CBSE Class 12 Mathematics 2018 - Set 65-3 - Compartment

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 100**

General Instructions

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 3 questions of four marks each and 3 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section A

1. Find the value of $[\hat{i}, \hat{k}, \hat{j}]$.

2. Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

3. Find the identity element in the set Q^+ of all positive rational numbers for the operation $*$ defined by $a * b = \frac{3ab}{2}$ for all $a, b \in Q^+$.

4. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{bmatrix}$ is a matrix satisfying $AA' = 9I$, find x.

Section B

5. The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$ in rupees. Find the marginal revenue when $x = 5$, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.

6. Differentiate $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ with respect to x.

7. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.

8. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 5$, $|\vec{b}| = 6$ and $|\vec{c}| = 9$, then find the angle between $\vec{a}$ and $\vec{b}$.

9. Solve the differential equation $\cos\left(\frac{dy}{dx}\right) = a$, $(a \in \mathbb{R})$.

10. Find : $\int \frac{3 - 5\sin x}{\cos^2 x}\, dx$.

11. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then find the value of k.

12. Prove that $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$, $x \in \left[\frac{1}{2}, 1\right]$.

Section C

13. Find x such that the four points $A(4, 4, 4)$, $B(5, x, 8)$, $C(5, 4, 1)$ and $D(7, 7, 2)$ are coplanar.

14. Find : $\int \frac{4}{(x-2)(x^2+4)}\,dx$.

15. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

16. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 metres. Find the dimensions of the window to admit maximum light through the whole opening. How having large windows help us in saving electricity and conserving environment ?

17. From a lot of 20 bulbs which include 5 defectives, a sample of 3 bulbs is drawn at random, one by one with replacement. Find the probability distribution of the number of defective bulbs. Also, find the mean of the distribution.

18(a). Solve the differential equation $(x^2 - y^2)\,dx + 2xy\,dy = 0$.

OR

18(b). Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$, given that $y = 0$ when $x = 1$.

19. Two groups are competing for the positions of the Board of Directors of a corporation. The probabilities that the first and second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

20. If $\sin y = x\cos(a+y)$, then show that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos a}$.

Also, show that $\frac{dy}{dx} = \cos a$, when $x = 0$.

21. Using properties of determinants, prove that $\begin{vmatrix} 5a & -2a+b & -2a+c \\ -2b+a & 5b & -2b+c \\ -2c+a & -2c+b & 5c \end{vmatrix} = 12(a+b+c)(ab+bc+ca)$.

22(a). Find the angle of intersection of the curves $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$, at the point in the first quadrant.

OR

22(b). Find the intervals in which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is

(i) Strictly increasing (ii) Strictly decreasing

23(a). If $x = a\sec^3\theta$ and $y = a\tan^3\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

OR

23(b). If $y = e^{\tan^{-1}x}$, prove that $(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$.

Section D

24. Using integration, find the area of the region : $\{(x, y) : 0 \leq 2y \leq x^2,\ 0 \leq y \leq x,\ 0 < x < 3\}$.

25(a). Given $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, compute $(AB)^{-1}$.

OR

25(b). Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ by using elementary row transformations.

26(a). Show that the relation R on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow (x - y)$ is divisible by 3, is an equivalence relation.

OR

26(b). A binary operation $*$ on the set $A = \{0, 1, 2, 3, 4, 5\}$ is defined as

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

Write the operation table for $a * b$ in A. Show that zero is the identity for this operation $*$ and each element 'a' $\neq 0$ of the set is invertible with $6 - a$, being the inverse of 'a'.

27. Find the vector equation of the line passing through $(1, 2, 3)$ and parallel to each of the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$. Also find the point of intersection of the line thus obtained with the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$.

28. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of B requires 1 g of silver and 2 g of gold. The company can use atmost 9 g of silver and 8 g of gold. If each unit of type A brings a profit of Rs.40 and that of type B Rs.50, find the number of units of each type that the company should produce to maximize the profit. Formulate and solve graphically the LPP and find the maximum profit.

29(a). Evaluate $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$.

OR

29(b). Evaluate $\int_1^3 (3x^2 + 2x + 1)\, dx$ as the limit of a sum.

- End of Question Paper -