi}.$$

Thus de Broglie's standing-wave condition reproduces Bohr's quantisation rule. A circumference containing a nonintegral number of wavelengths cannot sustain this stationary wave because successive portions do not reinforce in phase.

Step 3: (b) Calculate the energy absorbed and the photon frequency The initial level is $n_i = 1$ and the final level is $n_f = 4$. Hence

$$\begin{aligned}\Delta E &= E_4 - E_1 = 13.6 \left(1 - \frac{1}{16}\right) \text{ eV} \\ &= 12.75 \text{ eV} = 12.75(1.602 \times 10^{-19}) \text{ J} \\ &= 2.043 \times 10^{-18} \text{ J}.\end{aligned}$$

Using $h = 6.626 \times 10^{-34} \text{ J s}$,

$$\nu = \frac{\Delta E}{h} = \frac{2.043 \times 10^{-18}}{6.626 \times 10^{-34}} \text{ Hz} = 3.08 \times 10^{15} \text{ Hz}.$$

Step 4: Conclusion Stable orbits obey $m_e v r = nh/(2\pi)$, equivalently $2\pi r = n\lambda$. The photon absorbed in excitation from the ground state to $n = 4$ has frequency approximately $3.08 \times 10^{15} \text{ Hz}$.

Quick Tip to Solve: For excitation from the hydrogen ground state, use the positive energy gap $13.6(1 - 1/n^2) \text{ eV}$, then convert eV to joules before dividing by h .

19. (a) Explain the processes of nuclear fission and nuclear fusion by using the plot of binding energy per nucleon (BE/A) versus the mass number A .

(b) A radioactive isotope has a half-life of 10 years. How long will it take for the activity to reduce to 3.125% ? [3 marks]

Solution:

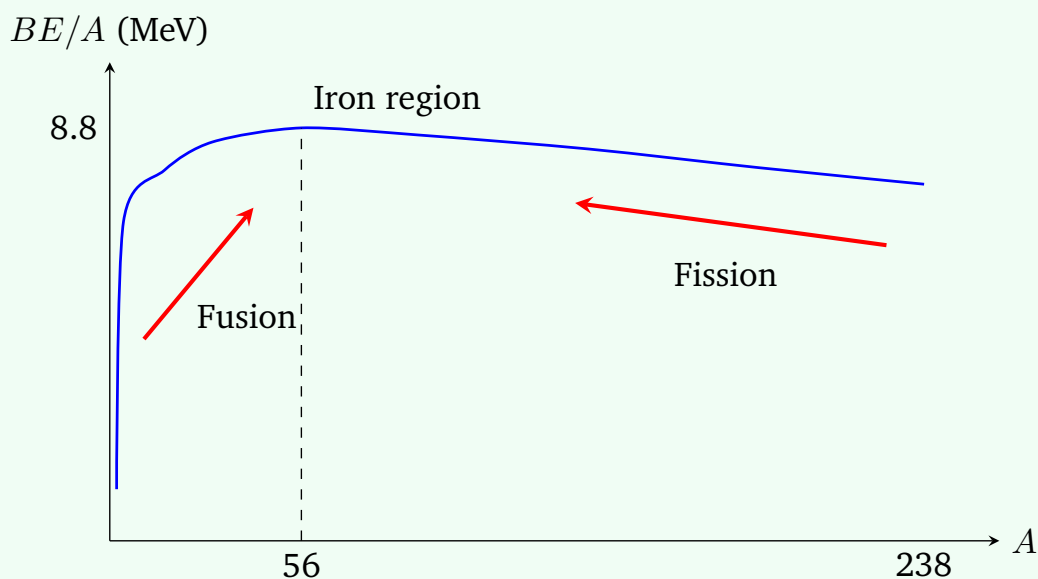
Concept:

- Binding energy per nucleon BE/A measures average nuclear binding. Larger BE/A indicates stronger binding and greater nuclear stability.
- The curve rises rapidly for light nuclei, reaches approximately 8.8 MeV near $A = 56$, and falls slowly for heavy nuclei. Energy is released when the total binding energy increases.
- Activity $\mathcal{A} = \lambda N$ obeys $\mathcal{A}/\mathcal{A}_0 = (1/2)^{t/T_{1/2}}$, with $T_{1/2}$ the half-life.

Step 1: (a) Interpret the binding-energy curve The schematic plot shows that medium-mass nuclei near iron have the highest binding energy per nucleon. Moving towards this region from either end of

Quick Tip to Solve: Convert the remaining percentage to a fraction first: $3.125\% = 1/32 = 2^{-5}$, so the elapsed time is exactly five half-lives.

the curve gives more tightly bound products. For a nuclear reaction the released energy is the increase in total binding energy, equal to the corresponding decrease in mass multiplied by c^2 . The curve therefore explains why both fusion of light nuclei and fission of very heavy nuclei can release energy.



Schematic binding energy per nucleon curve, showing the energy-releasing directions for fusion and fission.

Step 2: Explain nuclear fusion and fission In fusion, two light nuclei combine to form a heavier nucleus. On the rising left-hand part of the curve, the product generally has greater BE/A than the reactants, so its total binding energy is greater and energy is released. For example, fusion of hydrogen isotopes produces a more tightly bound helium nucleus.

In fission, a very heavy nucleus, such as uranium, splits into two medium-mass fragments, usually with accompanying neutrons. The fragments lie closer to the high part of the curve and have greater binding energy per nucleon than the parent. Their increased total binding energy appears as released energy, mainly kinetic energy of the fragments. The arrows in the plot indicate these movements towards higher binding, not the trajectory of a particular reaction.

Step 3: (b) Find the number of half-lives The final activity is 3.125% of its initial value. Thus

$$\frac{\mathcal{A}}{\mathcal{A}_0} = \frac{3.125}{100} = 0.03125 = \frac{1}{32} = \left(\frac{1}{2}\right)^5.$$

Comparing with $(1/2)^{t/T_{1/2}}$ gives $t/T_{1/2} = 5$. With the given half-life $T_{1/2} = 10$ years,

$$t = 5 \times 10 \text{ years} = 50 \text{ years}.$$

Activity halves together with the undecayed population because the decay constant of this isotope remains fixed.

Step 4: Conclusion Fusion of light nuclei and fission of heavy nuclei release energy by producing nuclei with higher binding energy per nucleon. The isotope's activity falls to 3.125% after 50 years, or five half-lives.

Quick Tip to Solve: Convert the remaining percentage to a fraction first: $3.125\% = 1/32 = 2^{-5}$, so the elapsed time is exactly five half-lives.

20. (a) A student wants to use two p-n junction diodes to convert alternating current into direct current. Draw the labelled circuit diagram she would use and explain how it works.

(b) Give the truth table and circuit symbol for NAND gate.

