

CBSE Class 12 Physics Question Paper 2018 with Solution PDF-Set 55/2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are **26** questions in all.
- (ii) This question paper has **five** sections : Section A, Section B, Section C, Section D and Section E.
- (iii) Section A contains **five** questions of **one** mark each, Section B contains **five** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains one value based question of **four** marks and Section E contains **three** questions of **five** marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in **one** question of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Draw graphs showing variation of photoelectric current with applied voltage for two incident radiations of equal frequency and different intensities. Mark the graph for the radiation of higher intensity. [1 mark]

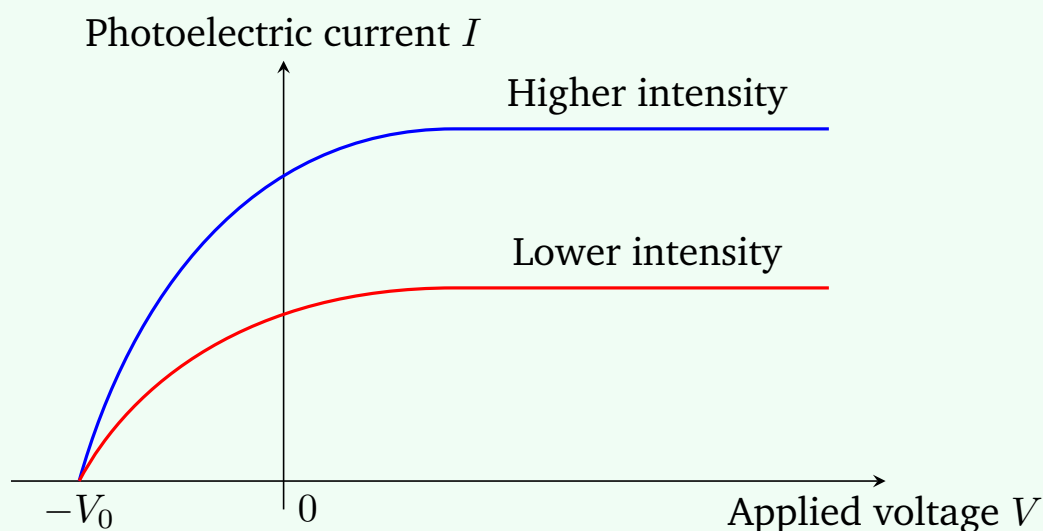
Solution:

Concept:

- At fixed frequency above threshold, increasing light intensity increases the number of photoelectrons emitted per second and hence the saturation current.
- The stopping potential is determined by $eV_0 = h\nu - \phi$, where e is the elementary charge, ν is frequency and ϕ is the work function.

Step 1: Compare saturation currents For the same emitter and equal incident frequency, higher intensity supplies more photons per second. Thus the upper curve has the larger saturation current and represents the radiation of higher intensity. At sufficiently large positive voltage, essentially all emitted electrons are collected.

Step 2: Compare stopping potentials Both radiations give photoelectrons the same maximum kinetic energy because their frequencies are equal. Consequently both curves reach zero current at the same negative applied voltage, $-V_0$. A larger intensity changes the current, but does not move this stopping-voltage intercept.



Photoelectric current for equal frequencies and different intensities

Step 3: Conclusion The higher-intensity radiation corresponds to the upper curve; both curves have the same stopping potential $-V_0$.

Quick Tip to Solve: Keep the negative voltage intercept common, and distinguish intensity by the heights of the saturation plateaus.

2. A proton and an electron travelling along parallel paths enter a region of uniform magnetic field, acting perpendicular to their paths. Which of them will move in a circular path with higher frequency ?

[1 mark]

Solution:

Concept:

- A uniform magnetic field perpendicular to velocity provides centripetal force: $|q|vB = mv^2/r$.
- The revolution frequency is $f = |q|B/(2\pi m)$, where m is particle mass and B is field magnitude.

Step 1: Obtain the frequency From the magnetic-force equation, $r = mv/(|q|B)$. The time for one revolution is therefore $T = 2\pi r/v = 2\pi m/(|q|B)$. Thus the frequency is independent of speed for the non-relativistic motion considered here.

Step 2: Compare electron and proton Both particles have the same magnitude of charge, e , and experience the same magnetic field. Hence $f_e/f_p = m_p/m_e \simeq 1836$. The electron has much smaller mass and therefore completes many more revolutions per second. The opposite charge signs reverse their directions of bending, but do not affect the magnitude of frequency.

Step 3: Conclusion The electron moves in a circular path with higher frequency.

Quick Tip to Solve: For a frequency comparison in a magnetic field, compare charge magnitude divided by mass, not the directions of bending.

3. Four nuclei of an element undergo fusion to form a heavier nucleus, with release of energy. Which of the two — the parent or the daughter nucleus — would have higher binding energy per nucleon ?

[1 mark]

Solution:

Concept:

- Binding energy is the energy needed to separate a nucleus into its constituent nucleons; higher binding energy per nucleon indicates more tightly bound nucleons.
- Energy-releasing fusion increases the total binding energy of the product relative to that of the initial nuclei.

Step 1: Use the released energy Let each parent nucleus have mass number A and binding energy B_p . The daughter has mass number $4A$. Since energy $Q > 0$ is released in the fusion described, the gain in binding energy is $B_d - 4B_p = Q$, where B_d is the daughter binding energy.

Step 2: Compare per nucleon Dividing by the conserved total number of nucleons gives $B_d/(4A) = B_p/A + Q/(4A)$. The positive final term establishes that the binding energy per nucleon of the daughter exceeds that of each parent nucleus. This conclusion follows from the stated release of energy, not merely from the daughter being heavier.

Step 3: Conclusion The daughter nucleus has higher binding energy per nucleon.

Quick Tip to Solve: Use the sign of the energy release: a positive fusion energy means a gain in binding energy per nucleon.

4. Which mode of propagation is used by short wave broadcast services ?

[1 mark]

Solution:

Concept:

- Sky-wave propagation uses the ionosphere to return suitable radio waves towards the Earth.
- Short-wave broadcast frequencies can undergo ionospheric refraction, enabling communication beyond the direct line of sight.

Step 1: Identify the propagation route Short-wave radiation is sent upward towards the ionosphere. The ionised layers refract the wave so that it can return to the Earth at a large distance from the transmitter; repeated returns can extend the communication range.

Step 2: Connect the route with broadcasting This Earth-ionosphere route is called sky-wave propagation. It is suitable for the short-wave broadcast services asked about here. A direct space-wave route is largely limited by the horizon, while ground-wave transmission is used chiefly at lower frequencies and suffers greater attenuation at short-wave frequencies.

Step 3: Conclusion Short-wave broadcast services use sky-wave propagation.

Quick Tip to Solve: Associate short-wave broadcasting with the ionosphere and hence with the sky-wave mode.

5. Name the electromagnetic radiations used for (a) water purification, and (b) eye surgery.

[1 mark]

Solution:

Concept:

- Ultraviolet radiation has photons energetic enough to damage microbial genetic material, making it useful for disinfection.
- Ultraviolet laser radiation can precisely remove a thin layer of corneal tissue in corrective eye surgery.

Step 1: (a) Water purification Ultraviolet rays are used to disinfect water. Exposure to suitable ultraviolet radiation inactivates microorganisms by damaging their genetic material. The radiation acts on the organisms; it does not mechanically filter suspended particles from the water.

Step 2: (b) Eye surgery Ultraviolet rays are also used in eye surgery, for example the ultraviolet radiation from an excimer laser used to reshape the cornea. Controlled removal of corneal tissue changes its curvature and helps correct refractive defects. The electromagnetic-spectrum band to name is ultraviolet in both applications.

Step 3: Conclusion (a) Ultraviolet rays. (b) Ultraviolet rays.

Quick Tip to Solve: Remember both syllabus applications together: ultraviolet disinfects water and is used in corneal eye surgery.

Section B (10 marks)

6. (a) Why are infra-red waves often called heat waves ? Explain.

(b) What do you understand by the statement, “Electromagnetic waves transport momentum” ?

[2 marks]

Solution:

Concept:

- Infra-red radiation is electromagnetic radiation emitted strongly by hot bodies. Its absorption can increase molecular vibrational energy and the internal energy of matter.
- An electromagnetic wave carries energy and momentum. For energy U travelling in vacuum, the associated momentum has magnitude $p = U/c$, where c is the speed of light.

Step 1: Explain the heating effect in part (a) Infra-red waves are readily absorbed by many materials. Absorption increases the vibrational motion of their molecules, converting the incident radiant energy into internal energy. The temperature of the absorbing body consequently rises. Their strong heating effect, together with their prominent emission from hot bodies, is why infra-red waves are often called heat waves.

Step 2: Interpret momentum transport in part (b) Electromagnetic radiation carries momentum in its direction of propagation. When the waves strike and are absorbed by a surface, they transfer momentum to that surface and exert a force, called radiation pressure when expressed per unit area. For a normally incident pulse of energy U completely absorbed by the surface, the momentum transferred is

$$\Delta p = \frac{U}{c}.$$

Thus radiation can push matter even though its propagation does not require a material medium.

Step 3: Conclusion (a) Infra-red waves are called heat waves because their absorption increases molecular motion and heats matter. (b) Momentum transport means that electromagnetic waves carry momentum and can transfer it to a body, exerting a force or radiation pressure.

Quick Tip to Solve: Link infra-red absorption to molecular vibration; interpret momentum transport by describing the force on an absorbing surface.

7. Two electric bulbs P and Q have their resistances in the ratio of 1 : 2. They are connected in series across a battery. Find the ratio of the power dissipation in these bulbs.

[2 marks]

Solution:

Concept:

- In a series circuit the same current I flows through every component, because there is only one path for charge flow.
- The electrical power dissipated in a resistance R is $P = I^2 R$. Thus, at a common current, power is directly proportional to resistance.

Quick Tip to Solve: For bulbs in series, use the common current and $I^2 R$; do not assume that each bulb has the full battery voltage across it.

Step 1: Identify the common quantity Let the resistances of bulbs P and Q be R_P and R_Q , respectively. They are connected in series across the battery, so their currents are equal. Write

$$R_P = R, \quad R_Q = 2R, \quad I_P = I_Q = I.$$

Although the currents are equal, the voltage drops are unequal: by Ohm's law, the bulb with twice the resistance has twice the voltage drop.

Step 2: Compare the dissipated powers Denote the powers dissipated in the two bulbs by P_P and P_Q . Using the common series current gives

$$\frac{P_P}{P_Q} = \frac{I^2 R_P}{I^2 R_Q} = \frac{R}{2R} = \frac{1}{2}.$$

The battery voltage and the current cancel, so their numerical values are not needed. The higher-resistance bulb Q dissipates twice the power of P in this connection.

Step 3: Conclusion The required ratio of power dissipation is $P_P : P_Q = 1 : 2$.

Quick Tip to Solve: For bulbs in series, use the common current and $I^2 R$; do not assume that each bulb has the full battery voltage across it.

8. If light of wavelength 412.5 nm is incident on each of the metals given below, which ones will show photoelectric emission and why?

Metal	Work Function (eV)
Na	1.92
K	2.15
Ca	3.20
Mo	4.17

[2 marks]

Solution:

Concept:

- The energy of one incident photon is $E = h\nu = hc/\lambda$, where h is Planck's constant, c is the speed of light and λ is the wavelength.
- Photoelectric emission requires $E \geq \phi$, where ϕ is the metal's work function. Einstein's equation is $K_{\max} = E - \phi$.

Step 1: Calculate the incident photon energy Convert the wavelength to SI units: $\lambda = 412.5 \times 10^{-9}$ m. With $h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^8$ m s⁻¹, and 1 eV = 1.60×10^{-19} J,

$$E = \frac{hc}{\lambda} = 4.82 \times 10^{-19} \text{ J} \simeq 3.01 \text{ eV}.$$

This is the energy available from a single photon for removing an electron and giving it kinetic energy.

Step 2: Compare with each work function For Na, $3.01 > 1.92$ eV, and for K, $3.01 > 2.15$ eV, so

Quick Tip to Solve: Compute the photon energy once and compare it with all four work functions in eV; emission requires a non-negative $E - \phi$.

both metals can emit photoelectrons. Their maximum kinetic energies are approximately 1.09 eV and 0.86 eV, respectively. For Ca, $3.01 < 3.20$ eV, and for Mo, $3.01 < 4.17$ eV. A photon cannot supply the minimum removal energy for either of these metals, so neither shows photoelectric emission.

Step 3: Conclusion Na and K show photoelectric emission; Ca and Mo do not. The photon energy, approximately 3.01 eV, exceeds only the work functions of Na and K.

Quick Tip to Solve: Compute the photon energy once and compare it with all four work functions in eV; emission requires a non-negative $E - \phi$.

9. A carrier wave of peak voltage 15 V is used to transmit a message signal. Find the peak voltage of the modulating signal in order to have a modulation index of 60%. [2 marks]

Solution:

Concept:

- In amplitude modulation, the modulation index is the ratio $m = A_m/A_c$, where A_m is the peak voltage of the modulating signal and A_c is the peak voltage of the unmodulated carrier.
- Modulation index is dimensionless because both amplitudes are voltages. A percentage modulation index is converted into its fractional value by dividing by 100.

Step 1: Express the given modulation index as a fraction The carrier peak voltage is $A_c = 15$ V. The required modulation index is 60%, so the fractional index to use in the amplitude-ratio formula is

$$m = \frac{60}{100} = 0.60.$$

This means the message-signal peak amplitude is sixty percent of the carrier peak amplitude. Both given and required quantities are peak voltages, so no conversion to rms voltage is needed.

