

CBSE Class 12 Physics Question Paper 2018 with Solution PDF-Set 55/3

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are **26** questions in all.
- (ii) This question paper has **five** sections : Section A, Section B, Section C, Section D and Section E.
- (iii) Section A contains **five** questions of **one** mark each, Section B contains **five** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains one value based question of **four** marks and Section E contains **three** questions of **five** marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in **one** question of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Which mode of propagation is used by short wave broadcast services ?

[1 mark]

Solution:

Concept:

- Sky-wave propagation uses the ionosphere to return radio waves towards the Earth's surface.
- Short-wave radio signals can travel beyond the horizon through this ionospheric return, allowing long-distance broadcasting.

Step 1: Identify the frequency band Short-wave broadcasting uses radio waves in the high-frequency band, approximately 3–30 MHz. Under suitable ionospheric conditions, these waves are bent back towards the Earth.

Step 2: Relate the path to the propagation mode The signal travels upwards from the transmitting antenna and returns from the ionosphere to a distant receiver. This Earth–ionosphere–Earth path is called sky-wave propagation and extends reception beyond the direct line of sight.

Step 3: Conclusion Short-wave broadcast services use **sky-wave propagation**.

Quick Tip to Solve: Associate short-wave broadcasting with ionospheric return and hence with sky-wave propagation.

2. Four nuclei of an element undergo fusion to form a heavier nucleus, with release of energy. Which of the two — the parent or the daughter nucleus — would have higher binding energy per nucleon ?

[1 mark]

Solution:

Concept:

- Binding energy is the energy needed to separate a nucleus into its constituent nucleons. Greater binding energy per nucleon means greater nuclear stability.
- Energy released in fusion equals the increase in total binding energy: $Q = B_{\text{final}} - B_{\text{initial}} > 0$.

Step 1: Compare total binding energies Let each parent nucleus contain A nucleons and have binding energy B_p . The four parent nuclei initially have total binding energy $4B_p$. Since energy is released on forming the daughter nucleus, its binding energy satisfies $B_d > 4B_p$.

Step 2: Compare the energy per nucleon The daughter contains the combined $4A$ nucleons. Dividing the binding-energy inequality by this common total gives

$$\frac{B_d}{4A} > \frac{4B_p}{4A} = \frac{B_p}{A}.$$

Thus fusion has made the nucleons more tightly bound on average.

Step 3: Conclusion The **daughter nucleus** has the higher binding energy per nucleon.

Quick Tip to Solve: For an energy-releasing fusion reaction, compare binding energy per nucleon: the product is more tightly bound.

3. Name the electromagnetic radiations used for (a) water purification, and (b) eye surgery. [1 mark]

Solution:

Concept:

- Ultraviolet radiation has wavelengths shorter than visible violet light and can produce strong photochemical effects.
- Its germicidal action and the precise action of ultraviolet laser beams are useful in sterilisation and certain surgical procedures.

Step 1: Identify the radiation for water purification Ultraviolet radiation is used to disinfect water because it damages the genetic material of microorganisms and prevents their reproduction. Here water purification refers to this germicidal treatment.

Step 2: Identify the radiation for eye surgery Ultraviolet radiation from an excimer laser is used in corrective eye surgery. Its focused beam removes very thin layers of corneal tissue precisely, allowing the corneal shape and hence its focusing power to be corrected.

Step 3: Conclusion (a) Water purification: **ultraviolet rays**. (b) Eye surgery: **ultraviolet rays**.

Quick Tip to Solve: Remember the paired ultraviolet uses: killing microbes in water and precisely reshaping the cornea with a laser.

4. Draw graphs showing variation of photoelectric current with applied voltage for two incident radiations of equal frequency and different intensities. Mark the graph for the radiation of higher intensity. [1 mark]

Solution:

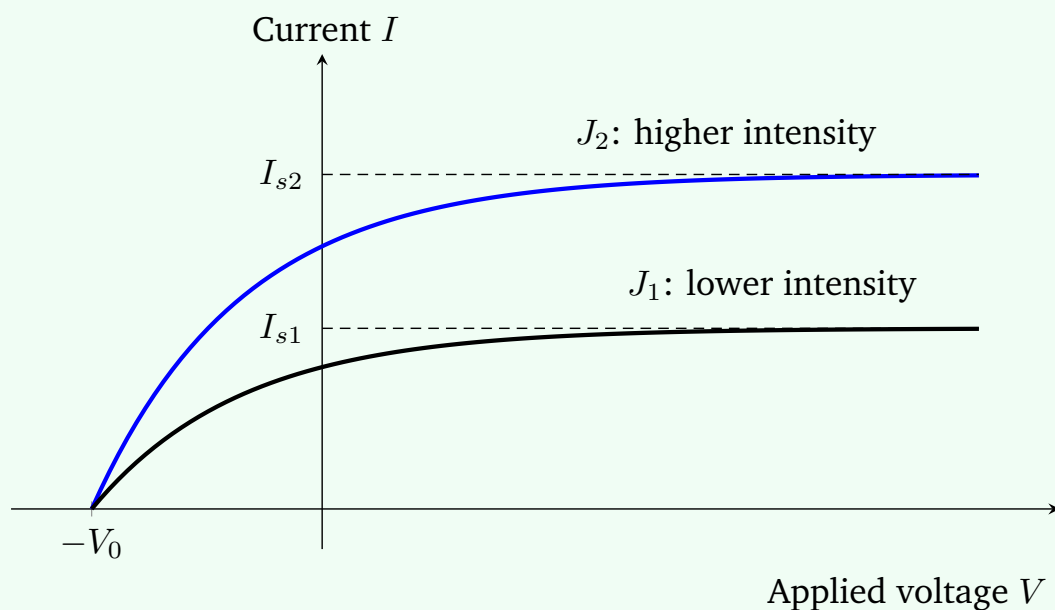
Concept:

- At a fixed incident frequency above threshold, increasing light intensity increases the number of photoelectrons emitted per second and hence the saturation current.
- The stopping potential V_0 depends on the maximum kinetic energy, with $eV_0 = h\nu - \phi$, where ν is frequency and ϕ is the metal's work function.

Step 1: Fix the common stopping potential Both radiations have the same frequency, so their photoelectrons have the same maximum kinetic energy. Both current curves therefore reach zero at the same negative voltage $-V_0$.

Step 2: Compare the saturation currents and sketch Let the intensities be $J_2 > J_1$. The higher intensity ejects more electrons per second, giving $I_{s2} > I_{s1}$. Each curve rises as the collector voltage increases and becomes nearly horizontal when all emitted electrons are collected. The upper curve is marked as the higher-intensity radiation.

Quick Tip to Solve: When frequency is unchanged, move the saturation-current plateau upwards for higher intensity while keeping the stopping-voltage intercept fixed.



Photoelectric current at equal frequency, with $J_2 > J_1$; axes are schematic.

Step 3: Conclusion The higher-intensity radiation has the **higher saturation-current plateau**; both curves have the **same stopping potential** $-V_0$.

Quick Tip to Solve: When frequency is unchanged, move the saturation-current plateau upwards for higher intensity while keeping the stopping-voltage intercept fixed.

5. A proton and an electron travelling along parallel paths enter a region of uniform magnetic field, acting perpendicular to their paths. Which of them will move in a circular path with higher frequency ? [1 mark]

Solution:

Concept:

- For velocity perpendicular to a uniform magnetic field, the magnetic force supplies the centripetal force: $|q|vB = mv^2/r$.
- The circular-motion frequency is $f = v/(2\pi r)$, where m is particle mass, $|q|$ its charge magnitude, and B the magnetic field strength.

Step 1: Express frequency in terms of mass From the force equation, the radius is $r = mv/(|q|B)$. Substituting it into the frequency expression gives

$$f = \frac{v}{2\pi} \frac{|q|B}{mv} = \frac{|q|B}{2\pi m}.$$

Thus the frequency is independent of speed in the usual non-relativistic treatment.

Step 2: Compare the electron and the proton The particles have equal charge magnitudes, $|q_e| = |q_p| = e$, and enter the same field. Hence $f_e/f_p = m_p/m_e \approx 1836$. Their opposite charge signs reverse

Quick Tip to Solve: For perpendicular motion in the same magnetic field, compare $|q|/m$; the larger ratio gives the higher circular frequency.

their senses of rotation but do not change this comparison of frequency magnitudes.

Step 3: Conclusion The **electron** moves in a circular path with the higher frequency because its mass is much smaller than the proton's.

Quick Tip to Solve: For perpendicular motion in the same magnetic field, compare $|q|/m$; the larger ratio gives the higher circular frequency.

Section B (10 marks)

6. A carrier wave of peak voltage 15 V is used to transmit a message signal. Find the peak voltage of the modulating signal in order to have a modulation index of 60%. [2 marks]

Solution:

Concept:

- In amplitude modulation, the modulation index is $m = A_m/A_c$, where A_m is the peak voltage of the message signal and A_c is the peak voltage of the unmodulated carrier.
- The modulation index is dimensionless. A percentage modulation must be divided by 100 before it is used in the amplitude-ratio formula.

Step 1: Express the modulation index as a ratio The carrier peak voltage is $A_c = 15\text{ V}$. The specified modulation is 60%, so

$$m = \frac{60}{100} = 0.60.$$

This means that the message amplitude is sixty percent of the carrier amplitude; it does not mean that the message voltage itself is 60 V.

Step 2: Calculate the message amplitude Rearrange the definition of modulation index and substitute the given carrier amplitude:

$$A_m = mA_c = 0.60 \times 15\text{ V} = 9.0\text{ V}.$$

Both amplitudes here are peak voltages, so no conversion to root-mean-square values is required. Also, $m < 1$, as expected for modulation without overmodulation.

Step 3: Conclusion The required peak voltage of the modulating signal is 9 V.

Quick Tip to Solve: Convert the percentage to a decimal, then multiply it by the carrier's peak voltage: here, take 60 percent of 15 V.

7. If light of wavelength 412.5 nm is incident on each of the metals given below, which ones will show photoelectric emission and why?

Metal	Work Function (eV)
Na	1.92
K	2.15
Ca	3.20
Mo	4.17

[2 marks]

Solution:

Concept:

- Each incident photon carries energy $E_\gamma = hc/\lambda$, where h is Planck's constant, c is the speed of light and λ is the wavelength.
- Photoelectric emission is possible when $E_\gamma \geq \phi$, where ϕ is the metal's work function. Einstein's equation then gives $K_{\max} = E_\gamma - \phi$.

Step 1: Calculate the incident photon energy Convert the wavelength to SI units: $\lambda = 412.5 \times 10^{-9}$ m. Using the given physical constants,

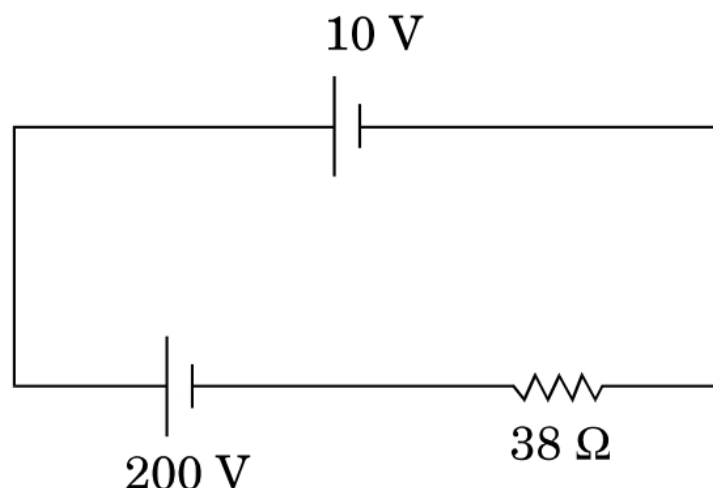
$$\begin{aligned} E_\gamma &= \frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{412.5 \times 10^{-9}} \\ &\simeq 4.82 \times 10^{-19} \text{ J} \\ &= \frac{4.82 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} \simeq 3.01 \text{ eV}. \end{aligned}$$

Step 2: Compare the photon energy with each work function For Na, $3.01 > 1.92$ eV, and for K, $3.01 > 2.15$ eV. In each case a photon supplies enough energy for an electron to escape, with the excess appearing as kinetic energy. For Ca, $3.01 < 3.20$ eV, and for Mo, $3.01 < 4.17$ eV. A photon cannot overcome either of these work functions, so these metals do not emit photoelectrons at the specified wavelength.

Step 3: Conclusion Na and K show photoelectric emission because their work functions are below the photon energy of about 3.01 eV. Ca and Mo do not.

Quick Tip to Solve: Compute the photon energy once in eV, then compare that same number with every entry in the work-function column.

8. A 10 V cell of negligible internal resistance is connected in parallel across a battery of emf 200 V and internal resistance $38\ \Omega$ as shown in the figure. Find the value of current in the circuit. [2 marks]



Solution:

Concept:

- Kirchhoff's loop rule states that the algebraic sum of potential changes around a closed circuit is zero.
- The longer plate of a cell symbol is its positive terminal. When like terminals of unequal sources are connected together, their emfs oppose around the loop; the larger emf drives the current.

Step 1: Read the cell polarities and resistance In the printed circuit, both positive terminals are on the left. The 200 V battery drives conventional current out of its positive terminal, up the left wire and from left to right through the 10 V cell. The smaller cell therefore opposes the driving battery and is being charged. The only non-negligible resistance in the loop is the battery's internal resistance, $r = 38\ \Omega$.

Step 2: Apply Kirchhoff's loop rule Traverse the loop in the direction driven by the larger battery. The rise across that battery is 200 V, the opposing drop across the ideal cell is 10 V, and the resistive drop is $38I$. Hence

$$200 - 10 - 38I = 0,$$
$$I = \frac{200 - 10}{38} = 5\text{ A}.$$

Equivalently, the ideal 10 V cell fixes the common terminal voltage at 10 V, so the larger battery has an internal voltage drop of $200 - 10 = 190\text{ V}$.