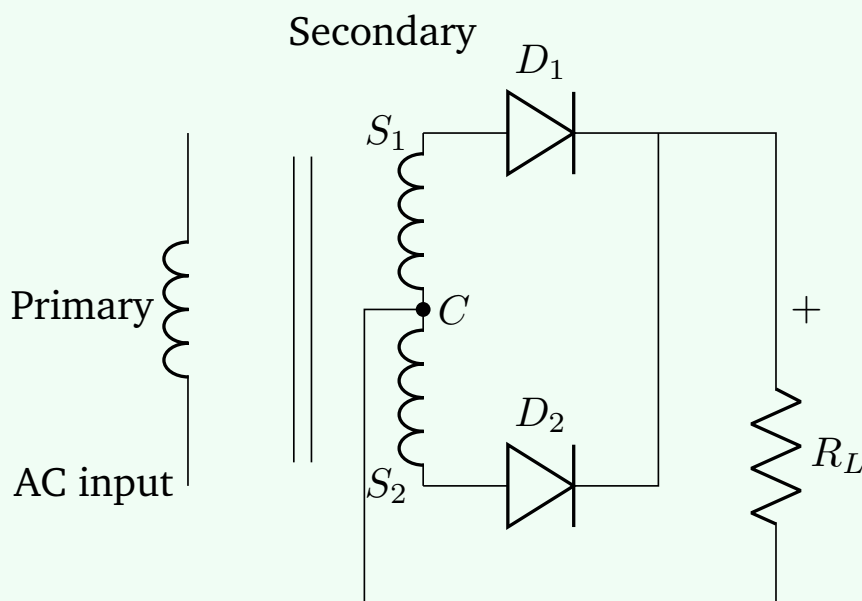
[3 marks]

Solution:

Concept:

- A p-n junction diode conducts when forward biased and blocks current when reverse biased. Two diodes and a centre-tapped transformer form a full-wave rectifier.
- A NAND gate gives the complement of the AND operation: $Y = \overline{AB}$. Its output is low only when both inputs are high.

Step 1: (a) Connect the two-diode full-wave rectifier Connect the anodes of D_1 and D_2 to the two ends S_1 and S_2 of a centre-tapped transformer secondary. Join their cathodes to the positive output terminal. Connect the load resistance R_L between this junction and the centre tap C , which is the return terminal. The transformer primary receives the alternating supply; the two secondary halves have opposite instantaneous polarities relative to C .



Two-diode full-wave rectifier; C is the centre tap of the secondary.

Step 2: Explain conduction in both half-cycles During the half-cycle in which S_1 is positive relative to C , D_1 is forward biased and D_2 is reverse biased. Current flows from S_1 , through D_1 and R_L , to C . In the next half-cycle S_2 becomes positive: D_2 conducts and D_1 blocks. Current then flows from S_2 , through D_2 and the same load, to C . Thus current through the load has the same direction in both half-cycles. The output is pulsating direct current, with ripple frequency twice the supply frequency; without a filter it is not constant in magnitude.

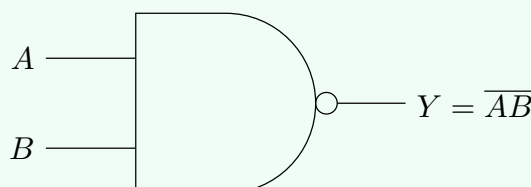
Step 3: (b) Write the NAND truth table and symbol The NAND symbol is an AND-gate shape followed

Quick Tip to Solve: Trace each conducting diode back to the centre tap: both paths must pass through the load in the same direction.

by a small inversion circle at its output. The circle denotes logical complementation. Its truth table is

A	B	$Y = \overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

In particular, even a single low input makes $AB = 0$, so the NAND output is high; two high inputs alone produce a low output.



NAND gate symbol with the output inversion circle.

Step 4: Conclusion (a) The labelled centre-tapped full-wave rectifier supplies unidirectional load current in both half-cycles. (b) For a NAND gate, $Y = \overline{AB}$, with outputs 1, 1, 1, 0 for inputs 00, 01, 10, 11, respectively.

Quick Tip to Solve: Trace each conducting diode back to the centre tap: both paths must pass through the load in the same direction.

21. Draw the typical input and output characteristics of an n-p-n transistor in CE configuration. Show how these characteristics can be used to determine (a) the input resistance (r_i), and (b) current amplification factor (β). [3 marks]

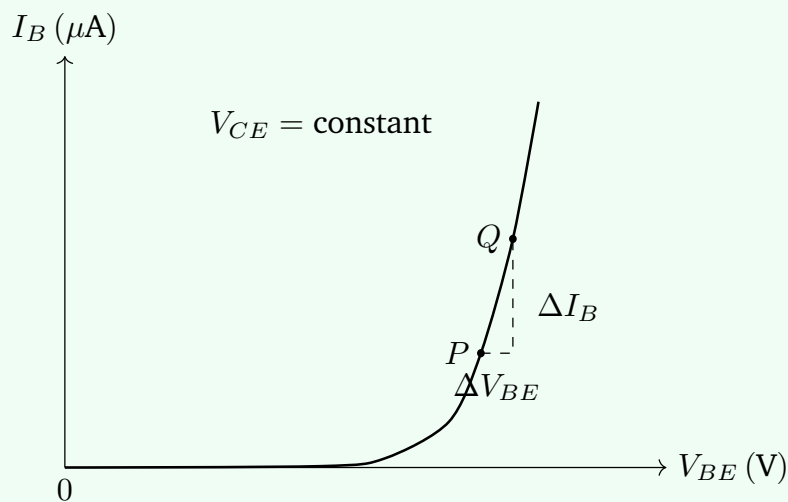
Solution:

Concept:

- In the common-emitter (CE) configuration, the emitter is common to the base input circuit and collector output circuit. The input is described by V_{BE} , I_B , and the output by V_{CE} , I_C .
- Small-signal parameters are determined from finite changes near an operating point: $r_i = (\Delta V_{BE} / \Delta I_B)_{V_{CE}}$ and $\beta = (\Delta I_C / \Delta I_B)_{V_{CE}}$.

Step 1: Draw the input characteristic Plot base current I_B on the vertical axis against base-emitter voltage V_{BE} on the horizontal axis, maintaining a fixed collector-emitter voltage V_{CE} . The curve resembles that of a forward-biased diode: the base current is very small at low voltage and increases rapidly once the base-emitter junction conducts appreciably. For a silicon transistor this rapid rise typically occurs near 0.6 to 0.7 V. The indicated points P, Q lie near the operating point on the same constant- V_{CE} curve.

Quick Tip to Solve: For beta, take readings on one vertical line through two output curves; for input resistance, use two nearby points on one input curve.



Typical CE input characteristic at fixed collector-emitter voltage.

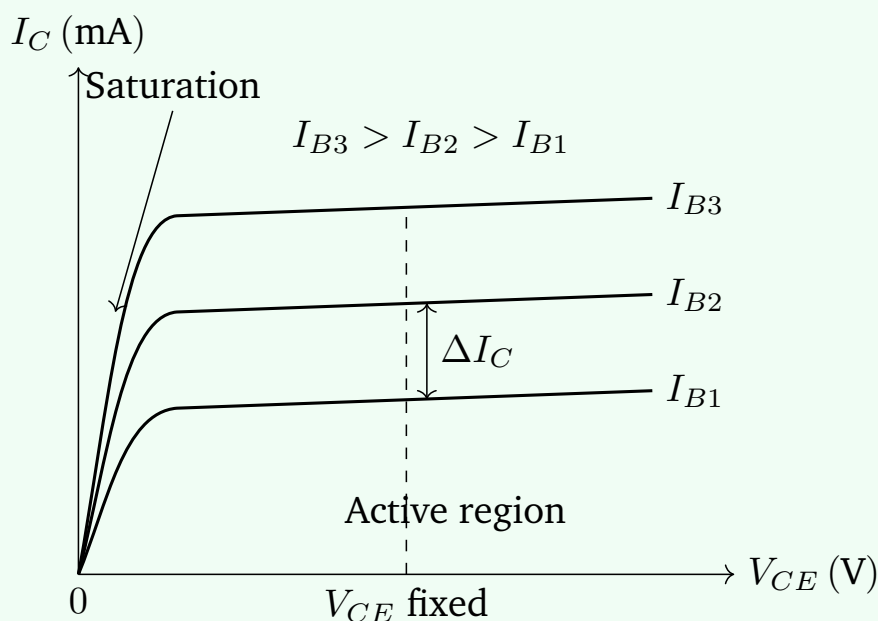
Step 2: (a) Determine input resistance Read the small voltage change and the corresponding base-current change between P and Q . The dynamic input resistance is

$$r_i = \left. \frac{\Delta V_{BE}}{\Delta I_B} \right|_{V_{CE} \text{ constant}}.$$

Equivalently, it is the reciprocal of the local slope of the I_B -versus- V_{BE} curve. Use volts and amperes to obtain ohms. This is a resistance for small changes about the chosen operating point, rather than the ratio V_{BE}/I_B from the origin.