Step 2: Determine the modulating-signal peak voltage Rearrange the defining relation to isolate the unknown amplitude:

$$A_m = mA_c.$$

Substituting the fractional index and carrier amplitude gives

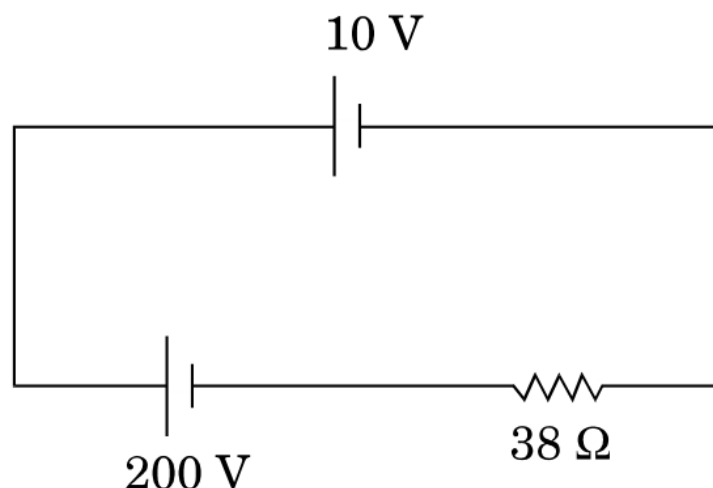
$$A_m = 0.60 \times 15 \text{ V} = 9.0 \text{ V}.$$

As a check, the ratio of this voltage to the carrier peak voltage is $9/15 = 0.60$, exactly the required modulation index. Since this ratio is less than unity, it also lies within the usual range without overmodulation.

Step 3: Conclusion The required peak voltage of the modulating signal is **9 V**.

Quick Tip to Solve: Convert 60% to 0.60 before multiplying by the carrier peak voltage; compare peak amplitude with peak amplitude.

10. A 10 V cell of negligible internal resistance is connected in parallel across a battery of emf 200 V and internal resistance $38\ \Omega$ as shown in the figure. Find the value of current in the circuit. [2 marks]



Solution:

Concept:

- An ideal cell fixes its terminal potential difference at its emf. Thus the two parallel terminals are maintained at $V = 10\text{ V}$ by the cell of negligible internal resistance.
- A battery delivering current I has terminal voltage $V = E - Ir$, where E is its emf and r is its internal resistance. Kirchhoff's loop law gives the same relation.

Step 1: Identify the opposing emfs in the loop In the printed circuit the longer plates of both sources are on the left: their positive terminals are connected together. The sources therefore oppose each other around the closed loop. The 200 V battery drives current through the 10 V cell, charging the latter. The available net emf is $200 - 10\text{ V}$, not the sum of the emfs.

Step 2: Apply the loop equation The only appreciable resistance in the loop is the battery's internal resistance, $38\ \Omega$. Traversing the loop in the direction driven by the larger emf,

$$200 - 10 - 38I = 0, \quad I = \frac{200 - 10}{38} = \frac{190}{38} = 5\text{ A}.$$

This also follows from $10 = 200 - 38I$: the battery's terminal voltage equals the voltage of the ideal cell connected across it.

Step 3: Conclusion The current is 5 A . It leaves the positive terminal of the 200 V battery and enters the positive terminal of the 10 V cell (clockwise in the printed figure).

Quick Tip to Solve: Check the long plates first: like terminals connected together mean subtract the emfs, then divide by the total loop resistance.

10. In a potentiometer arrangement for determining the emf of a cell, the balance point of the cell in open circuit is 350 cm. When a resistance of $9\ \Omega$ is used in the external circuit of the cell, the balance point shifts to 300 cm. Determine the internal resistance of the cell. [2 marks]

Solution:

Concept:

- At a potentiometer balance point no current is drawn from the test cell. With a fixed potential gradient k , a potential difference equals kl , where l is its balancing length.
- For a cell of emf E and internal resistance r , supplying an external resistance R , the terminal voltage is $V = ER/(R + r)$. Hence $r = R(E/V - 1)$.

Step 1: Relate the two balance lengths to cell voltages With the external circuit open, the cell supplies no current, so there is no internal voltage drop and its balance length measures the emf: $E = k(350\text{ cm})$. When the $9\ \Omega$ resistance is connected across the cell, the balance length measures its terminal voltage: $V = k(300\text{ cm})$. The potentiometer potential gradient is unchanged, so it cancels in the ratio:

$$\frac{E}{V} = \frac{l_1}{l_2} = \frac{350}{300} = \frac{7}{6}.$$

Step 2: Calculate the internal resistance The current through the external resistance is $I = V/R$, and the internal voltage drop satisfies $E - V = Ir$. Eliminating I ,

$$\begin{aligned} r &= R \left(\frac{E}{V} - 1 \right) = R \frac{l_1 - l_2}{l_2} \\ &= 9 \frac{350 - 300}{300} \Omega = 9 \frac{50}{300} \Omega = 1.5 \Omega. \end{aligned}$$

The shorter loaded balance length correctly indicates a terminal voltage below the emf and a positive internal resistance.

Step 3: Conclusion The internal resistance of the cell is $r = 1.5\ \Omega$.

Quick Tip to Solve: Use the loaded balance length in the denominator: internal resistance is external resistance times the fractional decrease from the open-circuit length.

Section C (36 marks)

11. (a) An iron ring of relative permeability μ_r has windings of insulated copper wire of n turns per metre. When the current in the windings is I , find the expression for the magnetic field in the ring.
- (b) The susceptibility of a magnetic material is 0.9853. Identify the type of magnetic material. Draw the modification of the field pattern on keeping a piece of this material in a uniform magnetic field.

[3 marks]

Solution:

Concept:

- Ampere's circuital law for a uniformly wound toroidal ring gives $H\ell = NI$, where H is magnetic field intensity, ℓ is the magnetic path length, and N is the total number of turns.
- The magnetic flux density is $B = \mu H = \mu_0 \mu_r H$. Here μ_0 is the permeability of free space and μ_r is the relative permeability of the core.
- Magnetic susceptibility χ_m relates magnetisation to field intensity by $M = \chi_m H$, and $\mu_r = 1 + \chi_m$. Paramagnetic substances have positive susceptibility and attract magnetic field lines.

Step 1: (a) Apply Ampere's law around the iron ring Take an Amperian path along the ring's mean circumference ℓ . For a uniformly wound closed ring the field is tangential and, in the usual thin-ring approximation, has a constant magnitude along this path. Since there are n turns per metre, the number of turns is $N = n\ell$. Therefore,

$$H\ell = NI = n\ell I, \quad H = nI.$$

Including the iron core's permeability gives

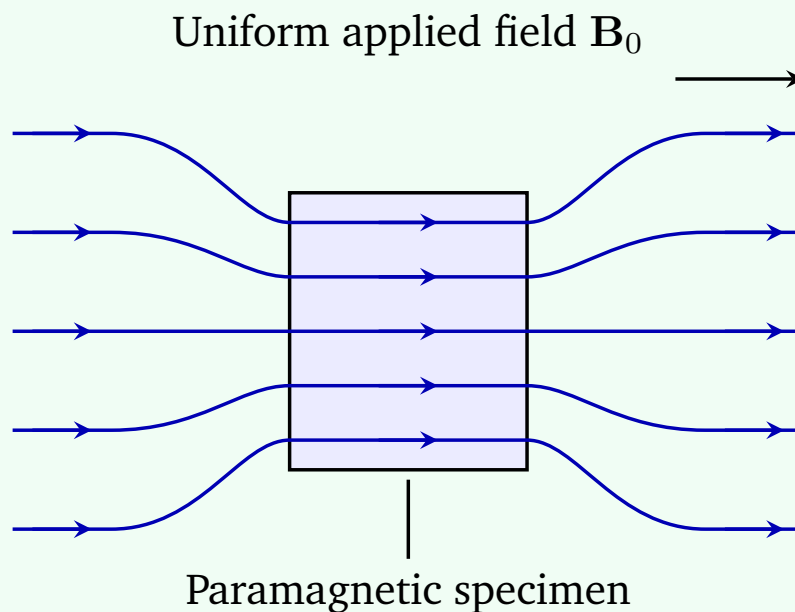
$$B = \mu_0 \mu_r H = \mu_0 \mu_r nI.$$

The field circulates around the ring, with its direction determined by the right-hand grip rule for the winding current.

Step 2: (b) Identify the material from its susceptibility The given susceptibility is $\chi_m = 0.9853$, which is positive and below unity. On the standard classification of magnetic materials this is a paramagnetic material. Its magnetisation is along the applied field, whereas a diamagnetic substance has negative susceptibility and magnetises oppositely. A ferromagnetic substance has a much larger positive susceptibility. The corresponding relative permeability is $\mu_r = 1 + 0.9853 = 1.9853 > 1$.

Step 3: (b) Show the modified field pattern Before insertion, the uniform field is represented by equally spaced parallel lines. With the paramagnetic specimen inserted, the lines bend towards it and become more closely spaced inside it than in the surrounding region. Their direction through the specimen remains the same as the applied field. The sketch shows continuous, non-intersecting field lines and arrows to the right; the increased line density within the specimen represents the enhanced magnetic flux density.

Quick Tip to Solve: Use turns per metre directly in the toroidal field formula, and use the sign of susceptibility to decide whether field lines enter or avoid the material.



Parallel uniform field lines become denser inside a paramagnetic specimen.

Step 4: Conclusion (a) The magnetic field in the iron ring is $B = \mu_0 \mu_r n I$. (b) The material is paramagnetic; magnetic field lines concentrate inside the specimen as shown.

Quick Tip to Solve: Use turns per metre directly in the toroidal field formula, and use the sign of susceptibility to decide whether field lines enter or avoid the material.

12. A bar magnet of magnetic moment 6 J/T is aligned at 60° with a uniform external magnetic field of 0.44 T . Calculate (a) the work done in turning the magnet to align its magnetic moment (i) normal to the magnetic field, (ii) opposite to the magnetic field, and (b) the torque on the magnet in the final orientation in case (ii). [3 marks]

Solution:

Concept:

- The potential energy of a magnetic dipole in a uniform field is $U = -\vec{m} \cdot \vec{B} = -mB \cos \theta$, where θ is the angle between its magnetic moment and the field.
- For a slow rotation by an external agent, the work done is the increase in potential energy: $W_{\text{ext}} = U_f - U_i = mB(\cos \theta_i - \cos \theta_f)$.
- The magnetic torque has magnitude $\tau = mB \sin \theta$ and tends to align the magnetic moment with the field.

Step 1: Establish the initial energy The magnetic moment is $m = 6 \text{ J T}^{-1}$, the field is $B = 0.44 \text{ T}$, and the initial angle is 60° . Thus $mB = 2.64 \text{ J}$, giving

$$U_i = -2.64 \cos 60^\circ = -1.32 \text{ J}.$$

Quick Tip to Solve: Use the original 60° angle in both work calculations; use cosine for energy and sine for torque.

Work done in turning the magnet means the work supplied by the external agent against the magnetic torque; the work done by the field itself has the opposite sign.

Step 2: (a)(i) Rotate the magnetic moment normal to the field Normal means $\theta_f = 90^\circ$, so $U_f = -mB \cos 90^\circ = 0$. Therefore

$$W_{\perp} = 0 - (-1.32) = 1.32 \text{ J.}$$

Both rotations start from the given initial orientation; the second calculation is not a continuation from the normal orientation.

Step 3: (a)(ii) Rotate the magnetic moment opposite to the field Opposite alignment corresponds to $\theta_f = 180^\circ$. The final energy is $U_f = -2.64 \cos 180^\circ = +2.64 \text{ J}$. Hence

$$W_{\text{opposite}} = 2.64 - (-1.32) = 3.96 \text{ J.}$$

Step 4: (b) Evaluate the torque in the opposite orientation At the final orientation of case (ii), the magnetic moment and field are antiparallel. Their cross product vanishes:

$$\tau = mB \sin 180^\circ = 0 \text{ N m.}$$

This is unstable equilibrium: the torque is zero exactly at 180° , although a small displacement produces a torque that turns the magnet away from this orientation toward alignment.

Step 5: Conclusion The required external work is 1.32 J for normal alignment and 3.96 J for opposite alignment. The final torque in case (ii) is 0 N m .

Quick Tip to Solve: Use the original 60° angle in both work calculations; use cosine for energy and sine for torque.

13. (a) Define the term ‘conductivity’ of a metallic wire. Write its SI unit.

(b) Using the concept of free electrons in a conductor, derive the expression for the conductivity of a wire in terms of number density and relaxation time. Hence obtain the relation between current density and the applied electric field $\vec{E}$. [3 marks]

Solution:

Concept:

- Conductivity σ measures a material's ability to carry current under an applied electric field; $\sigma = 1/\rho$, where ρ is resistivity.
- In the free-electron model, n is the number of conduction electrons per unit volume, e is the positive magnitude of the electron charge, m_e is electron mass, and τ is the mean relaxation time between successive collisions.
- Conventional current is opposite to electron motion, so the current density is $\vec{J} = -ne\vec{v}_d$, where $\vec{v}_d$ is the electron drift velocity.

Step 1: (a) Define conductivity and give its SI unit For an isotropic metallic wire obeying Ohm's law,

Quick Tip to Solve: Keep the electron charge as $-e$ in both acceleration and current density; the signs cancel in $\vec{J} = \sigma \vec{E}$.

conductivity is the current density produced per unit electric field:

$$\sigma = \frac{J}{E} = \frac{1}{\rho}.$$

Its SI unit is S m^{-1} , equivalently $\Omega^{-1} \text{m}^{-1}$.

Step 2: (b) Obtain the drift velocity of free electrons Random thermal electron velocities have zero vector average. In a uniform electric field $\vec{E}$, each electron experiences force $-e\vec{E}$ and acceleration

$$\vec{a} = -\frac{e\vec{E}}{m_e}.$$

Collisions randomise the electron velocities. Averaging the velocity acquired from the field over the relaxation time τ gives

$$\vec{v}_d = \vec{a}\tau = -\frac{e\tau}{m_e}\vec{E}.$$

Thus electron drift is opposite to the applied electric field.

Step 3: Derive conductivity and the current density relation For wire cross-sectional area A , the number of electrons crossing a section in time Δt is $nA|\vec{v}_d|\Delta t$. The current magnitude is therefore $I = neA|\vec{v}_d|$. Accounting for the negative electron charge, the vector current density becomes

$$\begin{aligned}\vec{J} &= -ne\vec{v}_d \\ &= \frac{ne^2\tau}{m_e}\vec{E}.\end{aligned}$$

Comparing this result with $\vec{J} = \sigma\vec{E}$ identifies the conductivity. The two negative signs cancel, so conventional current density points along the electric field.

Step 4: Conclusion Conductivity is $\sigma = 1/\rho$, with SI unit S m^{-1} . The free-electron model gives

$$\sigma = ne^2\tau/m_e \text{ and } \vec{J} = \sigma\vec{E}.$$

Quick Tip to Solve: Keep the electron charge as $-e$ in both acceleration and current density; the signs cancel in $\vec{J} = \sigma\vec{E}$.

