

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 100

General Instructions

- (i) This question paper contains **29 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 4 carry 1 mark each.
- (iii) Section B: Questions 5 to 12 carry 2 marks each.
- (iv) Section C: Questions 13 to 23 carry 4 marks each.
- (v) Section D: Questions 24 to 29 carry 6 marks each.
- (vi) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (vii) Use of calculators is not allowed.

Section A

1. If A is a square matrix of order 3 with $|A| = 4$, then write the value of $|-2A|$.

2. If $y = \sin^{-1} x + \cos^{-1} x$, find $\frac{dy}{dx}$.

3. Write the order and the degree of the differential equation $\left(\frac{d^4y}{dx^4}\right)^2 = \left[x + \left(\frac{dy}{dx}\right)^2\right]^3$.

4. If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines?

OR

Find the cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

Section B

5. If $*$ is defined on the set $\mathbb{R}$ of all real numbers by $a * b = \sqrt{a^2 + b^2}$, find the identity element, if it exists in $\mathbb{R}$ with respect to $*$.

6. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the values of k , a and b .

7. Find: $\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}} dx$, $0 < x < \pi/2$.

8. Find: $\int \frac{\sin(x-a)}{\sin(x+a)} dx$.

OR

Find: $\int (\log x)^2 dx$.

9. Form the differential equation representing the family of curves $y^2 = m(a^2 - x^2)$ by eliminating the arbitrary constants m and a .

10. Find a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

OR

Show that the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$, $-2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar.

11. Mother, father and son line up at random for a family photo. If A and B are two events given by A = Son on one end, B = Father in the middle, find $P(B/A)$.

12. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$. Find the probability distribution of X .

OR

A coin is tossed 5 times. Find the probability of getting (i) at least 4 heads, and (ii) at most 4 heads.

Section C

13. Show that the relation R on the set $\mathbb{Z}$ of all integers, given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$ is an equivalence relation.

OR

If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that $f \circ f(x) = x$ for all $x \neq \frac{2}{3}$. Also, find the inverse of f .

14. If $\tan^{-1} x - \cot^{-1} x = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$, $x > 0$, find the value of x and hence find the value of $\sec^{-1} \left(\frac{2}{x} \right)$.

15. Using properties of determinants, prove that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$

16. If $\sin y = x \sin(a+y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

OR

If $(\sin x)^y = x + y$, find $\frac{dy}{dx}$.

17. If $y = (\sec^{-1} x)^2$, $x > 0$, show that $x^2(x^2 - 1) \frac{d^2y}{dx^2} + (2x^3 - x) \frac{dy}{dx} - 2 = 0$.

18. Find the equations of the tangent and the normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.

19. Find: $\int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$.

20. Prove that $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$ and hence evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$.

21. Solve the differential equation: $\frac{dy}{dx} = \frac{x+y}{x-y}$.

OR

Solve the differential equation: $(1+x^2) dy + 2xy dx = \cot x dx$.

22. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $|\vec{c}| = 3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$; and $\vec{b}, \vec{c}$ are perpendicular to each other, then find $|3\vec{a} - 2\vec{b} + 2\vec{c}|$.

23. Find the value of λ for which the following lines are perpendicular to each other: $\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}$; $\frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$. Hence, find whether the lines intersect or not.

Section D

24. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, find A^{-1} . Hence, solve the following system of equations: $x + y + z = 6$, $y + 3z = 11$ and $x - 2y + z = 0$.

OR

Find the inverse of the following matrix, using elementary transformations: $A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$.

25. Show that the height of a cylinder, which is open at the top, having a given surface area and greatest volume, is equal to the radius of its base.

26. Find the area of the triangle whose vertices are $(-1, 1)$, $(0, 5)$ and $(3, 2)$, using integration.

OR

Find the area of the region bounded by the curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$, using integration.

27. Find the vector and cartesian equations of the plane passing through the points $(2, 5, -3)$, $(-2, -3, 5)$ and $(5, 3, -3)$. Also, find the point of intersection of this plane with the line passing through points $(3, 1, 5)$ and $(-1, -3, -1)$.

OR

Find the equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis. Hence, find the distance of the plane from x-axis.

28. There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and n black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$, find the value of n .

29. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. The profit is ₹ 50 each for type A and ₹ 60 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize profit? Formulate the above LPP and solve it graphically and also find the maximum profit.

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