Step 3: Draw the output characteristics Plot collector current I_C against collector-emitter voltage V_{CE} for several fixed values of base current I_B . Each curve rises steeply in the low-voltage saturation region and then becomes nearly horizontal in the active region. Larger fixed I_B gives a higher curve. The slight upward slope in the active region is typical of a real transistor. At approximately zero base current, the transistor is near cutoff apart from small leakage current.

Quick Tip to Solve: For beta, take readings on one vertical line through two output curves; for input resistance, use two nearby points on one input curve.



Typical CE output curves; gain is found at the same V_{CE} in the active region.

Step 4: (b) Determine current amplification factor Select a common V_{CE} in the active region and read the collector currents on two nearby curves whose base currents are I_{B1} and I_{B2} . Keeping this voltage fixed gives

$$\beta = \left. \frac{\Delta I_C}{\Delta I_B} \right|_{V_{CE} \text{ constant}} = \frac{I_{C2} - I_{C1}}{I_{B2} - I_{B1}}.$$

Convert both current differences to the same unit before division. The resulting small-signal current gain is dimensionless; it expresses the collector-current change caused by a small base-current change.

Step 5: Conclusion From the input curve, $r_i = \Delta V_{BE} / \Delta I_B$ at fixed V_{CE} . From the output family in the active region, $\beta = \Delta I_C / \Delta I_B$, again at fixed V_{CE} .

Quick Tip to Solve: For beta, take readings on one vertical line through two output curves; for input resistance, use two nearby points on one input curve.

22. (a) Give three reasons why modulation of a message signal is necessary for long distance transmission.

(b) Show graphically an audio signal, a carrier wave and an amplitude modulated wave. [3 marks]

Solution:

Concept:

- Modulation transfers a low-frequency message onto a high-frequency carrier. In amplitude modulation, the carrier amplitude varies with the instantaneous message voltage.
- For electromagnetic waves, $\lambda = c/f$; a practical transmitting antenna has a length of order $\lambda/4$. The carrier frequency f_c is much larger than the message frequency f_m .

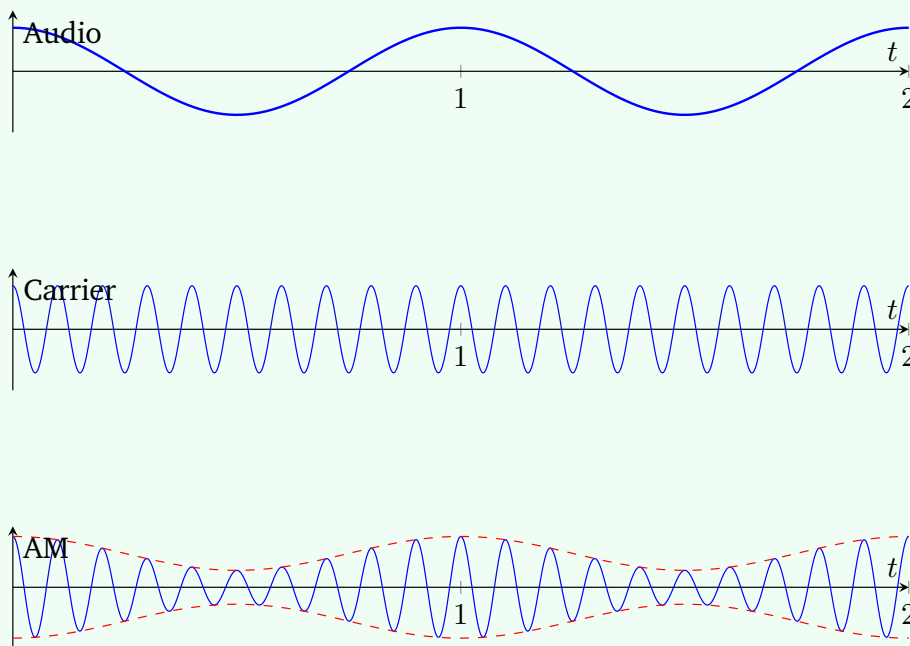
Quick Tip to Solve: Draw the slow audio curve first, then use its shape for the two envelopes of the rapidly oscillating AM wave.

Step 1: (a) Three reasons for modulation First, modulation makes the antenna size practical. An audio signal of frequency 1 kHz has wavelength 3×10^5 m, requiring a quarter-wave antenna about 75 km long. A much higher carrier frequency gives a much smaller wavelength and antenna length. Second, a high-frequency carrier permits efficient radiation of energy: for a fixed short antenna, radiated power increases as the ratio of antenna length to wavelength increases. Direct radiation of the long-wavelength audio signal is inefficient and cannot provide useful long-distance transmission. Third, assigning different carrier frequencies to different transmitters prevents their message signals from mixing indiscriminately. A receiver can tune to one carrier and select the required station, although the original audio-frequency ranges of the stations overlap.

Step 2: (b) Audio, carrier and amplitude-modulated waveforms Let the audio signal be $m(t) = A_m \cos(2\pi f_m t)$ and the carrier be $c(t) = A_c \cos(2\pi f_c t)$. An amplitude-modulated signal is

$$s(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t),$$

where $\mu = A_m/A_c$ is the modulation index for the chosen scaling. The graph uses $\mu = 0.5$, so the envelope never crosses zero. The rapid oscillations retain the carrier frequency; their upper and lower envelopes follow the audio waveform. All three plots use the same time scale, with the carrier making many cycles during one audio cycle.



Audio signal, carrier and AM wave on a common time scale; dashed curves show the AM envelopes.

Step 3: Conclusion Modulation enables a practical antenna, efficient radiation and separation of different stations. In AM, the envelope reproduces the message while the carrier oscillation frequency remains unchanged.

Quick Tip to Solve: Draw the slow audio curve first, then use its shape for the two envelopes of the rapidly oscillating AM wave.

Section D (4 marks)

23. The teachers of Geeta's school took the students on a study trip to a power generating station, located nearly 200 km away from the city. The teacher explained that electrical energy is transmitted over such a long distance to their city, in the form of alternating current (ac) raised to a high voltage. At the receiving end in the city, the voltage is reduced to operate the devices. As a result, the power loss is reduced. Geeta listened to the teacher and asked questions about how the ac is converted to a higher or lower voltage.