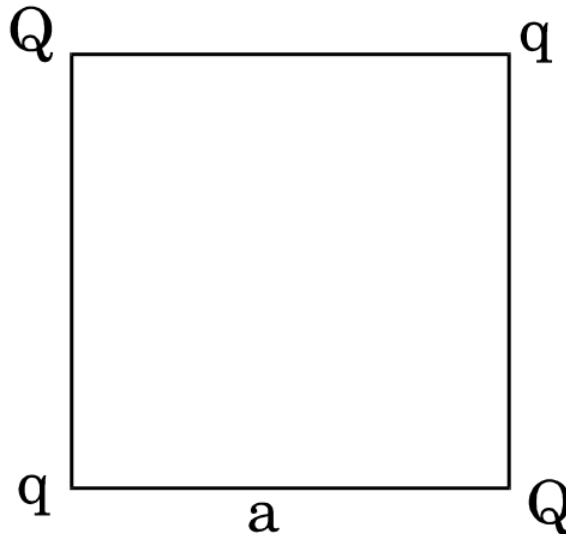
14. Four point charges Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure.

Find the

(a) resultant electric force on a charge Q , and

(b) potential energy of this system.

[3 marks]



Solution:

Concept:

- Coulomb's law gives the force between point charges: $F = k|q_1q_2|/r^2$, where $k = 1/(4\pi\epsilon_0)$; force is repulsive for like signs and attractive for unlike signs.
- The resultant force is the vector sum of forces due to all other charges. Adjacent corners of the square are separated by a , while opposite corners are separated by $\sqrt{2}a$.
- The electrostatic potential energy, with zero energy at infinite separation, is $U = \sum_{i<j} kq_iq_j/r_{ij}$; each distinct charge pair must be counted once.

Step 1: (a) Resolve the forces from the adjacent charges Choose the upper-left charge Q , as in the printed square. Put it at the origin, with $\hat{x}$ rightward and $\hat{y}$ upward. The adjacent charges q are then at $(a, 0)$ and $(0, -a)$, while the other Q is at $(a, -a)$. Using signed charge products gives

$$\vec{F}_{\text{upper right}} = -\frac{kQq}{a^2}\hat{x}, \quad \vec{F}_{\text{lower left}} = \frac{kQq}{a^2}\hat{y}.$$

Their sum is $(kQq/a^2)(-\hat{x} + \hat{y})$. For positive Qq , these two equal perpendicular forces have resultant magnitude $\sqrt{2}kQq/a^2$, along the outward diagonal through the chosen corner.

Step 2: Include the opposite-corner charge The opposite Q is at distance $\sqrt{2}a$. Its repulsive force has magnitude $kQ^2/(2a^2)$ and is along the same outward diagonal. Define its unit vector as $\hat{u} = (-\hat{x} + \hat{y})/\sqrt{2}$. Then

$$\vec{F}_Q = \frac{k}{a^2} \left(\sqrt{2}Qq + \frac{Q^2}{2} \right) \hat{u}.$$

Quick Tip to Solve: For the force, combine the two side forces before adding the diagonal force; for energy, count four sides and two diagonals.

For like-signed charges, the coefficient is positive and the force points away from the square's centre. If Q and q have opposite signs, the same signed expression applies: its sign determines whether the resultant points outward or inward. The magnitude in general is the absolute value of that coefficient. The force on the other Q is equal in magnitude and opposite in direction by symmetry.

Step 3: (b) Sum the energies of the six distinct pairs The four side pairs each contain Q and q , with separation a , so their total contribution is $4kQq/a$. The two diagonal pairs are the Q, Q pair and the q, q pair, each at separation $\sqrt{2}a$. Hence

$$U = 4 \frac{kQq}{a} + \frac{kQ^2}{\sqrt{2}a} + \frac{kq^2}{\sqrt{2}a}$$

$$= \frac{k}{a} \left(4Qq + \frac{Q^2 + q^2}{\sqrt{2}} \right).$$

No extra factor of two is needed because all six pairs have already been counted exactly once.

Step 4: Conclusion Along the outward diagonal through the chosen Q , the signed force is

$$\vec{F}_Q = \frac{1}{4\pi\epsilon_0 a^2} (\sqrt{2}Qq + Q^2/2) \hat{u}. \text{ The system's energy is } U = \frac{1}{4\pi\epsilon_0 a} [4Qq + (Q^2 + q^2)/\sqrt{2}].$$

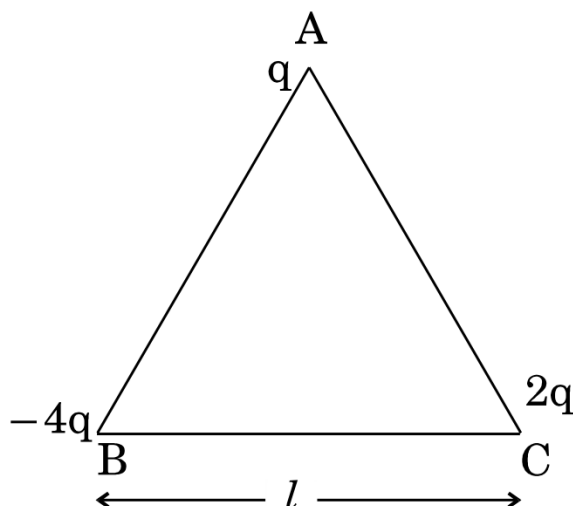
Quick Tip to Solve: For the force, combine the two side forces before adding the diagonal force; for energy, count four sides and two diagonals.

OR

14. (a) Three point charges q , $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side ' l ' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q .

(b) Find out the amount of the work done to separate the charges at infinite distance.

[3 marks]



Solution:

Concept:

- Coulomb's law gives the magnitude of the force between charges separated by l : $F = k|q_1q_2|/l^2$, where $k = 1/(4\pi\epsilon_0)$. Unlike charges attract and like charges repel.
- The electrostatic potential energy is the sum over distinct charge pairs: $U = k \sum_{i < j} q_i q_j / r_{ij}$. Work done by an external agent in separating the charges slowly is $W_{\text{ext}} = U_f - U_i$.

Step 1: (a) Identify the two forces on the charge at A The printed figure places q at A , $-4q$ at B , and $2q$ at C . The force due to B acts from A towards B , with magnitude $F_B = 4kq^2/l^2$. The force due to C acts along CA , away from C , with magnitude $F_C = 2kq^2/l^2$. Although $\angle BAC = 60^\circ$, the angle between these force vectors is $180^\circ - 60^\circ = 120^\circ$, because the second force is repulsive.

Step 2: Add the forces vectorially Apply the cosine rule to the force vectors, using their actual directions:

$$\begin{aligned} F^2 &= F_B^2 + F_C^2 + 2F_B F_C \cos 120^\circ \\ &= \left(\frac{kq^2}{l^2}\right)^2 (16 + 4 - 8) \\ &= 12 \left(\frac{kq^2}{l^2}\right)^2. \end{aligned}$$

Taking the positive square root gives the required magnitude. Using 60° between the forces would incorrectly treat the repulsive force as attractive.

Step 3: (b) Calculate the energy change on separation Each distinct pair is initially separated by l , so

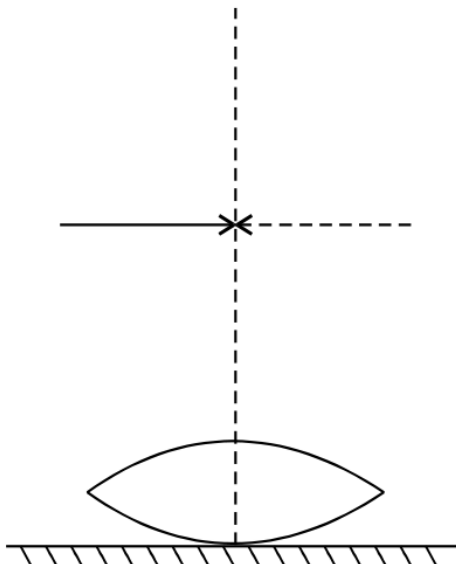
$$U_i = \frac{k}{l} [q(-4q) + q(2q) + (-4q)(2q)] = -\frac{10kq^2}{l}.$$

When all mutual separations tend to infinity, each pair energy tends to zero and $U_f = 0$. Therefore the external work needed for quasistatic separation is $W_{\text{ext}} = 0 - U_i = 10kq^2/l$. It is positive because this configuration has negative binding energy. The electric field itself does the opposite work, $-10kq^2/l$.

Step 4: Conclusion (a) The resultant force magnitude is $F = \frac{2\sqrt{3}}{4\pi\epsilon_0} \frac{q^2}{l^2}$. (b) The work required to separate the charges to infinity is $W_{\text{ext}} = \frac{10}{4\pi\epsilon_0} \frac{q^2}{l}$.

Quick Tip to Solve: Draw attraction and repulsion arrows before selecting the angle; sum each charge pair only once when calculating potential energy.

15. A symmetric biconvex lens of radius of curvature R and made of glass of refractive index 1.5, is placed on a layer of liquid placed on top of a plane mirror as shown in the figure. An optical needle with its tip on the principal axis of the lens is moved along the axis until its real, inverted image coincides with the needle itself. The distance of the needle from the lens is measured to be x . On removing the liquid layer and repeating the experiment, the distance is found to be y . Obtain the expression for the refractive index of the liquid in terms of x and y . [3 marks]



Solution:

Concept:

- In the lens-plane-mirror method, coincidence of the needle with its inverted returned image places the needle in the front focal plane of the effective lens. Rays emerge parallel, reflect from the plane mirror, and return to the same object plane.
- For thin lenses in contact, powers add: $1/F = 1/f_g + 1/f_\ell$. The lens-maker formula is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, with radii signed along the incident direction.

Step 1: Identify the two measured focal lengths Let μ be the refractive index of the liquid relative to air. With the liquid in place, the needle distance x is the focal length F of the combined glass lens and the liquid lens formed beneath it. Without the liquid, the needle distance y is the focal length f_g of the glass lens alone. The mirror returns parallel rays through the lens; it does not introduce a finite focusing power. Thus the readings are focal lengths, not twice the focal lengths.

Step 2: Use the glass lens to eliminate the curvature radius For light travelling down towards the mirror, the symmetric biconvex glass lens has $R_1 = +R$ and $R_2 = -R$. Its refractive index is 1.5, so

$$\frac{1}{y} = (1.5 - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{1}{R}.$$

Consequently $R = y$. This is why the curvature radius need not appear in the final expression.

Step 3: Calculate the liquid-lens power and combine the powers The liquid occupies the gap between the convex lower surface of the glass lens and the plane mirror. It forms a plano-concave lens. Its

Quick Tip to Solve: The trapped liquid is a diverging plano-concave lens: subtract its power from the glass power, and use the dry reading to obtain $R = y$.

upper curved boundary has radius $-R$ for downward light and its lower boundary is plane, so

$$\frac{1}{f_l} = (\mu - 1) \left(-\frac{1}{R} - 0 \right) = -\frac{\mu - 1}{R}.$$

Therefore, in the thin-lens approximation,

$$\begin{aligned} \frac{1}{x} &= \frac{1}{y} - \frac{\mu - 1}{R} \\ &= \frac{1}{y} - \frac{\mu - 1}{y} = \frac{2 - \mu}{y}. \end{aligned}$$

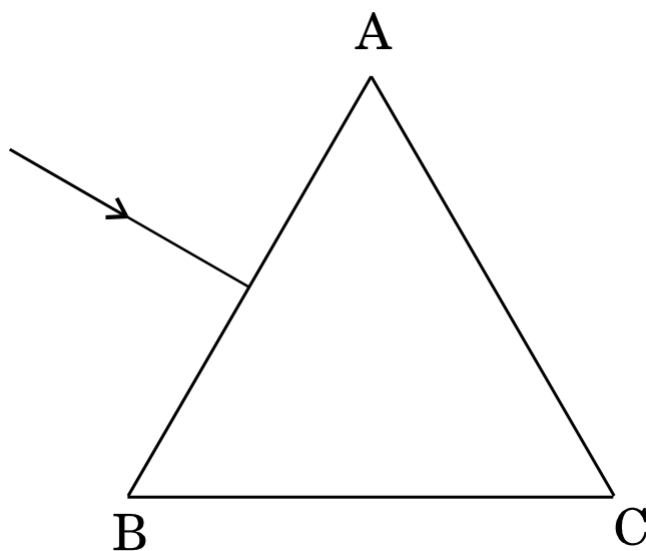
Rearranging this equation expresses the unknown liquid index using only the two measured distances. The negative liquid power also explains why an ordinary liquid with $1 < \mu < 2$ gives $x > y$.

Step 4: Conclusion The refractive index of the liquid is $\mu = 2 - \frac{y}{x}$, where x is measured with the liquid and y without it.

Quick Tip to Solve: The trapped liquid is a diverging plano-concave lens: subtract its power from the glass power, and use the dry reading to obtain $R = y$.

16. (a) Show using a proper diagram how unpolarised light can be linearly polarised by reflection from a transparent glass surface.