Step 3: Conclusion The circuit current is **5 A**, clockwise in the printed figure, entering the positive terminal of the 10 V cell.

Quick Tip to Solve: Inspect the long plates before combining emfs: like terminals connected in parallel make the emfs subtract around this loop.

8. In a potentiometer arrangement for determining the emf of a cell, the balance point of the cell in open circuit is 350 cm. When a resistance of $9\ \Omega$ is used in the external circuit of the cell, the balance point shifts to 300 cm. Determine the internal resistance of the cell. [2 marks]

Solution:

Concept:

- With a constant potential gradient k , a potentiometer measures a potential difference as $V = kl$, where l is its balancing length.
- For a cell of emf E and internal resistance r supplying an external resistance R , its terminal voltage is $V = ER/(R + r)$. Thus $r = R(E/V - 1)$.

Step 1: Relate the two balance lengths to voltages In the open circuit, no current is drawn from the cell at balance, so the balancing length $l_1 = 350$ cm measures its emf: $E = kl_1$. With $R = 9\ \Omega$ connected across the cell, the balance length $l_2 = 300$ cm measures its terminal voltage: $V = kl_2$. Since the potentiometer's potential gradient is unchanged,

$$\frac{E}{V} = \frac{l_1}{l_2} = \frac{350}{300}.$$

Step 2: Calculate the internal resistance The cell current through the external resistor is $I = V/R$. The internal voltage drop is $E - V = Ir$, giving

$$\begin{aligned} r &= \frac{E - V}{V/R} = R \left(\frac{E}{V} - 1 \right) \\ &= 9 \left(\frac{350}{300} - 1 \right) \Omega \\ &= 9 \left(\frac{50}{300} \right) \Omega = 1.5\ \Omega. \end{aligned}$$

The decrease in balancing length is consistent with a terminal voltage lower than the open-circuit emf.

Step 3: Conclusion The internal resistance of the cell is $1.5\ \Omega$.

Quick Tip to Solve: Use $r = R(l_1 - l_2)/l_2$: the loaded balance length belongs in the denominator, and both lengths may stay in centimetres.

9. (a) Why are infra-red waves often called heat waves ? Explain.

(b) What do you understand by the statement, "Electromagnetic waves transport momentum" ?

[2 marks]

Solution:

Concept:

- Infra-red radiation is electromagnetic radiation strongly absorbed by many materials, especially through molecular vibrational and rotational modes.

Quick Tip to Solve: Explain heating through absorption, then explain momentum through the force exerted on an absorbing or reflecting surface.

- An electromagnetic wave carries energy and momentum. In vacuum, a directed radiation pulse of energy U has momentum $p = U/c$, where c is the speed of light.

Step 1: Explain the heating action of infra-red radiation (a) Infra-red waves are readily absorbed by water molecules and many other molecules present in matter. Absorption increases molecular motion and hence the internal energy of the material. Its temperature therefore rises. Because of this pronounced heating effect, infra-red waves are commonly called heat waves. Hot bodies are also strong sources of infra-red radiation.

Step 2: Relate transported momentum to radiation pressure (b) The statement means that electromagnetic radiation carries linear momentum in its direction of propagation and can transfer that momentum to an object on which it falls. A surface that absorbs a normally incident pulse of energy U receives momentum U/c . If the pulse is perfectly reflected back, the momentum transfer is $2U/c$. Transfer of momentum per unit time produces a force on the surface and therefore radiation pressure.

Step 3: Conclusion Infra-red waves are called heat waves because their absorption raises a body's internal energy and temperature. Electromagnetic waves transport momentum, so their absorption or reflection can exert a mechanical force on matter.

Quick Tip to Solve: Explain heating through absorption, then explain momentum through the force exerted on an absorbing or reflecting surface.

10. Two electric bulbs P and Q have their resistances in the ratio of 1 : 2. They are connected in series across a battery. Find the ratio of the power dissipation in these bulbs. [2 marks]

Solution:

Concept:

- Components in series carry the same current because there is a single path for charge flow.
- The electrical power dissipated in a resistance R carrying current I is $\mathcal{P} = I^2 R$. Here $\mathcal{P}$ denotes power, to distinguish it from the bulb named P.

Step 1: Identify the common current through the bulbs Let the resistances of bulbs P and Q be $R_P = R$ and $R_Q = 2R$, respectively. Since the bulbs are connected in series, the same current I flows through both. The battery voltage is shared by the two bulbs; it is not the voltage across each bulb separately.

Step 2: Form the ratio using Joule's law The powers dissipated in the two bulbs are

$$\mathcal{P}_P = I^2 R, \quad \mathcal{P}_Q = I^2 (2R).$$

Dividing cancels the common factor I^2 :

$$\frac{\mathcal{P}_P}{\mathcal{P}_Q} = \frac{I^2 R}{I^2 (2R)} = \frac{1}{2}.$$

Thus, for this series arrangement, the bulb with twice the resistance dissipates twice the power. A numerical battery voltage is unnecessary because the common current cancels from the ratio.

Quick Tip to Solve: For bulbs in series use $I^2 R$: equal current makes the power ratio equal to the resistance ratio.

Step 3: Conclusion The ratio of the power dissipation in bulbs P and Q is $\mathcal{P}_P : \mathcal{P}_Q = 1 : 2$.

Quick Tip to Solve: For bulbs in series use $I^2 R$: equal current makes the power ratio equal to the resistance ratio.

Section C (36 marks)

11. Draw the typical input and output characteristics of an n-p-n transistor in CE configuration. Show how these characteristics can be used to determine (a) the input resistance (r_i), and (b) current amplification factor (β). [3 marks]

Solution:

Concept:

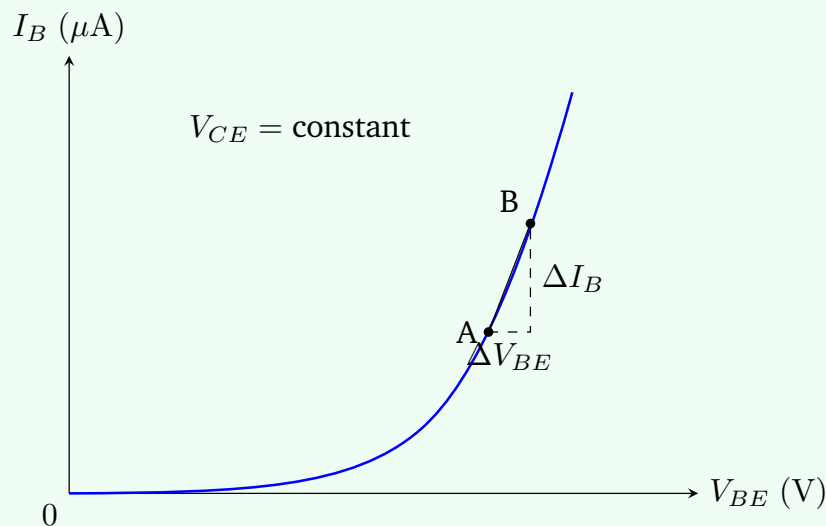
- In the common-emitter (CE) configuration, the emitter is common to the input base-emitter circuit and the output collector-emitter circuit. In the active region, the base-emitter junction is forward biased and the collector-base junction is reverse biased.
- The input characteristic plots base current I_B against base-emitter voltage V_{BE} at constant collector-emitter voltage V_{CE} . The output characteristics plot collector current I_C against V_{CE} for different fixed values of I_B .
- Small-signal input resistance and current gain are $r_i = (\Delta V_{BE} / \Delta I_B)_{V_{CE}}$ and $\beta = (\Delta I_C / \Delta I_B)_{V_{CE}}$, respectively, evaluated near the operating point.

Step 1: Draw the input characteristic and determine input resistance Hold V_{CE} fixed, increase V_{BE} , and record I_B . The curve resembles that of a forward-biased diode: the base current is initially small and then rises rapidly. Choose two nearby points A and B about the operating point on the same constant- V_{CE} curve. Read their voltage and current differences using the small triangle shown:

$$r_i = \left. \frac{\Delta V_{BE}}{\Delta I_B} \right|_{V_{CE} \text{ constant}} = \frac{V_{BE,B} - V_{BE,A}}{I_{B,B} - I_{B,A}}.$$

Because current is on the vertical axis, this resistance is the reciprocal of the local slope of the input curve. Convert the base current difference from microamperes to amperes to obtain r_i in ohms.

Quick Tip to Solve: Keep V_{CE} fixed for both readings: use nearby points on one input curve for r_i , and two output curves for β .



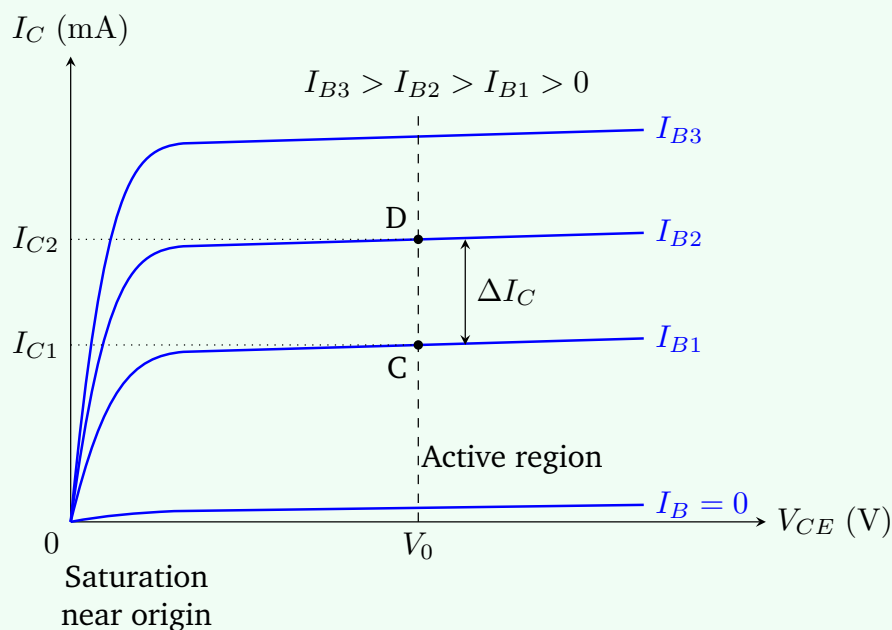
CE input characteristic at fixed V_{CE} ; the small triangle gives the local increments.

Step 2: Draw the output characteristics and determine current gain For each fixed base current, vary V_{CE} and record I_C . In the active region the curves are almost horizontal, showing that I_C depends mainly on I_B . Larger base current gives a higher curve. Near the origin the transistor is in saturation; the $I_B = 0$ curve represents cut-off apart from a small leakage current. Choose a fixed $V_{CE} = V_0$ in the active region. The vertical line intersects the curves for I_{B1} and I_{B2} at C and D. Read the corresponding collector currents I_{C1} and I_{C2} , and calculate

$$\beta = \left. \frac{\Delta I_C}{\Delta I_B} \right|_{V_{CE} \text{ constant}} = \frac{I_{C2} - I_{C1}}{I_{B2} - I_{B1}}.$$

Express both current differences in the same units; β is dimensionless. This gives the incremental current gain, whereas the dc current gain at a point is I_C/I_B .

Quick Tip to Solve: Keep V_{CE} fixed for both readings: use nearby points on one input curve for r_i , and two output curves for β .



CE output characteristics; at V_0 , the separation CD is ΔI_C for $\Delta I_B = I_{B2} - I_{B1}$.

Step 3: Conclusion From the CE input curve, $r_i = (\Delta V_{BE} / \Delta I_B)_{V_{CE}}$. From two nearby output curves at the same V_{CE} , $\beta = (\Delta I_C / \Delta I_B)_{V_{CE}}$.

Quick Tip to Solve: Keep V_{CE} fixed for both readings: use nearby points on one input curve for r_i , and two output curves for β .

12. (a) A student wants to use two p-n junction diodes to convert alternating current into direct current. Draw the labelled circuit diagram she would use and explain how it works.

(b) Give the truth table and circuit symbol for NAND gate.

[3 marks]

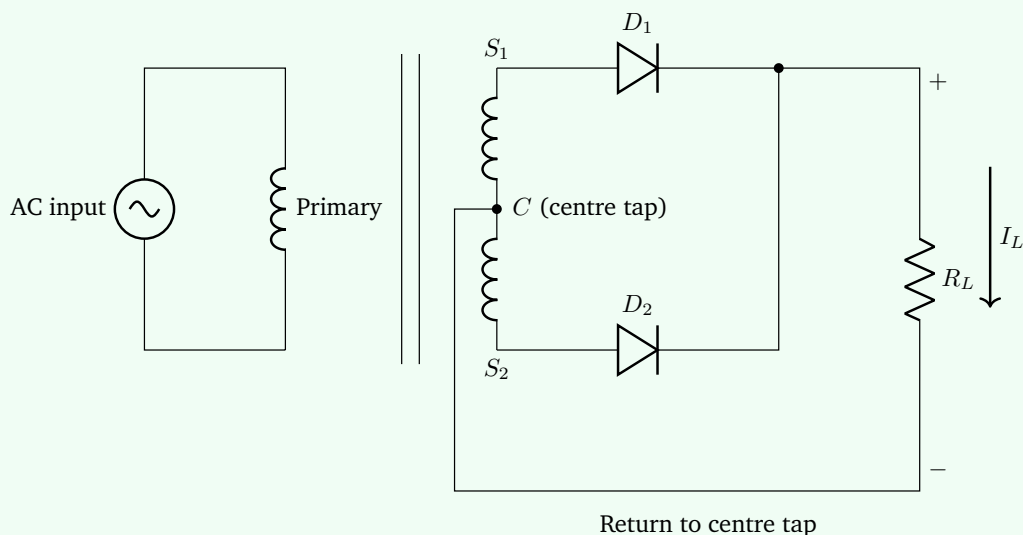
Solution:

Concept:

- A p-n junction diode conducts appreciably when forward biased and blocks current when reverse biased. Two diodes with a centre-tapped transformer form a full-wave rectifier.
- A NAND gate performs AND followed by NOT: $Y = \overline{A \cdot B}$. Its output is LOW only when both inputs are HIGH.