(a) Name the device used to change the alternating voltage to a higher or lower value. State one cause for power dissipation in this device.

(b) Explain with an example, how power loss is reduced if the energy is transmitted over long distances as an alternating current rather than a direct current.

(c) Write two values each shown by the teachers and Geeta.

[4 marks]

Solution:

Concept:

- A transformer changes alternating voltage by mutual induction. For an ideal transformer, $V_s/V_p = N_s/N_p$, where N_s, N_p are secondary and primary turns.
- Transmission-line heating is $P_{\text{loss}} = I^2 R$. At fixed transmitted power P , increasing the voltage reduces current and therefore the heating loss.

Step 1: (a) Device and a cause of dissipation The device is a transformer. A step-up transformer, with $N_s > N_p$, raises the alternating voltage at the generating station; a step-down transformer, with $N_s < N_p$, lowers it near the consumer. One cause of power dissipation is Joule heating in its windings: the copper wire has finite resistance, so the primary and secondary currents produce $I^2 R$ heat. Eddy-current heating in the iron core is another possible cause. The transformer operates through a changing magnetic flux linking the coils; a steady direct current does not provide this continuously changing flux.

Step 2: (b) Why high-voltage ac reduces transmission loss In the conventional transformer-based system described, ac can readily be stepped up to a high voltage before transmission and stepped down afterwards. For a resistive load or unity power factor, $P = VI$, so the line loss is

$$P_{\text{loss}} = \left(\frac{P}{V}\right)^2 R.$$

For example, transmit 100 kW through a line of resistance 10 Ω . At 1 kV, the current is 100 A, giving a loss $100^2 \times 10 = 100$ kW. If the voltage is raised to 10 kV, the current becomes 10 A, giving only $10^2 \times 10 = 1$ kW. Increasing voltage tenfold reduces the loss by a factor of one hundred. Ordinary transformers cannot step up steady dc, so low-voltage dc transmitted directly would carry the larger current. The advantage in this example comes from high voltage and low current; ac does not intrinsically have less resistive loss than dc at the same current. High-voltage dc transmission using suitable conversion equipment is also possible.

Step 3: (c) Values shown by the teachers and Geeta The teachers show dedication to students' education by arranging a visit that connects classroom physics with a working power station. They

Quick Tip to Solve: Keep transmitted power fixed when comparing losses: multiplying voltage by a factor divides the line loss by the square of that factor.

also show concern for energy conservation by explaining how transmission losses can be reduced. Geeta shows curiosity by asking how the voltage is changed and attentiveness by listening carefully to the explanation before seeking further understanding.

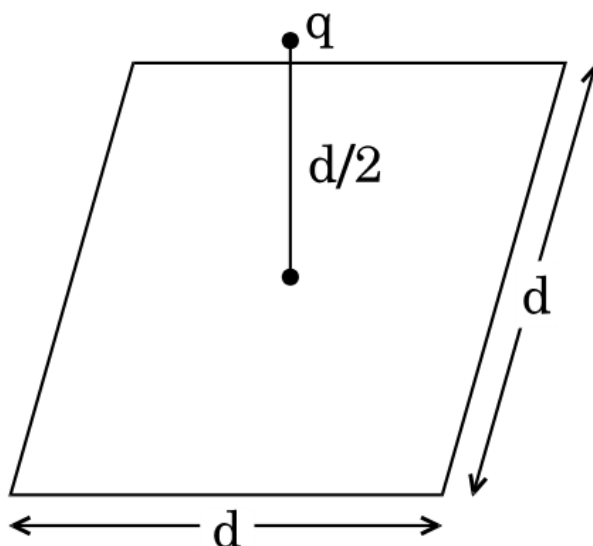
Step 4: Conclusion Transformers raise and lower ac voltage; winding resistance causes heat loss. High-voltage transmission reduces current and I^2R losses. The teachers display educational dedication and concern for energy conservation, while Geeta displays curiosity and attentiveness.

Quick Tip to Solve: Keep transmitted power fixed when comparing losses: multiplying voltage by a factor divides the line loss by the square of that factor.

Section E (15 marks)

24. (a) Define electric flux. Is it a scalar or a vector quantity? A point charge q is at a distance of $d/2$ directly above the centre of a square of side d , as shown in the figure. Use Gauss' law to obtain the expression for the electric flux through the square.

(b) If the point charge is now moved to a distance ' d ' from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected. [5 marks]



Solution:

Concept:

- Electric flux through an oriented surface is $\Phi_E = \int_S \mathbf{E} \cdot d\mathbf{A}$, where $d\mathbf{A}$ is normal to the surface. For a uniform field on a plane, $\Phi_E = EA \cos \theta$. It measures the field passing through the surface and is a signed scalar quantity.
- Gauss' law states $\oint \mathbf{E} \cdot d\mathbf{A} = q_{\text{enclosed}}/\epsilon_0$, where ϵ_0 is the permittivity of free space. It applies to a closed surface, so the given open square must first be made part of a suitable closed surface.
- A point charge at the centre of a cube has identical geometrical relations to all six faces. Their outward electric fluxes are therefore equal, even though the field is not uniform over any individual face.

Quick Tip to Solve: Compare the perpendicular distance with half the square's side: if they are equal, complete a cube with the charge at its centre and divide its total flux by six.

Step 1: (a) Define flux and its nature For a small surface element, $d\Phi_E = \mathbf{E} \cdot d\mathbf{A}$; adding these contributions gives the surface integral in the concept block. The dot product makes flux a scalar, not a vector. Its SI unit is Nm^2C^{-1} . The sign depends on the chosen surface normal: field along that normal gives positive flux, while field opposite to it gives negative flux.

Step 2: (a) Construct a closed Gaussian cube Regard the given square of side d as one face of a cube of side d , constructed on the side containing the charge. The centre of this cube lies on the normal through the centre of the square, at distance $d/2$ from that face. Thus the given point charge lies exactly at the cube's centre. No other charge is enclosed, and Gauss' law gives

$$\Phi_{\text{cube}} = \frac{q}{\varepsilon_0}.$$

This construction is essential: Gauss' law cannot be applied directly to the open square as if it were a complete closed surface.

Step 3: (a) Use symmetry to select one face Rotating the cube about its centre interchanges its six faces without changing the centrally placed charge or its electric field. Consequently, each face receives the same integrated outward flux. With the square's normal chosen outward from this cube,

$$6\Phi_{\text{square}} = \frac{q}{\varepsilon_0}, \quad \Phi_{\text{square}} = \frac{q}{6\varepsilon_0}.$$

This is the signed expression for charge q ; the flux magnitude is $|q|/(6\varepsilon_0)$. Reversing the square's normal reverses its sign. We have used equality of the integrated fluxes, not an assumption that the electric field is constant over a face.

Step 4: (b) Repeat the construction for the enlarged square The new side is $2d$, and the charge is a distance d from its centre along the same perpendicular. Build a cube of side $2d$ on that square. Its centre is at a distance $(2d)/2 = d$ from the face, so once again the charge lies at the cube's centre. The total enclosed charge remains q , and symmetry again divides its total flux equally among six faces. Hence

$$\Phi'_{\text{square}} = \frac{q}{6\varepsilon_0}.$$

Equivalently, the enlarged square subtends the same solid angle at the charge: both all linear dimensions and the perpendicular distance have doubled. Corresponding field strengths decrease by a factor of four while corresponding area elements increase by a factor of four, leaving the flux integral unchanged.