(b) The figure shows a ray of light falling normally on the face AB of an equilateral glass prism having refractive index $\frac{3}{2}$, placed in water of refractive index $\frac{4}{3}$. Will this ray suffer total internal reflection on striking the face AC ? Justify your answer. [3 marks]

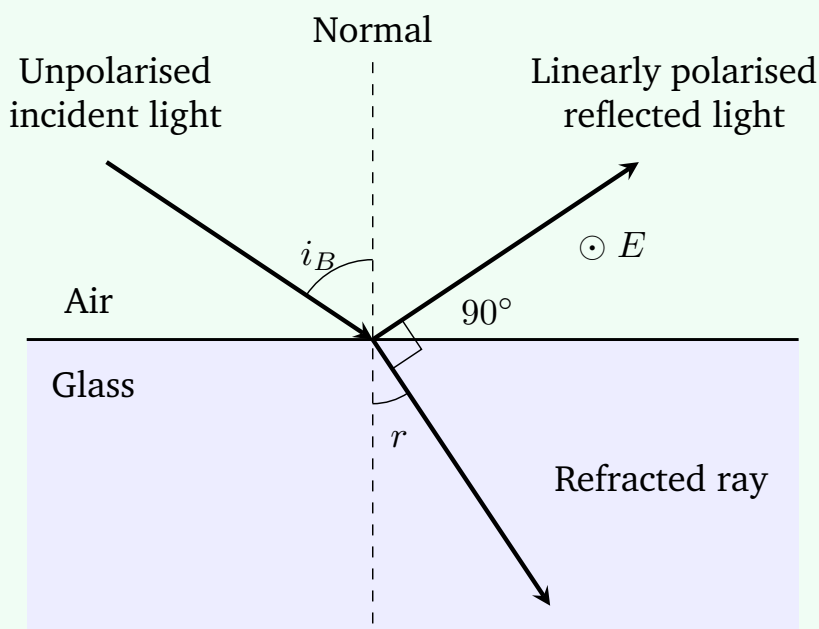


Solution:

Concept:

- Brewster's law is $\tan i_B = n_g/n_a$, where i_B is the polarising angle and n_g, n_a are the refractive indices of glass and air. At this angle the reflected and refracted rays are perpendicular.
- Total internal reflection requires travel from an optically denser medium to a rarer medium and incidence greater than the critical angle c , where $\sin c = n_{\text{rarer}}/n_{\text{denser}}$.

Step 1: (a) Polarise the reflected light at Brewster's angle Direct unpolarised light from air onto the glass surface at i_B . Its incident electric field has components both in and perpendicular to the plane of incidence. At this special angle the component parallel to the plane of incidence has zero reflected amplitude. Hence the reflected beam contains only the electric field perpendicular to that plane and is completely linearly polarised. In the diagram the plane of incidence is the page, so a circled dot denotes an electric field perpendicular to the page. The transmitted beam is generally only partially polarised. Brewster's condition gives $i_B + r = 90^\circ$ and $\tan i_B = n_g/n_a$. Reflection at an arbitrary incidence angle need not give complete polarisation.



Polarisation by reflection at Brewster's angle; the reflected and refracted rays are perpendicular.

Step 2: (b) Find the incidence angle at face AC The incident ray strikes face AB normally, so it enters the glass without changing direction and travels along the normal to AB . The angle between the normals of the two prism faces equals the prism angle, 60° . Therefore the angle of incidence, measured from the normal at AC , is $i = 60^\circ$, not 30° , which is its angle with the face itself.

Step 3: Compare the incidence with the glass-water critical angle Glass has index $n_g = 3/2$, while the surrounding water has index $n_w = 4/3$. The ray does travel from denser glass into rarer water, but that condition alone is insufficient. The critical angle satisfies

$$\sin c = \frac{n_w}{n_g} = \frac{4/3}{3/2} = \frac{8}{9}, \quad c = \sin^{-1} \left(\frac{8}{9} \right) \simeq 62.7^\circ.$$

Quick Tip to Solve: Measure incidence from the normal and use the surrounding water's index in the critical-angle ratio; the air-glass critical angle is irrelevant here.

Since $i = 60^\circ < c$, the ray is transmitted into the water with ordinary partial reflection. Equivalently, Snell's law gives $\sin r = (9/8) \sin 60^\circ = 9\sqrt{3}/16 \simeq 0.9743 < 1$, confirming that a refracted ray is possible.

Step 4: Conclusion (a) At Brewster's angle, the reflected beam is linearly polarised with its electric field perpendicular to the plane of incidence. (b) The ray does not undergo total internal reflection at AC, because $60^\circ < \sin^{-1}(8/9) \simeq 62.7^\circ$.

Quick Tip to Solve: Measure incidence from the normal and use the surrounding water's index in the critical-angle ratio; the air-glass critical angle is irrelevant here.

17. (a) State Bohr's postulate to define stable orbits in hydrogen atom. How does de Broglie's hypothesis explain the stability of these orbits ?

(b) A hydrogen atom initially in the ground state absorbs a photon which excites it to the $n = 4$ level. Estimate the frequency of the photon. [3 marks]

Solution:

Concept:

- Bohr's stationary orbits have quantised orbital angular momentum: $m_e v r = n\hbar = nh/(2\pi)$, where $n = 1, 2, 3, \dots$, m_e is the electron mass and r is the orbital radius. An electron in a stationary orbit does not radiate energy.
- The de Broglie wavelength of an electron of momentum $p = m_e v$ is $\lambda = h/p$. A stable matter wave must fit an integral number of wavelengths around the orbit.
- Hydrogen energy levels are $E_n = -13.6/n^2$ eV. A photon absorbed in a transition supplies $h\nu = E_f - E_i$, where ν is its frequency.

Step 1: State the stationary-orbit condition (a) Only those circular orbits in which the electron's angular momentum is an integral multiple of $\hbar/(2\pi)$ are allowed. Bohr postulated that an electron revolving in such a stationary orbit neither emits nor absorbs radiation; its energy therefore remains fixed. Radiation is exchanged only during a transition between stationary states.

Step 2: Fit the electron wave around the orbit According to de Broglie, the electron is associated with a matter wave. A wave that returns to its starting point in phase produces a standing wave along the circumference, so

$$2\pi r = n\lambda = n \frac{h}{m_e v} \Rightarrow m_e v r = \frac{nh}{2\pi}.$$

This reproduces Bohr's quantisation condition. If the circumference is not an integral multiple of λ , the wave fails to join continuously in phase and cannot form an allowed standing-wave state. Thus de Broglie's hypothesis explains the allowed stable orbits through constructive interference of the electron wave.

Step 3: Calculate the photon energy and frequency (b) The atom starts at $n = 1$ and ends at $n = 4$.

Quick Tip to Solve: For hydrogen excitation from the ground state, use $\Delta E = 13.6(1 - 1/n^2)$ eV, then convert electron volts to joules before dividing by h .

The final energy is less negative, so the required absorbed energy is positive:

$$E_1 = -13.6 \text{ eV}, \quad E_4 = -\frac{13.6}{16} = -0.85 \text{ eV},$$

$$\Delta E = E_4 - E_1 = 12.75 \text{ eV}$$

$$= 12.75(1.6 \times 10^{-19}) = 2.04 \times 10^{-18} \text{ J},$$

$$\nu = \frac{\Delta E}{h} = \frac{2.04 \times 10^{-18}}{6.63 \times 10^{-34}} \simeq 3.08 \times 10^{15} \text{ Hz}.$$

Step 4: Conclusion Stable orbits obey $m_e v r = n h / (2\pi)$, equivalently $2\pi r = n\lambda$; the electron matter wave is a standing wave. The absorbed photon has frequency approximately $3.08 \times 10^{15} \text{ Hz}$.

Quick Tip to Solve: For hydrogen excitation from the ground state, use $\Delta E = 13.6(1 - 1/n^2) \text{ eV}$, then convert electron volts to joules before dividing by h .

18. (a) If one of two identical slits producing interference in Young's experiment is covered with glass, so that the light intensity passing through it is reduced to 50%, find the ratio of the maximum and minimum intensity of the fringe in the interference pattern.

(b) What kind of fringes do you expect to observe if white light is used instead of monochromatic light ?

[3 marks]

Solution:

Concept:

- For two coherent waves of individual intensities I_1 and I_2 , the resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, where ϕ is the phase difference.
- The extremes are $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$. Wave amplitude is proportional to the square root of intensity.
- In Young's experiment the fringe width is $\beta = \lambda D / d$, where D is the screen distance and d the slit separation. Different wavelengths in white light therefore produce different fringe widths.

Step 1: Find the amplitude ratio after attenuation (a) Let the intensity due to each initially identical slit be I_0 . After glass covers one slit, the two individual intensities are $I_1 = I_0$ and $I_2 = I_0/2$. Their amplitudes are consequently in the ratio

$$\frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{2}.$$

A phase shift due to glass displaces the pattern but does not change this intensity ratio.

Step 2: Evaluate the maximum-to-minimum intensity ratio At constructive interference the amplitudes

Quick Tip to Solve: A reduction to half the intensity means an amplitude factor $1/\sqrt{2}$, not $1/2$. Take square roots before adding or subtracting the slit contributions.

add, while at destructive interference they subtract. Hence

$$\begin{aligned}\frac{I_{\max}}{I_{\min}} &= \frac{(\sqrt{I_0} + \sqrt{I_0/2})^2}{(\sqrt{I_0} - \sqrt{I_0/2})^2} \\ &= \left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right)^2 = (3 + 2\sqrt{2})^2 \\ &= 17 + 12\sqrt{2} \simeq 33.97.\end{aligned}$$

Since the amplitudes are unequal, the minimum has nonzero intensity: the waves cannot cancel completely.

Step 3: Describe the white-light pattern (b) With white light in the usual equal-path Young's arrangement, the central fringe is white: zero path difference makes all visible wavelengths constructively interfere at the same position. On either side, a few coloured fringes are visible. Violet lies nearer the centre and red farther away because β increases with wavelength. At larger distances, maxima of different colours overlap, so distinct fringes become indistinguishable and the screen appears almost uniformly illuminated.

Step 4: Conclusion The intensity ratio is $I_{\max} : I_{\min} = (17 + 12\sqrt{2}) : 1 \simeq 34 : 1$. White light gives a white central fringe and a few coloured fringes on both sides.

Quick Tip to Solve: A reduction to half the intensity means an amplitude factor $1/\sqrt{2}$, not $1/2$. Take square roots before adding or subtracting the slit contributions.

19. Draw the typical input and output characteristics of an n-p-n transistor in CE configuration. Show how these characteristics can be used to determine (a) the input resistance (r_i), and (b) current amplification factor (β). [3 marks]

Solution:

Concept:

- In common-emitter (CE) configuration, the emitter is common to the input base-emitter circuit and the output collector-emitter circuit. For an n-p-n transistor in the active region, the base-emitter junction is forward biased and the collector-base junction is reverse biased.
- The input characteristic plots base current I_B against base-emitter voltage V_{BE} at constant collector-emitter voltage V_{CE} . The small-signal input resistance is $r_i = (\Delta V_{BE} / \Delta I_B)_{V_{CE}}$.
- The output characteristics plot collector current I_C against V_{CE} for different fixed values of I_B . The small-signal current gain is $\beta = (\Delta I_C / \Delta I_B)_{V_{CE}}$.

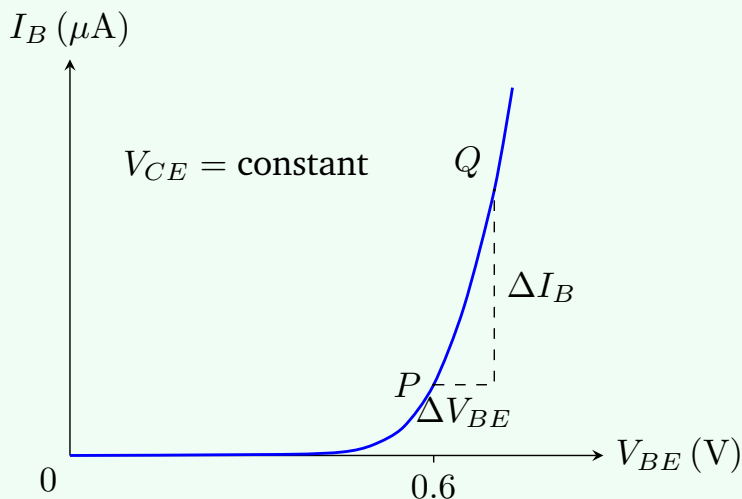
Step 1: Draw the input characteristic and measure its slope (a) Keep V_{CE} fixed, vary V_{BE} , and record I_B . The input curve resembles the forward characteristic of a diode: the base current is small initially and rises rapidly after the junction begins conducting appreciably. The graph below shows a typical silicon transistor curve; the actual turn-on voltage depends on the device. Choose nearby points P and Q around the operating point on the same curve. Read their voltage difference along the horizontal

Quick Tip to Solve: For r_i , select two nearby points on one input curve. For β , select two nearby output curves at the same V_{CE} , and convert both current differences to the same unit.

axis and their current difference along the vertical axis:

$$r_i = \left. \frac{\Delta V_{BE}}{\Delta I_B} \right|_{V_{CE} \text{ constant}}.$$

Thus input resistance is the reciprocal of the local slope of the I_B -versus- V_{BE} curve. Convert a current difference in microamperes to amperes to obtain resistance in ohms.



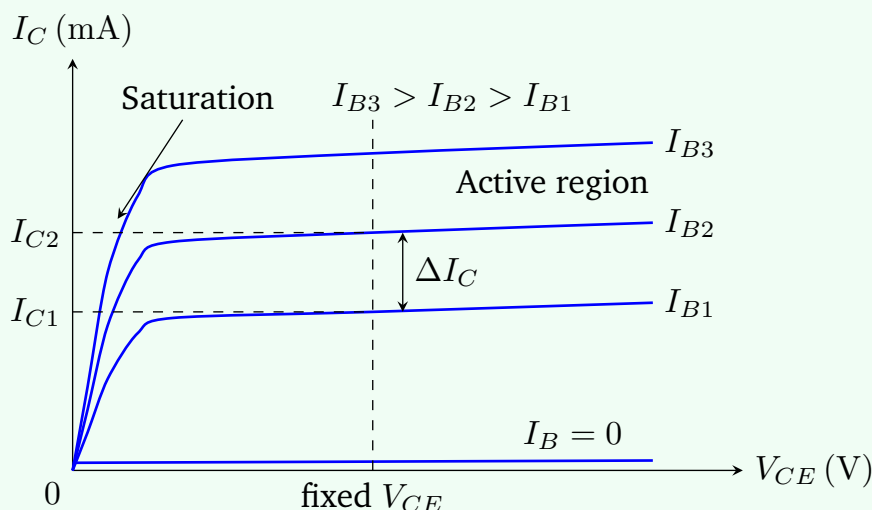
Typical CE input characteristic at fixed V_{CE} ; the dashed triangle gives the local input resistance.

Step 2: Draw the output family and determine current gain (b) Set a fixed base current, vary V_{CE} , and record I_C ; repeat for several values of I_B . Each curve first rises sharply in the saturation region and then becomes nearly horizontal in the active region. Larger base current gives a larger collector current. A real transistor shows a small residual slope in the active region. To determine β , draw a vertical line at a chosen constant V_{CE} in the active region. Read I_{C1} and I_{C2} on the curves for two nearby base currents I_{B1} and I_{B2} . Then

$$\beta = \left. \frac{I_{C2} - I_{C1}}{I_{B2} - I_{B1}} \right|_{V_{CE} \text{ constant}} = \left. \frac{\Delta I_C}{\Delta I_B} \right|_{V_{CE} \text{ constant}}.$$

Both current differences must be expressed in the same units. This is the incremental current gain; the dc ratio I_C/I_B is a different measurement.