Step 1: (a) Connect a full-wave rectifier Connect the two ends S_1 and S_2 of the centre-tapped secondary to the anodes of D_1 and D_2 . Join their cathodes to the positive end of the load resistance R_L , and connect the other end of the load to the centre tap C . Apply the alternating input to the transformer primary. The two secondary ends have opposite instantaneous polarities relative to C .

Quick Tip to Solve: Trace each diode-to-load path: both must drive the load in the same direction. For NAND, invert every entry of the AND truth table.

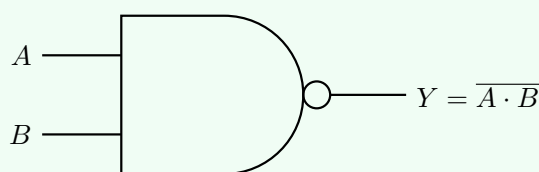


Centre-tapped full-wave rectifier; both diode cathodes feed the positive load terminal.

Step 2: Explain conduction during the two half cycles When S_1 is positive relative to C , D_1 is forward biased and D_2 is reverse biased. Current follows $S_1 \rightarrow D_1 \rightarrow R_L \rightarrow C$. During the next half cycle, S_2 is positive, so D_2 conducts and D_1 blocks; current follows $S_2 \rightarrow D_2 \rightarrow R_L \rightarrow C$. Thus current through R_L has the same direction in both half cycles. The output is unidirectional pulsating DC, with ripple frequency twice the input frequency; a filter would be needed for a smoother output.

Step 3: (b) Give the NAND symbol and truth table The small circle at the output of the AND-shaped symbol denotes inversion. For inputs A, B , first form $A \cdot B$, then reverse that value to obtain Y . Thus

A	B	$Y = \overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0



Two-input NAND gate, with output inversion shown by the circle.

Step 4: Conclusion The two-diode centre-tapped circuit rectifies both half cycles to give pulsating DC. A NAND gate gives $Y = 1$ for 00, 01, 10, and $Y = 0$ for 11.

Quick Tip to Solve: Trace each diode-to-load path: both must drive the load in the same direction. For NAND, invert every entry of the AND truth table.

13. (a) Explain the processes of nuclear fission and nuclear fusion by using the plot of binding energy per nucleon (BE/A) versus the mass number A .

(b) A radioactive isotope has a half-life of 10 years. How long will it take for the activity to reduce to 3.125%?

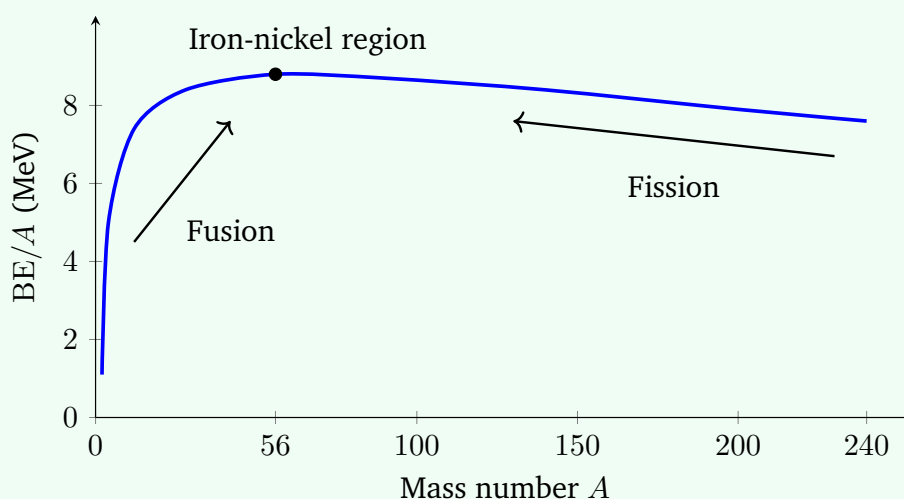
[3 marks]

Solution:

Concept:

- Binding energy per nucleon, BE/A , measures the average binding of a nucleon in a nucleus. A larger value generally corresponds to greater nuclear stability.
- Energy is released when the total binding energy of the reaction products exceeds that of the initial nuclei: $Q = BE_{\text{products}} - BE_{\text{initial}} > 0$.
- Radioactive activity $R = \lambda N$ follows $R/R_0 = (1/2)^{t/T_{1/2}}$, where $T_{1/2}$ is the half-life and λ is the decay constant.

Step 1: (a) Read the binding-energy curve The binding energy per nucleon rises rapidly for light nuclei, reaches about 8.8 MeV in the iron-nickel region near $A \approx 56$, and then decreases slowly for heavy nuclei. Nuclei in the middle of the curve are more tightly bound than either very light or very heavy nuclei. The sketch shows the overall trend; small variations among individual light nuclei are omitted.



Schematic binding energy per nucleon curve; both indicated processes lead towards more tightly bound nuclei.

Step 2: Explain fission and fusion using the curve In **nuclear fission**, a heavy nucleus such as uranium splits into two medium-mass nuclei, usually with a few emitted neutrons. The fragments lie closer to the maximum of the curve and have greater binding energy per nucleon than the original heavy nucleus. The resulting increase in total binding energy is released as energy, mainly as kinetic energy of the products.

In **nuclear fusion**, two very light nuclei combine to form a heavier, more tightly bound nucleus. On the rising portion of the curve, the product has greater binding energy per nucleon, so the increase in total binding energy is released. Very high temperatures help the positively charged nuclei overcome their mutual electrostatic repulsion and approach within the short range of the nuclear force.

Step 3: (b) Convert the remaining activity into half-lives The activity is reduced to 3.125% of its

Quick Tip to Solve: For activity percentages, repeatedly halve 100 until the target is reached. Here 3.125 corresponds to five halvings, so multiply the half-life by five.

initial value, so the remaining fraction is

$$\frac{R}{R_0} = \frac{3.125}{100} = 0.03125 = \frac{1}{32} = \left(\frac{1}{2}\right)^5.$$

Comparing this with $R/R_0 = (1/2)^{t/10}$, with time measured in years, gives

$$\frac{t}{10} = 5, \quad t = 5 \times 10 = 50 \text{ years}.$$

The successive percentages are 50, 25, 12.5, 6.25, 3.125, confirming five half-lives.

Step 4: Conclusion Both fission of heavy nuclei and fusion of very light nuclei release energy by moving the products towards greater binding energy per nucleon. The activity reaches 3.125% after 50 years.

Quick Tip to Solve: For activity percentages, repeatedly halve 100 until the target is reached. Here 3.125 corresponds to five halvings, so multiply the half-life by five.

14. (a) Give three reasons why modulation of a message signal is necessary for long distance transmission.

(b) Show graphically an audio signal, a carrier wave and an amplitude modulated wave. [3 marks]

Solution:

Concept:

- Modulation impresses a low-frequency message on a high-frequency carrier. In amplitude modulation (AM), the carrier amplitude follows the message, while its frequency remains fixed.
- An efficient transmitting antenna has a length of the order of $\lambda/4$, where $\lambda = c/f$. The carrier frequency f_c is much greater than the message frequency f_m .

Step 1: (a) Three reasons for modulation First, modulation makes the antenna size practical. For a 1 kHz audio signal, $\lambda/4 = c/(4f) = 75 \text{ km}$, whereas a 1 MHz carrier needs a length of about 75 m. Second, it enables efficient radiation: for a given short antenna, radiated power increases as $(l/\lambda)^2$, so direct radiation of a long-wavelength audio signal is extremely weak. A high-frequency carrier radiates much more efficiently. Third, assigning different carrier frequencies to different transmitters separates their messages into distinct frequency bands. A receiver can tune to one band, avoiding the mixing of audio signals from many transmitters.

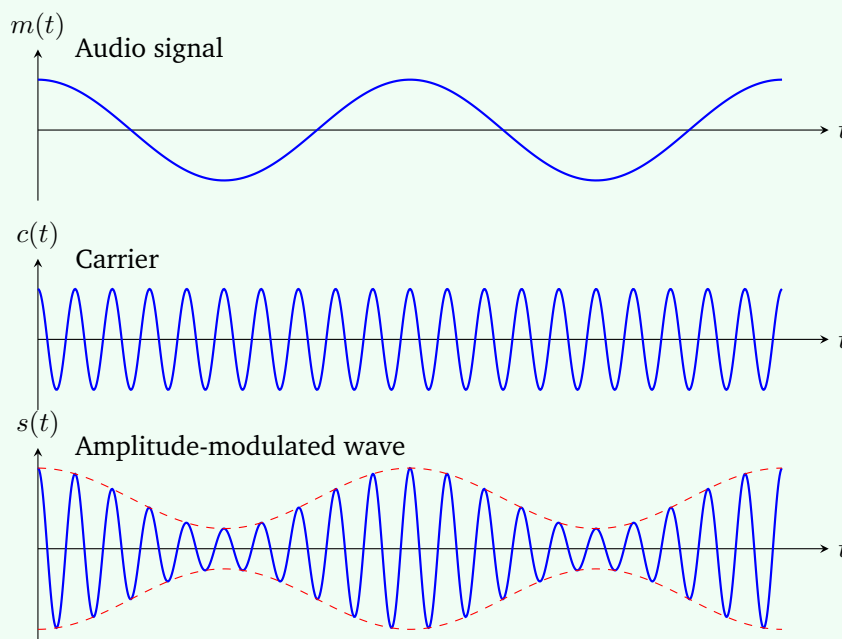
Step 2: (b) Audio, carrier and AM waveforms Take a sinusoidal message $m(t) = A_m \cos \omega_m t$ and carrier $c(t) = A_c \cos \omega_c t$. For modulation index μ , the AM wave is

$$s(t) = A_c[1 + \mu \cos \omega_m t] \cos \omega_c t, \quad 0 < \mu < 1.$$

The first graph shows the slow audio oscillation; the second has rapid oscillations of constant amplitude. The third has the same rapid carrier oscillations, but their amplitude changes slowly with the message. Its dashed upper and lower envelopes are $\pm A_c[1 + \mu \cos \omega_m t]$. The shared horizontal

Quick Tip to Solve: Draw many carrier cycles within each audio cycle and make the AM peaks touch the slowly varying dashed envelope.

scale emphasizes that $f_c \gg f_m$.



Audio signal, carrier and amplitude-modulated wave on a common time scale; dashed curves show the AM envelope.

Step 3: Conclusion Modulation permits a practical antenna, efficient radiation and separation of different messages. In AM, the envelope reproduces the audio waveform about the constant carrier-amplitude level.

Quick Tip to Solve: Draw many carrier cycles within each audio cycle and make the AM peaks touch the slowly varying dashed envelope.

15. (a) If one of two identical slits producing interference in Young's experiment is covered with glass, so that the light intensity passing through it is reduced to 50%, find the ratio of the maximum and minimum intensity of the fringe in the interference pattern.

(b) What kind of fringes do you expect to observe if white light is used instead of monochromatic light ?

[3 marks]

Solution:

Concept:

- The intensity of a light wave is proportional to the square of its amplitude: $I \propto a^2$. Halving intensity therefore reduces amplitude by a factor of $\sqrt{2}$.
- For two coherent waves, constructive and destructive interference give $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$.
- In Young's experiment, the fringe width is $\beta = \lambda D/d$, where D is the screen distance and d is slit separation. Different wavelengths produce different fringe widths.

Step 1: (a) Convert transmitted intensities into amplitudes Let the intensity due to either uncovered slit alone be I_0 . After covering one slit, $I_1 = I_0$ and $I_2 = I_0/2$. Hence the amplitude ratio is

Quick Tip to Solve: Take square roots of the slit intensities before adding or subtracting; interference combines amplitudes, not intensities.

$\sqrt{I_1}/\sqrt{I_2} = \sqrt{2}$, rather than 2. A fixed phase delay introduced by the glass can shift the pattern, but does not change these maximum and minimum values.

Step 2: Calculate the ratio of the fringe intensities Substitution in the interference formulae gives

$$I_{\max} = I_0 \left(1 + \frac{1}{\sqrt{2}} \right)^2,$$

$$I_{\min} = I_0 \left(1 - \frac{1}{\sqrt{2}} \right)^2,$$

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right)^2 = (3 + 2\sqrt{2})^2 = 17 + 12\sqrt{2}.$$

The minimum is nonzero because the two waves have unequal amplitudes and cannot cancel completely.

Step 3: (b) Describe the usual white-light interference pattern With white light in the usual equal-path arrangement, the central bright fringe is white: all wavelengths interfere constructively at zero path difference. On either side are a few coloured fringes. In the first-order spectrum, violet lies nearer the centre and red farther away because $\beta \propto \lambda$. Farther out, fringes of different wavelengths and orders overlap, so the coloured fringes lose contrast and merge into nearly uniform illumination. If the glass is retained, its additional wavelength-dependent optical path can shift and colour the central region; the familiar white centre refers to equal optical paths.

Step 4: Conclusion The intensity ratio is $I_{\max} : I_{\min} = (17 + 12\sqrt{2}) : 1 \approx 33.97 : 1$. White light ordinarily gives a white central fringe with a few coloured fringes on both sides.

Quick Tip to Solve: Take square roots of the slit intensities before adding or subtracting; interference combines amplitudes, not intensities.

16. (a) State Bohr's postulate to define stable orbits in hydrogen atom. How does de Broglie's hypothesis explain the stability of these orbits ?