Step 5: Conclusion Electric flux is a signed scalar. For the outward normal of the constructed cube, the flux through the original square is $q/(6\varepsilon_0)$. Doubling the side and the charge's perpendicular distance leaves this flux unchanged.

Quick Tip to Solve: Compare the perpendicular distance with half the square's side: if they are equal, complete a cube with the charge at its centre and divide its total flux by six.

24. (a) Use Gauss' law to derive the expression for the electric field ($\vec{E}$) due to a straight uniformly charged infinite line of charge density λ C/m.
 (b) Draw a graph to show the variation of E with perpendicular distance r from the line of charge.
 (c) Find the work done in bringing a charge q from perpendicular distance r_1 to r_2 ($r_2 > r_1$). [5 marks]

Solution:**Concept:**

- Gauss' law states $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where ϵ_0 is the permittivity of free space.
- Cylindrical symmetry makes the field of an infinite uniform line radial, independent of axial position and azimuth, and dependent only on perpendicular distance r .
- For slow displacement of a test charge, external work is the potential-energy change: $W_{\text{ext}} = q\Delta V = -q \int \vec{E} \cdot d\vec{r}$. Work done by the electric field has the opposite sign.

Step 1: (a) Select a cylindrical Gaussian surface Take a cylinder of radius r and length L , coaxial with the charged line. Its enclosed charge is $Q_{\text{enc}} = \lambda L$. Rotational symmetry rules out an azimuthal field component, and reflection symmetry along the infinite line rules out an axial component. Hence $\vec{E} = E_r(r)\hat{r}$. On the curved surface the signed radial component E_r is constant and parallel to its outward area vector. On the two end caps the radial field is perpendicular to the axial area vectors, so their flux is zero. This use of symmetry is essential: Gauss' law alone does not make an arbitrary surface field uniform.

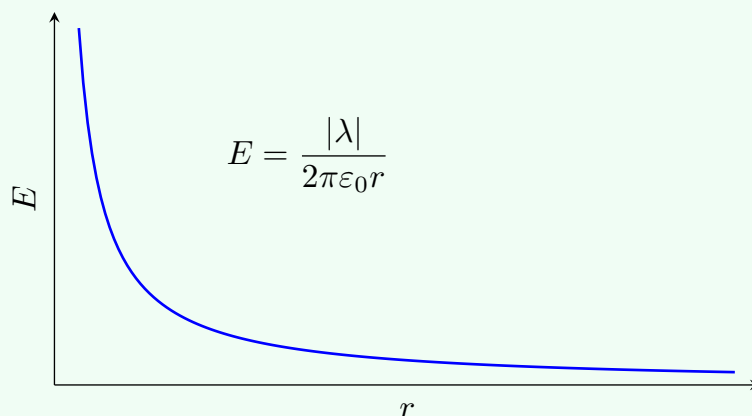
Step 2: Apply Gauss' law and obtain the field Only the curved area $2\pi rL$ contributes. Thus

$$E_r(2\pi rL) = \frac{\lambda L}{\epsilon_0}, \quad \vec{E}(r) = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r}.$$

The length L cancels, as expected for an infinite line. The magnitude is $E = |\lambda|/(2\pi\epsilon_0 r)$. The field points outward for positive λ , inward for negative λ , and its units are N C^{-1} . The result applies outside the ideal line, at $r > 0$.

Step 3: (b) Plot the distance dependence For fixed nonzero line-charge density, $E \propto 1/r$. Therefore the magnitude-versus-distance curve is a decreasing rectangular hyperbola in the first quadrant. Doubling the distance halves the field. In the ideal infinite-line model, the field grows without bound as $r \rightarrow 0^+$ and tends to zero as $r \rightarrow \infty$; the axes are asymptotes. The sketch plots magnitude, so it applies to either sign of charge density.

Quick Tip to Solve: Use a coaxial cylinder so the end-cap flux vanishes, and check the work sign by asking whether the field assists the outward displacement.



Electric-field magnitude decreases inversely with perpendicular distance.

Step 4: (c) Integrate the work from the first distance to the second Choose a radial path from r_1 to r_2 ; electrostatic work is path independent. The signed radial field gives

$$\begin{aligned} V(r_2) - V(r_1) &= - \int_{r_1}^{r_2} \frac{\lambda}{2\pi\epsilon_0 r} dr \\ &= - \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right), \\ W_{\text{ext}} &= q[V(r_2) - V(r_1)] \\ &= - \frac{q\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right). \end{aligned}$$

Here 'bringing' is interpreted as work by an external agent in a slow move with no net kinetic-energy change. Work by the electric field is instead $W_{\text{field}} = q\lambda \ln(r_2/r_1)/(2\pi\epsilon_0)$. Since $r_2 > r_1$, the logarithm is positive: for $q\lambda > 0$, outward motion is assisted by the field and external work is negative. Only a potential difference is needed; setting potential zero at infinity is inappropriate for an infinite line.

Step 5: Conclusion The line field is $\vec{E} = \lambda\hat{r}/(2\pi\epsilon_0 r)$, and its magnitude follows a decreasing $1/r$ hyperbola. The external work to bring q from r_1 to r_2 is $-q\lambda \ln(r_2/r_1)/(2\pi\epsilon_0)$; field work is its negative.

Quick Tip to Solve: Use a coaxial cylinder so the end-cap flux vanishes, and check the work sign by asking whether the field assists the outward displacement.

25. (a) State the principle of an ac generator and explain its working with the help of a labelled diagram. Obtain the expression for the emf induced in a coil having N turns each of cross-sectional area A , rotating with a constant angular speed ' ω ' in a magnetic field $\vec{B}$, directed perpendicular to the axis of rotation.

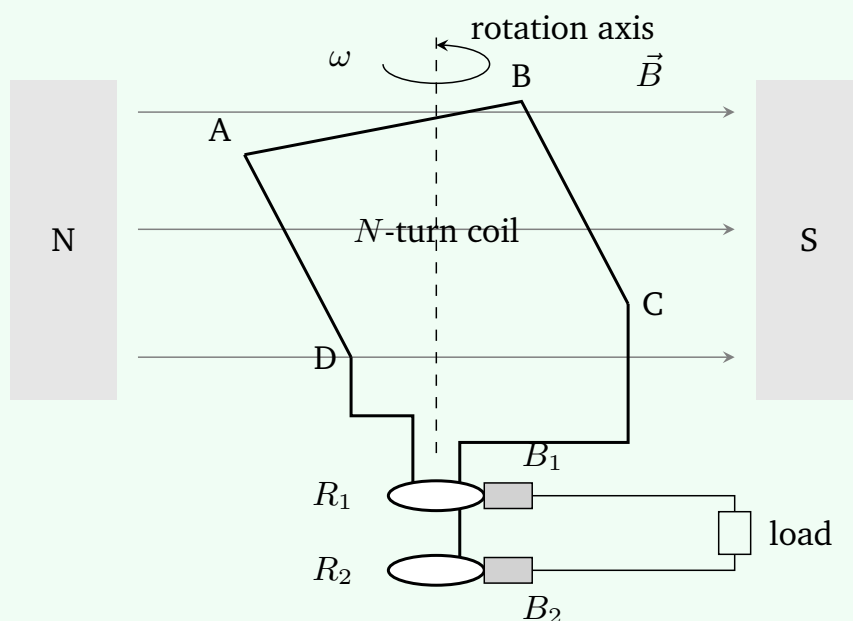
(b) An aeroplane is flying horizontally from west to east with a velocity of 900 km/hour. Calculate the potential difference developed between the ends of its wings having a span of 20 m. The horizontal component of the Earth's magnetic field is 5×10^{-4} T and the angle of dip is 30° . [5 marks]

Solution:

Concept:

- Faraday's law: the induced emf is $\varepsilon = -d\lambda/dt$, where $\lambda = N\Phi$ is the flux linkage of an N -turn coil. The negative sign expresses Lenz's law.
- For a uniform field, $\Phi = BA \cos \theta$, where A is the area of one turn and θ is the angle between its normal and $\vec{B}$.
- Motional emf across a conductor is $\varepsilon = (\vec{v} \times \vec{B}) \cdot \vec{l}$. The field component perpendicular to both the velocity and the conductor determines the potential difference.