Quick Tip to Solve: For r_i , select two nearby points on one input curve. For β , select two nearby output curves at the same V_{CE} , and convert both current differences to the same unit.



Typical CE output characteristics; at fixed V_{CE} , compare collector currents for two nearby base-current values.

Step 3: Conclusion From the input characteristic, $r_i = \Delta V_{BE} / \Delta I_B$ at fixed V_{CE} , in ohms. From the output family, $\beta = \Delta I_C / \Delta I_B$ at fixed V_{CE} in the active region; β is dimensionless.

Quick Tip to Solve: For r_i , select two nearby points on one input curve. For β , select two nearby output curves at the same V_{CE} , and convert both current differences to the same unit.

20. (a) Give three reasons why modulation of a message signal is necessary for long distance transmission.

(b) Show graphically an audio signal, a carrier wave and an amplitude modulated wave. [3 marks]

Solution:

Concept:

- Modulation transfers a low-frequency message onto a high-frequency carrier. In amplitude modulation, the carrier amplitude varies in accordance with the instantaneous message while its frequency remains fixed.
- For efficient radiation, the transmitting antenna should have a length of the order of $\lambda/4$, where $\lambda = c/f$. Thus raising the transmitted frequency reduces the required antenna length.

Step 1: (a) Three reasons for modulation Practical antenna size: Direct transmission of audio would require an impractically long antenna. For example, at $f = 1 \text{ kHz}$, $\lambda = c/f = 3 \times 10^5 \text{ m}$, so $\lambda/4 = 75 \text{ km}$. A high-frequency carrier makes a practical antenna possible.

Efficient radiation and longer range: For an antenna of fixed length, radiation becomes more efficient as the ratio of antenna length to wavelength increases (in the short-antenna regime). A low-frequency audio signal is poorly radiated; using a high-frequency carrier enables efficient transmission over long distances.

Avoiding mixing of messages: Audio signals from different transmitters occupy similar frequency ranges. Modulating them onto different carrier frequencies separates their channels, allowing a

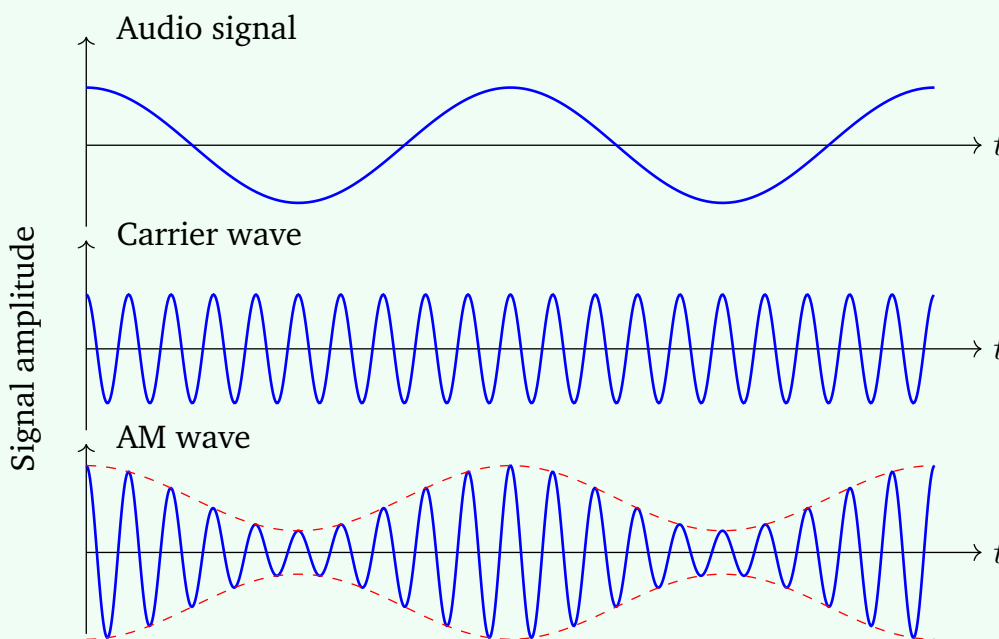
Quick Tip to Solve: Draw many carrier oscillations within one audio cycle, then vary only their amplitude so that their peaks meet the audio-shaped envelopes.

receiver to select one station by tuning.

Step 2: (b) Audio, carrier and amplitude-modulated waveforms Take a sinusoidal message $m(t) = A_m \cos \omega_m t$ and carrier $c(t) = A_c \cos \omega_c t$, with $\omega_c \gg \omega_m$. The amplitude-modulated signal is

$$s(t) = A_c[1 + \mu \cos \omega_m t] \cos \omega_c t, \quad \mu = \frac{A_m}{A_c} \leq 1.$$

The message changes slowly, whereas the unmodulated carrier oscillates rapidly with constant amplitude. In the AM wave, the rapid oscillations retain the carrier frequency and their envelope reproduces the message variation, shifted upward by the carrier amplitude. The dashed curves show the upper and lower envelopes; the sketch uses $\mu = 0.6$ to avoid overmodulation.



Audio signal, carrier wave and AM wave on the same time scale.

Step 3: Conclusion Modulation gives a practical antenna size, efficient radiation for long-distance transmission and separate selectable channels. The sketches show a slow audio signal, a constant-amplitude high-frequency carrier and an AM wave whose envelope follows the audio signal.

Quick Tip to Solve: Draw many carrier oscillations within one audio cycle, then vary only their amplitude so that their peaks meet the audio-shaped envelopes.

21. (a) Explain the processes of nuclear fission and nuclear fusion by using the plot of binding energy per nucleon (BE/A) versus the mass number A .

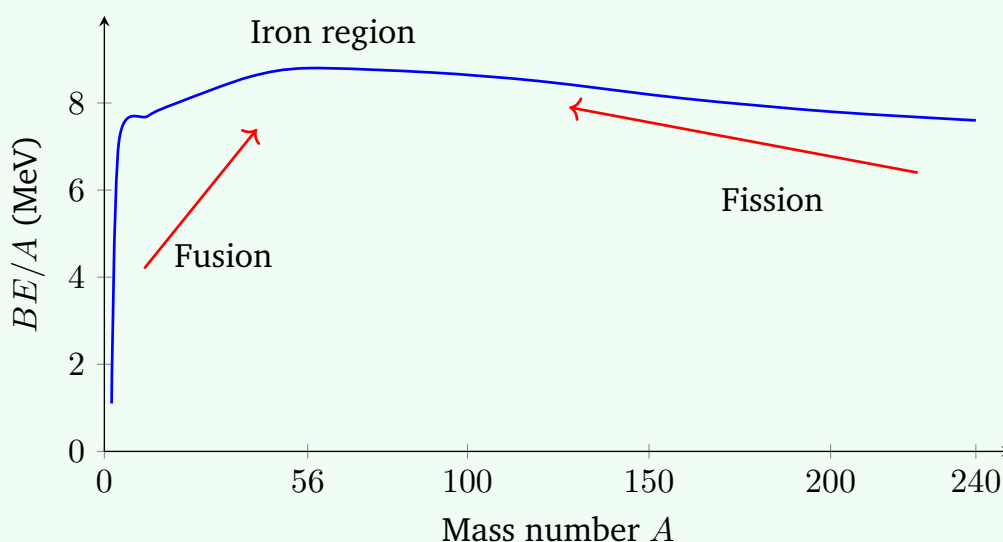
(b) A radioactive isotope has a half-life of 10 years. How long will it take for the activity to reduce to 3.125% ? [3 marks]

Solution:

Concept:

- Binding energy per nucleon, BE/A , measures the average energy needed to separate a nucleon from a nucleus. A higher value generally means greater nuclear stability.
- Energy is released when a nuclear reaction increases the total binding energy of its products: $Q = BE_{\text{products}} - BE_{\text{reactants}} > 0$. The released energy corresponds to the decrease in rest mass.
- Radioactive activity R is proportional to the number of undecayed nuclei: $R/R_0 = (1/2)^{t/T_{1/2}}$, where R_0 is initial activity and $T_{1/2}$ is the half-life.

Step 1: (a) Interpret the binding-energy curve The curve rises steeply for light nuclei, reaches a broad maximum of about 8.8 MeV per nucleon near the iron region ($A \simeq 56$), and decreases slowly for heavy nuclei. Thus nuclei of intermediate mass are more tightly bound per nucleon than very light or very heavy nuclei. The schematic plot shows the overall trend; small variations among individual light nuclei are omitted.



Schematic binding energy per nucleon versus mass number; both reactions move toward greater binding.

Step 2: Explain fission and fusion using the curve **Fission:** A heavy nucleus, such as uranium, splits into two lighter nuclei of intermediate mass, usually with a few neutrons. These fragments lie closer to the maximum of the curve and have a greater binding energy per nucleon. Since the total number of nucleons is conserved, the increase in total binding energy appears as released energy, mainly kinetic energy of the products.

Fusion: Two light nuclei combine to form a heavier nucleus that still lies on the rising part of the curve. The product is more tightly bound per nucleon, so its greater total binding energy also releases energy. High temperature is needed to overcome the electrostatic repulsion between the positively charged nuclei. Both processes therefore move toward the more tightly bound intermediate-mass

Quick Tip to Solve: Convert the remaining percentage to a fraction first: $3.125\% = 1/32 = 2^{-5}$, so the elapsed time is immediately five half-lives.

region, from opposite sides of the curve.

Step 3: (b) Calculate the number of half-lives The required fraction of the original activity is

$$\frac{R}{R_0} = \frac{3.125}{100} = 0.03125 = \frac{1}{32} = \left(\frac{1}{2}\right)^5.$$

Using $R/R_0 = (1/2)^{t/T_{1/2}}$, compare the exponents to obtain $t/T_{1/2} = 5$. With $T_{1/2} = 10$ years, the required elapsed time is $t = 5 \times 10$ years. The successive remaining percentages are 50, 25, 12.5, 6.25, 3.125, confirming five halvings.

Step 4: Conclusion Fission of heavy nuclei and fusion of light nuclei release energy because their products have greater total binding energy, moving toward the maximum of the BE/A curve. The activity reaches 3.125% of its initial value after 50 years.

Quick Tip to Solve: Convert the remaining percentage to a fraction first: $3.125\% = 1/32 = 2^{-5}$, so the elapsed time is immediately five half-lives.

22. (a) A student wants to use two p-n junction diodes to convert alternating current into direct current. Draw the labelled circuit diagram she would use and explain how it works.

(b) Give the truth table and circuit symbol for NAND gate.

[3 marks]

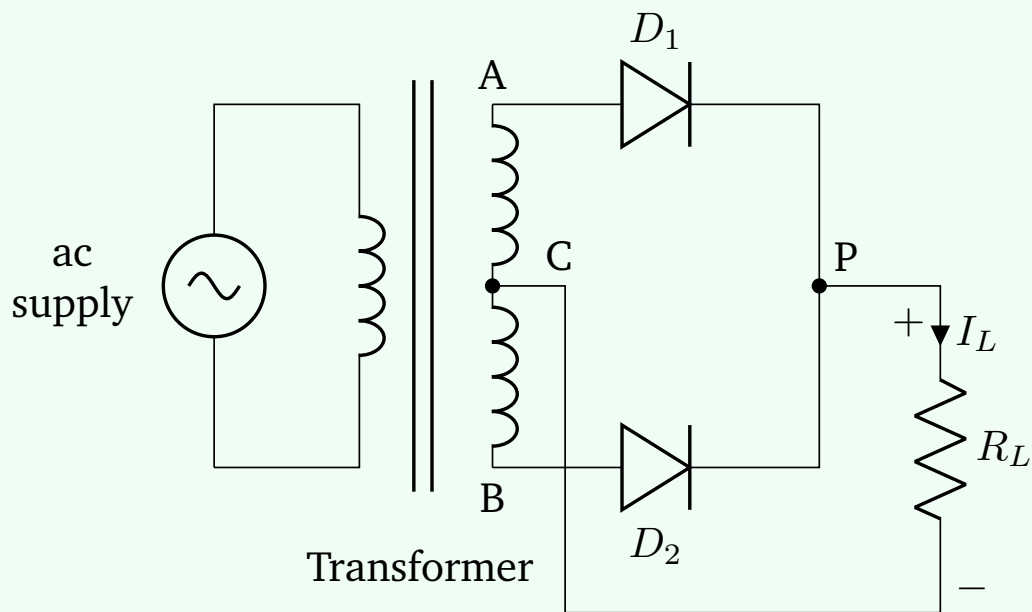
Solution:

Concept:

- A p-n junction diode conducts when forward biased and blocks current when reverse biased. Rectification uses this directional conduction to produce unidirectional load current.
- A centre-tapped transformer and two diodes form a full-wave rectifier: the ends of the secondary have opposite instantaneous polarities relative to the centre tap.
- A NAND gate gives the complement of the AND output: $Y = \overline{A \cdot B}$. Its output is low only when both inputs are high.

Step 1: (a) Connect a two-diode full-wave rectifier Connect the secondary ends A and B of a centre-tapped transformer to the anodes of diodes D_1 and D_2 . Join their cathodes at P , and connect the load resistance R_L between P and the centre tap C . The primary is connected to the ac supply. The diagram labels the positive output at P and the return at C .

Quick Tip to Solve: In the rectifier, trace each conducting path through the load: both must enter its positive terminal. For NAND, write zero only for input 11.



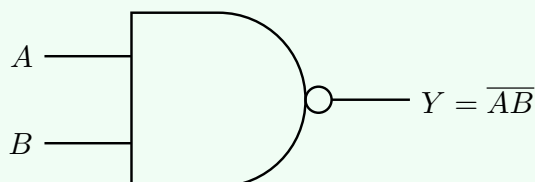
Centre-tapped transformer full-wave rectifier; both diode cathodes meet at P.