(b) A hydrogen atom initially in the ground state absorbs a photon which excites it to the $n = 4$ level. Estimate the frequency of the photon. [3 marks]

Solution:

Concept:

- Bohr's stationary orbits have quantized angular momentum, $m_e v r = n\hbar = nh/(2\pi)$, where $n = 1, 2, 3, \dots$. An electron in a stationary orbit does not radiate energy.
- De Broglie's wavelength is $\lambda = h/(m_e v)$. A stable orbit accommodates an integral number of wavelengths: $2\pi r = n\lambda$.
- Hydrogen energy levels are $E_n = -13.6/n^2$ eV. Absorption of a photon obeys $h\nu = E_f - E_i$, where ν is the photon frequency.

Step 1: (a) State the postulate for stationary orbits The electron can revolve only in certain permitted stationary orbits without continuously losing energy by radiation. These orbits satisfy the angular-

Quick Tip to Solve: Use the difference between the two negative energy levels, then convert electronvolts to joules before dividing by Planck's constant.

momentum condition

$$m_e v r = \frac{nh}{2\pi}, \quad n = 1, 2, 3, \dots$$

Thus its orbit radius and energy cannot take arbitrary values. Radiation is absorbed or emitted during a transition between stationary states, with the photon energy equal to their energy difference.

Step 2: Explain the condition using the electron's matter wave According to de Broglie, the moving electron is associated with a wave of wavelength $\lambda = h/(m_e v)$. For this wave to join smoothly onto itself after one circuit, the orbit circumference must contain an integer number of wavelengths. Successive circuits then reinforce to form a standing wave. Thus

$$2\pi r = n\lambda = \frac{nh}{m_e v} \implies m_e v r = \frac{nh}{2\pi}.$$

A nonintegral number of wavelengths does not give a single-valued, self-reinforcing standing wave around the orbit. This wave condition explains Bohr's selection of allowed stationary orbits.

Step 3: (b) Find the absorbed energy and photon frequency Initially $n_i = 1$, and finally $n_f = 4$. The absorbed energy is positive:

$$\begin{aligned}\Delta E &= E_4 - E_1 = 13.6 \left(1 - \frac{1}{16}\right) \text{ eV} \\ &= 12.75 \text{ eV} = 12.75(1.602 \times 10^{-19}) \text{ J} \\ &= 2.043 \times 10^{-18} \text{ J}.\end{aligned}$$

Using $h = 6.626 \times 10^{-34} \text{ J s}$,

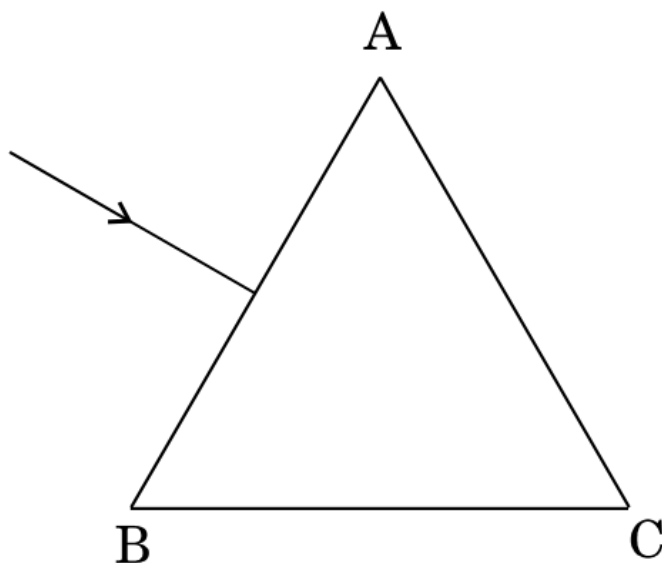
$$\nu = \frac{\Delta E}{h} = \frac{2.043 \times 10^{-18}}{6.626 \times 10^{-34}} \text{ s}^{-1} \simeq 3.08 \times 10^{15} \text{ Hz}.$$

Step 4: Conclusion Stable orbits satisfy $m_e v r = nh/(2\pi)$, equivalently $2\pi r = n\lambda$. Excitation from the ground state to $n = 4$ requires a photon of frequency approximately $3.08 \times 10^{15} \text{ Hz}$.

Quick Tip to Solve: Use the difference between the two negative energy levels, then convert electronvolts to joules before dividing by Planck's constant.

17. (a) Show using a proper diagram how unpolarised light can be linearly polarised by reflection from a transparent glass surface.

(b) The figure shows a ray of light falling normally on the face AB of an equilateral glass prism having refractive index $\frac{3}{2}$, placed in water of refractive index $\frac{4}{3}$. Will this ray suffer total internal reflection on striking the face AC ? Justify your answer. [3 marks]



Solution:

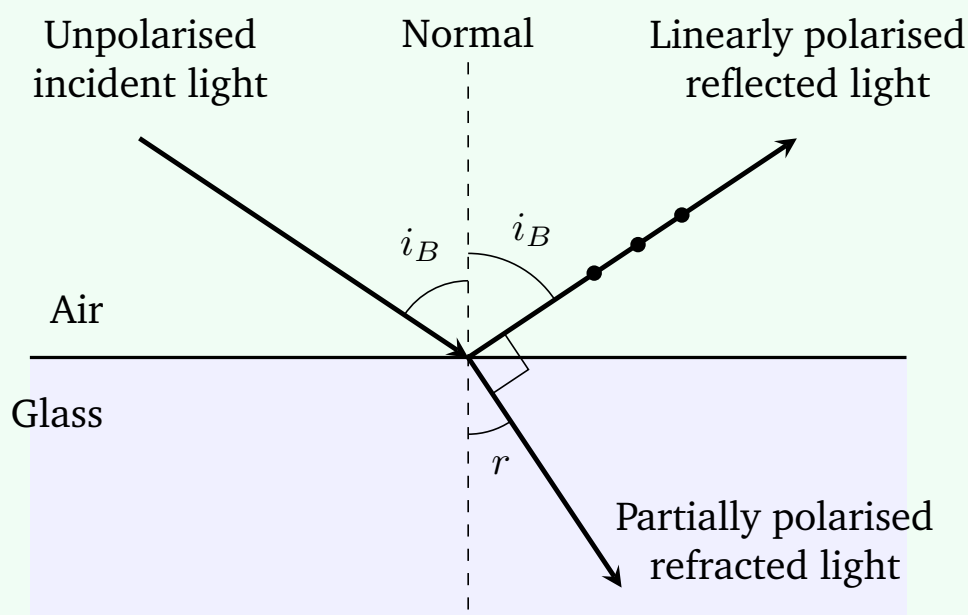
Concept:

- At Brewster's angle i_B , the reflected light is completely plane polarised and $\tan i_B = n_2/n_1$. Its electric field is perpendicular to the plane of incidence.
- Total internal reflection requires passage from a denser to a rarer medium and incidence greater than the critical angle C , where $\sin C = n_{\text{rarer}}/n_{\text{denser}}$.

Step 1: (a) Set the incidence to Brewster's angle Direct unpolarised light from air onto a transparent glass surface at the polarising angle i_B . Resolve the incident electric field into components parallel and perpendicular to the plane of incidence. At this angle the parallel component is absent from the reflected beam: its remaining vibrations are perpendicular to that plane, so it is linearly polarised. The refracted beam remains partially polarised. In the diagram the plane of incidence is the page, and dots indicate electric vibrations perpendicular to it.

$$i_B + r = 90^\circ, \quad \tan i_B = \frac{n_{\text{glass}}}{n_{\text{air}}}.$$

Quick Tip to Solve: Normal entry fixes the second-face incidence at the prism angle; use the surrounding water's index when finding the critical angle.



Polarisation by reflection at Brewster's angle; dots denote electric vibrations perpendicular to the page.

Step 2: (b) Find the angle at the second face The ray enters face AB normally, so its refraction angle there is zero and it travels along the normal to AB . For a prism, the internal refraction angles satisfy $r_1 + r_2 = A$. Since the equilateral prism has $A = 60^\circ$, its angle of incidence on AC is

$$i_{AC} = r_2 = 60^\circ.$$

Step 3: Compare with the glass-water critical angle The relevant rarer medium is water, not air. Hence

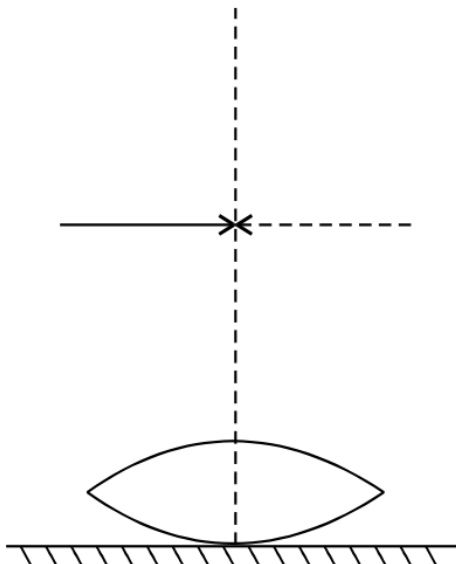
$$\sin C = \frac{4/3}{3/2} = \frac{8}{9}, \quad C \simeq 62.7^\circ.$$

Although light passes from glass to water, the incidence 60° is below the critical angle. Equivalently, Snell's law gives $\sin r = (9/8) \sin 60^\circ \simeq 0.974 < 1$, so a refracted ray can emerge into water.

Step 4: Conclusion (a) Reflection at Brewster's angle produces linearly polarised light with electric vibrations perpendicular to the incidence plane. (b) The ray does **not** undergo total internal reflection at AC , since $60^\circ < 62.7^\circ$.

Quick Tip to Solve: Normal entry fixes the second-face incidence at the prism angle; use the surrounding water's index when finding the critical angle.

18. A symmetric biconvex lens of radius of curvature R and made of glass of refractive index 1.5, is placed on a layer of liquid placed on top of a plane mirror as shown in the figure. An optical needle with its tip on the principal axis of the lens is moved along the axis until its real, inverted image coincides with the needle itself. The distance of the needle from the lens is measured to be x . On removing the liquid layer and repeating the experiment, the distance is found to be y . Obtain the expression for the refractive index of the liquid in terms of x and y . [3 marks]



Solution:

Concept:

- In the lens-plane-mirror method, coincidence of the needle and its real inverted image places the needle at the front focal point of the optical combination. Rays then reach the mirror parallel to the principal axis.
- For a thin lens in air, $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, with signed radii R_1, R_2 . Thin lenses in contact have additive powers: $1/F = 1/f_1 + 1/f_2$.

Step 1: Interpret the two coincidence distances With the liquid present, the outgoing rays from a needle at the focal point become parallel before reflection from the plane mirror. On returning through the combination they form an image at the needle position. Thus the effective focal length of the glass-liquid combination is $F = x$. Without the liquid, the same condition gives the focal length of the glass lens alone as $f_g = y$. We use the usual thin-lens approximation, neglecting separation between the glass lens and the thin liquid layer.

Step 2: Relate the glass curvature to the focal length The symmetric biconvex glass lens has signed radii $R_1 = +R$, $R_2 = -R$, and refractive index 1.5. Therefore

$$\frac{1}{y} = (1.5 - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{1}{R}.$$

It follows that $R = y$. This eliminates the unmeasured radius in favour of the second experimental distance.

Step 3: Find the power contributed by the liquid The liquid between the convex lower glass surface

Quick Tip to Solve: Treat the trapped liquid as a negative plano-concave lens; coincidence distances are focal lengths, so add their reciprocals as powers.

and the plane mirror forms a plano-concave lens. Its curved surface has radius $-R$ for downward light, while the plane surface has infinite radius. If its refractive index is μ_ℓ , its power is

$$\frac{1}{f_\ell} = (\mu_\ell - 1) \left(-\frac{1}{R} - 0 \right) = -\frac{\mu_\ell - 1}{R}.$$

Add the glass and liquid powers, then substitute $R = y$:

$$\begin{aligned} \frac{1}{x} &= \frac{1}{y} - \frac{\mu_\ell - 1}{R}, \\ \frac{y}{x} &= 1 - (\mu_\ell - 1) = 2 - \mu_\ell. \end{aligned}$$

Step 4: Conclusion The refractive index of the liquid is $\mu_\ell = 2 - \frac{y}{x}$. For the usual liquid with $1 < \mu_\ell < 2$, its negative lens power makes $x > y$, consistent with this expression.

Quick Tip to Solve: Treat the trapped liquid as a negative plano-concave lens; coincidence distances are focal lengths, so add their reciprocals as powers.

- 19. (a)** An iron ring of relative permeability μ_r has windings of insulated copper wire of n turns per metre. When the current in the windings is I , find the expression for the magnetic field in the ring.
- (b)** The susceptibility of a magnetic material is 0.9853. Identify the type of magnetic material. Draw the modification of the field pattern on keeping a piece of this material in a uniform magnetic field.

[3 marks]

Solution:

Concept:

- For a uniformly wound toroidal core, Ampere's law gives $H\ell = NI$. Here H is magnetic field intensity, ℓ the magnetic path length and N the number of turns linking it.
- The magnetic flux density is $B = \mu_0\mu_r H$, where μ_0 is vacuum permeability and μ_r relative permeability. For a linear isotropic material, $M = \chi_m H$ and $\mu_r = 1 + \chi_m$.

Step 1: (a) Apply Ampere's law around the ring Choose a circular magnetic path within the uniformly wound iron ring. Neglect leakage and take n to be the number of turns per unit length along this path. Then $N = n\ell$. The magnetic field intensity follows the circular path and has essentially constant magnitude along it, so

$$H\ell = NI = (n\ell)I, \quad H = nI.$$

Multiplying by the permeability of the iron yields

$$B = \mu_0\mu_r nI.$$

The field is tangential to the ring, with direction given by the right-hand grip rule applied to the winding current. The formula uses the given effective relative permeability of the iron.