Step 1: (a) Principle and construction of the ac generator An ac generator works on electromagnetic induction: changing the magnetic flux linked with a rotating coil induces an emf. Mechanical energy supplied to rotate the coil is converted into electrical energy. A rectangular armature coil $ABCD$, containing N turns, rotates between the north and south poles of a magnet. Its axis of rotation is perpendicular to the magnetic field. The two ends of the coil are connected separately to two continuous slip rings R_1 and R_2 . Stationary carbon brushes B_1 and B_2 touch these rings and connect the coil to an external load. The labelled schematic shows these essential parts; the leads from the two coil ends remain electrically separate.



Labelled schematic of an ac generator: rotating coil, separate slip rings, stationary brushes and external load.

Step 2: (a) Working and emf derivation Let the coil normal be parallel to the field at $t = 0$. At time t ,

Quick Tip to Solve: For horizontal flight, first draw the velocity and wing-span directions: only the vertical field contributes, so use $B_V = B_H \tan \delta$.

its angle with the field is $\theta = \omega t$. The flux linkage and the induced emf are therefore

$$\begin{aligned}\lambda(t) &= NBA \cos(\omega t), \\ \varepsilon(t) &= -\frac{d}{dt}[NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t).\end{aligned}$$

The peak emf is $\varepsilon_0 = NBA\omega$, and its frequency is $f = \omega/(2\pi)$. When the coil normal is parallel to the field, the flux is maximum but its rate of change is zero, so the emf is zero. A quarter-turn later, the normal is perpendicular to the field: the flux is zero but its rate of change is maximum, giving peak emf. After half a revolution, the emf changes sign because the coil sides reverse their direction of motion through the field. Continuous slip rings maintain connection to the load while allowing this polarity reversal, so the output is alternating. If the circuit is closed, an alternating current flows through the load.

Step 3: (b) Select the relevant component of the Earth's field The aeroplane moves eastward and its wings extend horizontally along the north-south direction. The vertical component of the Earth's field produces a magnetic force along the wings. The horizontal component produces a vertical force and therefore does not contribute to the potential difference between the wing tips. With horizontal component B_H and dip angle δ ,

$$B_V = B_H \tan \delta = 5 \times 10^{-4} \tan 30^\circ = \frac{5 \times 10^{-4}}{\sqrt{3}} \text{ T}.$$

Charges separate until the electric force balances the magnetic force along the span. Thus the magnitude of the wing-tip potential difference is $\Delta V = B_V lv$, where l is the full wing span.

Step 4: (b) Convert the speed and calculate In SI units the speed is

$$v = 900 \times \frac{1000}{3600} = 250 \text{ m s}^{-1}.$$

Taking $l = 20 \text{ m}$,

$$\Delta V = \frac{5 \times 10^{-4}}{\sqrt{3}} \times 20 \times 250 = \frac{2.5}{\sqrt{3}} \text{ V} \simeq 1.44 \text{ V}.$$

This uses the vertical component derived from the supplied horizontal component; treating the given B_H as the total or vertical field would give an incorrect result.

Step 5: Conclusion (a) The generator produces $\varepsilon(t) = NBA\omega \sin(\omega t)$, with peak emf $NBA\omega$. (b) The potential difference between the wing tips is 1.44 V.

Quick Tip to Solve: For horizontal flight, first draw the velocity and wing-span directions: only the vertical field contributes, so use $B_V = B_H \tan \delta$.

25. A device X is connected across an ac source of voltage $V = V_0 \sin \omega t$. The current through X is given as $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$.

- (a) Identify the device X and write the expression for its reactance.
 (b) Draw graphs showing variation of voltage and current with time over one cycle of ac, for X .
 (c) How does the reactance of the device X vary with frequency of the ac? Show this variation graphically.
 (d) Draw the phasor diagram for the device X .

[5 marks]

Solution:

Concept:

- For a capacitor of capacitance C , the charge is $q = CV$ and the charging current is $I = dq/dt = C dV/dt$.
- Capacitive reactance is the ratio of voltage amplitude to current amplitude: $X_C = V_0/I_0 = 1/(\omega C) = 1/(2\pi fC)$, where f is frequency and $\omega = 2\pi f$.
- In a pure capacitor, current leads voltage by $\pi/2$ radians, corresponding to a quarter of the period $T = 1/f$.

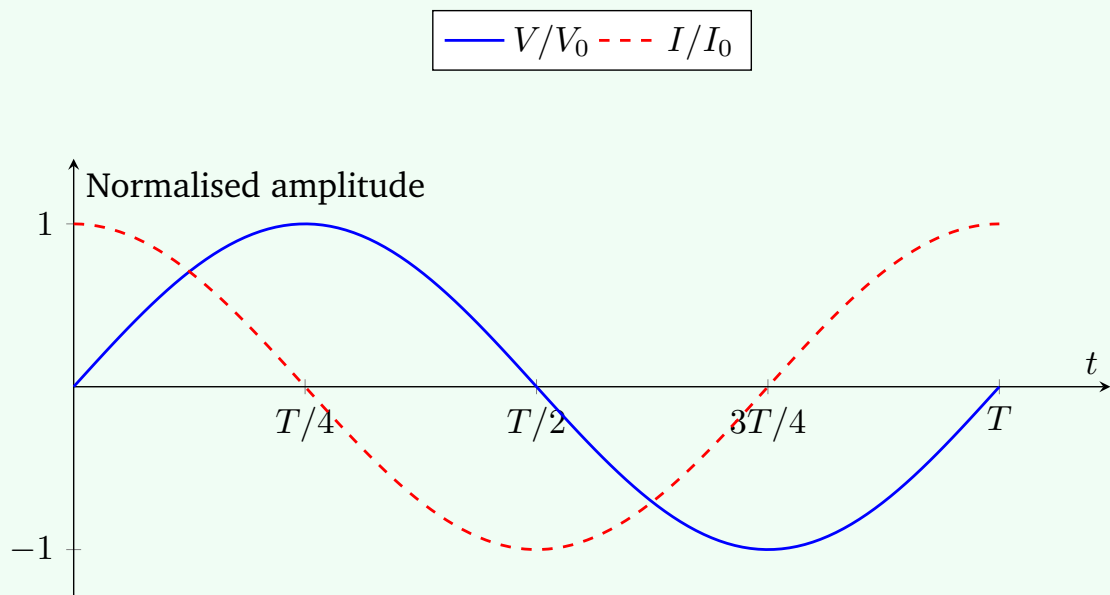
Step 1: (a) Identify the device and derive its reactance The given current is $I = I_0 \sin(\omega t + \pi/2) = I_0 \cos \omega t$. Hence the current leads the voltage by 90° . This is the characteristic phase relation of an ideal capacitor. A resistor would have current in phase with voltage, while an inductor would have current lagging voltage by 90° . For a capacitor connected to the given source,

$$I = C \frac{dV}{dt} = \omega C V_0 \cos \omega t.$$

Comparing amplitudes gives $I_0 = \omega C V_0$; therefore the reactance is $X_C = V_0/I_0 = 1/(\omega C)$, measured in Ω .