Step 2: Explain conduction in the two half-cycles In the half-cycle when A is positive relative to C, B is negative: D_1 is forward biased and conducts, while D_2 is reverse biased. The current path is $A \rightarrow D_1 \rightarrow P \rightarrow R_L \rightarrow C$. In the next half-cycle, B is positive relative to C: D_2 conducts and D_1 blocks. The path becomes $B \rightarrow D_2 \rightarrow P \rightarrow R_L \rightarrow C$. Thus current passes through the load from P to C in both half-cycles. Both halves of the input waveform are used, giving pulsating dc with ripple frequency twice the supply frequency. The diodes alone do not provide perfectly smooth dc; a filter would be needed for smoothing.

Step 3: (b) Give the NAND symbol and truth table The small circle at the output of the AND-shaped symbol denotes inversion. Applying $Y = \overline{A \cdot B}$ to all four possible input combinations gives

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

For either input equal to zero, the AND result is zero and inversion makes the NAND output one. When both inputs equal one, the AND result is one and inversion makes the output zero.



NAND gate: the output circle indicates logical inversion.

Step 4: Conclusion (a) Use the labelled centre-tapped full-wave rectifier; alternate diode conduction

Quick Tip to Solve: In the rectifier, trace each conducting path through the load: both must enter its positive terminal. For NAND, write zero only for input 11.

keeps the load current in the same direction. (b) The NAND output is $Y = \overline{AB}$, with outputs 1, 1, 1, 0 for inputs 00, 01, 10, 11, respectively.

Quick Tip to Solve: In the rectifier, trace each conducting path through the load: both must enter its positive terminal. For NAND, write zero only for input 11.

Section D (4 marks)

23. The teachers of Geeta's school took the students on a study trip to a power generating station, located nearly 200 km away from the city. The teacher explained that electrical energy is transmitted over such a long distance to their city, in the form of alternating current (ac) raised to a high voltage. At the receiving end in the city, the voltage is reduced to operate the devices. As a result, the power loss is reduced. Geeta listened to the teacher and asked questions about how the ac is converted to a higher or lower voltage.

(a) Name the device used to change the alternating voltage to a higher or lower value. State one cause for power dissipation in this device.

(b) Explain with an example, how power loss is reduced if the energy is transmitted over long distances as an alternating current rather than a direct current.

(c) Write two values each shown by the teachers and Geeta.

[4 marks]

Solution:

Concept:

- A transformer changes ac voltage by mutual induction. For an ideal transformer, $V_s/V_p = N_s/N_p$, where N_p, N_s are the primary and secondary turns.
- For fixed transmitted power P , with unity power factor for this comparison, the line current is $I = P/V$. Resistive heating in transmission wires of total resistance R is $P_{\text{loss}} = I^2 R$.
- A step-up transformer reduces the current needed for a given transmitted power; a step-down transformer then supplies a suitable voltage to consumers.

Step 1: (a) Name the device and one source of dissipation The device is a transformer. A step-up transformer has $N_s > N_p$ and increases the ac voltage; a step-down transformer has $N_s < N_p$ and decreases it. One cause of power dissipation is Joule heating in the copper windings: because their resistance is finite, current produces heat at the rate $I^2 R$. The alternating primary current produces a changing magnetic flux linking the secondary and hence induces an emf there. A steady dc supply does not produce this continuously changing flux, so an ordinary transformer cannot continuously step up or step down steady dc.

Step 2: (b) Explain the reduction in transmission loss The advantage in the arrangement described is that ac can easily be raised to a high voltage using a transformer before transmission. For the same power, a higher voltage means a smaller line current, and therefore smaller $I^2 R$ loss. If the voltage is increased by a factor k , the current becomes I/k , and the line loss becomes $I^2 R/k^2$. At the receiving end, a step-down transformer restores the voltage needed by appliances. Thus the reduction follows from high-voltage, low-current transmission; alternating current at the same voltage and power would not inherently have a lower resistive loss than direct current.

Quick Tip to Solve: Keep transmitted power fixed when comparing losses: multiplying voltage by k divides current by k and divides wire heating by k squared.

Step 3: Demonstrate with a numerical example Suppose 100 kW is sent through wires having a total resistance of $1\ \Omega$, neglecting transformer losses and taking unity power factor. At 1 kV, the line current and loss are

$$I_1 = \frac{100000}{1000} = 100\text{ A}, \quad P_{\text{loss},1} = 100^2 \times 1 = 10000\text{ W}.$$

After stepping the voltage up to 10 kV, they become

$$I_2 = \frac{100000}{10000} = 10\text{ A}, \quad P_{\text{loss},2} = 10^2 \times 1 = 100\text{ W}.$$

A tenfold voltage increase therefore reduces the loss to one hundredth of its previous value. In the conventional comparison with low-voltage dc, dc cannot be raised by an ordinary transformer, whereas ac permits this convenient high-voltage transmission.

Step 4: (c) Identify two values shown by each The teachers show responsibility towards students' education by arranging a useful study visit, and commitment to practical learning by explaining the power station and transmission system. Geeta shows scientific curiosity by asking how the voltage is changed, and attentiveness by listening carefully to the explanation before asking relevant questions.

Step 5: Conclusion (a) A transformer changes ac voltage; winding resistance causes heat loss. (b) Raising the transmission voltage lowers the current and hence the I^2R loss, as the example demonstrates. (c) The teachers show educational responsibility and commitment to practical learning; Geeta shows scientific curiosity and attentiveness.

Quick Tip to Solve: Keep transmitted power fixed when comparing losses: multiplying voltage by k divides current by k and divides wire heating by k squared.

Section E (15 marks)

24. (a) Draw a ray diagram to show image formation when the concave mirror produces a real, inverted and magnified image of the object.

(b) Obtain the mirror formula and write the expression for the linear magnification.

(c) Explain two advantages of a reflecting telescope over a refracting telescope.

[5 marks]

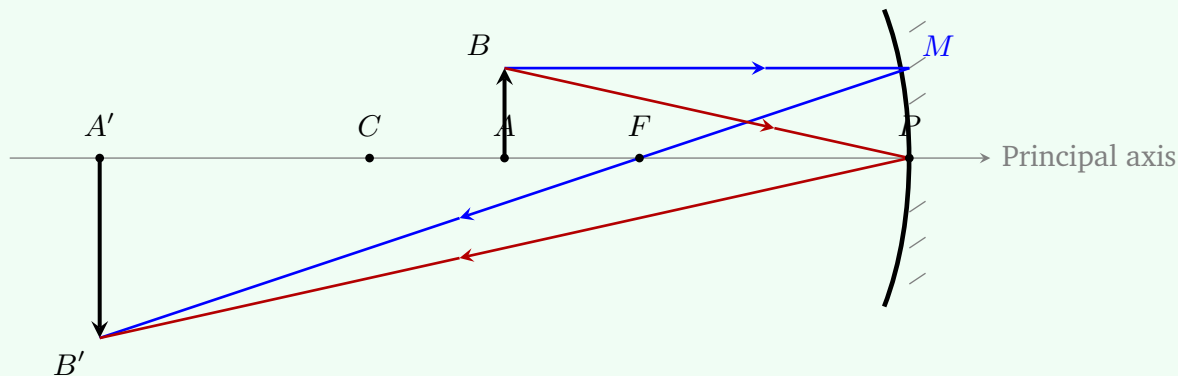
Solution:

Concept:

- A concave mirror forms a real, inverted, magnified image when the object lies between its principal focus F and centre of curvature C .
- A ray parallel to the principal axis is reflected through F ; a ray incident at the pole obeys the law of reflection, with equal angles to the principal axis.
- The mirror formula applies to a spherical mirror for paraxial rays and a small aperture. In the Cartesian convention, distances are measured from the pole P , with the direction of incident light taken as positive.
- Reflection is independent of wavelength, whereas a refracting objective disperses light. A larger objective aperture collects more light and improves angular resolution.

Quick Tip to Solve: For a magnified real image in a concave mirror, first put the object between F and C ; verify that the image is beyond C and that the signed magnification is negative with magnitude greater than one.

Step 1: (a) Locate the object and construct the image Place the upright object AB between C and F . From its tip B , draw one ray parallel to the principal axis; after reflection it passes through F . Draw a second ray to the pole P , reflecting it with equal angles to the principal axis. The reflected rays actually meet at B' beyond C , giving image $A'B'$. Since B' lies below the axis and $A'B' > AB$, the image is real, inverted and magnified. The arrows show actual light propagation towards the mirror and then away from it.



Concave mirror: object between C and F , image beyond C (paraxial construction).

Step 2: (b) Derive the mirror formula using similar triangles Let $U = AP$, $V = A'P$, $F_0 = FP$ be positive distance magnitudes in the arrangement shown, and let $h = AB$, $H = A'B'$ be positive height magnitudes. The ray reflected at the pole gives equal angles with the axis. Thus the right triangles ABP and $A'B'P$ are similar, so

$$\frac{H}{h} = \frac{V}{U}.$$

Let M be the point where the parallel ray strikes the mirror. For a small aperture, its axial position is approximately that of P , and its height is h . The reflected ray MFB' makes the right triangles with axial lengths FP and FA' similar. Hence

$$\frac{H}{h} = \frac{V - F_0}{F_0}.$$

Equating these two ratios and rearranging gives

$$\begin{aligned}\frac{V}{U} &= \frac{V - F_0}{F_0}, \\ F_0 V &= UV - UF_0, \\ UV &= F_0(U + V), \\ \frac{1}{F_0} &= \frac{1}{U} + \frac{1}{V}.\end{aligned}$$

Here the incident light travels to the right, whereas object, real image and focus lie to the left of P . Therefore $u = -U$, $v = -V$, $f = -F_0$. Substitution yields the signed mirror formula $1/f = 1/u + 1/v$. The general signed formula also covers virtual images when their appropriate Cartesian signs are used.

Step 3: (b) Obtain the linear magnification The object height is $h_o = h > 0$, while the inverted image

Quick Tip to Solve: For a magnified real image in a concave mirror, first put the object between F and C ; verify that the image is beyond C and that the signed magnification is negative with magnitude greater than one.

has signed height $h_i = -H$. Using the pole-ray triangle relation,

$$m = \frac{h_i}{h_o} = -\frac{H}{h} = -\frac{V}{U} = -\frac{v}{u}.$$

For this arrangement $F_0 < U < 2F_0$, and the mirror formula gives $V = F_0U/(U - F_0) > 2F_0$. Thus $m < 0$ and $|m| > 1$, which checks the inversion and enlargement in the ray diagram.

Step 4: (c) Explain two advantages of a reflecting telescope **Freedom from chromatic aberration:** A reflecting telescope uses a concave mirror as its objective. The law of reflection is the same for all wavelengths, so different colours have the same objective focus. A lens objective has a wavelength-dependent refractive index and can produce colour fringes unless corrected.

Practical construction of a large aperture: The objective mirror can be supported from its back, allowing a large aperture with less deformation than a large lens supported at its rim. Such a mirror also does not require a large, homogeneous optical-glass volume through which the light must pass. A larger aperture gathers more light from faint astronomical objects and gives better angular resolving power.

Step 5: Conclusion The object lies between F and C , and its image forms beyond C , real, inverted and enlarged. The required relations are $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ and $m = \frac{h_i}{h_o} = -\frac{v}{u}$. Reflecting telescopes avoid objective chromatic aberration and permit practical large-aperture objectives for greater light collection and resolution.

Quick Tip to Solve: For a magnified real image in a concave mirror, first put the object between F and C ; verify that the image is beyond C and that the signed magnification is negative with magnitude greater than one.

OR

- 24. (a) Define a wavefront. Using Huygens' principle, verify the laws of reflection at a plane surface.**
(b) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band ? Explain.
(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the obstacle. Explain why. [5 marks]

Solution:

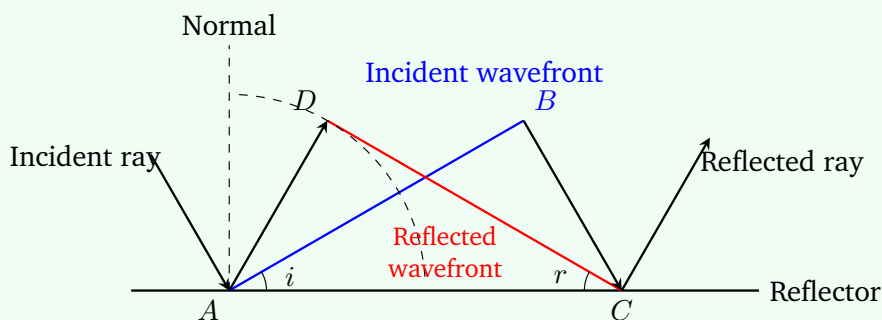
Concept:

- A wavefront is the locus of all points of a wave that oscillate in the same phase at a given instant. Rays in a homogeneous isotropic medium are normal to the wavefront.
- Huygens' principle: each point on a wavefront acts as a source of secondary wavelets; their forward envelope gives the new wavefront. In the same medium, each wavelet travels a distance vt in time t .
- For a slit of width a , the first diffraction minima satisfy $a \sin \theta = \pm \lambda$. For small angles and a screen distance D , the central maximum has width $W = 2D\lambda/a$.
- At the central maximum, contributions from the whole slit arrive in phase. With fixed incident

Quick Tip to Solve: For a slit-width change, use width proportional to $1/a$ and central intensity proportional to a^2 ; apply the two scalings separately.

illumination, resultant amplitude is proportional to slit width and intensity is proportional to the square of this amplitude.

Step 1: (a) Construct the reflected wavefront using Huygens' principle Consider a plane incident wavefront AB meeting the plane reflecting surface AC first at A . Point B travels along the incident ray BC , perpendicular to AB , and reaches the surface at C after time t . Thus $BC = vt$. Meanwhile the secondary reflected wavelet originating at A has radius $AD = vt$. Draw the tangent CD from C to this wavelet; the tangent touches it at D , so $AD \perp CD$. The envelope CD is the reflected wavefront. Consequently AD is along a reflected ray. The incident and reflected waves remain in the same medium, so their speeds are equal and $BC = AD$.



Huygens' construction for reflection: $BC = AD = vt$, $AB \perp BC$, and $CD \perp AD$.