Step 2: (b) Classify the material using its susceptibility The given susceptibility is positive and

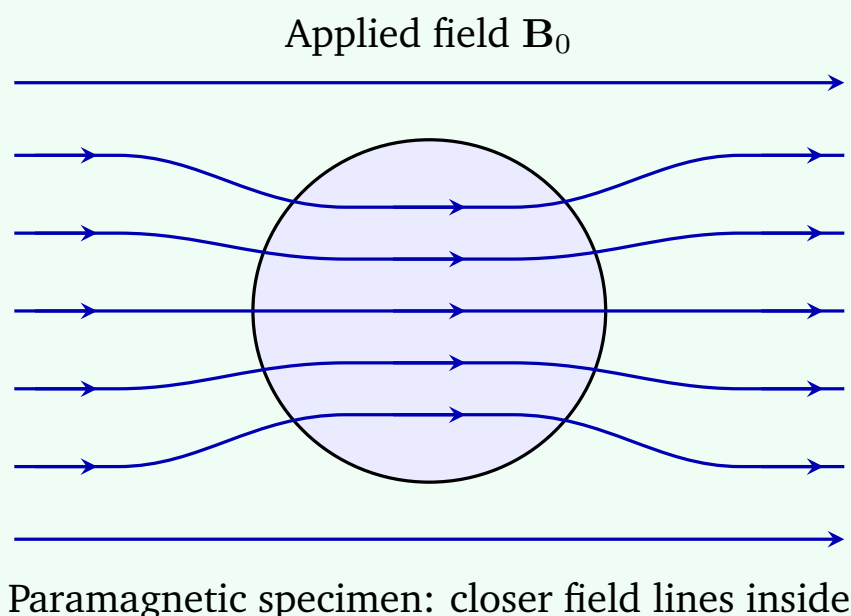
Quick Tip to Solve: Use $H = nI$ for the winding and then multiply by $\mu_0\mu_r$; for the specimen, positive susceptibility means magnetisation along the applied field.

less than unity: $\chi_m = 0.9853$. In the standard classification this is a paramagnetic material. Its magnetisation is parallel to the applied field, unlike the opposite response of a diamagnetic material. Its relative permeability is

$$\mu_r = 1 + 0.9853 = 1.9853 > 1.$$

It is therefore attracted towards stronger regions of a nonuniform magnetic field. A ferromagnet would instead have a very large positive susceptibility in this elementary classification.

Step 3: Draw the field enhancement inside the specimen Place the specimen in a field directed to the right. Field lines bend towards the material and are closer together inside it than in the surrounding space, showing the increased magnetic flux density. The arrows retain the direction of the applied field. The sketch represents a finite paramagnetic specimen; field lines remain continuous and neither begin nor end on its surface.



A paramagnetic specimen concentrates the initially uniform magnetic field.

Step 4: Conclusion (a) The magnetic field in the iron ring is $B = \mu_0 \mu_r nI$. (b) The material is **paramagnetic**; its relative permeability is 1.9853, and magnetic field lines concentrate inside it as shown.

Quick Tip to Solve: Use $H = nI$ for the winding and then multiply by $\mu_0 \mu_r$; for the specimen, positive susceptibility means magnetisation along the applied field.

20. A bar magnet of magnetic moment 6 J/T is aligned at 60° with a uniform external magnetic field of 0.44 T . Calculate (a) the work done in turning the magnet to align its magnetic moment (i) normal to the magnetic field, (ii) opposite to the magnetic field, and (b) the torque on the magnet in the final orientation in case (ii). [3 marks]

Solution:

Concept:

- A magnetic dipole of moment m at angle θ to a uniform field B has potential energy $U = -mB \cos \theta$.
- The work done by an external agent in turning the magnet slowly is $W_{\text{ext}} = \Delta U = mB(\cos \theta_i - \cos \theta_f)$. Its magnetic torque has magnitude $\tau = mB \sin \theta$.

Step 1: Identify the initial orientation and energy The initial angle is $\theta_i = 60^\circ$, the magnetic moment is $m = 6 \text{ J T}^{-1}$, and $B = 0.44 \text{ T}$. Thus,

$$mB = 6(0.44) = 2.64 \text{ J}, \quad U_i = -2.64 \cos 60^\circ = -1.32 \text{ J}.$$

Both requested rotations begin at this same initial orientation. The external work is positive when the magnet's potential energy increases; the field itself then does negative work.

Step 2: Calculate the work for the normal orientation In part (a)(i), the moment is turned normal to the magnetic field, so $\theta_f = 90^\circ$ and $U_f = 0$. Consequently,

$$W_{\text{ext}} = 2.64(\cos 60^\circ - \cos 90^\circ) = 2.64(0.5 - 0) = 1.32 \text{ J}.$$

Step 3: Calculate the work and torque for the opposite orientation In part (a)(ii), opposite alignment means $\theta_f = 180^\circ$, for which $U_f = +2.64 \text{ J}$. The work from the original 60° position is therefore

$$W_{\text{ext}} = 2.64[0.5 - (-1)] = 3.96 \text{ J}.$$

For part (b), evaluate torque at this final angle:

$$\tau = mB \sin 180^\circ = 0 \text{ N m}.$$

Although the torque vanishes exactly at opposite alignment, this is unstable equilibrium: a slight angular displacement makes the magnet turn away from this maximum-energy orientation.

Step 4: Conclusion The external work is 1.32 J for (a)(i) and 3.96 J for (a)(ii). The final torque in case (ii) is 0 N m .

Quick Tip to Solve: Use $\Delta(-mB \cos \theta)$ for work and $mB \sin \theta$ for torque; both rotations start at 60° , not one after the other.

21. (a) Define the term ‘conductivity’ of a metallic wire. Write its SI unit.

(b) Using the concept of free electrons in a conductor, derive the expression for the conductivity of a wire in terms of number density and relaxation time. Hence obtain the relation between current density and the applied electric field $\vec{E}$. [3 marks]

Solution:

Concept:

- Conductivity σ is the reciprocal of resistivity ρ , and measures a material’s ability to conduct electric current: $\sigma = 1/\rho$.
- In a metal, free electrons acquire a mean drift velocity under an electric field. If their number density is n , their current density is $\vec{J} = -ne\vec{v}_d$, where an electron has charge $-e$.

Step 1: Define conductivity and give its SI unit For a metallic wire in the ohmic regime, conductivity is the ratio of current density to applied electric field, $\sigma = J/E$; equivalently, it is the reciprocal of resistivity. A larger conductivity gives a larger current density for the same field. Since resistivity has SI unit $\Omega \text{ m}$, the SI unit of conductivity is

$$(\Omega \text{ m})^{-1} = \text{S m}^{-1}.$$

Here S denotes siemens. Conductivity is a property of the material at a specified temperature, rather than the conductance of a particular length of wire.

Step 2: Obtain the electron drift velocity Let m_e be the electron mass and τ the relaxation time, the mean time between successive collisions. In a uniform field $\vec{E}$, an electron experiences force $-e\vec{E}$, giving acceleration

$$\vec{a} = -\frac{e}{m_e}\vec{E}.$$

The random thermal velocities average to zero. In the relaxation-time model, averaging the field-induced velocity gained between collisions gives

$$\vec{v}_d = \vec{a}\tau = -\frac{e\tau}{m_e}\vec{E}.$$

Thus electrons drift opposite to the applied field.

Step 3: Derive the conductivity and current-density relation For electron number density n , the charge carried through unit area per unit time gives $\vec{J} = -ne\vec{v}_d$. Substituting the drift velocity,

$$\vec{J} = -ne\left(-\frac{e\tau}{m_e}\vec{E}\right) = \frac{ne^2\tau}{m_e}\vec{E}.$$

Comparing this expression with $\vec{J} = \sigma\vec{E}$ identifies the conductivity. The two negative signs cancel because conventional current is opposite to electron motion, so $\vec{J}$ is along $\vec{E}$.

Step 4: Conclusion Conductivity is reciprocal resistivity, with SI unit S m^{-1} . For a metallic conductor,

$$\sigma = ne^2\tau/m_e \text{ and } \vec{J} = \sigma\vec{E}.$$

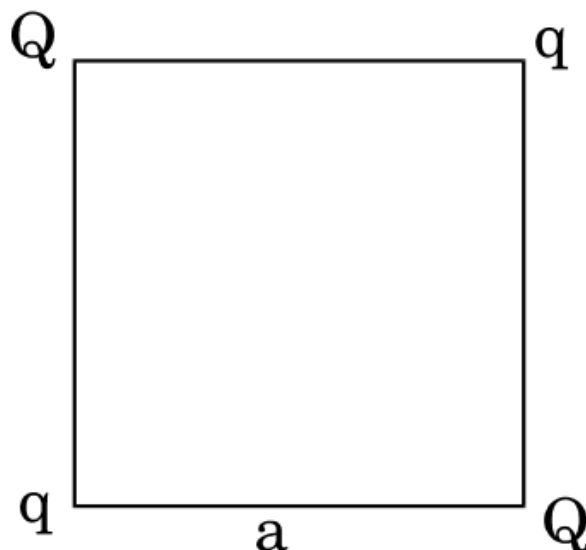
Quick Tip to Solve: Keep electron charge as $-e$ in both force and current density; the signs cancel and conventional current points along the field.

22. Four point charges Q , q , Q and q are placed at the corners of a square of side 'a' as shown in the figure.

Find the

- (a) resultant electric force on a charge Q , and
(b) potential energy of this system.

[3 marks]



Solution:

Concept:

- Coulomb's law gives the pairwise electrostatic force, and the net force is the vector sum of all three forces on the selected charge. Write $k = 1/(4\pi\epsilon_0)$.
- The potential energy of a system of point charges is $U = \sum_{i < j} kq_i q_j / r_{ij}$, with zero energy at infinite separation. Each distinct pair is counted once, retaining its charge signs.

Step 1: Add the two forces due to adjacent charges Select the Q at the upper-left corner. The adjacent q charges are each at distance a ; the other Q lies across the diagonal at distance $\sqrt{2}a$. Let $\hat{n}$ point along the diagonal from the square's centre towards the selected Q . For like-signed Q and q , the two adjacent forces have equal magnitude kQq/a^2 , act at right angles, and combine along $\hat{n}$. Their resultant is

$$\vec{F}_{\text{adj}} = \frac{\sqrt{2}kQq}{a^2} \hat{n}.$$

This vector expression also applies to opposite signs: a negative Qq reverses the resultant towards the centre.

Step 2: Include the force from the opposite corner The other Q repels the selected charge along the same outward diagonal. Its force is

$$\vec{F}_{\text{diag}} = \frac{kQ^2}{(\sqrt{2}a)^2} \hat{n} = \frac{kQ^2}{2a^2} \hat{n}.$$

Quick Tip to Solve: For force, combine the two perpendicular side contributions first; for energy, count four side pairs and two diagonal pairs without adding vectors.

Hence the net force is

$$\vec{F}_Q = \frac{k}{a^2} \left(\sqrt{2}Qq + \frac{Q^2}{2} \right) \hat{n}.$$

The force on the other Q has the same magnitude and the opposite direction by symmetry. For like-signed charges, the coefficient is positive and the force points away from the centre; for arbitrary signs, the coefficient determines the direction.

Step 3: Sum all six pair energies The four sides each join a Q and a q , contributing kQq/a per side. One diagonal joins the two Q charges and the other joins the two q charges. Both diagonals have length $\sqrt{2}a$. Therefore,

$$\begin{aligned} U &= 4 \frac{kQq}{a} + \frac{kQ^2}{\sqrt{2}a} + \frac{kq^2}{\sqrt{2}a} \\ &= \frac{k}{a} \left[4Qq + \frac{Q^2 + q^2}{\sqrt{2}} \right]. \end{aligned}$$

There is no extra factor of one-half here because each of the six distinct pairs has already been included exactly once.

Step 4: Conclusion $\vec{F}_Q = \frac{1}{4\pi\epsilon_0 a^2} \left(\sqrt{2}Qq + \frac{Q^2}{2} \right) \hat{n}$, with $\hat{n}$ outward along the diagonal through the selected Q . Its magnitude is the absolute value of the scalar coefficient. The system energy is

$$U = \frac{1}{4\pi\epsilon_0 a} \left[4Qq + \frac{Q^2 + q^2}{\sqrt{2}} \right].$$

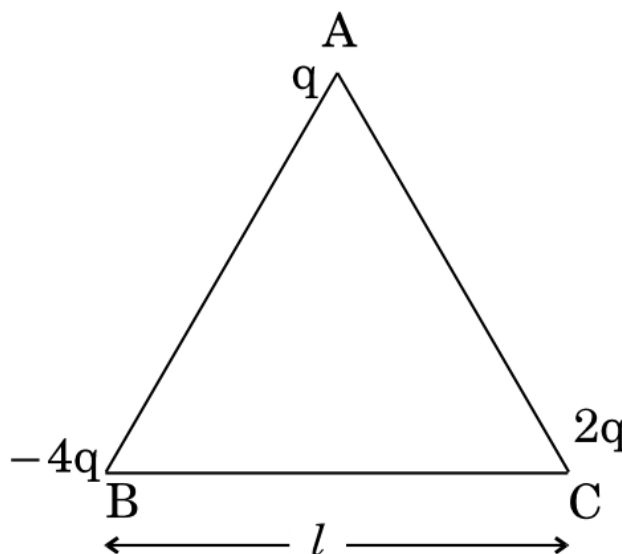
Quick Tip to Solve: For force, combine the two perpendicular side contributions first; for energy, count four side pairs and two diagonal pairs without adding vectors.

OR

22. (a) Three point charges q , $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side ' l ' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q .

(b) Find out the amount of the work done to separate the charges at infinite distance.

[3 marks]



Solution:

Concept:

- Coulomb's law gives the magnitude of the force between two point charges as $F = k|q_1q_2|/r^2$, where $k = 1/(4\pi\epsilon_0)$. Like charges repel and unlike charges attract.
- Electrostatic forces obey vector superposition. The potential energy of a system is $U = \sum_{i<j} kq_iq_j/r_{ij}$, counting each distinct pair once.
- The external work in separating the charges slowly, without changing their kinetic energy, is $W_{\text{ext}} = U_f - U_i$. At infinite pair separations, $U_f = 0$.

