

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 100

General Instructions

- (i) This question paper contains **29 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 4 carry 1 mark each.
- (iii) Section B: Questions 5 to 12 carry 2 marks each.
- (iv) Section C: Questions 13 to 23 carry 4 marks each.
- (v) Section D: Questions 24 to 29 carry 6 marks each.
- (vi) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (vii) Use of calculators is not allowed.

Section A

1. Find the differential equation representing the family of curves $y = ae^{2x} + 5$, where a is an arbitrary constant.

2. If $y = \cos(\sqrt{3x})$, then find $\frac{dy}{dx}$.

3. If A is a square matrix satisfying $A'A = I$, write the value of $|A|$.

4. Find the direction cosines of a line which makes equal angles with the coordinate axes.

OR

A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.

Section B

5. Show that the points $A(-2\hat{i} + 3\hat{j} + 5\hat{k})$, $B(\hat{i} + 2\hat{j} + 3\hat{k})$ and $C(7\hat{i} - \hat{k})$ are collinear.

OR

Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = -3\hat{i} + 5\hat{j} - 2\hat{k}$.

6. Find: $\int \frac{x-5}{(x-3)^3} e^x dx$.

7. Examine whether the operation $*$ defined on $\mathbb{R}$, the set of all real numbers, by $a * b = \sqrt{a^2 + b^2}$ is a binary operation or not, and if it is, find whether it is associative or not.

8. If $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$, show that $(A - 2I)(A - 3I) = 0$.

9. Find: $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$.

OR

Find: $\int \frac{x-3}{(x-1)^3} e^x dx$.

10. A coin is tossed 5 times. What is the probability of getting (i) 3 heads, (ii) at most 3 heads?

OR

Find the probability distribution of X , the number of heads in a simultaneous toss of two coins.

11. If $P(\text{not } A) = 0.7$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then find $P(A/B)$.

12. Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitrary constants.

Section C

13. Solve for x : $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$.

14. If $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$, then prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

OR

Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x .

15. Find: $\int \frac{2 \cos x}{(1 - \sin x)(2 - \cos^2 x)} dx$.

16. Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

OR

Let $f : \mathbb{N} \rightarrow Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in \mathbb{N} : y = 4x + 3, \text{ for some } x \in \mathbb{N}\}$. Show that f is invertible. Find its inverse.

17. Using properties of determinants, show that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca).$$

18. If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that

$$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}, \text{ is a constant independent of } a \text{ and } b.$$

19. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point $(-1, 4)$.

20. Prove that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ and hence evaluate $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$.

21. Solve the differential equation: $x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right)$.

OR

Solve the differential equation: $\frac{dy}{dx} = -\left[\frac{x+y \cos x}{1+\sin x}\right]$.

22. If the lines $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{2} = \frac{z-6}{-5}$ are perpendicular, find the value of λ . Hence find whether the lines are intersecting or not.

23. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

Section D

24. Show that for the matrix $A =$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}, A^3 - 6A^2 + 5A + 11I = 0.$$

Hence, find A^{-1} .

OR

Using matrix method, solve the following system of equations: $3x - 2y + 3z = 8$, $2x + y - z = 1$, $4x - 3y + 2z = 4$.

25. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

26. A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement) both of which are found to be red. Find the probability that the balls are drawn from the second bag.

27. Using method of integration, find the area of the triangle whose vertices are $(1, 0)$, $(2, 2)$ and $(3, 1)$.

OR

Using method of integration, find the area of the region enclosed between two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

28. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of Rs. 40 and that of type B Rs. 50, find the number of units of each type that the company should produce to maximize profit. Formulate the above problem as an LPP and solve it graphically. Also find the maximum profit.

29. Find the vector and cartesian equations of the plane passing through the points having position vectors $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$. Write the equation of a plane passing through a point $(2, 3, 7)$ and parallel to the plane obtained above. Hence, find the distance between the two parallel planes.

OR

Find the equation of the line passing through $(2, -1, 2)$ and $(5, 3, 4)$, and of the plane passing through $(2, 0, 3)$, $(1, 1, 5)$ and $(3, 2, 4)$. Also find their point of intersection.

- End of Question Paper -