Step 2: (a) Verify both laws of reflection Triangles ABC and ADC are right angled at B and D , respectively. They have the same hypotenuse AC and equal sides $BC = AD$; hence they are congruent. Therefore

$$\angle BAC = \angle DCA.$$

The incident ray is perpendicular to AB , and the surface normal is perpendicular to AC . Thus the angle of incidence i equals $\angle BAC$. Similarly the reflected ray is perpendicular to CD , so the angle of reflection r equals $\angle DCA$. Hence $i = r$. The construction takes place in the plane containing the incident ray and the normal to the reflecting surface. The reflected ray, being normal to the reflected wavefront in this construction, lies in this same plane. This verifies the second law: the incident ray, reflected ray and normal at the point of incidence are coplanar.

Step 3: (b) Find the change in central-band width Keep the wavelength, screen distance and incident illumination unchanged. The central band extends between the first minima on either side of the axis. Its angular width is approximately $2\lambda/a$, and its linear width is $W = 2D\lambda/a$. When the slit width becomes $a' = 2a$,

$$W' = \frac{2D\lambda}{2a} = \frac{W}{2}.$$

Thus the central band becomes half as wide. Physically, a wider aperture produces less angular spreading by diffraction.

Step 4: (b) Find the change in central intensity At the centre of the diffraction pattern there is no path difference between contributions from different parts of the slit, so they add coherently. If the original central amplitude is A_0 , doubling the slit width doubles the number of contributing secondary sources under the same illumination, giving $A'_0 = 2A_0$. Since intensity is proportional to amplitude

Quick Tip to Solve: For a slit-width change, use width proportional to $1/a$ and central intensity proportional to a^2 ; apply the two scalings separately.

squared,

$$\frac{I'_0}{I_0} = \left(\frac{A'_0}{A_0} \right)^2 = 4.$$

Therefore the peak intensity of the central band becomes four times its original value. This refers to the intensity at the centre, rather than the total light integrated over the band.

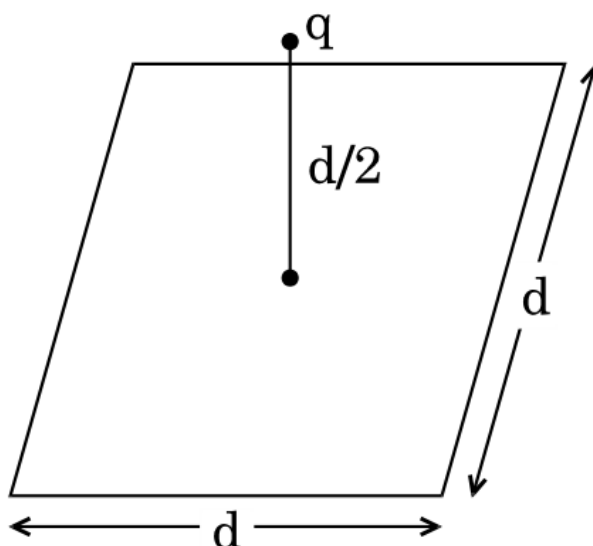
Step 5: (c) Explain the bright spot behind the circular obstacle The distant source supplies an approximately plane wavefront. Light diffracts around the rim of the tiny circular obstacle. A point on the symmetry axis at the centre of its geometrical shadow is equidistant from all points of the circular rim. Secondary wavelets from the rim therefore reach this point with equal optical path lengths and in the same phase. Their constructive interference produces a bright spot, called the Poisson or Arago spot, at the centre of the shadow on a screen behind the obstacle. The spot is in the shadow region behind the obstacle; it is not a luminous point on the opaque obstacle itself.

Step 6: Conclusion (a) A wavefront is a surface of constant phase; Huygens' construction gives $i = r$ and coplanarity of the incident ray, reflected ray and normal. (b) Doubling the slit width halves the central-band width and makes its peak intensity four times as large, under unchanged illumination. (c) Equal-path diffracted waves from the circular rim interfere constructively at the centre of the shadow, producing the Poisson spot.

Quick Tip to Solve: For a slit-width change, use width proportional to $1/a$ and central intensity proportional to a^2 ; apply the two scalings separately.

25. (a) Define electric flux. Is it a scalar or a vector quantity ? A point charge q is at a distance of $d/2$ directly above the centre of a square of side d , as shown in the figure. Use Gauss' law to obtain the expression for the electric flux through the square.

(b) If the point charge is now moved to a distance ' d ' from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected. [5 marks]



Solution:

Concept:

- Electric flux measures the electric field passing through an oriented surface: $\Phi_E = \int_S \vec{E} \cdot d\vec{A}$, where $d\vec{A} = \hat{n} dA$ is the area element along the chosen normal.
- Gauss' law states $\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\epsilon_0$, where q_{enc} is the enclosed charge and ϵ_0 is the permittivity of free space.
- A charge at the centre of a cube produces equal integrated flux through its six faces by symmetry, although its field is not uniform over any one face.

Step 1: (a) Define electric flux and its nature Electric flux through a surface is the surface integral of the component of the electric field normal to that surface. For a uniform field and a plane surface of area A ,

$$\Phi_E = EA \cos \theta,$$

where θ is the angle between $\vec{E}$ and the area normal. In general, $\Phi_E = \int_S \vec{E} \cdot d\vec{A}$. Flux is a scalar quantity because the integrand is a dot product. It can be positive, negative or zero according to the chosen orientation of the surface; this sign does not make it a vector. Its SI unit is $\text{N m}^2 \text{C}^{-1}$.

Step 2: (a) Complete a Gaussian cube around the charge Regard the given square of side d as one face of a cube of side d , extending from the square towards the charge. The centre of this cube lies on the perpendicular through the square's centre, at a distance $d/2$ from the square. Thus the given charge q is exactly at the cube's centre. The closed cube encloses the charge, so Gauss' law gives

$$\Phi_{\text{cube}} = \oint_{\text{cube}} \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}.$$

The six faces are equivalent relative to the central charge. Therefore their integrated outward fluxes are equal. Writing the outward flux through the given square as Φ_{square} ,

$$6\Phi_{\text{square}} = \frac{q}{\epsilon_0}, \quad \Phi_{\text{square}} = \frac{q}{6\epsilon_0}.$$

Here the outward normal on the original face points away from the charge. Reversing that normal reverses the signed flux. We do not set E constant on a face: it is the equal total flux through each face that follows from symmetry.

Step 3: (b) Apply the same construction after both distances double The new square has side $2d$, and the charge is a perpendicular distance d above its centre. Complete a new cube of side $2d$ on this square. Its centre is a distance $(2d)/2 = d$ from this face, so the charge is again at the cube's centre. The enclosed charge is still q ; consequently the closed-surface flux remains q/ϵ_0 , shared equally among the six faces:

$$\Phi'_{\text{square}} = \frac{1}{6} \frac{q}{\epsilon_0} = \Phi_{\text{square}}.$$

Geometrically, the enlarged square subtends the same solid angle at the charge because all relevant lengths double together. Equivalently, corresponding area elements grow by a factor of four while the field at them falls by a factor of four and their angles stay the same. Thus the flux remains unchanged.

Step 4: Conclusion (a) Electric flux is the scalar $\int_S \vec{E} \cdot d\vec{A}$. With the square oriented as an outward

Quick Tip to Solve: Check whether the perpendicular charge distance is half the square's side; then complete a cube and take one-sixth of its total flux.

face of the constructed cube, $\Phi_E = q/(6\epsilon_0)$. (b) Doubling both the square's side and the perpendicular charge distance leaves this flux unchanged.

Quick Tip to Solve: Check whether the perpendicular charge distance is half the square's side; then complete a cube and take one-sixth of its total flux.

OR

25. (a) Use Gauss' law to derive the expression for the electric field ($\vec{E}$) due to a straight uniformly charged infinite line of charge density λ C/m.

(b) Draw a graph to show the variation of E with perpendicular distance r from the line of charge.

(c) Find the work done in bringing a charge q from perpendicular distance r_1 to r_2 ($r_2 > r_1$). [5 marks]

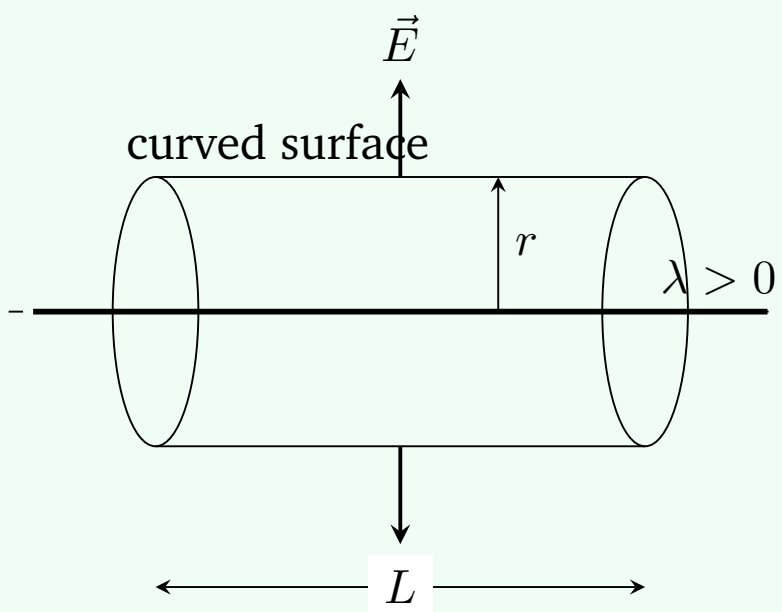
Solution:

Concept:

- Gauss' law states that $\oint_S \vec{E} \cdot d\vec{S} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the charge enclosed by a closed surface and ϵ_0 is the permittivity of free space.
- An infinite uniform line charge has cylindrical symmetry: its field is radial, has no component along the line, and has the same magnitude at every point at a given perpendicular distance r .
- For quasistatic motion, the work done by an external agent is the change in potential energy: $W_{\text{ext}} = q(V_2 - V_1) = -q \int_1^2 \vec{E} \cdot d\vec{r}$. Work done by the electric field has the opposite sign.

Step 1: (a) Choose a cylindrical Gaussian surface Take a closed cylinder of radius r and length L , coaxial with the straight infinite line charge of linear density λ . The charge inside it is $Q_{\text{enc}} = \lambda L$. By translational and rotational symmetry, the electric field is perpendicular to the line and constant in magnitude on the curved surface. For positive λ , it points radially outward; for negative λ , it points radially inward. On the curved surface, $\vec{E}$ is parallel to its outward area vector. On both flat end caps, the field is perpendicular to the area vector, so their flux is zero.

Quick Tip to Solve: Choose a coaxial cylinder so that only its curved surface contributes flux; for outward motion of like charges, check that external work is negative before finalising the logarithm.



Coaxial Gaussian cylinder; radial field for a positive line charge.

Step 2: (a) Evaluate the flux and apply Gauss' law Writing the signed radial component as E_r , the curved surface has area $2\pi rL$. Thus

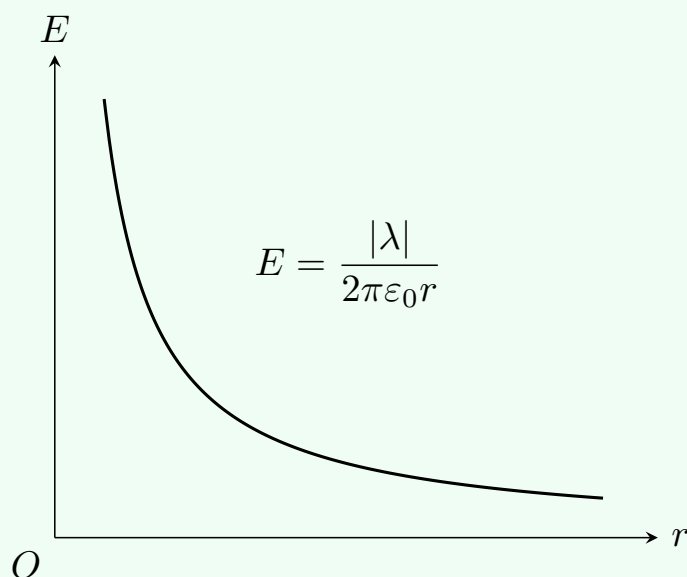
$$\oint_S \vec{E} \cdot d\vec{S} = E_r(2\pi rL) + 0 + 0 = \frac{\lambda L}{\epsilon_0},$$

$$E_r = \frac{\lambda}{2\pi\epsilon_0 r}.$$

Hence $\vec{E}(r) = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r}$, where $\hat{r}$ is the unit vector directed away from the line. Its magnitude is $E = |\lambda|/(2\pi\epsilon_0 r)$. The cancellation of L confirms that the result is independent of the arbitrary length of the Gaussian cylinder.

Step 3: (b) Sketch the variation with distance For a fixed nonzero line-charge density, $E \propto 1/r$. The magnitude decreases monotonically as r increases: doubling r halves E . The graph is a rectangular hyperbola in the positive E - r quadrant. For the ideal infinitesimally thin line, E diverges as $r \rightarrow 0^+$, while $E \rightarrow 0$ as $r \rightarrow \infty$; the axes are asymptotes. The singularity at zero belongs to the ideal line-charge model.

Quick Tip to Solve: Choose a coaxial cylinder so that only its curved surface contributes flux; for outward motion of like charges, check that external work is negative before finalising the logarithm.



Field magnitude versus perpendicular distance from an infinite line charge.

Step 4: (c) Find the potential difference and external work Move the charge quasistatically from perpendicular distance r_1 to r_2 , with $r_2 > r_1 > 0$. Only the radial displacement contributes to $\vec{E} \cdot d\vec{r}$. The potential difference is finite even though an infinite line does not admit a finite potential referenced to infinity:

$$\begin{aligned} V(r_2) - V(r_1) &= - \int_{r_1}^{r_2} \frac{\lambda}{2\pi\epsilon_0 r} dr \\ &= - \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right). \end{aligned}$$

Therefore the work done by the external agent is

$$W_{\text{ext}} = \Delta U = - \frac{q\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right) = \frac{q\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_1}{r_2}\right).$$

If $q\lambda > 0$, this work is negative: the repulsive electric force assists the outward motion, and the external agent removes energy while preventing acceleration. The electric field itself does positive work $W_{\text{field}} = \frac{q\lambda}{2\pi\epsilon_0} \ln(r_2/r_1)$. These explicit signs distinguish the two possible meanings of work done.