Step 1: (a) Identify the two forces on the charge at A In the printed triangle, q is at A, $-4q$ at B and $2q$ at C. All separations are l . The force due to B is attractive, along AB towards B, whereas the force due to C is repulsive, along CA away from C. Their magnitudes are

$$F_B = \frac{4kq^2}{l^2}, \quad F_C = \frac{2kq^2}{l^2}.$$

Although the internal angle BAC is 60° , the angle between these two force vectors is $180^\circ - 60^\circ = 120^\circ$, because the second force points away from C.

Step 2: Combine the forces by the vector triangle rule Put $f = kq^2/l^2$. The squared magnitude of the resultant is

$$\begin{aligned} F^2 &= F_B^2 + F_C^2 + 2F_B F_C \cos 120^\circ \\ &= (4f)^2 + (2f)^2 + 2(4f)(2f)(-1/2) \\ &= 12f^2. \end{aligned}$$

The negative cross term reflects the partially opposing directions; adding the two magnitudes directly would incorrectly treat the forces as parallel.

Step 3: (b) Calculate the initial potential energy and the separation work There are three distinct charge pairs, AB, AC and BC. Retain their algebraic signs when adding their energies:

$$\begin{aligned} U_i &= \frac{k}{l} [q(-4q) + q(2q) + (-4q)(2q)] \\ &= \frac{kq^2}{l} (-4 + 2 - 8) = -\frac{10kq^2}{l}. \end{aligned}$$

At infinite separation all three interaction terms vanish. Hence $W_{\text{ext}} = 0 - U_i$. The positive required work raises the initially negative potential energy to zero; the work done by the electric forces has the opposite sign.

Step 4: Conclusion (a) The resultant force magnitude is $F = \frac{2\sqrt{3}q^2}{4\pi\epsilon_0 l^2}$. (b) The external work required is $W_{\text{ext}} = \frac{10q^2}{4\pi\epsilon_0 l}$; the electrostatic field does the negative of this work.

Quick Tip to Solve: Use 120° , not the triangle's 60° , between one attractive and one repulsive force. For energy, add the three signed pair products once each.

Section D (4 marks)

23. The teachers of Geeta's school took the students on a study trip to a power generating station, located nearly 200 km away from the city. The teacher explained that electrical energy is transmitted over such a long distance to their city, in the form of alternating current (ac) raised to a high voltage. At the receiving end in the city, the voltage is reduced to operate the devices. As a result, the power loss is reduced. Geeta listened to the teacher and asked questions about how the ac is converted to a higher or lower voltage.

(a) Name the device used to change the alternating voltage to a higher or lower value. State one cause for power dissipation in this device.

(b) Explain with an example, how power loss is reduced if the energy is transmitted over long distances as an alternating current rather than a direct current.

(c) Write two values each shown by the teachers and Geeta.

[4 marks]

Solution:

Concept:

- A transformer changes an alternating voltage by mutual induction. Ideally, $V_s/V_p = N_s/N_p$, where N_p, N_s are the primary and secondary turns and V_p, V_s their rms voltages.
- For a fixed transmitted power P , taking power factor as unity, $I = P/V$. The heating loss in transmission lines of total resistance R is $P_{\text{loss}} = I^2 R = P^2 R/V^2$.

Step 1: (a) Name the device and a cause of its power loss The device is a **transformer**. A step-up transformer has $N_s > N_p$ and raises the voltage for transmission; a step-down transformer has $N_s < N_p$ and lowers it at the receiving end. The alternating primary current produces changing magnetic flux in the core, which induces the secondary emf. One cause of dissipation is **copper loss**: the primary and secondary windings have nonzero resistance, so current produces heat at rates $I_p^2 R_p$ and $I_s^2 R_s$. Thus a real transformer transfers slightly less power to its load than it takes from its supply.

Step 2: (b) Explain the advantage of raising the transmission voltage To transmit the same power at a larger voltage, a smaller current is needed. Since line heating depends on the square of the current, increasing voltage by a factor n reduces the line loss by a factor n^2 , for unchanged resistance and power factor. AC can readily be stepped up before transmission and stepped down for safe use by ordinary transformers. A steady DC supply does not produce the continuously changing flux needed by an ordinary transformer, so it cannot be stepped up in this way. The reduction in loss arises from the higher voltage and lower current; AC and DC carrying the same current through the same resistance have the same resistive heating loss.

Step 3: Illustrate the reduction with numerical values For illustration, transmit $P = 100 \text{ kW}$ through lines having total resistance $R = 10 \Omega$, with unity power factor. At $V_1 = 10 \text{ kV}$,

$$I_1 = \frac{100000}{10000} = 10 \text{ A}, \quad P_{\text{loss},1} = 10^2(10) = 1000 \text{ W}.$$

If a step-up transformer raises the voltage to $V_2 = 100 \text{ kV}$,

$$I_2 = \frac{100000}{100000} = 1 \text{ A}, \quad P_{\text{loss},2} = 1^2(10) = 10 \text{ W}.$$

Quick Tip to Solve: Keep transmitted power fixed when comparing line losses: multiplying voltage by n divides current by n and resistive loss by n^2 .

Thus raising the voltage tenfold reduces the line loss to one hundredth of its previous value. The voltage is then reduced near consumers using a step-down transformer.

Step 4: (c) Identify the values shown in the incident The teachers show **commitment to practical education**, by arranging a visit that connects classroom physics with a working power station, and **responsibility towards their students**, by explaining how electrical energy reaches homes and devices. Geeta shows **attentiveness**, by listening carefully to the explanation, and **scientific curiosity**, by asking how the alternating voltage is raised or lowered.

Step 5: Conclusion A transformer changes the AC voltage; winding resistance causes copper loss. High-voltage transmission reduces current and therefore I^2R loss, as the example reduces it from 1000 W to 10 W. The teachers display commitment to practical education and responsibility, while Geeta displays attentiveness and scientific curiosity.

Quick Tip to Solve: Keep transmitted power fixed when comparing line losses: multiplying voltage by n divides current by n and resistive loss by n^2 .

Section E (15 marks)

24. (a) State the principle of an ac generator and explain its working with the help of a labelled diagram. Obtain the expression for the emf induced in a coil having N turns each of cross-sectional area A rotating with a constant angular speed ' ω ' in a magnetic field $\vec{B}$, directed perpendicular to the axis of rotation.

(b) An aeroplane is flying horizontally from west to east with a velocity of 900 km/hour. Calculate the potential difference developed between the ends of its wings having a span of 20 m. The horizontal component of the Earth's magnetic field is 5×10^{-4} T and the angle of dip is 30° . [5 marks]

Solution:

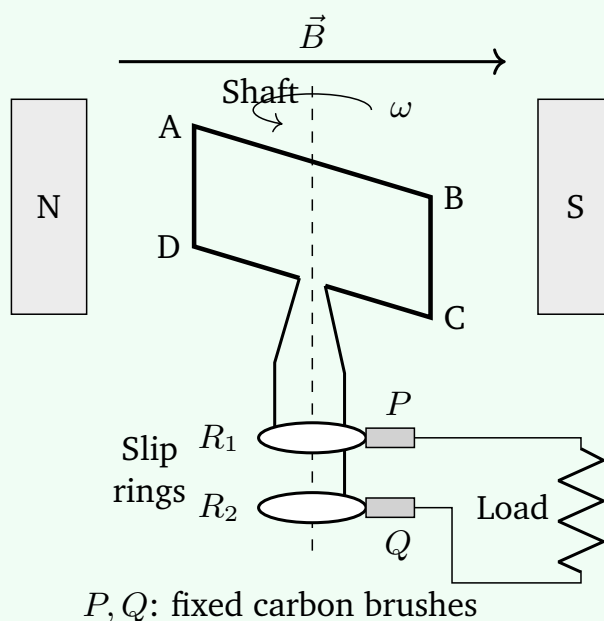
Concept:

- An ac generator uses electromagnetic induction: a changing magnetic flux through a rotating coil produces an emf. Faraday's law gives $\varepsilon = -d(N\Phi)/dt$, where $N\Phi$ is the flux linkage.
- For a uniform field of magnitude B , the flux through one turn of area A is $\Phi = BA \cos \theta$, where θ is the angle between the field and the normal to the coil.
- A conductor of length l moving perpendicular to a magnetic field component $B_\perp$, with its length perpendicular to the velocity, develops motional emf $\varepsilon = B_\perp lv$. The Earth's vertical field is $B_V = B_H \tan \delta$, where δ is the angle of dip.

Step 1: (a) Describe the generator and its principle A rectangular conducting coil ABCD of N turns is rotated mechanically between the north and south poles of a magnet. Its shaft is perpendicular to the magnetic field. The two coil ends connect to two separate, continuous slip rings R_1 and R_2 , which rotate with the coil. Fixed carbon brushes P and Q touch the rings and connect them to the external load. The labelled diagram shows these connections schematically.

As the coil rotates, its orientation relative to the field changes, so its magnetic flux changes. An emf is therefore induced by Faraday's law. Mechanical energy supplied to the shaft is converted into electrical energy in the external circuit.

Quick Tip to Solve: Measure the generator's angle from the coil normal. For horizontal wings, first use $B_V = B_H \tan \delta$; using B_H directly gives the wrong wing-tip emf.



AC generator: rotating coil, separate slip rings, fixed brushes and external load (schematic).

Step 2: Derive the instantaneous induced emf Choose $t = 0$ when the normal to the coil is parallel to $\vec{B}$. For constant angular speed ω , the angle between this normal and the field is $\theta = \omega t$. Hence the flux linkage is

$$N\Phi = NBA \cos(\omega t).$$

Applying Faraday's law and differentiating with respect to time,

$$\begin{aligned}\varepsilon &= -\frac{d}{dt}[NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t) = \varepsilon_0 \sin(\omega t), \\ \varepsilon_0 &= NBA\omega.\end{aligned}$$

Here ε_0 is the peak emf. The overall sign depends on the selected terminal polarity; the sinusoidal form and peak value are unchanged.

Step 3: Explain the alternating output When the coil's normal is parallel or antiparallel to the field, flux has an extremum and the instantaneous emf is zero. When the normal is perpendicular to the field, flux is zero but changes most rapidly, giving maximum emf magnitude. After each half-turn, the motion of each coil side relative to the field reverses and so does the induced emf. The slip rings maintain each coil end's connection to its own brush, allowing this reversal to appear across the load. Thus the output alternates with period $T = 2\pi/\omega$ and frequency $f = \omega/(2\pi)$.

Step 4: (b) Select the effective field and convert the speed The aeroplane moves east, while its horizontal wings extend north-south. The vertical component of the Earth's field produces a magnetic force along the wings. The horizontal field component produces a vertical force and therefore does

Quick Tip to Solve: Measure the generator's angle from the coil normal. For horizontal wings, first use $B_V = B_H \tan \delta$; using B_H directly gives the wrong wing-tip emf.

not contribute to the potential difference between the wing tips. With the given angle of dip,

$$B_V = B_H \tan 30^\circ = \frac{5 \times 10^{-4}}{\sqrt{3}} \text{ T.}$$

Convert the speed into SI units:

$$v = 900 \times \frac{1000}{3600} = 250 \text{ m s}^{-1}, \quad l = 20 \text{ m.}$$

Step 5: Calculate the wing-tip potential difference Free charges in the wings experience the magnetic force $q(\vec{v} \times \vec{B})$. Charge separation builds an electric field until the electric and magnetic forces balance. Hence the magnitude of the potential difference is

$$\begin{aligned} |\Delta V| &= B_V l v \\ &= \frac{5 \times 10^{-4}}{\sqrt{3}} \times 20 \times 250 \\ &= \frac{2.5}{\sqrt{3}} \text{ V} \simeq 1.44 \text{ V.} \end{aligned}$$

For a downward vertical field, $\vec{v} \times \vec{B}$ points north, so the northern wing tip is positive relative to the southern tip.

Step 6: Conclusion The generator produces $\varepsilon = NBA\omega \sin(\omega t)$, with peak emf $\varepsilon_0 = NBA\omega$, for the stated initial orientation. The potential difference between the aeroplane's wing tips has magnitude 1.44 V .

Quick Tip to Solve: Measure the generator's angle from the coil normal. For horizontal wings, first use $B_V = B_H \tan \delta$; using B_H directly gives the wrong wing-tip emf.

OR

24. A device X is connected across an ac source of voltage $V = V_0 \sin \omega t$. The current through X is given as $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$.

- Identify the device X and write the expression for its reactance.
- Draw graphs showing variation of voltage and current with time over one cycle of ac, for X .
- How does the reactance of the device X vary with frequency of the ac? Show this variation graphically.
- Draw the phasor diagram for the device X . [5 marks]

Solution:

Concept:

- A capacitor obeys $q = CV$ and $I = dq/dt = C dV/dt$, where C is its capacitance, q its charge, V the applied voltage, and I the charging current.
- Capacitive reactance is $X_C = V_0/I_0 = 1/(\omega C) = 1/(2\pi f C)$, measured in Ω . Here f is frequency

Quick Tip to Solve: Differentiate the given voltage: a sine becomes a cosine, immediately identifying a capacitor and showing that current leads by one quarter-cycle.

and $\omega = 2\pi f$ is angular frequency.

- In a purely capacitive ac circuit, current leads voltage by $\pi/2$ radians, or one quarter of the time period $T = 1/f$. Phasors rotate anticlockwise at angular speed ω .