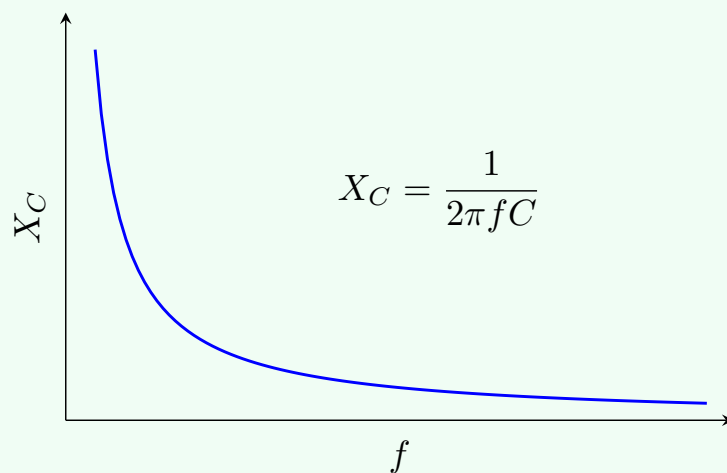
Step 2: (b) Plot voltage and current over one cycle To show both waveforms clearly on the same graph, use their normalised values $V/V_0 = \sin \omega t$ and $I/I_0 = \cos \omega t$. At $t = 0$, voltage is zero and increasing, whereas current has its positive maximum. At $T/4$, voltage is maximum and current is zero; at $T/2$, voltage is zero and current has its negative maximum. At $3T/4$, voltage has its negative maximum and current is zero. Both return to their initial values at T . The current curve reaches each corresponding point $T/4$ earlier than the voltage curve.

Quick Tip to Solve: Rewrite $\sin(\omega t + \pi/2)$ as $\cos \omega t$: a current maximum when voltage crosses zero immediately identifies a capacitor.



Normalised voltage and current over one cycle; current leads by $T/4$.

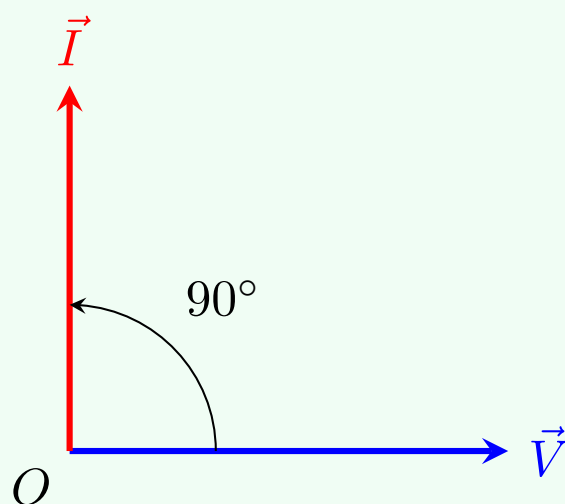
Step 3: (c) Show how reactance depends on frequency Keeping capacitance fixed, $X_C = 1/(2\pi fC)$, so $X_C \propto 1/f$. Doubling the frequency halves the reactance. The graph in the positive f - X_C quadrant is a decreasing rectangular hyperbola, approaching both axes asymptotically. As $f \rightarrow 0$, $X_C \rightarrow \infty$, consistent with an ideal capacitor blocking steady current. As $f \rightarrow \infty$, $X_C \rightarrow 0$. No finite positive frequency gives zero reactance.



Capacitive reactance varies inversely with frequency.

Step 4: (d) Draw the phasor diagram Take the voltage phasor $\vec{V}$ along the positive horizontal reference direction. The current phasor $\vec{I}$ is then vertical upward, 90° ahead of voltage in the counterclockwise sense of rotation. Equivalently, taking current as reference would put voltage 90° clockwise from it. Phasor lengths represent amplitudes on their respective scales; their perpendicular directions express the phase difference.

Quick Tip to Solve: Rewrite $\sin(\omega t + \pi/2)$ as $\cos \omega t$: a current maximum when voltage crosses zero immediately identifies a capacitor.



Current phasor leads voltage phasor by 90° .

Step 5: Conclusion The device X is an ideal capacitor, with $X_C = 1/(\omega C) = 1/(2\pi fC)$. Its current leads voltage by 90° , or $T/4$; its reactance decreases inversely with frequency. The waveform, reactance and phasor diagrams show these relations.

Quick Tip to Solve: Rewrite $\sin(\omega t + \pi/2)$ as $\cos \omega t$: a current maximum when voltage crosses zero immediately identifies a capacitor.

26. (a) Draw a ray diagram to show image formation when the concave mirror produces a real, inverted and magnified image of the object.
- (b) Obtain the mirror formula and write the expression for the linear magnification.
- (c) Explain two advantages of a reflecting telescope over a refracting telescope. [5 marks]

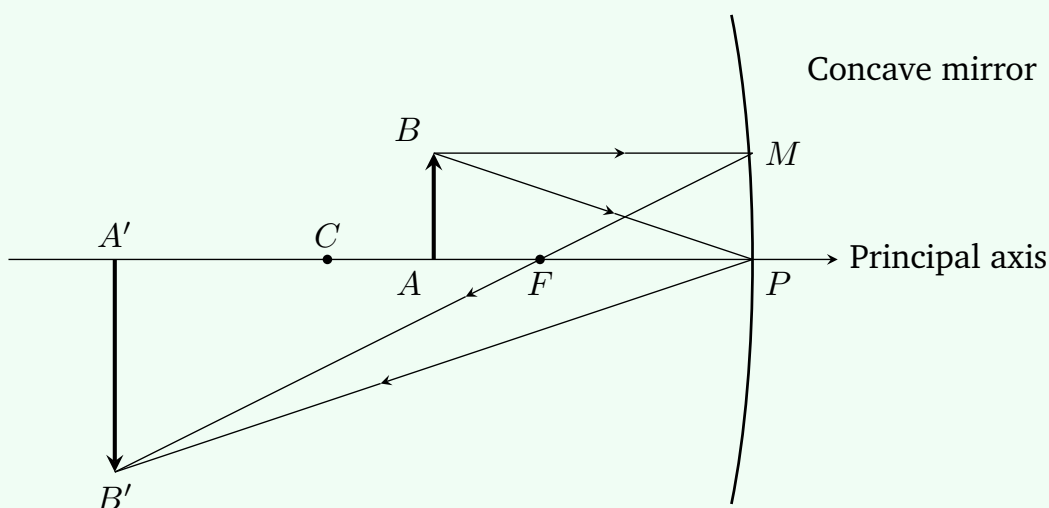
Solution:

Concept:

- A concave mirror forms a real, inverted and enlarged image when the object lies between its principal focus F and centre of curvature C . The image lies beyond C .
- For paraxial rays, a ray parallel to the principal axis reflects through F ; a ray incident at the pole P obeys the law of reflection.
- The Cartesian convention measures distances from P , positive along the direction of incident light. Here u , v and f are negative. Linear magnification is the signed image height divided by the signed object height.