Step 5: Conclusion (a) $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r}$. (b) The magnitude follows the decreasing hyperbola $E \propto 1/r$.
(c) The work required by an external agent is $W_{\text{ext}} = \frac{q\lambda}{2\pi\epsilon_0} \ln(r_1/r_2)$; the work done by the electric field is its negative.

Quick Tip to Solve: Choose a coaxial cylinder so that only its curved surface contributes flux; for outward motion of like charges, check that external work is negative before finalising the logarithm.

26. (a) State the principle of an ac generator and explain its working with the help of a labelled diagram. Obtain the expression for the emf induced in a coil having N turns each of cross-sectional area A , rotating with a constant angular speed ' ω ' in a magnetic field $\vec{B}$, directed perpendicular to the axis of rotation.

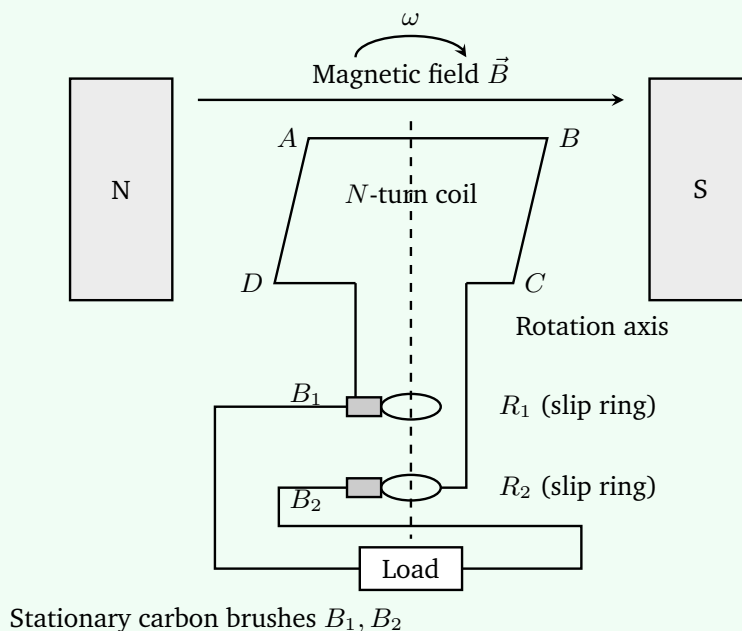
(b) An aeroplane is flying horizontally from west to east with a velocity of 900 km/hour. Calculate the potential difference developed between the ends of its wings having a span of 20 m. The horizontal component of the Earth's magnetic field is 5×10^{-4} T and the angle of dip is 30° . [5 marks]

Solution:

Concept:

- An ac generator converts mechanical energy into electrical energy by electromagnetic induction. Faraday's law gives $e = -d\Phi_{\text{link}}/dt$, where Φ_{link} is the total magnetic flux linked with the coil.
- For a coil of N turns, each of area A , in a uniform field of magnitude B , $\Phi_{\text{link}} = NBA \cos \theta$, where θ is the angle between the field and the normal to the coil.
- The motional potential difference across a conductor of length l perpendicular to both its velocity and the relevant magnetic field is $\Delta V = Blv$. At dip angle δ , the vertical component of Earth's field is $B_V = B_H \tan \delta$.

Step 1: (a) Principle and labelled arrangement The generator works on Faraday's law: a changing magnetic flux through a coil induces an emf. A rectangular armature coil $ABCD$, having N turns, rotates mechanically between the poles of a magnet. Its axis of rotation is perpendicular to the magnetic field directed from the north pole to the south pole. The two coil terminals connect separately to two insulated slip rings R_1 and R_2 , which rotate with the coil. Stationary carbon brushes B_1 and B_2 touch the respective rings and connect the coil to an external load. Slip rings maintain these connections throughout rotation.



Schematic ac generator: rotating coil, two separate slip rings, stationary brushes and external load.

Step 2: (a) Working and derivation of the induced emf As the coil turns, the angle between its normal

Quick Tip to Solve: For horizontal east-west flight, use the vertical field $B_H \tan \delta$ in Blv , and convert the speed from kilometres per hour to metres per second first.

and the field changes, so its flux linkage changes. The induced current is collected by the brushes and flows through the load. During successive half revolutions, the motion of each active side relative to the field reverses. Consequently the induced emf, and hence the current in a resistive load, reverses direction every half revolution, producing alternating current. Choose $t = 0$ when the coil normal is parallel to the field. For constant angular speed ω , $\theta = \omega t$, and therefore

$$\begin{aligned}\Phi_{\text{link}}(t) &= NBA \cos(\omega t), \\ e(t) &= -\frac{d}{dt} [NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t).\end{aligned}$$

Thus the amplitude is $e_0 = NBA\omega$, and the frequency is $f = \omega/(2\pi)$. The emf is zero when the flux is maximum or minimum, and its magnitude is maximum when the coil normal is perpendicular to the field. A different initial orientation changes only the phase of the sinusoidal expression.

Step 3: (b) Select the field component perpendicular to the wings and velocity The aircraft velocity is horizontal towards the east, while the wings lie along the north-south direction. The vertical component of the magnetic field therefore produces magnetic force along the wings. The horizontal component does not produce a force along their span, so the relevant component is

$$B_V = B_H \tan 30^\circ = \frac{5 \times 10^{-4}}{\sqrt{3}} \text{ T}.$$

Charges in the conducting wings experience $q(\vec{v} \times \vec{B})$ and separate until the electric force balances the magnetic force. This gives an electric field of magnitude vB_V along the wings and a potential difference $B_V lv$ across their ends.

Step 4: (b) Substitute the speed and wing span Convert the speed to SI units before substitution:

$$v = 900 \times \frac{1000}{3600} = 250 \text{ m s}^{-1}.$$

With $l = 20 \text{ m}$, the magnitude of the potential difference is

$$\begin{aligned}\Delta V &= B_V lv \\ &= \frac{5 \times 10^{-4}}{\sqrt{3}} \times 20 \times 250 \text{ V} \\ &= \frac{2.5}{\sqrt{3}} \text{ V} \approx 1.44 \text{ V}.\end{aligned}$$

The dip factor is essential: using the horizontal component directly would count a component that cannot separate charges along the wings.

Step 5: Conclusion (a) The ac generator operates by electromagnetic induction and produces $e = NBA\omega \sin(\omega t)$, with peak emf $NBA\omega$. (b) The potential difference between the wing tips is approximately 1.44 V.

Quick Tip to Solve: For horizontal east-west flight, use the vertical field $B_H \tan \delta$ in Blv , and convert the speed from kilometres per hour to metres per second first.

26. A device X is connected across an ac source of voltage $V = V_0 \sin \omega t$. The current through X is given as $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$.

- (a) Identify the device X and write the expression for its reactance.
 (b) Draw graphs showing variation of voltage and current with time over one cycle of ac, for X .
 (c) How does the reactance of the device X vary with frequency of the ac? Show this variation graphically.
 (d) Draw the phasor diagram for the device X .

[5 marks]

Solution:

Concept:

- For an ideal capacitor of capacitance C , the charge is $q = CV$ and the instantaneous current is $I = dq/dt = C dV/dt$.
- Capacitive reactance is $X_C = 1/(\omega C) = 1/(2\pi fC)$, where ω is the angular frequency and f is the frequency. Reactance is measured in ohms.
- In a purely capacitive ac circuit the current leads the voltage by $\pi/2$ radians, corresponding to one quarter of the period $T = 2\pi/\omega$.

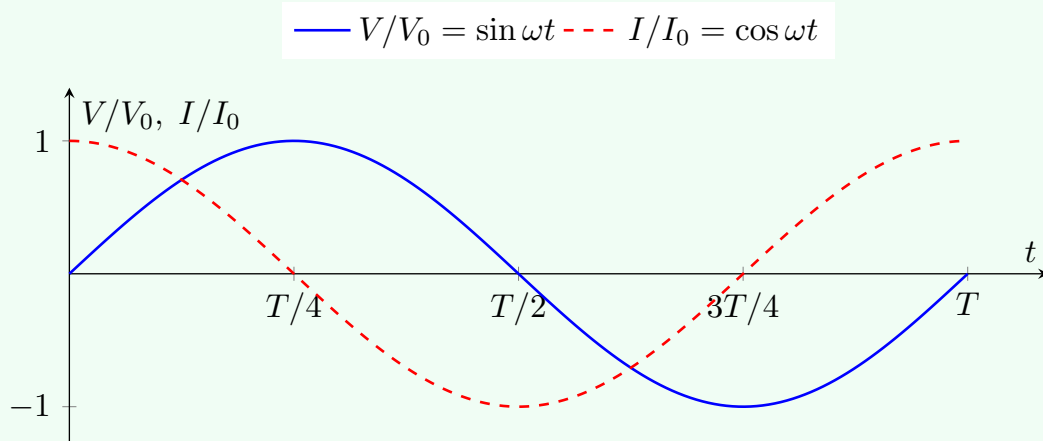
Step 1: (a) Identify the device and derive its reactance The given current is $I = I_0 \sin(\omega t + \pi/2) = I_0 \cos \omega t$. Thus the current reaches its positive maximum a quarter-cycle before the voltage. This is the phase relation of an ideal capacitor, so X is a capacitor. Using the supplied voltage,

$$\begin{aligned} q &= CV_0 \sin \omega t, \\ I &= \frac{dq}{dt} = \omega CV_0 \cos \omega t, \\ I_0 &= \omega CV_0, \\ X_C &= \frac{V_0}{I_0} = \frac{1}{\omega C} = \frac{1}{2\pi fC}. \end{aligned}$$

The amplitude ratio V_0/I_0 , equivalently the rms ratio $V_{\text{rms}}/I_{\text{rms}}$, defines capacitive reactance. An ideal resistor would give current in phase with voltage, whereas an ideal inductor would make current lag by $\pi/2$.

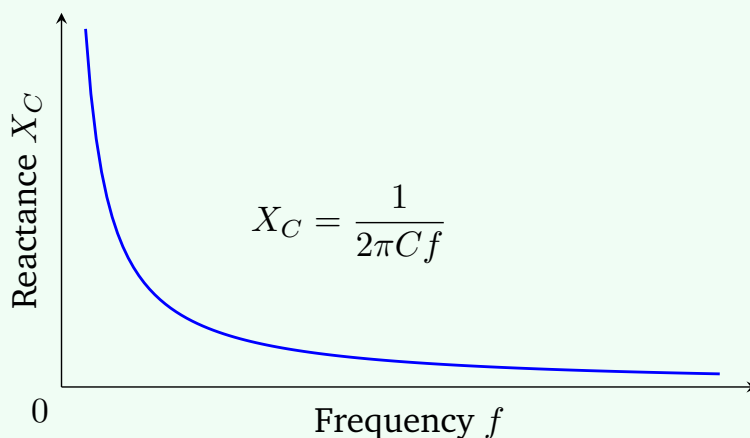
Step 2: (b) Plot voltage and current for one complete cycle The following graph uses normalized ordinates V/V_0 and I/I_0 , so both waveforms can be compared on the same axes despite having different physical units. The voltage is a sine curve and the current is a cosine curve. At $t = 0$, voltage is zero and increasing, while current is at its positive maximum. At $T/4$, voltage is maximum and current is zero; at $T/2$, voltage is zero and current is at its negative maximum; at $3T/4$, voltage is minimum and current is zero. At T , the initial values repeat. These points show that the current curve is shifted to the left of the voltage curve by $T/4$.

Quick Tip to Solve: Differentiate the supplied sine voltage: its cosine derivative immediately identifies a capacitor and fixes the current lead of one quarter-cycle.



Voltage and current over one cycle; normalized amplitudes show the quarter-cycle current lead.

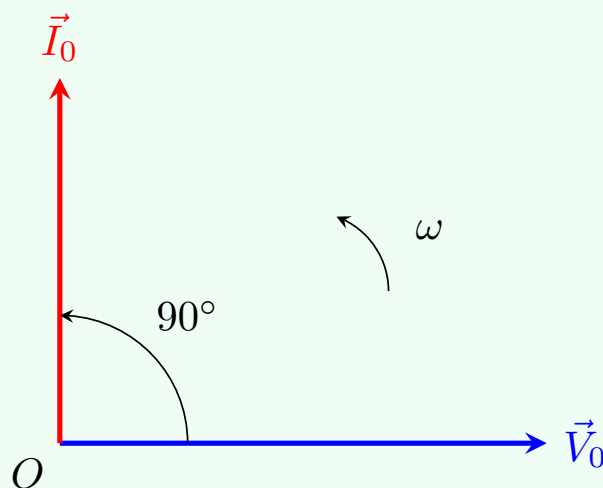
Step 3: (c) Show how reactance varies with frequency For a fixed capacitance, $X_C = 1/(2\pi C f)$, and hence $X_C \propto 1/f$. Doubling the frequency halves the reactance. The graph of X_C against positive frequency is therefore a decreasing rectangular hyperbola, not a straight line. As $f \rightarrow 0^+$, $X_C \rightarrow \infty$, which corresponds to an ideal capacitor blocking steady direct current after charging. As $f \rightarrow \infty$, $X_C \rightarrow 0$. The curve approaches both coordinate axes asymptotically and never reaches zero at any finite frequency.



Capacitive reactance versus frequency for a fixed capacitance.

Step 4: (d) Draw and interpret the phasor diagram Choose the voltage phasor along the positive horizontal axis as the reference. Draw the current phasor vertically upward, at an angle $+\pi/2$ from the voltage phasor in the counterclockwise direction. Both phasors rotate counterclockwise with the same angular speed ω , preserving this phase difference. Their lengths represent their respective amplitudes; they need not be numerically equal. Thus the current phasor is ahead of the voltage phasor by 90° , consistent with $I = C dV/dt$ and with the plotted waveforms.

Quick Tip to Solve: Differentiate the supplied sine voltage: its cosine derivative immediately identifies a capacitor and fixes the current lead of one quarter-cycle.



The current phasor leads the voltage phasor by 90° .

Step 5: Conclusion The device is an ideal capacitor with $X_C = 1/(\omega C) = 1/(2\pi fC)$. Its current leads voltage by $\pi/2$, or $T/4$, and its reactance decreases hyperbolically with increasing frequency. The waveform, reactance and phasor graphs show these relations.

Quick Tip to Solve: Differentiate the supplied sine voltage: its cosine derivative immediately identifies a capacitor and fixes the current lead of one quarter-cycle.