Step 1: (a) Identify the device and derive its reactance The specified current is $I = I_0 \sin(\omega t + \pi/2) = I_0 \cos \omega t$. Its phase is ahead of the applied voltage $V = V_0 \sin \omega t$ by $\pi/2$. This is the phase relation for an ideal capacitor. To verify it directly, differentiate the charge on a capacitor:

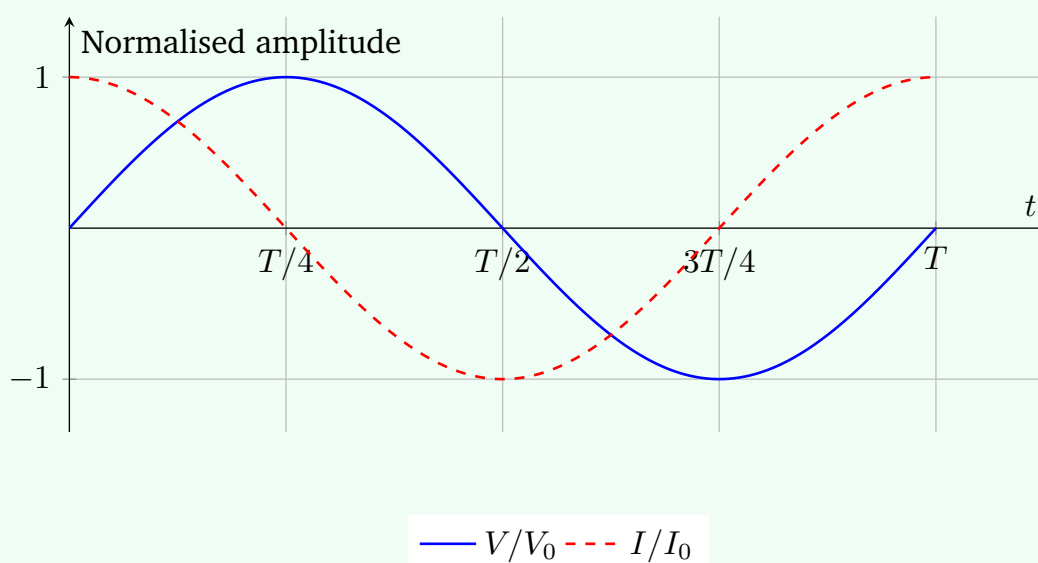
$$I = C \frac{dV}{dt} = \omega C V_0 \cos \omega t.$$

Comparing amplitudes gives $I_0 = \omega C V_0$, so

$$X_C = \frac{V_0}{I_0} = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

A resistor would give current in phase with voltage, while an ideal inductor would give current lagging voltage; neither matches the printed phase relation.

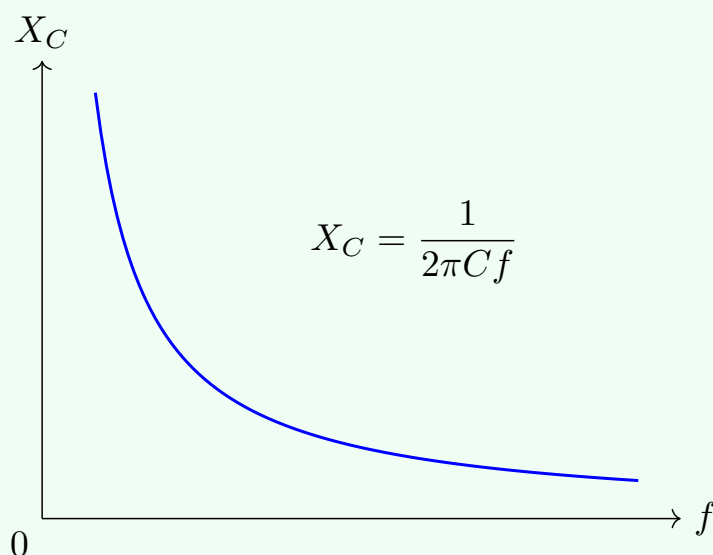
Step 2: (b) Plot voltage and current for one complete cycle Plot $V/V_0 = \sin(2\pi t/T)$ and $I/I_0 = \cos(2\pi t/T)$ on a common time axis. Normalising each curve by its own amplitude allows quantities with different units to be compared without implying that their actual amplitudes are equal. At $t = 0$, voltage is zero and increasing whereas current is at its positive maximum. At $T/4$, voltage is maximum and current is zero; at $T/2$, voltage is zero and current is at its negative maximum. At $3T/4$, voltage is at its negative maximum and current is zero. At T , the initial values recur. Thus each current maximum occurs $T/4$ before the corresponding voltage maximum.



One full cycle: normalised voltage and current; $T = 1/f$.

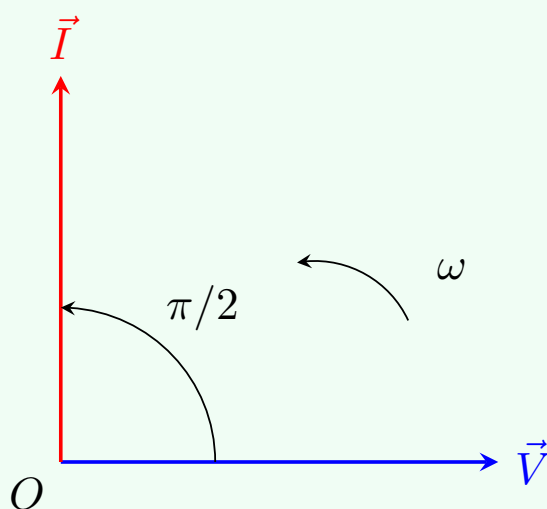
Step 3: (c) Show how reactance varies with frequency For fixed capacitance, $X_C = 1/(2\pi C f)$, hence $X_C \propto 1/f$. Doubling the frequency halves the reactance. The graph in the positive f - X_C quadrant is a decreasing rectangular hyperbola, with the two coordinate axes as asymptotes. As $f \rightarrow 0^+$, $X_C \rightarrow \infty$; as $f \rightarrow \infty$, $X_C \rightarrow 0$. An ideal capacitor consequently blocks steady dc after charging, while offering progressively smaller reactance at higher ac frequencies.

Quick Tip to Solve: Differentiate the given voltage: a sine becomes a cosine, immediately identifying a capacitor and showing that current leads by one quarter-cycle.



Capacitive reactance decreases inversely with frequency for constant C .

Step 4: (d) Construct the phasor diagram Choose the voltage phasor $\vec{V}$ along the positive horizontal reference axis. Place the current phasor $\vec{I}$ at 90° anticlockwise from it. Since phasors rotate anticlockwise, the current phasor is ahead of the voltage phasor by $\pi/2$. The angular relation is the physical information in this schematic; the lengths of voltage and current arrows use their own scales. Taking their vertical projections after a rotation through ωt reproduces $V_0 \sin \omega t$ and $I_0 \sin(\omega t + \pi/2)$, respectively.



Capacitor phasors: current leads voltage by $\pi/2$.

Step 5: Conclusion X is an ideal capacitor, with $X_C = 1/(\omega C) = 1/(2\pi f C)$. Current leads voltage by 90° , or $T/4$. The time curves are a sine and an advanced sine (cosine), the reactance-frequency curve is a decreasing hyperbola, and the current phasor is 90° ahead of the voltage phasor.

Quick Tip to Solve: Differentiate the given voltage: a sine becomes a cosine, immediately identifying a capacitor and showing that current leads by one quarter-cycle.

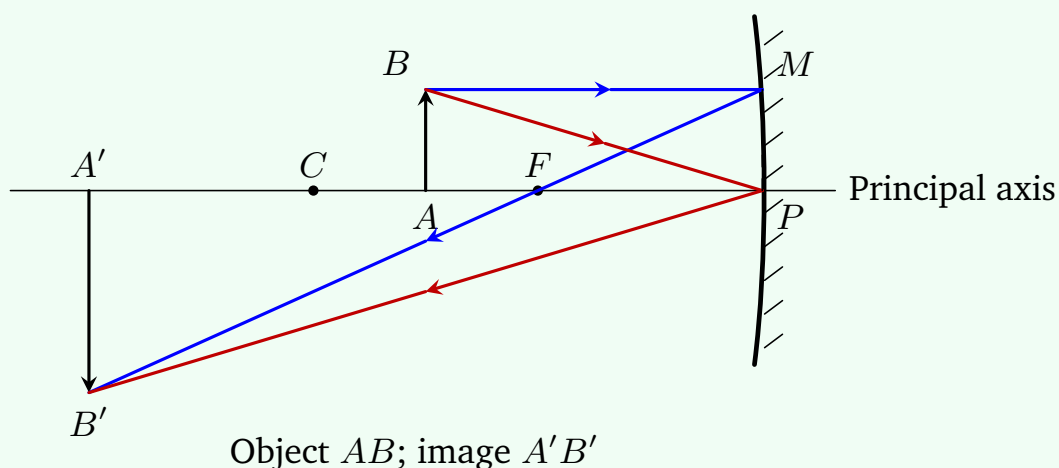
25. (a) Draw a ray diagram to show image formation when the concave mirror produces a real, inverted and magnified image of the object.
- (b) Obtain the mirror formula and write the expression for the linear magnification.
- (c) Explain two advantages of a reflecting telescope over a refracting telescope. [5 marks]

Solution:

Concept:

- For a concave spherical mirror, a paraxial ray parallel to the principal axis is reflected through the principal focus F ; at the pole P , the incident and reflected rays make equal angles with the principal axis.
- The mirror formula relates signed object distance u , image distance v and focal length f , measured from the pole. Linear magnification is the signed height ratio $m = h_i/h_o$.
- Reflection does not disperse light into colours. A telescope's light-gathering power increases with the area of its objective, while its angular resolution improves as the aperture increases.

Step 1: (a) Place the object between the focus and centre of curvature Draw the upright object AB between F and C . From its tip B , draw one ray parallel to the principal axis, meeting the mirror at M ; its reflected ray passes through F . Draw a second ray from B to P , reflected with its angle of reflection equal to its angle of incidence. The two reflected rays actually intersect at B' , beyond C and below the axis. Thus $A'B'$ is real, inverted and larger than AB . The sketch uses the usual paraxial approximation near the pole.



Paraxial ray diagram: object between C and F ; enlarged inverted image beyond C .

Step 2: (b) Relate image and object heights using the pole ray First use positive lengths $U = AP$, $V = A'P$, $F_0 = FP$, $h = AB$ and $H = A'B'$. The triangles ABP and $A'B'P$ are similar: both are right angled and their angles at P are equal by the law of reflection. Hence

$$\frac{H}{h} = \frac{V}{U}.$$

Here H is the magnitude of the downward image height; its signed value will be negative.

Step 3: Use the parallel ray to obtain the mirror relation For a mirror of small aperture and paraxial rays, M lies approximately in the plane through P perpendicular to the principal axis, so $MP \simeq h$.

Quick Tip to Solve: Place the object between F and C , then intersect a parallel ray reflected through F with the pole ray. Keep positive lengths separate from Cartesian distances until the final sign conversion.

The right triangles MPF and $B'A'F$ are similar because the reflected ray passes through F . Therefore

$$\frac{H}{h} = \frac{V - F_0}{F_0}.$$

Equating the two height ratios and rearranging gives

$$\begin{aligned}\frac{V}{U} &= \frac{V - F_0}{F_0}, \\ VF_0 &= UV - UF_0, \\ UV &= F_0(U + V), \\ \frac{1}{F_0} &= \frac{1}{U} + \frac{1}{V}.\end{aligned}$$

Step 4: Apply Cartesian signs and state the magnification Take the incident-light direction, from left to right, as positive. In this real-image arrangement the object, image and focus are all to the left of P . Thus $u = -U$, $v = -V$ and $f = -F_0$. Substitution yields the mirror formula

$$\boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}.$$

Since the object is upright and the image inverted, $h_o = h$ and $h_i = -H$. Therefore

$$m = \frac{h_i}{h_o} = -\frac{H}{h} = -\frac{V}{U} = \boxed{-\frac{v}{u}}.$$

With $F_0 < U < 2F_0$, the derived relation gives $V > 2F_0$, so $m < 0$ and $|m| > 1$, consistent with the sketch. With Cartesian signs the mirror formula also describes the other paraxial object positions.

Step 5: (c) Explain two advantages of a reflecting telescope **Freedom from chromatic aberration:** the reflecting objective focuses different wavelengths by reflection, whose law is independent of wavelength. Its focal length therefore does not vary with colour, unlike that of a simple refracting objective; coloured fringes caused by the objective's dispersion are avoided.

Practical large aperture: a mirror can be supported over its back, whereas a refracting objective must transmit light and is supported at its rim. A large reflecting objective is consequently easier to construct and support. Its larger collecting area gathers more light from faint objects, and its larger diameter improves resolving power.

Step 6: Conclusion A concave mirror forms a real, inverted, magnified image beyond C when the object is between F and C . Its paraxial formula is $1/f = 1/u + 1/v$, and its linear magnification is $m = -v/u$. Reflecting telescopes avoid objective chromatic aberration and permit large supported apertures for better light collection and resolution.

Quick Tip to Solve: Place the object between F and C , then intersect a parallel ray reflected through F with the pole ray. Keep positive lengths separate from Cartesian distances until the final sign conversion.

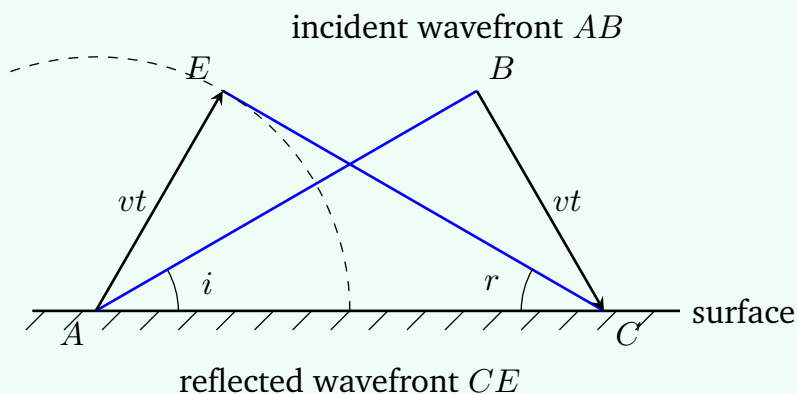
25. (a) Define a wavefront. Using Huygens' principle, verify the laws of reflection at a plane surface.
 (b) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band ? Explain.
 (c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the obstacle. Explain why. [5 marks]

Solution:

Concept:

- A wavefront is the locus of points of a wave that vibrate in the same phase. Rays are normal to the wavefront in an isotropic medium.
- Huygens' principle: each point of a wavefront emits secondary wavelets; their forward common envelope gives the subsequent wavefront. In time t , a wavelet travels a distance vt , where v is the wave speed.
- For a slit of width a , the first diffraction minima obey $a \sin \theta = \pm \lambda$. The central maximum has approximate width $w = 2D\lambda/a$ on a distant screen at distance D .
- Coherent contributions of equal phase add in amplitude, and intensity is proportional to the square of the resultant amplitude.