Step 1: (a) Construct the real, inverted and magnified image Place the upright object AB between C and F . From its top B , draw a ray parallel to the principal axis; after reflection it passes through F . Draw a second ray from B to the pole P ; its reflected ray makes the same angle with the axis as the incident ray. The reflected rays meet below the axis at B' . The image $A'B'$ is beyond C , real because actual reflected rays meet, inverted because B' is below the axis, and magnified because $A'B' > AB$.

Quick Tip to Solve: For an enlarged real concave-mirror image, put the object between F and C , then check that the ray intersection is beyond C and below the principal axis.



Object between C and F; reflected rays meet beyond C to give an inverted, magnified image.

Step 2: (b) Derive the mirror formula using similar triangles Let the positive distance magnitudes be $U = AP$, $V = A'P$, $F_0 = FP$, and let the positive object and image-height magnitudes be $h = AB$ and $H = A'B'$. The ray reflected at the pole makes equal angles with the axis on the two sides of that axis. Thus triangles ABP and $A'B'P$ are similar, giving

$$\frac{H}{h} = \frac{V}{U}.$$

Let M be the incidence point of the parallel ray. In the paraxial approximation, its perpendicular projection on the axis may be taken as P , so its height is h and its axial distance from F is F_0 . Similar triangles formed by the reflected ray through F then give

$$\frac{H}{h} = \frac{V - F_0}{F_0}.$$

Equating the two expressions and rearranging,

$$\begin{aligned} \frac{V}{U} &= \frac{V}{F_0} - 1, \\ \frac{1}{U} + \frac{1}{V} &= \frac{1}{F_0}. \end{aligned}$$

Since $u = -U$, $v = -V$ and $f = -F_0$, the Cartesian form follows by substitution. The signed image height is $h_i = -H$, while $h_o = h$, so $h_i/h_o = -V/U = -v/u$. This derivation assumes a small aperture and paraxial rays.

Step 3: (c) Explain two advantages of reflecting telescopes First, the objective is a mirror, and reflection does not separate different colours as refraction through a lens does. A reflecting telescope therefore avoids chromatic aberration of the objective: different wavelengths have the same geometrical focus.

Second, a large mirror can be supported from its back, whereas a large objective lens has to be supported at its edge. Mirrors of much larger aperture can therefore be made and supported

Quick Tip to Solve: For an enlarged real concave-mirror image, put the object between F and C , then check that the ray intersection is beyond C and below the principal axis.

conveniently. Their larger collecting area gathers more light from faint celestial objects, and the larger aperture also improves angular resolving power. These are consequences of the practical large-aperture advantage.

Step 4: Conclusion The object must be between F and C , producing a real, inverted, magnified image beyond C . The mirror formula and linear magnification are

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}, \quad m = \frac{h_i}{h_o} = -\frac{v}{u}.$$

Reflecting telescopes avoid objective chromatic aberration and permit conveniently supported large apertures.

Quick Tip to Solve: For an enlarged real concave-mirror image, put the object between F and C , then check that the ray intersection is beyond C and below the principal axis.

OR

26. (a) Define a wavefront. Using Huygens' principle, verify the laws of reflection at a plane surface.
 (b) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band? Explain.
 (c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the obstacle. Explain why. [5 marks]

Solution:

Concept:

- A wavefront is the locus of points of a wave having the same phase at a given instant. Rays in an isotropic medium are normal to the wavefront.
- By Huygens' principle, each point of a wavefront emits secondary wavelets; their forward envelope gives the subsequent wavefront.
- For a slit of width a , the first diffraction minima satisfy $a \sin \theta = \pm \lambda$. At small angles the central width on a screen a distance D away is $W = 2D\lambda/a$.
- Coherent contributions add as amplitudes. At the centre of the geometrical shadow of a circular obstacle, symmetry gives equal optical paths from all points of its rim.

Step 1: (a) Construct the reflected wavefront using Huygens' principle Let a plane incident wavefront AB meet the reflecting plane first at A . During a time t , its other end B travels along the incident ray to a point C on the mirror, so $BC = vt$, where v is the speed of light in the surrounding medium. In that same time the reflected secondary wavelet from A has radius vt . Draw its tangent from C , touching the wavelet at D . The tangent CD is the reflected wavefront and $AD = vt$ is normal to it. All intermediate secondary wavelets have this common envelope. Thus $BC = AD$; also $AB \perp BC$ and $AD \perp CD$.

Step 2: (a) Verify both laws of reflection Right triangles ABC and ADC have the common hypotenuse AC and the equal sides $BC = AD$. Hence they are congruent and $\angle BAC = \angle ACD$. The angle

Quick Tip to Solve: Use width proportional to $1/a$, central amplitude proportional to a , and intensity proportional to a^2 ; keep peak intensity distinct from total transmitted power.

between the incident wavefront AB and the mirror AC equals the angle i between the incident ray and the mirror normal, since both pairs of lines are mutually perpendicular. Similarly, the angle between the reflected wavefront CD and the mirror equals the reflection angle r . Consequently $i = r$. The construction lies in the plane containing the incident propagation direction and the surface normal; the reflected direction also lies in this plane. Thus the incident ray, reflected ray and normal at the point of incidence are coplanar, verifying the second law as well.

Step 3: (b) Calculate the change in the central band's size and intensity Keep wavelength, screen distance and incident illumination per unit slit area unchanged. The first minima flank the central maximum, so its angular width is approximately $2\lambda/a$ and its linear width is $W = 2D\lambda/a$. Doubling the slit width gives

$$W' = \frac{2D\lambda}{2a} = \frac{W}{2}.$$

At the central point the contributions from the whole slit are in phase. For fixed slit height and uniform illumination, the resultant amplitude E_0 is proportional to a ; therefore $E'_0 = 2E_0$. Since intensity is proportional to squared amplitude,

$$\frac{I'_0}{I_0} = \left(\frac{E'_0}{E_0}\right)^2 = 4.$$

This factor refers to peak central intensity; it is not the intensity at every position across the narrowed band. The increased transmitted power is consistent with a narrower, taller central maximum.

Step 4: (c) Explain the bright spot behind the circular obstacle The small circular obstacle diffracts light into its geometrical shadow. For a distant source the incident wavefront is approximately plane and reaches all points of the circular rim with the same phase. At the axial centre of the shadow, contributions diffracted around the rim travel equal optical distances to the observation point. Their phase differences therefore vanish and their amplitudes reinforce constructively. A bright central spot, called the Poisson or Arago spot, results. The spot is observed on a screen behind the obstacle at the centre of its shadow; it is not light transmitted through the opaque obstacle itself.

Step 5: Conclusion A wavefront joins points of equal phase. Huygens' construction gives $i = r$ and coplanarity of the incident ray, reflected ray and normal. Doubling the slit width halves the central band's width and makes its peak intensity four times as large under unchanged incident illumination. Constructive interference of diffracted light produces the bright central spot in the circular obstacle's shadow.

Quick Tip to Solve: Use width proportional to $1/a$, central amplitude proportional to a , and intensity proportional to a^2 ; keep peak intensity distinct from total transmitted power.