Step 1: (a) Construct the incident and reflected wavefronts Let AB be a plane incident wavefront approaching a plane reflecting surface, with A touching it first. During time t , point B reaches the surface at C , so $BC = vt$. Meanwhile the secondary wavelet originating at A grows to radius $AE = vt$. Draw the tangent CE from C to this wavelet. It is the reflected wavefront: the wavelets emitted at the intermediate points of the surface have the same common tangent. The incident ray BC is perpendicular to AB , and the reflected ray AE is perpendicular to CE . Thus ABC and AEC are right triangles.



Huygens' construction: AB is incident, CE is reflected, and $BC = AE = vt$.

Step 2: (a) Verify both laws of reflection The right triangles have the same hypotenuse AC , and $BC = AE = vt$. If i and r denote the angles the rays make with the surface normal, the angles between their respective wavefronts and the surface are also i and r . Hence

$$\sin i = \frac{BC}{AC} = \frac{vt}{AC}, \quad \sin r = \frac{AE}{AC} = \frac{vt}{AC}.$$

Quick Tip to Solve: For a slit-width change, use two separate scalings: central width varies as $1/a$, but central amplitude varies as a , so peak intensity varies as a^2 .

Both angles lie between 0° and 90° , giving $i = r$. The construction lies in the plane containing the incident ray and the surface normal; the reflected ray, being normal to CE within that plane, lies in it too. This verifies that the incident ray, reflected ray and normal are coplanar, as well as equality of the two angles.

Step 3: (b) Compare the widths and central intensities The central diffraction band extends between the first minima on either side. At fixed wavelength and screen distance, for the usual small diffraction angles,

$$w = \frac{2D\lambda}{a}, \quad w' = \frac{2D\lambda}{2a} = \frac{w}{2}.$$

At its centre, the path difference between contributions from all parts of the slit is zero. With unchanged incident illumination and slit height, the resultant central amplitude is proportional to slit width. Doubling that width therefore gives $A'_0 = 2A_0$, so

$$\frac{I'_0}{I_0} = \left(\frac{A'_0}{A_0}\right)^2 = 4.$$

This intensity ratio refers to the peak intensity at the centre; the intensity varies across the band. More light passes through the wider slit, while the diffraction pattern becomes narrower.

Step 4: (c) Explain the central bright spot behind the obstacle Light diffracts around the edge of the tiny circular obstacle into its geometrical shadow. For an axial distant source, points around the circular rim are reached in the same phase and are equally distant from the centre of the shadow on a screen behind the obstacle. Their diffracted contributions therefore reach that point in phase and interfere constructively, producing a bright Poisson (Arago) spot. The stated centre is the centre of the obstacle's geometrical shadow, where ray optics alone would predict darkness.

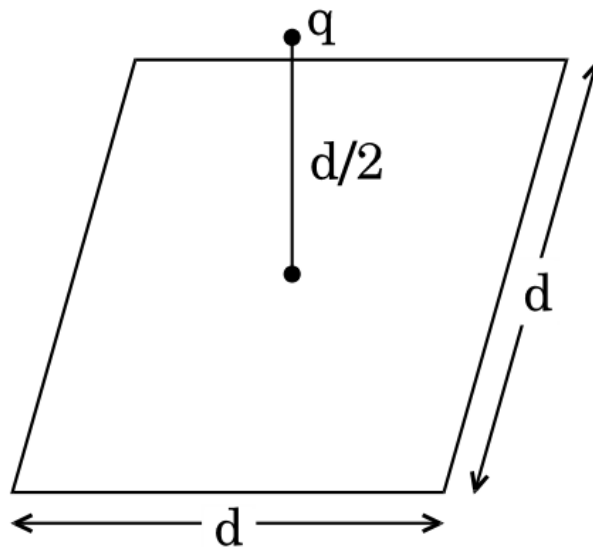
Step 5: Conclusion (a) A wavefront joins equal-phase points; Huygens' construction gives $i = r$ and coplanarity of the incident ray, reflected ray and normal. (b) Doubling slit width halves the central-band width and makes its peak intensity four times as large. (c) Constructive interference of light diffracted around the circular rim produces the central bright spot in the shadow.

Quick Tip to Solve: For a slit-width change, use two separate scalings: central width varies as $1/a$, but central amplitude varies as a , so peak intensity varies as a^2 .

26. (a) Define electric flux. Is it a scalar or a vector quantity?

A point charge q is at a distance of $d/2$ directly above the centre of a square of side d , as shown in the figure. Use Gauss' law to obtain the expression for the electric flux through the square.

(b) If the point charge is now moved to a distance ' d ' from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected. [5 marks]



Solution:

Concept:

- Electric flux through a surface measures the electric field passing normally through it: $\Phi_E = \int_S \vec{E} \cdot d\vec{A}$, where $d\vec{A}$ is the directed area element. It is a scalar because the integrand is a dot product; its SI unit is $\text{N m}^2/\text{C}$.
- Gauss' law states $\oint_S \vec{E} \cdot d\vec{A} = q_{\text{enclosed}}/\epsilon_0$ for a closed surface, where ϵ_0 is the permittivity of free space. Symmetry can divide this total flux among equivalent faces.

Step 1: (a) Define electric flux and its nature For a uniform field through a flat area, electric flux is $\Phi_E = EA \cos \theta$, where θ is the angle between the field and the chosen area normal. More generally, the field may vary over the surface, so the dot product must be integrated. Flux is a scalar, although it may be positive, negative or zero depending on the orientation of the normal. Here the field of the point charge is not uniform over the square, so substituting the field at its centre into EA would not give the flux.

Step 2: (a) Construct a closed Gaussian surface Imagine a cube of side d with the given square as one face, extending towards the charge. Its centre lies on the perpendicular through the centre of the square, at a distance $d/2$ from that face. Thus the charge q is exactly at the centre of this imaginary cube. The cube encloses the charge, and Gauss' law gives

$$\Phi_{\text{cube}} = \frac{q}{\epsilon_0}.$$

The cube is only a mathematical closed surface; no conducting material or additional charges are introduced.

Quick Tip to Solve: Compare the charge's height with half the square's side: equality places the charge at the centre of a cube, so one face receives one sixth of the total flux.

Step 3: (a) Use symmetry to obtain the square's flux All six faces are identical and are symmetrically situated about the central charge. Rotating the cube to interchange any two faces leaves the charge and its radial field unchanged. Therefore the flux through each face, measured along its outward normal, is the same. If this flux is Φ_{square} , then

$$6\Phi_{\text{square}} = \frac{q}{\varepsilon_0}, \quad \Phi_{\text{square}} = \frac{q}{6\varepsilon_0}.$$

This choice of normal points away from the charge through the given face. Reversing the chosen normal reverses the signed flux; the magnitude is $|q|/(6\varepsilon_0)$.

Step 4: (b) Apply the same construction after both lengths double The new square has side $2d$, while the charge is at perpendicular distance d from its centre. It is therefore again at the centre of a cube constructed on the square, now of side $2d$, because half the new side is d . The enclosed charge is still q , and symmetry again shares the total flux equally among six faces:

$$\Phi'_{\text{square}} = \frac{1}{6} \frac{q}{\varepsilon_0} = \Phi_{\text{square}}.$$

Equivalently, corresponding distances from the charge double, reducing the field to one quarter, while corresponding area elements increase fourfold and their angles to the field stay the same. Hence the integral of $\vec{E} \cdot d\vec{A}$ is unchanged.

Step 5: Conclusion Electric flux is a scalar, $\Phi_E = \int_S \vec{E} \cdot d\vec{A}$. With the outward normal of the constructed cube, the flux through the original square is $\boxed{q/(6\varepsilon_0)}$. Doubling both the charge's height and the side of the square leaves this flux unchanged.

Quick Tip to Solve: Compare the charge's height with half the square's side: equality places the charge at the centre of a cube, so one face receives one sixth of the total flux.

OR

26. (a) Use Gauss' law to derive the expression for the electric field ($\vec{E}$) due to a straight uniformly charged infinite line of charge density λ C/m.

(b) Draw a graph to show the variation of E with perpendicular distance r from the line of charge.

(c) Find the work done in bringing a charge q from perpendicular distance r_1 to r_2 ($r_2 > r_1$). [5 marks]

Solution:

Concept:

- Gauss' law is $\oint_S \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\varepsilon_0$, where Q_{enc} is the charge enclosed by the closed surface and ε_0 is the permittivity of free space.
- An infinite uniform line has cylindrical symmetry. Its field is radial, and its magnitude depends only on the perpendicular distance r from the line; λ denotes charge per unit length.
- For slow motion with no change in kinetic energy, external work is the potential-energy change: $W_{\text{ext}} = q[V(r_2) - V(r_1)] = -q \int_{r_1}^{r_2} \vec{E} \cdot d\vec{r}$. The work done by the electric field has the opposite sign.

Quick Tip to Solve: Use a coaxial cylinder so that only its curved surface contributes flux. For the work, integrate $1/r$ to obtain a logarithm, and state whether the work is by the external agent or by the field.

Step 1: (a) Choose a cylindrical Gaussian surface Take a closed cylinder of radius r and length L , with its axis along the charged line. Translational symmetry along the line and rotational symmetry about it imply that $\vec{E}$ is radial and has the same magnitude at every point of the curved surface. For $\lambda > 0$, the field points outward. The outward area vectors of the two flat end caps are parallel to the line and perpendicular to $\vec{E}$, so their flux is zero. On the curved surface, $\vec{E}$ is parallel to the outward area vector; therefore its flux is $E(2\pi rL)$.

Step 2: (a) Apply Gauss' law and obtain the field The length of charged line enclosed is L , so the enclosed charge is λL . For a positive line charge, Gauss' law gives

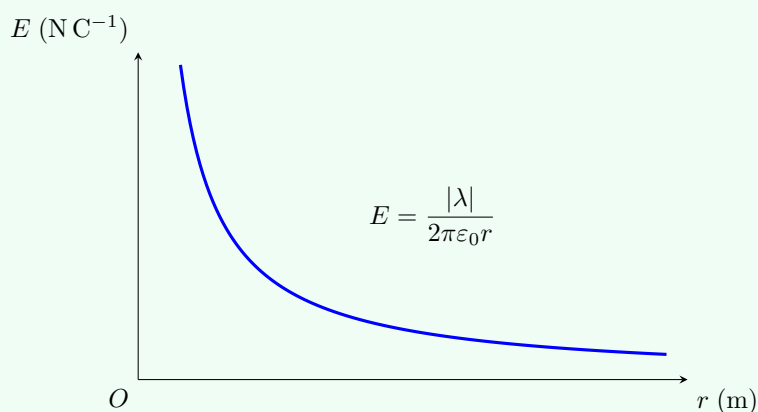
$$E(2\pi rL) = \frac{\lambda L}{\epsilon_0}, \quad E = \frac{\lambda}{2\pi\epsilon_0 r}.$$

Writing the result with the signed charge density gives the general vector expression

$$\vec{E}(r) = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r},$$

where $\hat{r}$ is directed radially away from the line. Thus the field is outward for positive λ and inward for negative λ . Its magnitude is $|\lambda|/(2\pi\epsilon_0 r)$, in N C^{-1} . The arbitrary length L cancels, as required for an infinite line.

Step 3: (b) Plot the variation with perpendicular distance For fixed line-charge density, the field magnitude obeys $E \propto 1/r$. Hence its graph against positive r is a decreasing rectangular hyperbola: doubling the distance halves the field. For the ideal line, E becomes unbounded as $r \rightarrow 0^+$, and approaches zero as $r \rightarrow \infty$. Neither axis is crossed. The following qualitative plot shows the field magnitude, so it applies to either sign of the line charge.



Field magnitude versus distance from an infinite uniformly charged line (qualitative).

Step 4: (c) Calculate the work between the two distances Move the charge slowly from r_1 to $r_2 > r_1$. Using the outward radial coordinate, the potential difference is

$$\begin{aligned} V(r_2) - V(r_1) &= - \int_{r_1}^{r_2} \frac{\lambda}{2\pi\epsilon_0 r} dr \\ &= - \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right). \end{aligned}$$

Quick Tip to Solve: Use a coaxial cylinder so that only its curved surface contributes flux. For the work, integrate $1/r$ to obtain a logarithm, and state whether the work is by the external agent or by the field.

Therefore the work done by an external agent in bringing the charge is

$$W_{\text{ext}} = \Delta U = \frac{q\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_1}{r_2}\right).$$

For $q\lambda > 0$, this is negative because the repulsive electric field does positive work during outward motion. If work done by the electric field is intended, it is instead

$$W_{\text{field}} = -W_{\text{ext}} = \frac{q\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right).$$

Both expressions are in joules. Only a potential difference between finite distances is used: the potential of an infinite line cannot be assigned a finite zero at infinity.

Step 5: Conclusion The field is $\vec{E} = \lambda\hat{r}/(2\pi\epsilon_0 r)$, with magnitude varying as $1/r$. The external work is $W_{\text{ext}} = [q\lambda/(2\pi\epsilon_0)] \ln(r_1/r_2)$; the electric field does the opposite work.

Quick Tip to Solve: Use a coaxial cylinder so that only its curved surface contributes flux. For the work, integrate $1/r$ to obtain a logarithm, and state whether the work is by the external agent or by the field.